INTRODUCTION TO

DEEP LEARNING

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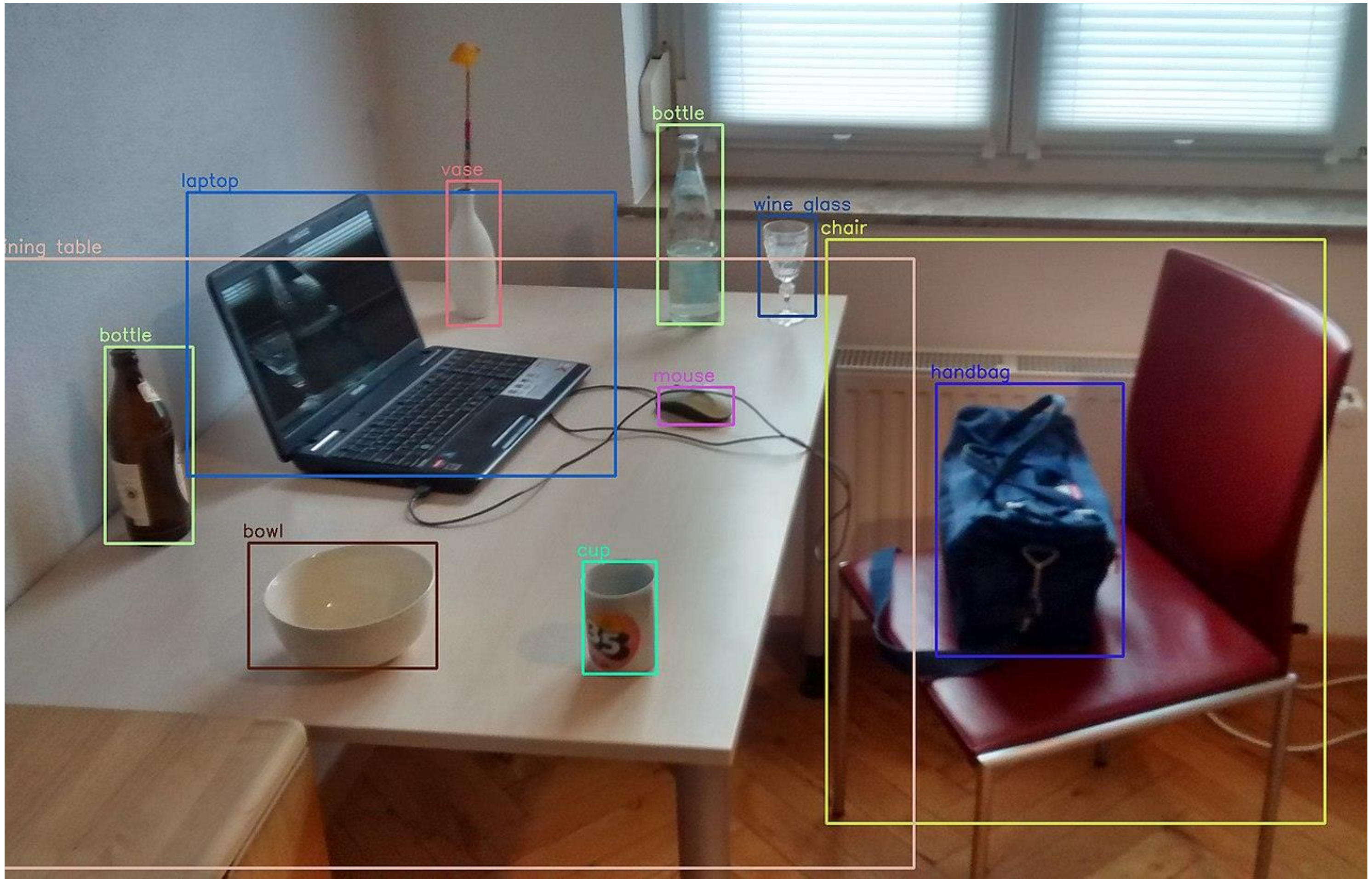
AGENDA

- State-of-the-art
- What changed?
- Fundamentals
- Deep Learning
- Examples / Practice

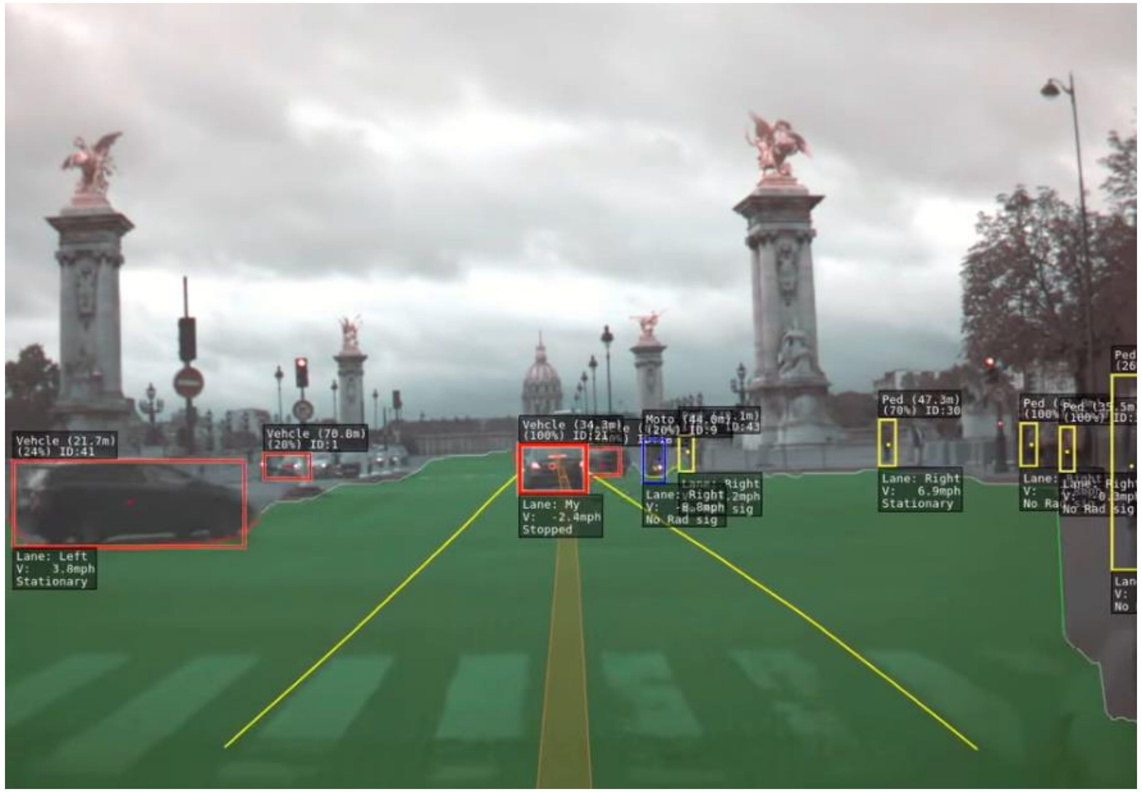
State of the art

COMPUTER VISION

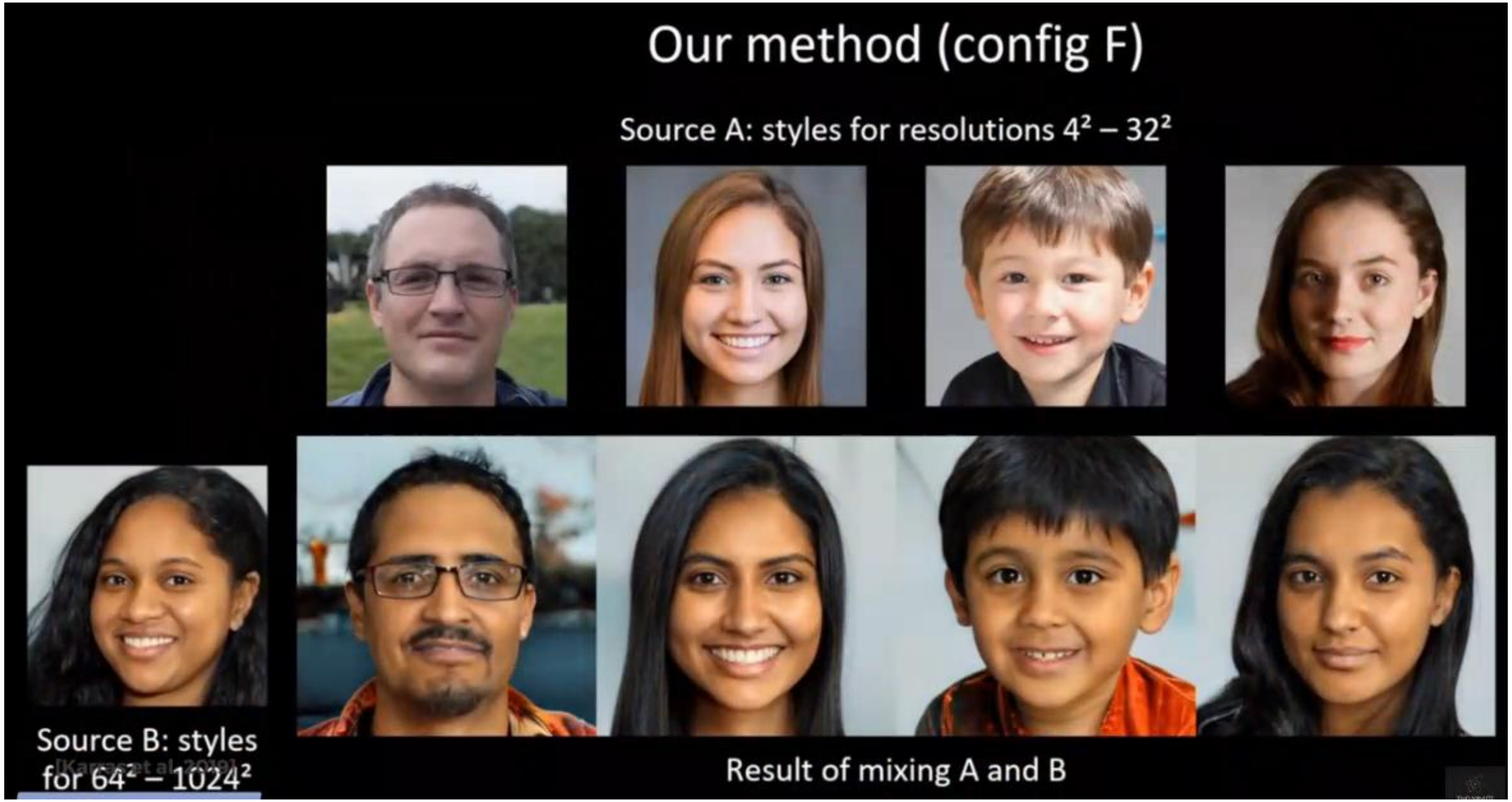
SEMANTIC SEGMENTATION AND OBJECT DETECTION



SEMANTIC SEGMENTATION AND OBJECT DETECTION



GENERATIVE ADVERSIAL NETWORKS (GANs)



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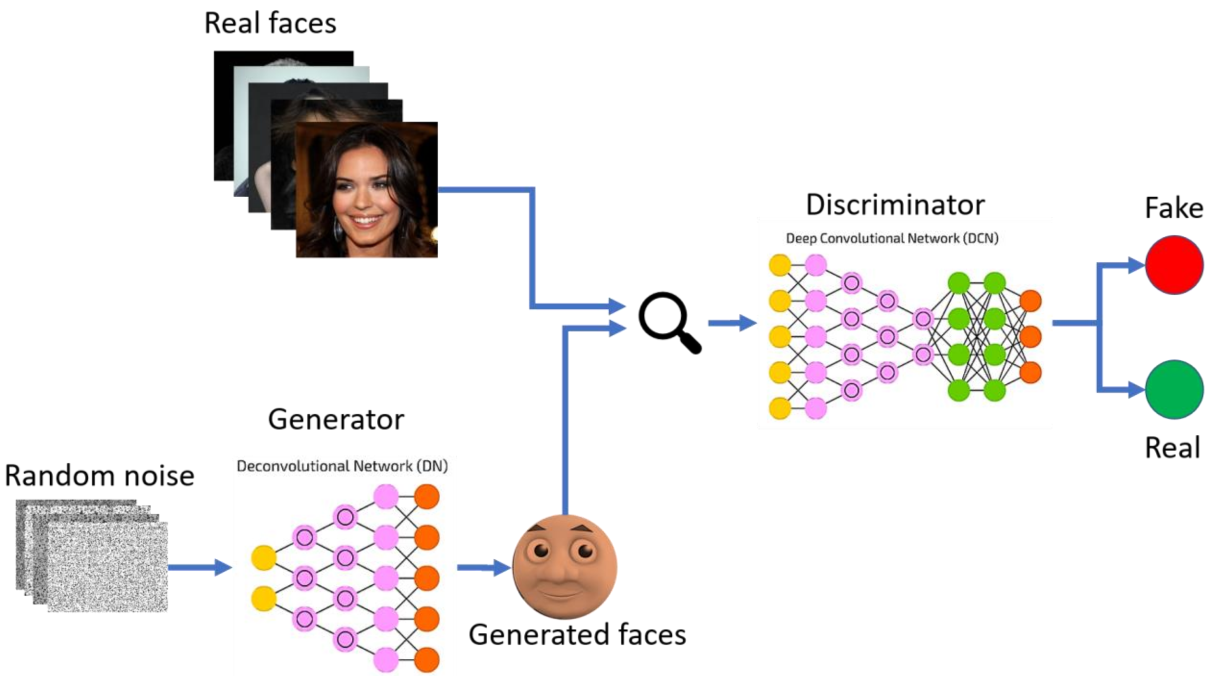
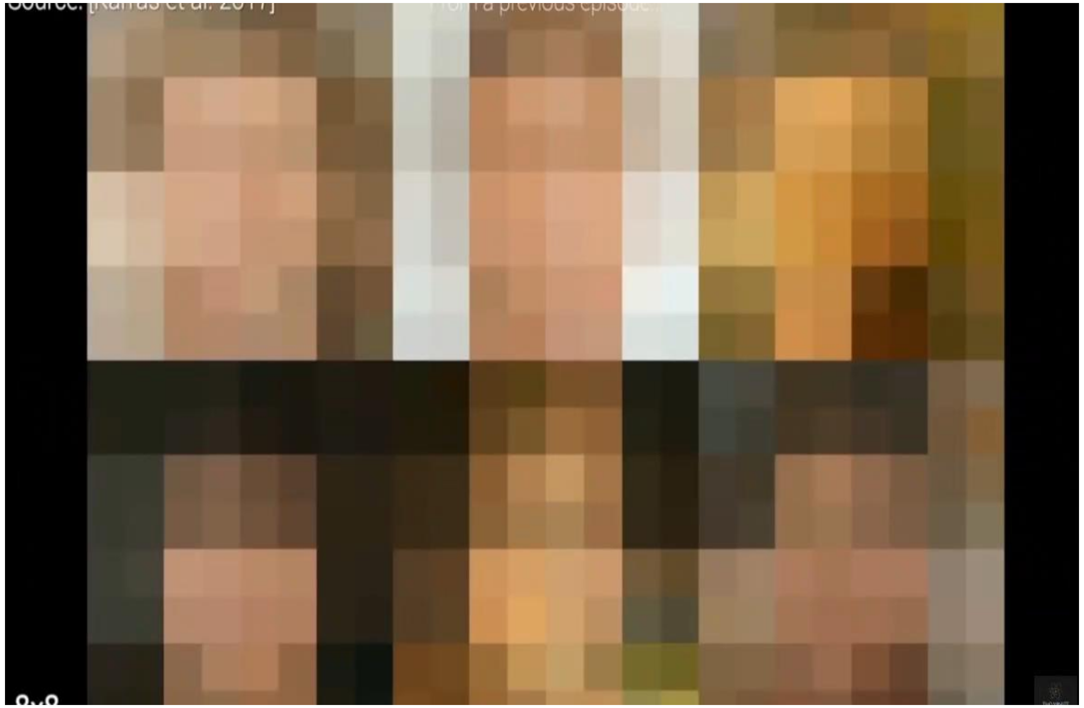
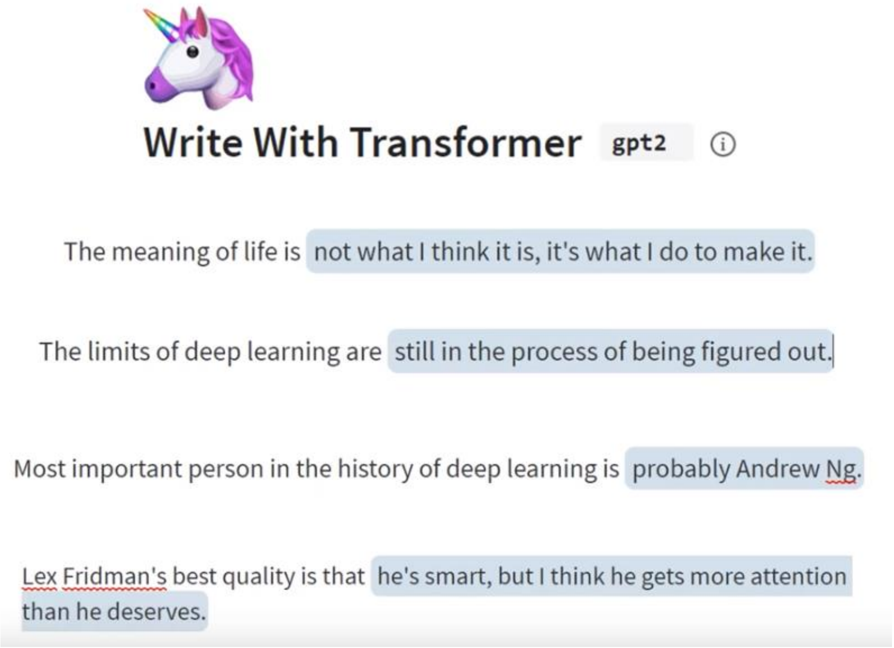




GENERATIVE ADVERSIAL NETWORKS (GANs)

NATURAL LANGUAGE PROCESSING

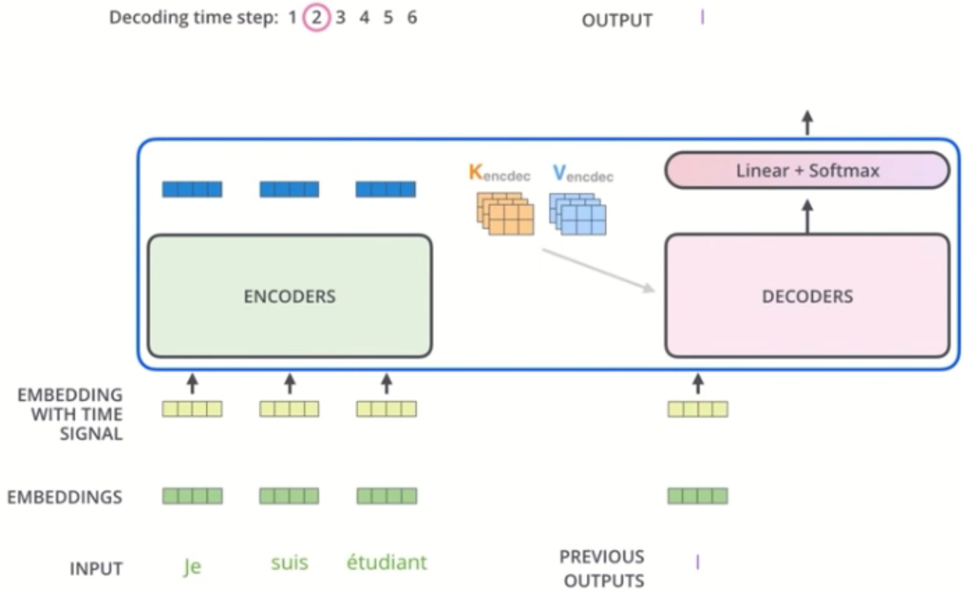
NATURAL LANGUAGE PROCESSING (NLP)



NATURAL LANGUAGE PROCESSING (NLP)

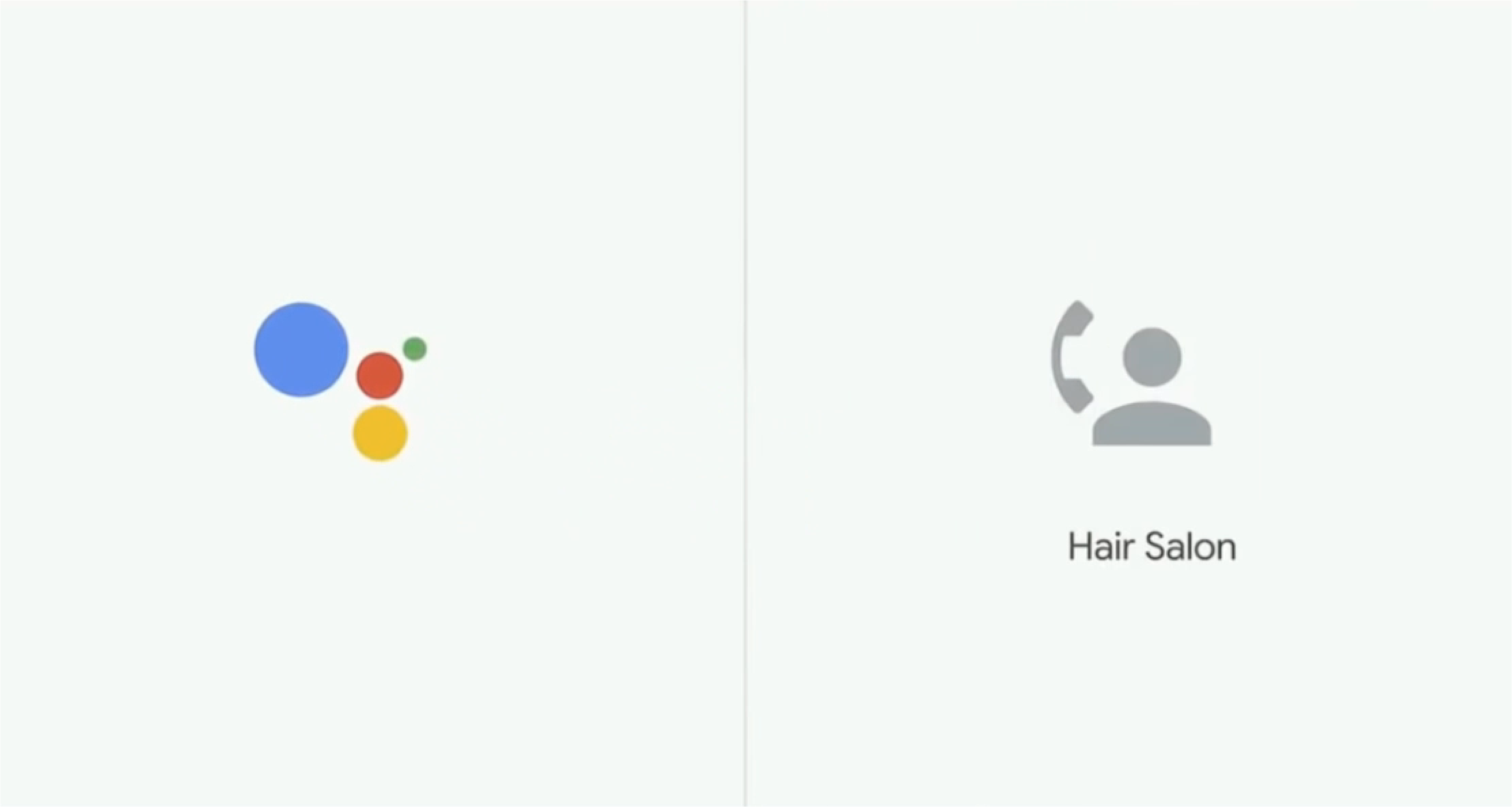
Question:	While eating a hamburger with friends, what are people trying to do?
Choices:	**have fun**, tasty, or indigestion
CoS-E:	Usually a hamburger with friends indicates a good time.
Question:	After getting drunk people couldn't understand him,it was because of his what?
Choices:	lower standards,**slurred speech**, or falling down
CoS-E:	People who are drunk have difficulty speaking.
Question:	People do what during their time off from work?
Choices:	**take trips**, brow shorter, or become hysterical
CoS-E:	People usually do something relaxing, such as taking trips,when they don't need to work.

Transformer



Vaswani et al. "Attention is all you need." *Advances in Neural Information Processing Systems*. 2017.

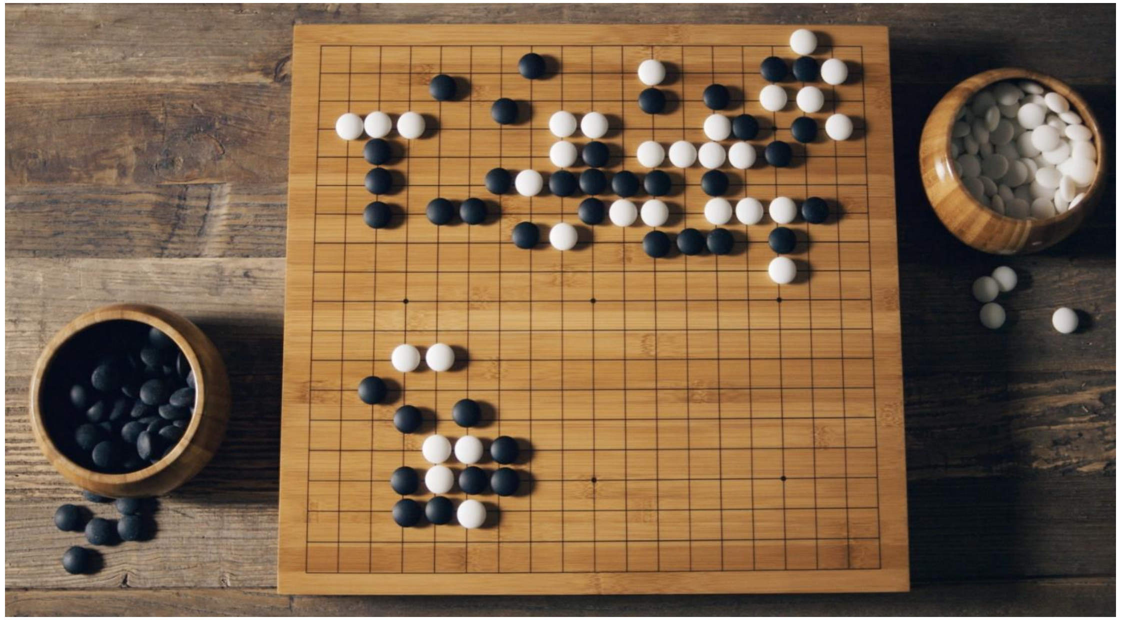
NATURAL LANGUAGE PROCESSING (NLP)

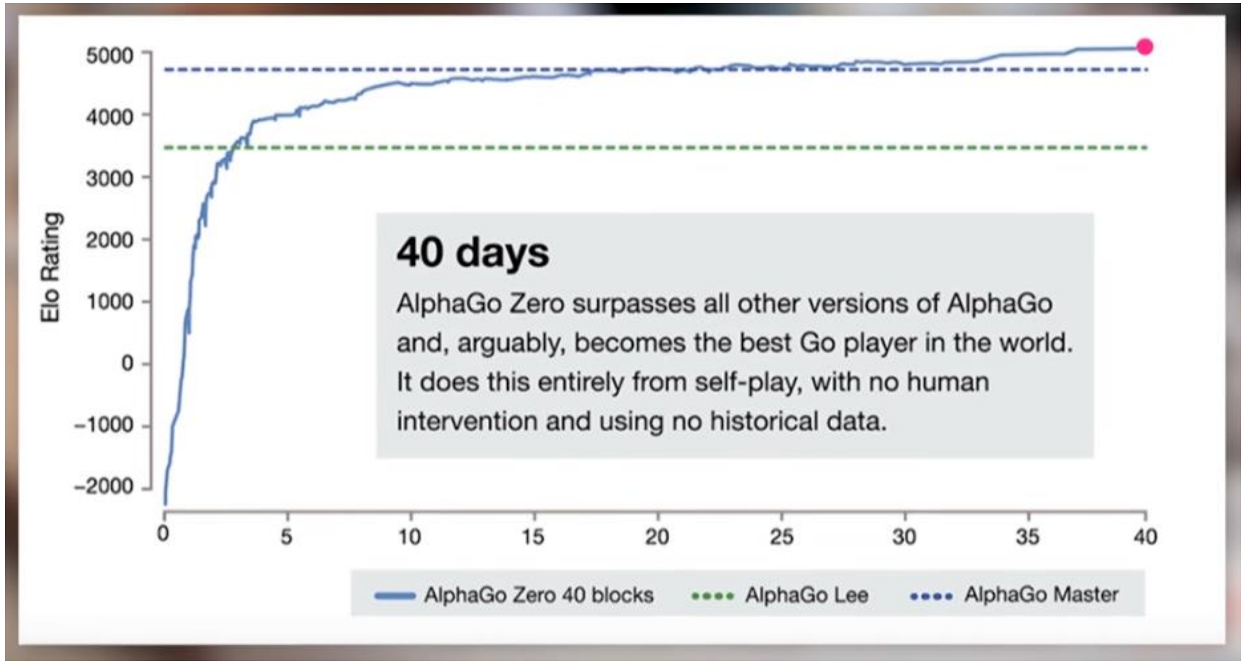


REINFORCEMENT LEARNING

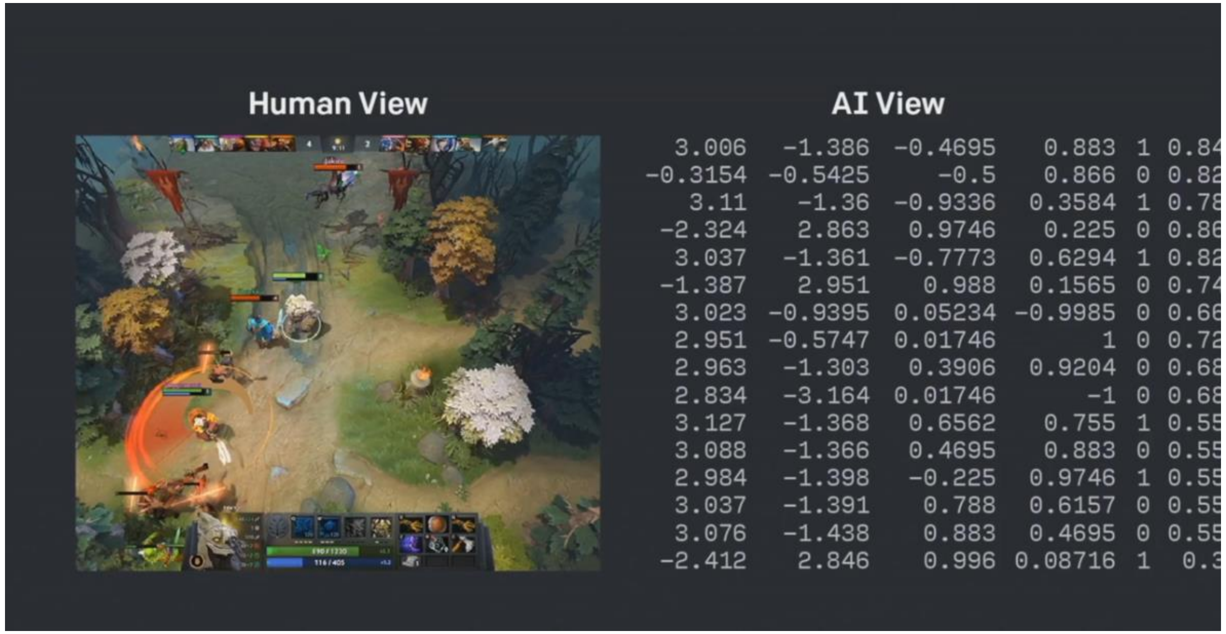
REINFORCEMENT LEARNING

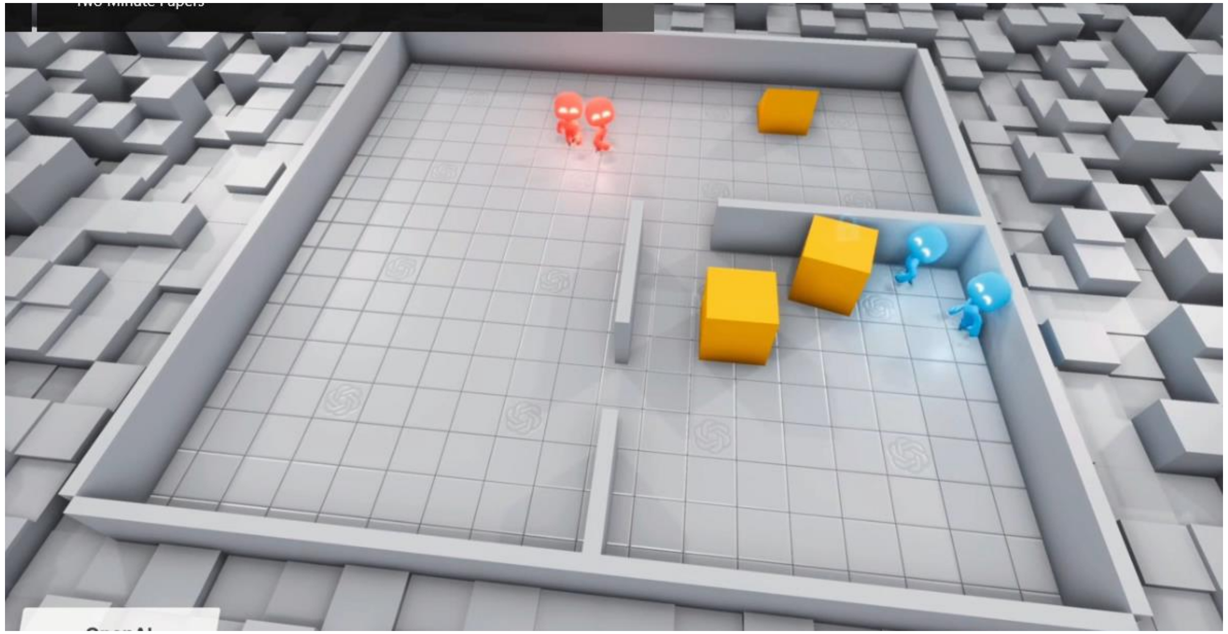
AlphaGo





REINFORCEMENT LEARNING





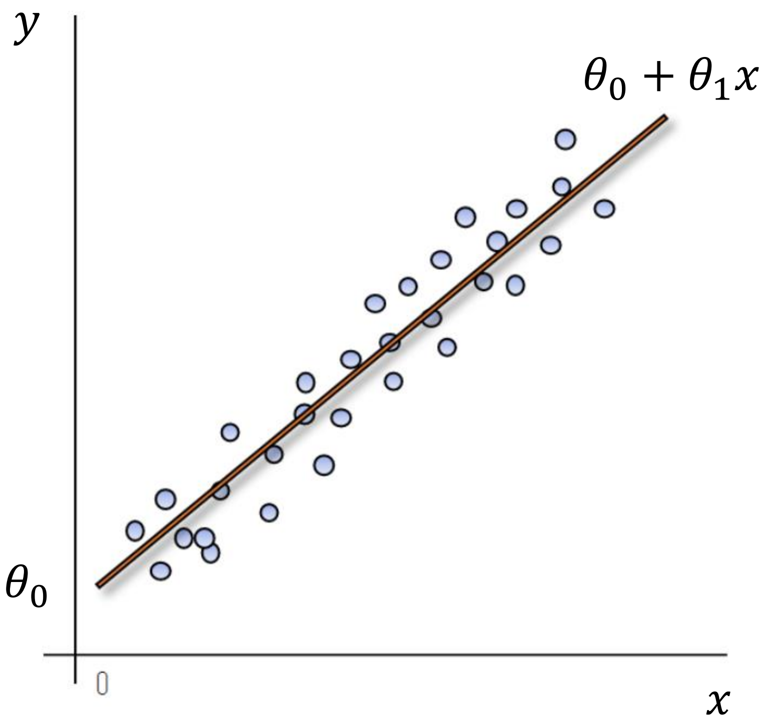
Why Now?

WHAT CHANGED?

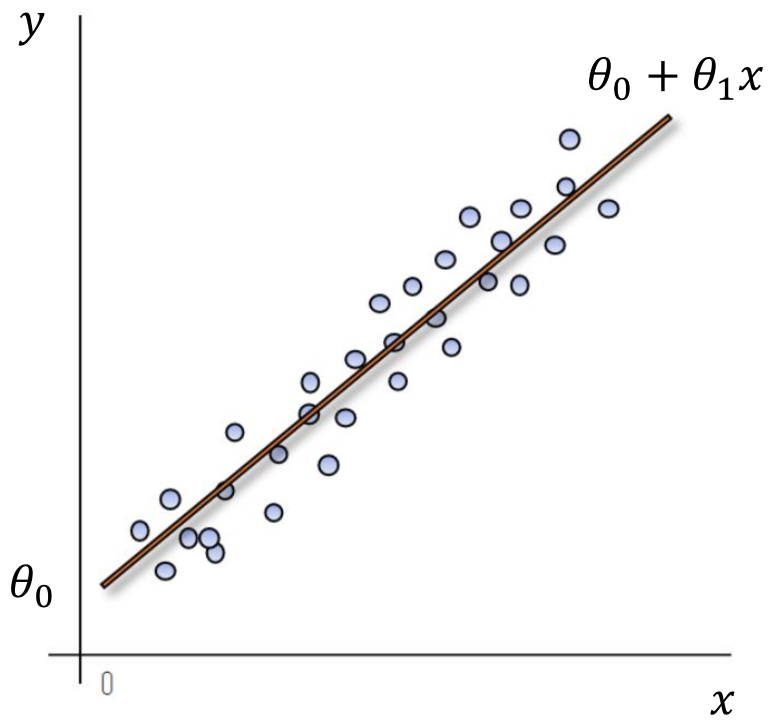
- Availability of high quality labelled data
- Quantity of data
- Advances in computational resources
- Better understanding of deep neural networks
- Solved issues in DNN learning process
  - Dropout/regularization
  - Batch Normalization
  - Vanishing/Exploding Gradients

Gradient Descent

LINEAR REGRESSION



LINEAR REGRESSION



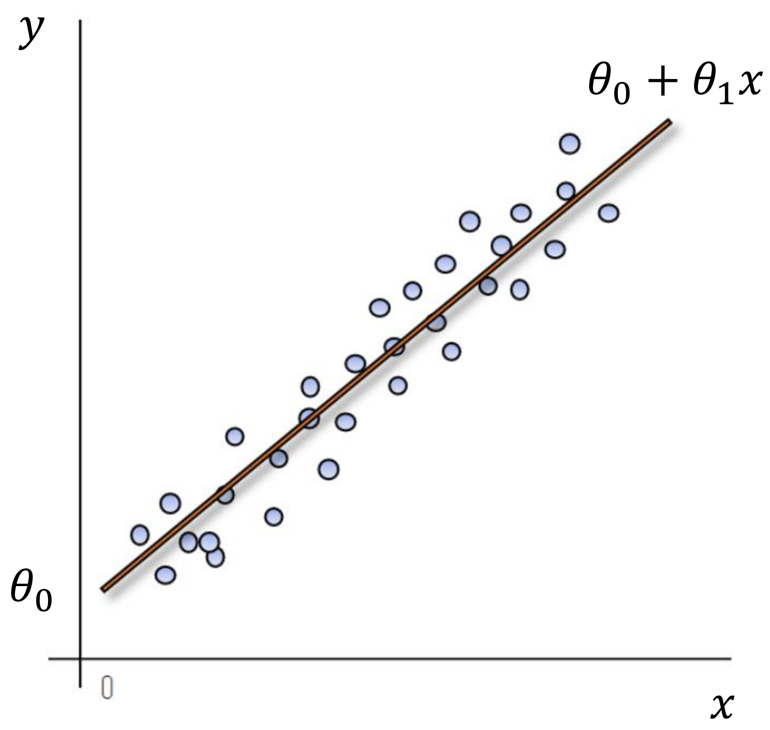
Univariate

$$h_\theta(x) = \theta_0 + \theta_1 x$$

Multivariate

$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \ldots + \theta_n x_n$$

LINEAR REGRESSION



Univariate

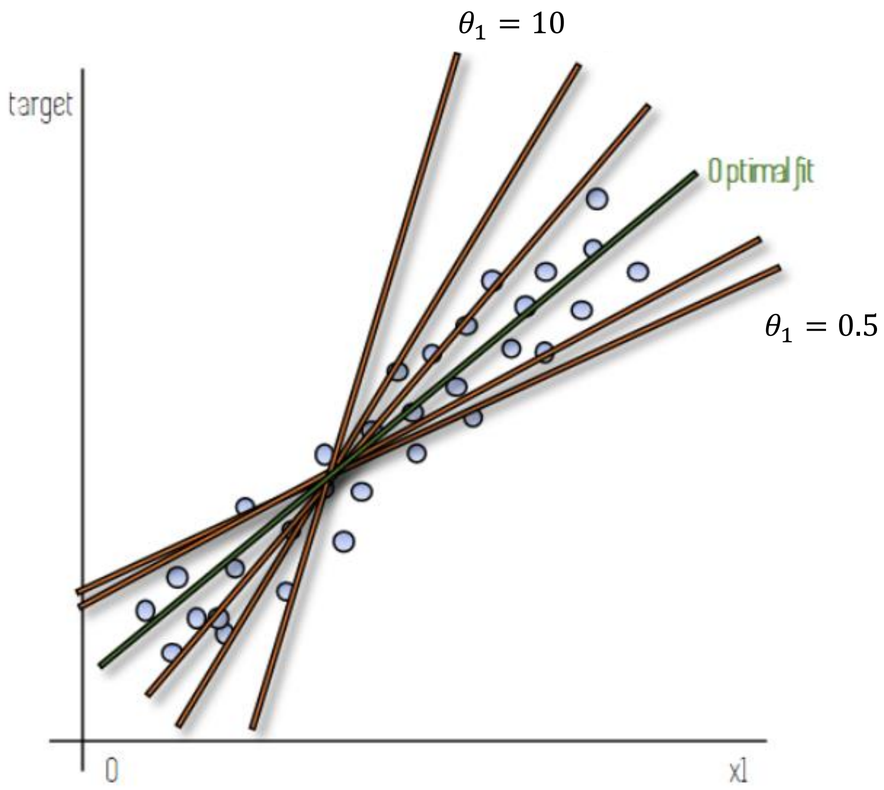
$$h_\theta(x) = \theta_0 + \theta_1 x$$

Multivariate

$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \ldots + \theta_n x_n$$

$$h_\theta(x) = \theta_0 + \theta . x, \text{ with } \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix} \text{ and } x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

LINEAR REGRESSION



Graphical representation of the different iterations of a linear regression model with one feature (x1)

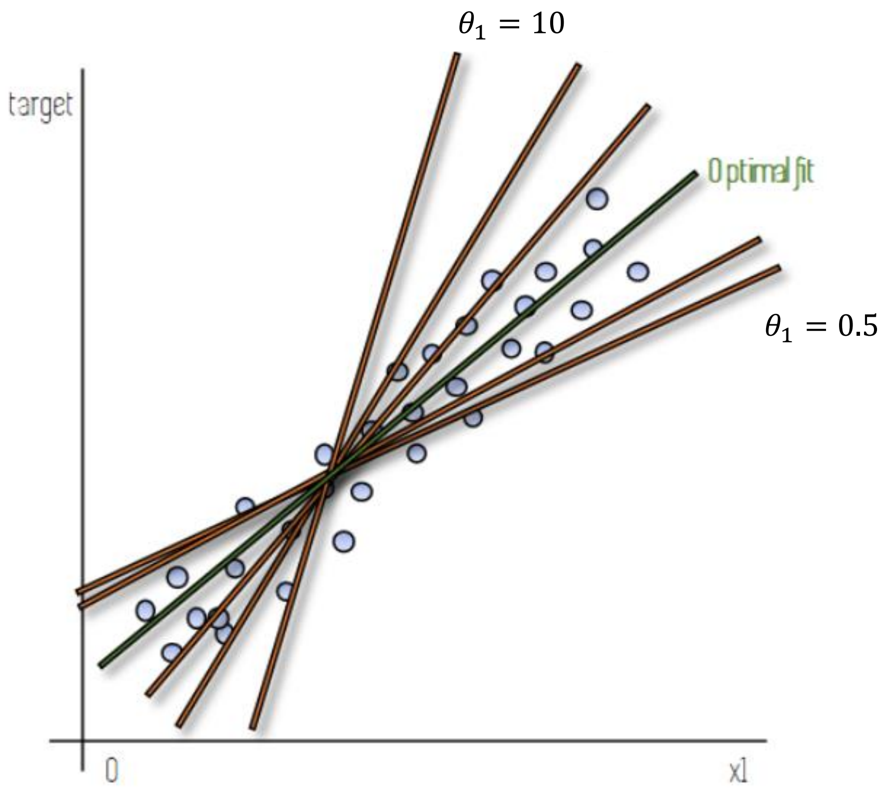
$$h_\theta(x) = \theta_0 + \theta_1 x$$

$$\theta_1 \ search\ space \in [0.5;\ 10]$$

Iteratively explore all options in space and compare them using an evaluation metric (for example Mean Absolute Error)

$$optimal\ \theta_1 = 1$$

LINEAR REGRESSION



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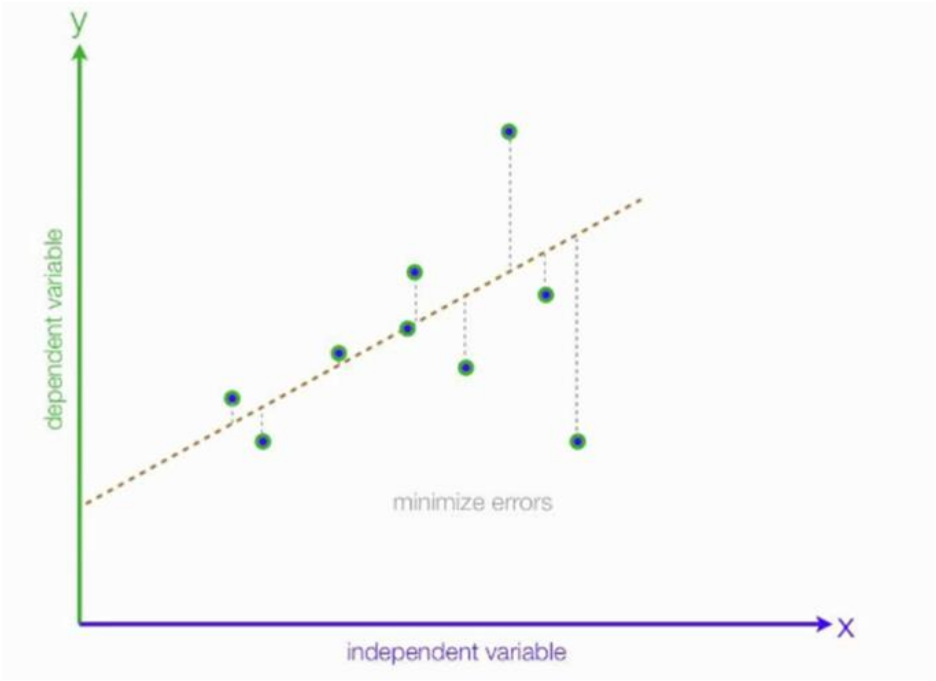
Optimization Problem!

GRADIENT DESCENT
OBJECTIVE FUNCTION

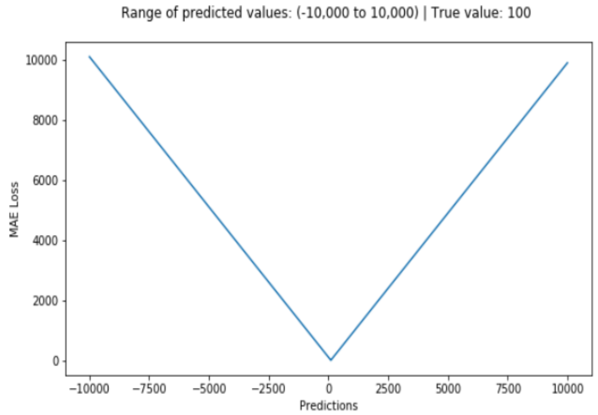
$$MAE = \frac{1}{m}\sum_{i=1}^{m}|h_\theta(x_i) - y_i| \qquad MSE = \frac{1}{m}\sum_{i=1}^{m}|h_\theta(x_i) - y_i|^2$$

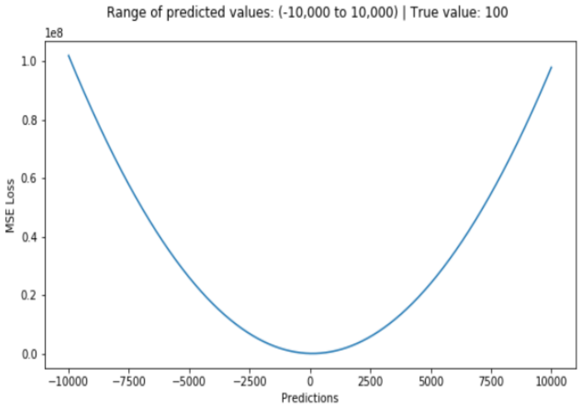
GRADIENT DESCENT

OBJECTIVE FUNCTION



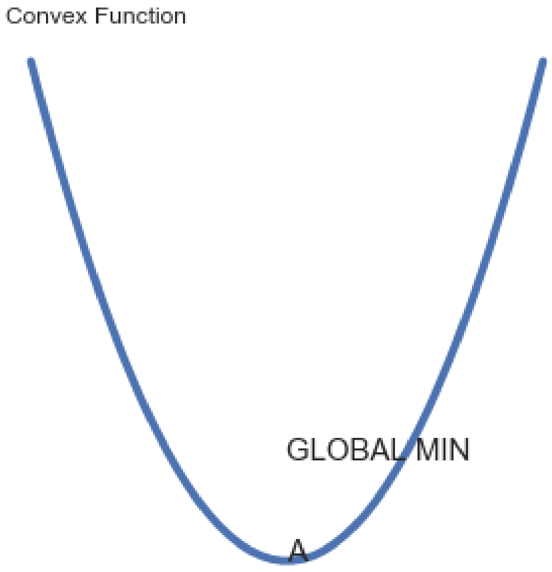
$$MAE = \frac{1}{m}\sum_{i=1}^{m}\left|h_\theta(x_i) - y_i\right| \qquad MSE = \frac{1}{m}\sum_{i=1}^{m}\left(h_\theta(x_i) - y_i\right)^2$$





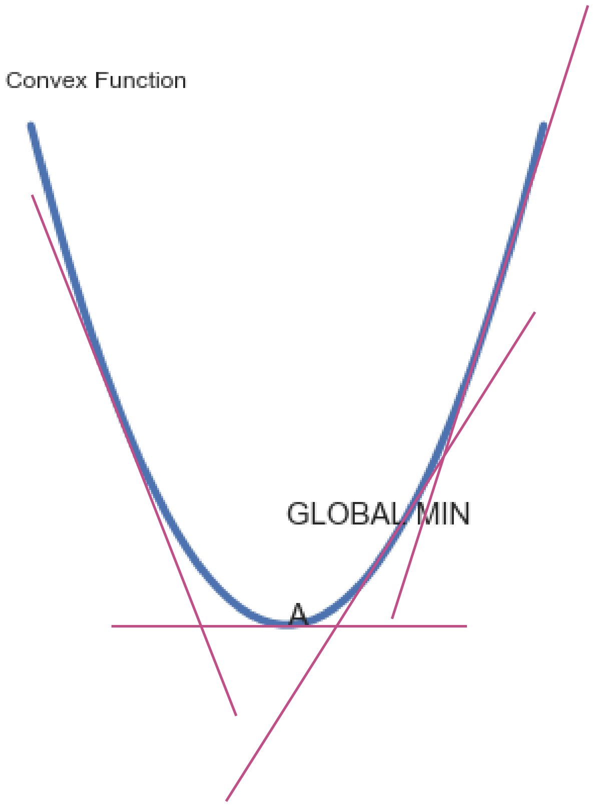
GRADIENT DESCENT

WEIGHTS UPDATING



$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m}(h_\theta(x_i) - y_i)^2$$

GRADIENT DESCENT
WEIGHTS UPDATING



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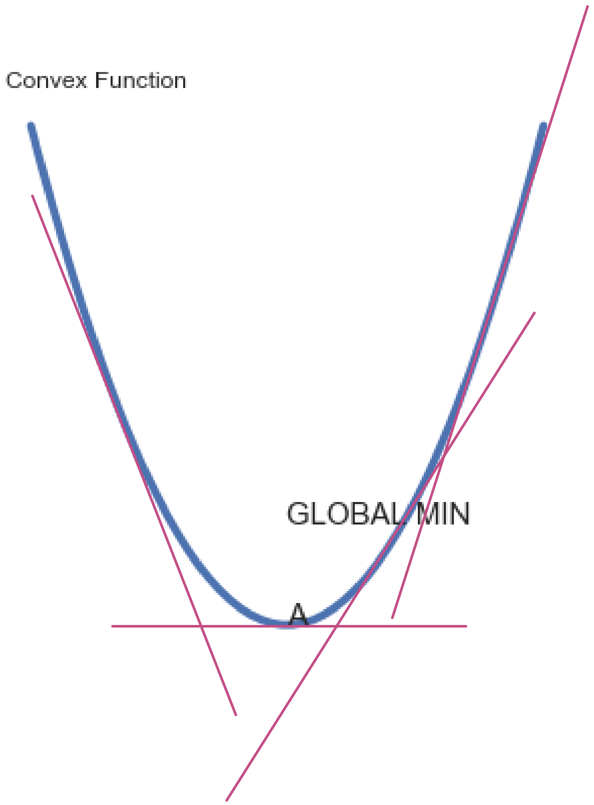
Minimum when $\frac{\partial J(\theta)}{\partial \theta_i} = 0$

GRADIENT DESCENT

WEIGHTS UPDATING

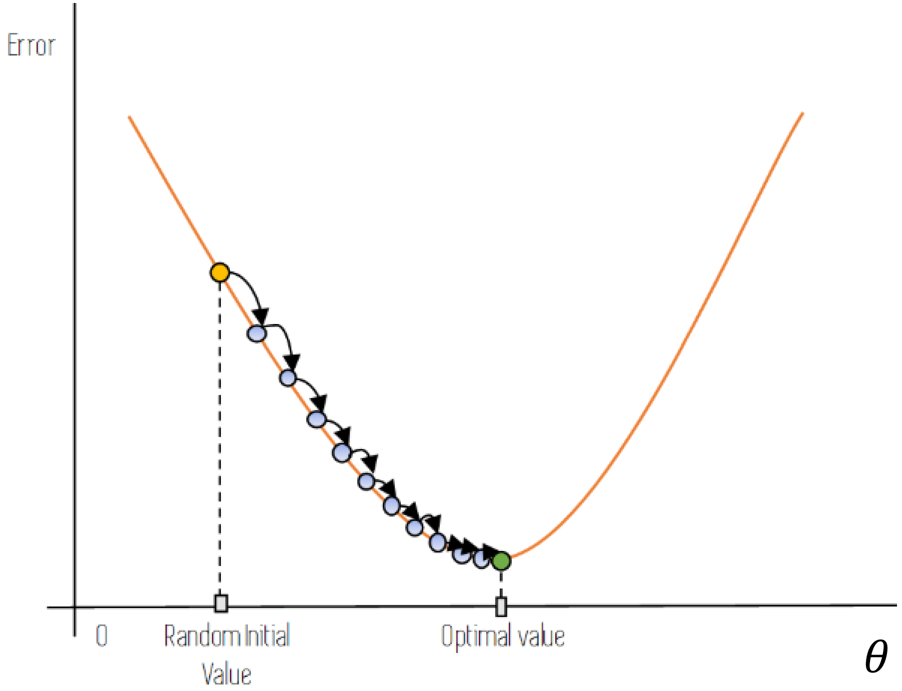


$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m}(h_\theta(x_i) - y_i)^2$$

Minimum when $\frac{\partial J(\theta)}{\partial \theta_i} = 0$

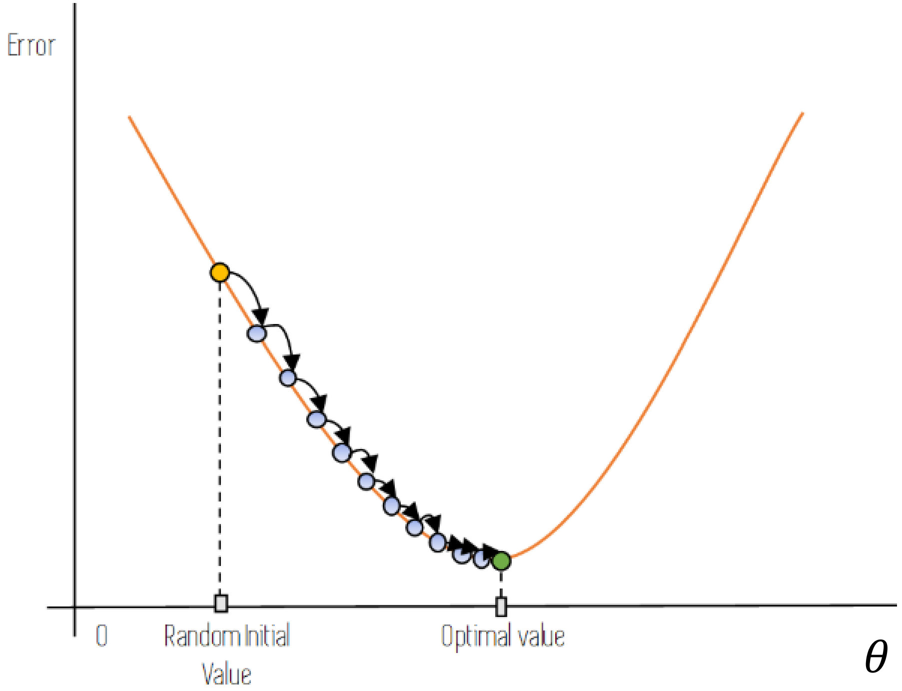
minimize $J(\theta) \equiv$ minimize $\text{const} * J(\theta) \equiv$ minimize $\frac{1}{\mathbf{2}m}\sum_{i=1}^{m}(h_\theta(x_i) - y_i)^2$

GRADIENT DESCENT
WEIGHTS UPDATING



Iteratively update θ and compute $J(\theta)$ until hopefully we get convergence on the global minimum

GRADIENT DESCENT
WEIGHTS UPDATING



Iteratively update θ and compute $J(\theta)$ until hopefully we get convergence on the global minimum

Gradient descent can converge to a local minimum

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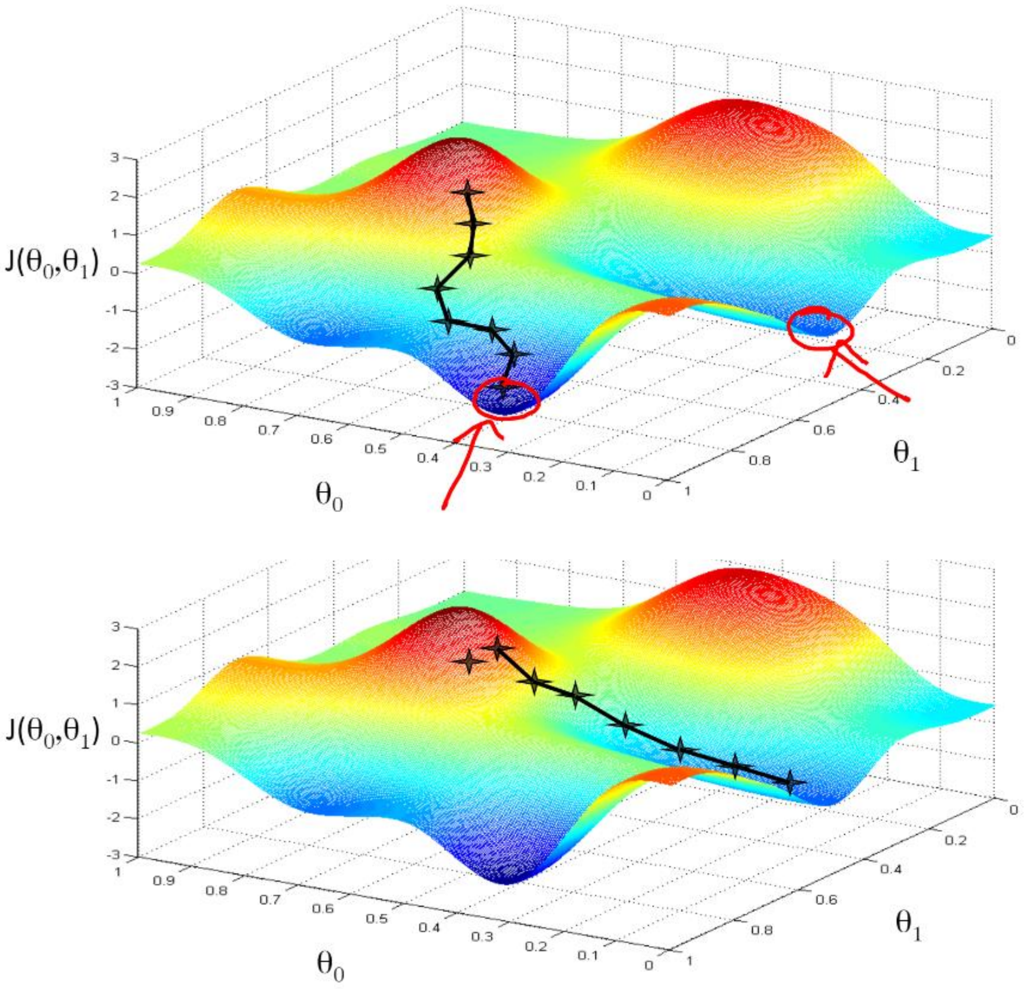
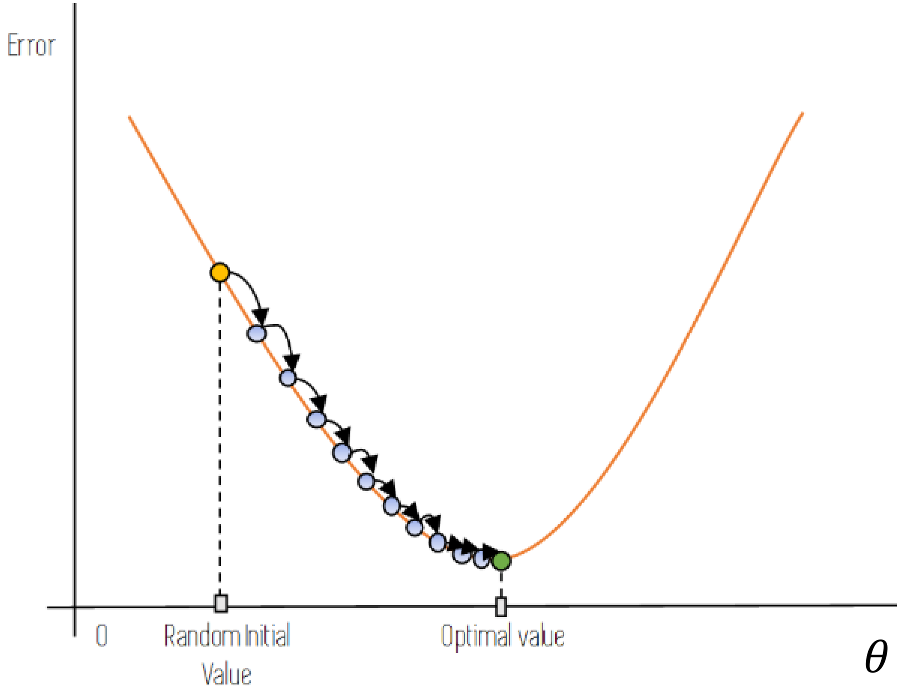


Iteratively update θ and compute $J(\theta)$ until hopefully we get convergence on the global minimum

Gradient descent can converge to a local minimum

Weights initialization (and optimizer parameters) must be chosen carefully

GRADIENT DESCENT
WEIGHTS UPDATING



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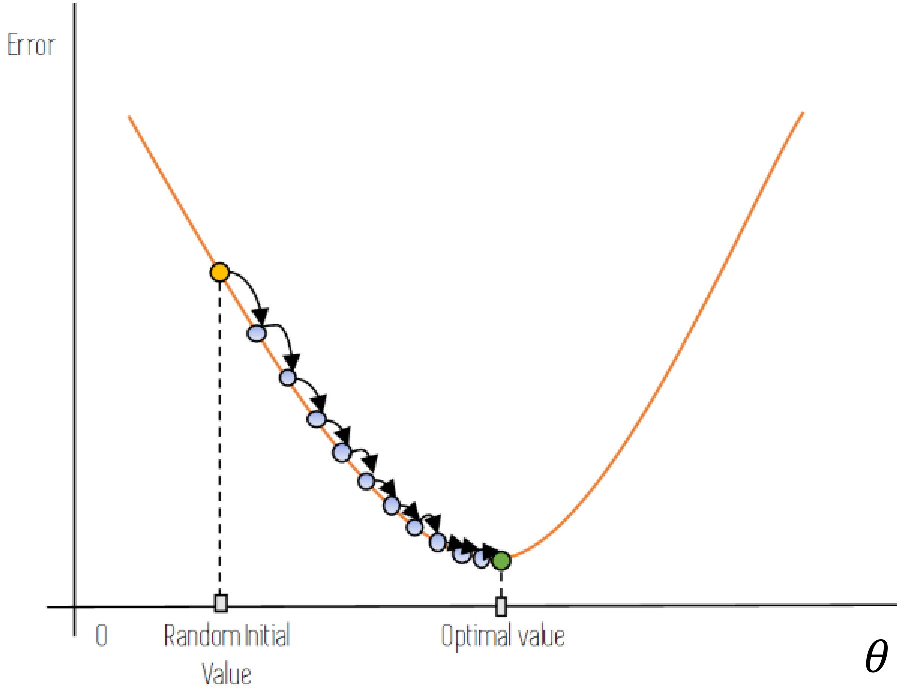
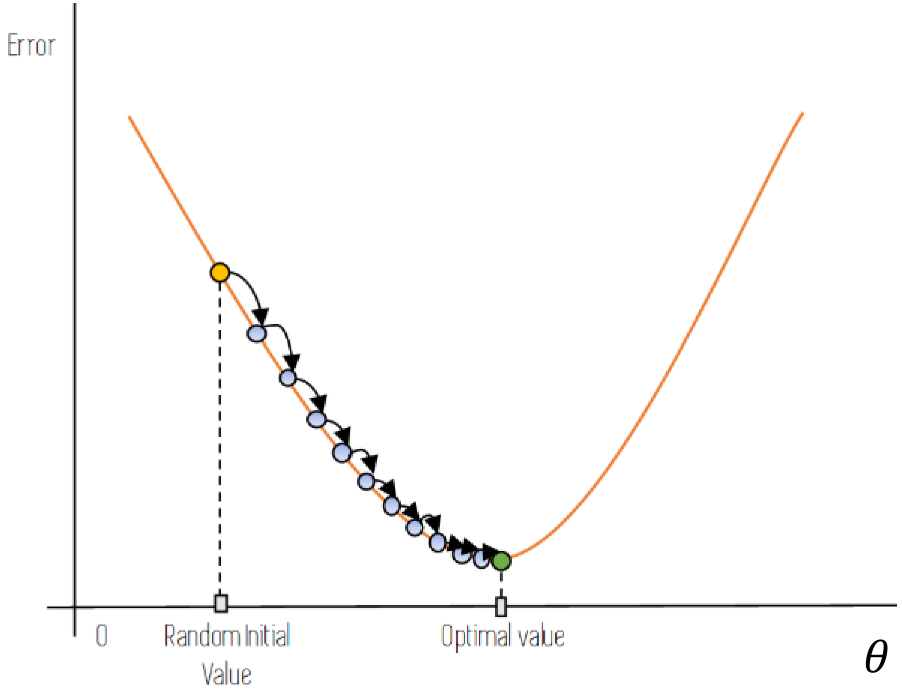


$$\boxed{\theta_j \coloneqq \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)}$$

GRADIENT DESCENT

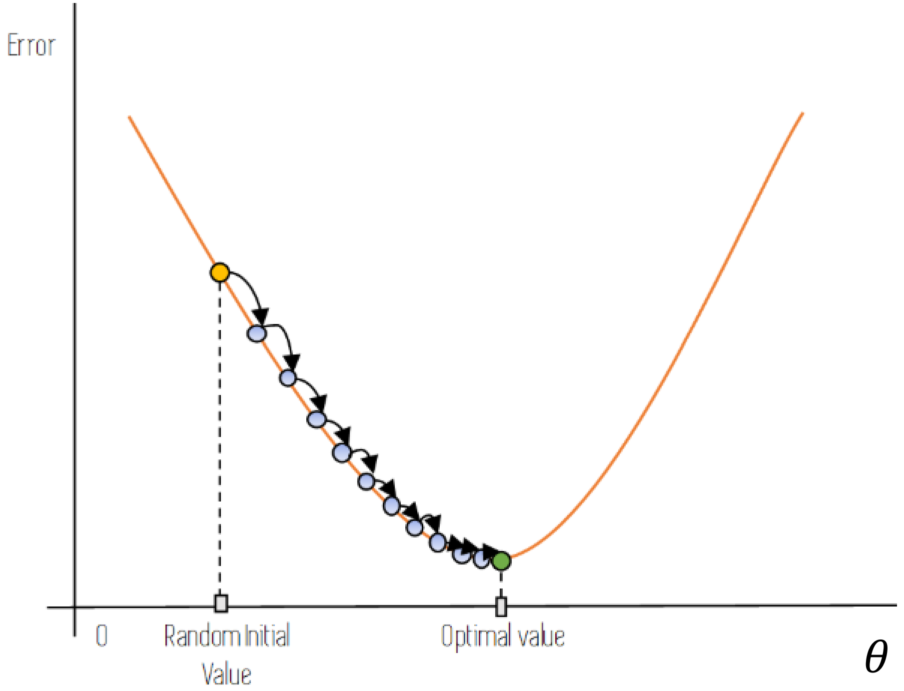
WEIGHTS UPDATING



learning rate

$$\boxed{\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)}$$

GRADIENT DESCENT
WEIGHTS UPDATING



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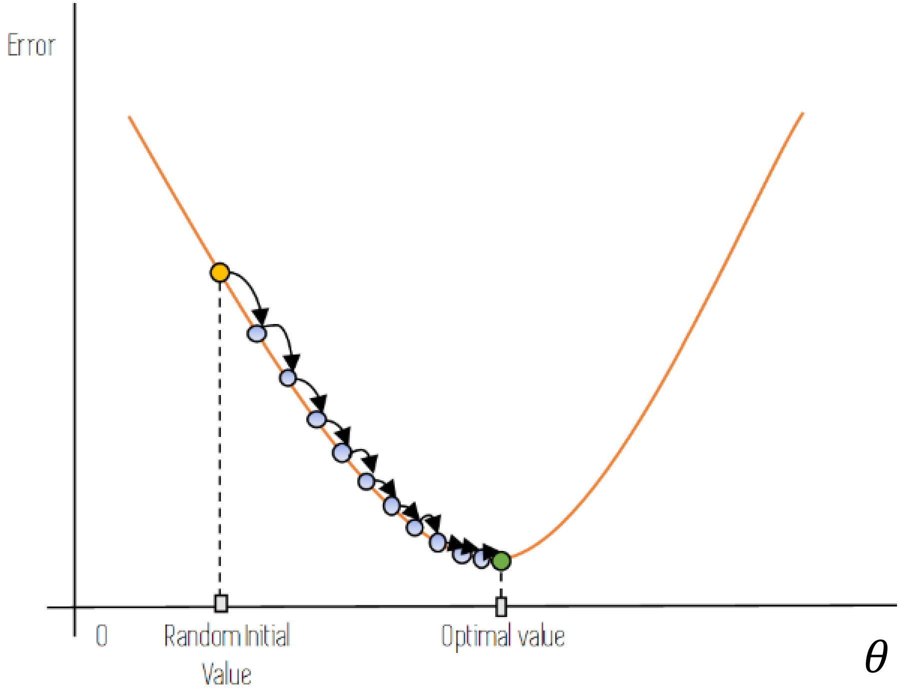
All θ_j must be updated simultaneous, otherwise the calculation of $\frac{\partial}{\partial \theta_j} J(\theta)$ will change within the same optimization step

GRADIENT DESCENT

WEIGHTS UPDATING



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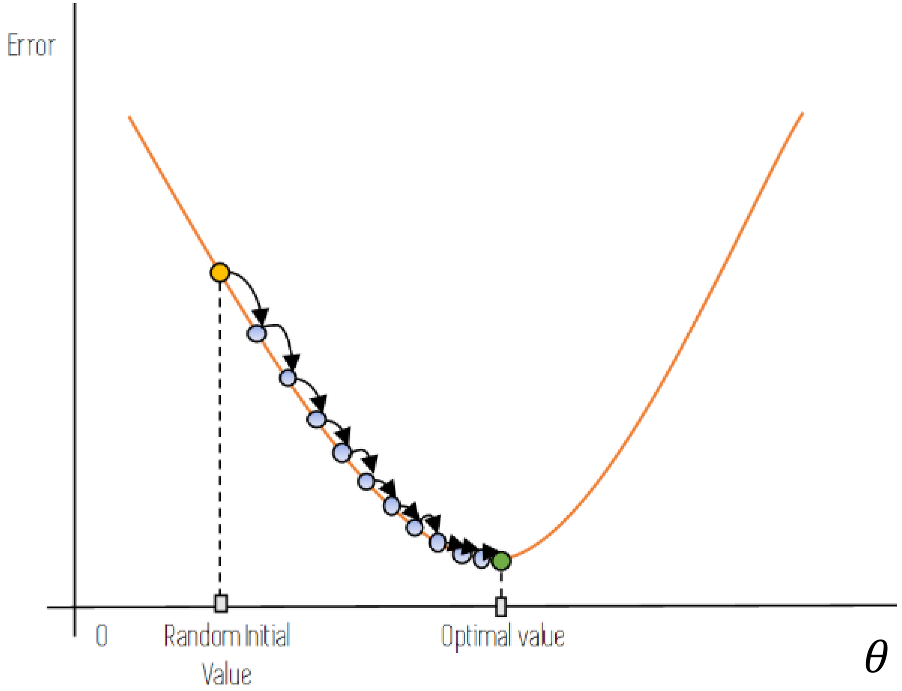
Derivative term decreases with closeness to the minimum, adaptively reducing the intensity of the updates

This allows for the learning rate to be fixed

GRADIENT DESCENT

WEIGHTS UPDATING



learning rate

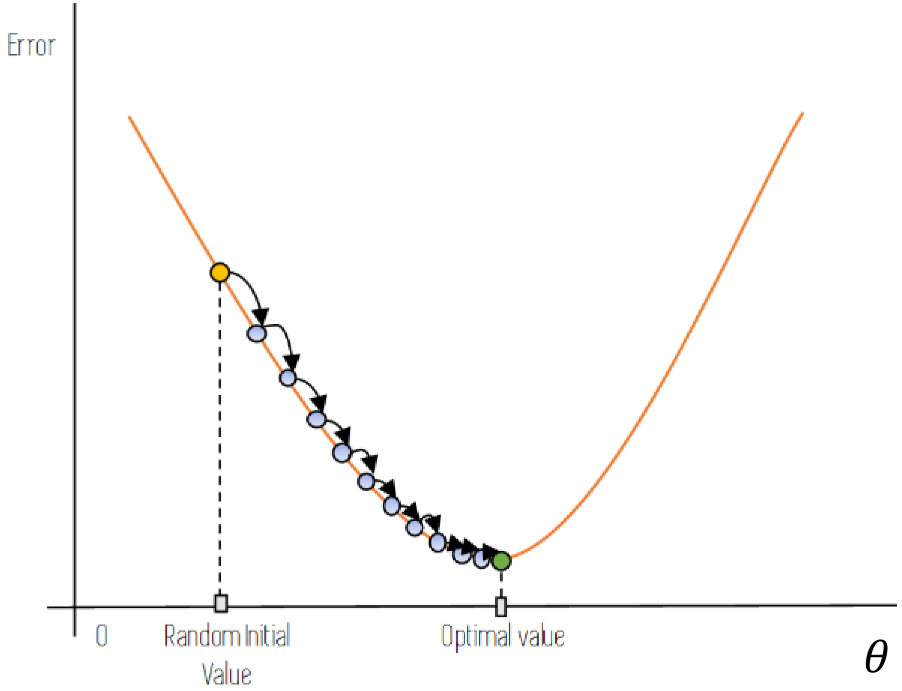
$$\boxed{\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)}$$

Learning rate choice is very important!

Too small step size and algorithm will be slow

Too big step size and algorithm will overshoot, which can lead to not reaching the minimum or even diverge

GRADIENT DESCENT
WEIGHTS UPDATING



learning rate

$$\boxed{\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)}$$

$$\theta_j := \theta_j - \alpha \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right).\, x_j^{(i)}$$

SO FAR

- Machine learning model is about fitting a function to several examples
- To fit this function, it is necessary to solve an optimization problem regarding the weights of our function
- To solve the optimization problem, a loss/objective function must be defined, as well as an updating strategy
- Gradient descent is a specific strategy that can be used to train machine learning models
- Gradient descent (and other learning strategies) can be susceptible to local minimums
- Good initializations of weights and parameters can be crucial to achieve a good outcome

Logistic Regression

LOGISTIC REGRESSION

$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \theta_4 x_4$$

$h_\theta(x) = \theta^T x$, with $x_0 = 1$

LOGISTIC REGRESSION

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$h_\theta(x) = \theta^T x$, with $x_0 = 1$

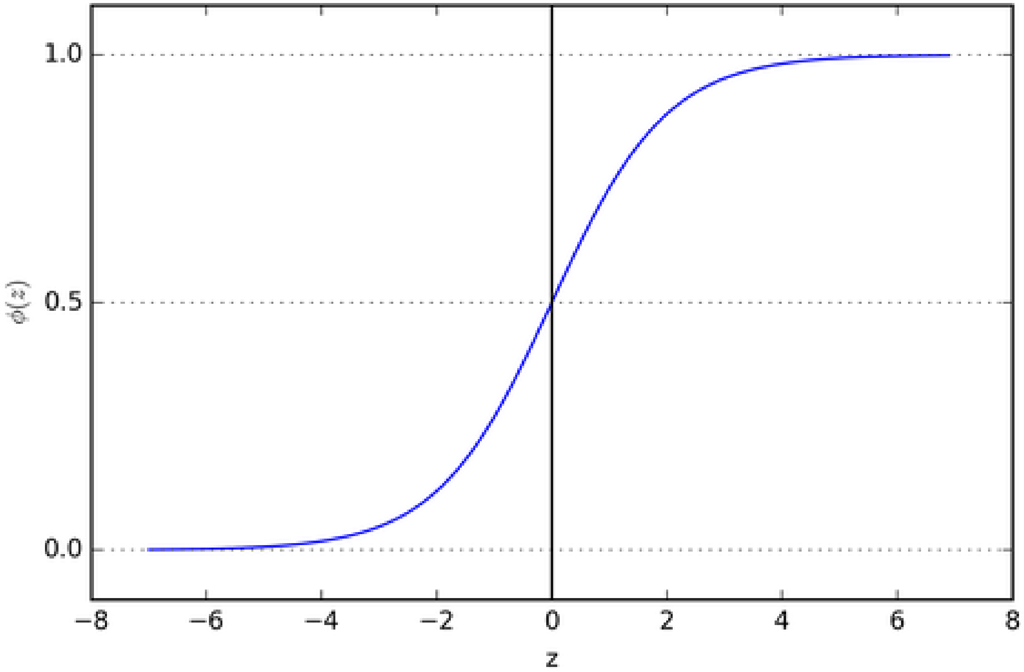
For classification problems it is useful that the results of $h_\theta(x)$ be either 0 or 1 (doesn't belong or belong to class)

Then we need to change our hypothesis.

LOGISTIC REGRESSION

SIGMOID

Sigmoid function: $g(z) = \frac{1}{1+e^{-z}}$



LOGISTIC REGRESSION

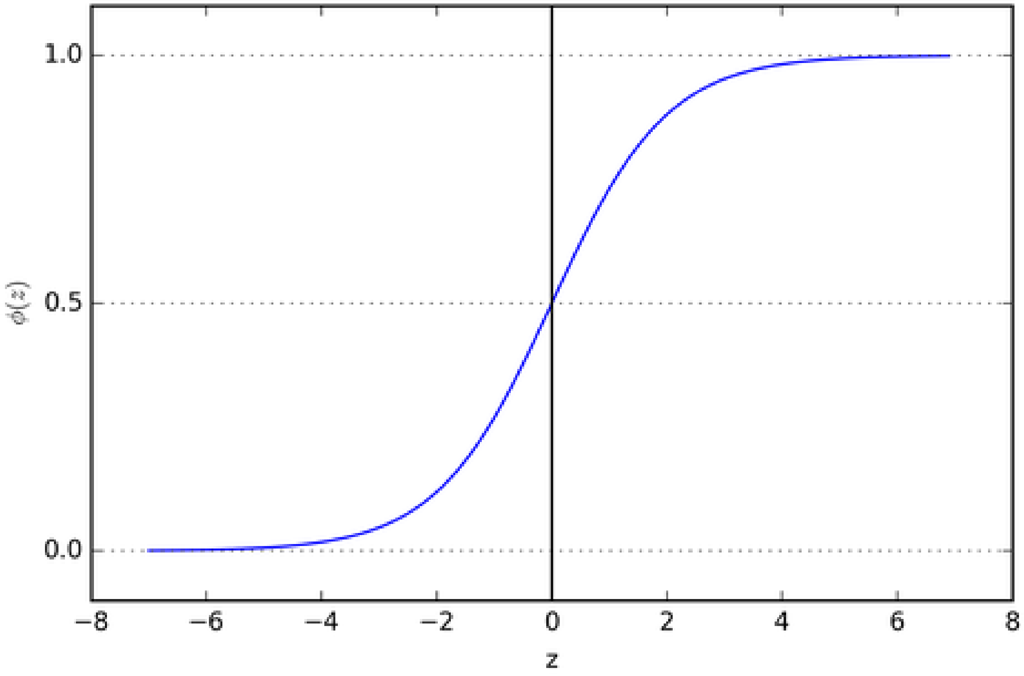
SIGMOID

Sigmoid function: $g(z) = \frac{1}{1+e^{-z}}$

Logistic regression: $h_\theta(x) = g(\theta^T x)$

$$h_\theta(x) = \frac{1}{1 + e^{-\theta^T x}}$$

$$0 \le h_\theta(x) \le 1$$



LOGISTIC REGRESSION
SIGMOID

Sigmoid function: $g(z) = \frac{1}{1+e^{-z}}$

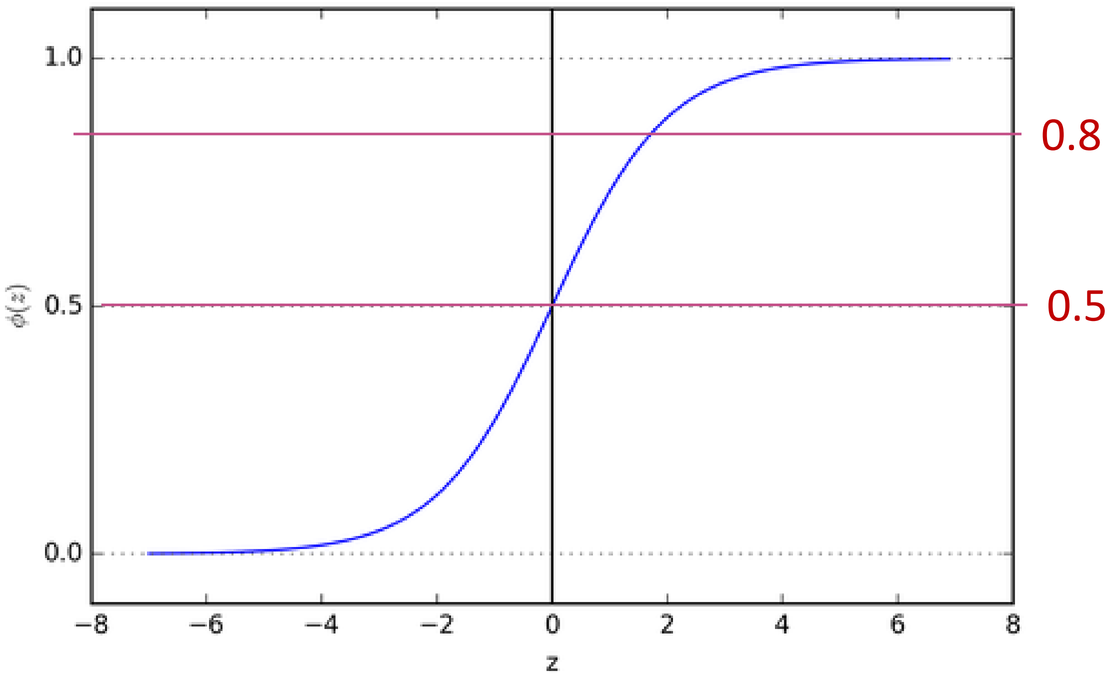
Logistic regression: $h_\theta(x) = g(\theta^T x)$

$$h_\theta(x) = \frac{1}{1 + e^{-\theta^T x}}$$

$$0 \leq h_\theta(x) \leq 1$$

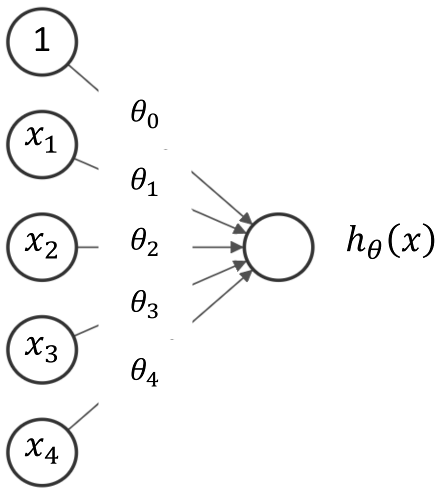
Predict 1 if $h_\theta(x) \geq 0.5$ defines similar probability for both events

Predict 1 if $h_\theta(x) \geq 0.8$ states that we only predict 1 if probability above 80%



LOGISTIC REGRESSION
INTUITION

$$h_\theta(x) = \theta^T x = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \theta_4 x_4$$

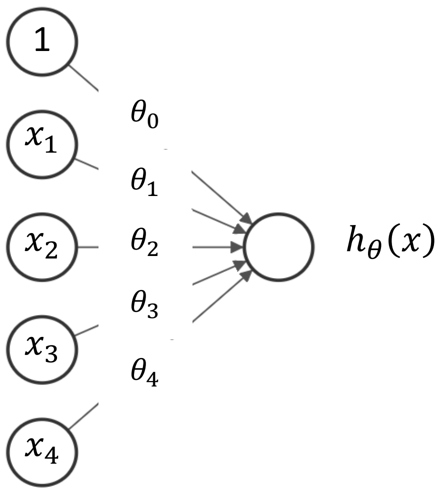


LOGISTIC REGRESSION
INTUITION

$$h_\theta(x) = \theta^T x = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \theta_4 x_4$$

Also valid for logistic regression!

$$h_\theta(x) = g(\theta^T x) = \frac{1}{1 + e^{-\theta^T x}}$$

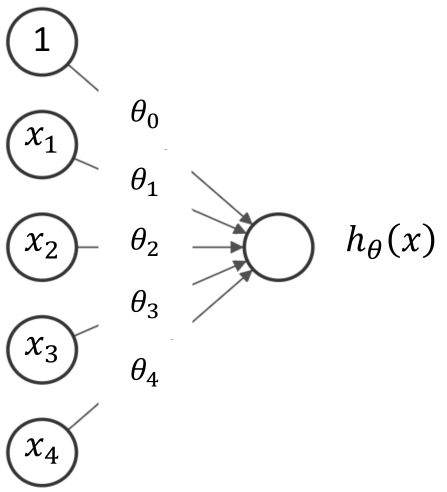


$$h_\theta(x) = \theta^T x = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \theta_4 x_4$$

Also valid for logistic regression!

$$h_\theta(x) = g(\theta^T x) = \frac{1}{1 + e^{-\theta^T x}}$$

And $h_\theta(x)$ defines then the probability of the neuron to be activated



LOGISTIC REGRESSION
OBJECTIVE FUNCTION

Can logistic regression be optimized in the same fashion as linear regression?

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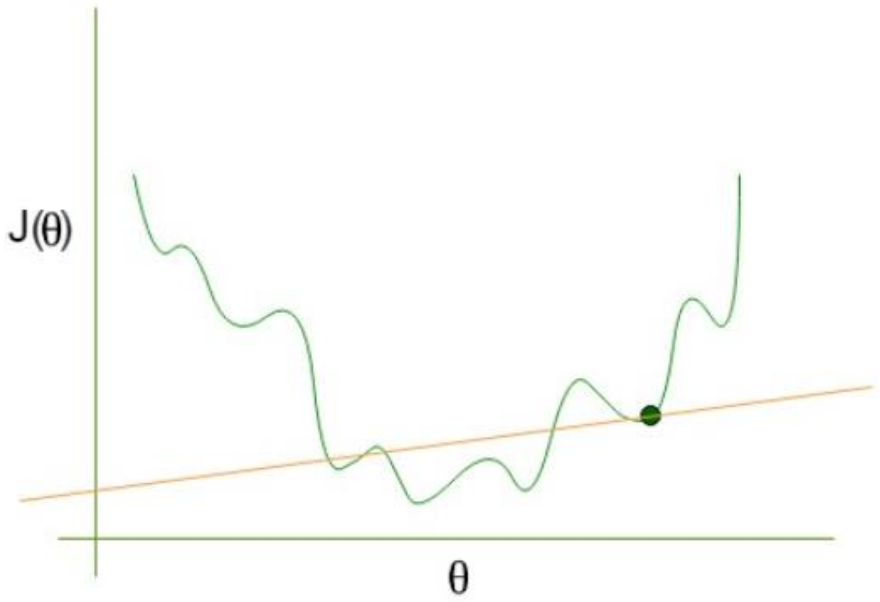
Application of linear regression cost function will result in a non-convex function.

What should be the cost function for logistic regression then?



LOGISTIC REGRESSION
OBJECTIVE FUNCTION

$$Cost(h_\theta(x), y) = \begin{cases} -\log\big(h_\theta(x)\big), & if\ y = 1 \\ -\log\big(1 - h_\theta(x)\big), & if\ y = 0 \end{cases}$$

LOGISTIC REGRESSION
OBJECTIVE FUNCTION

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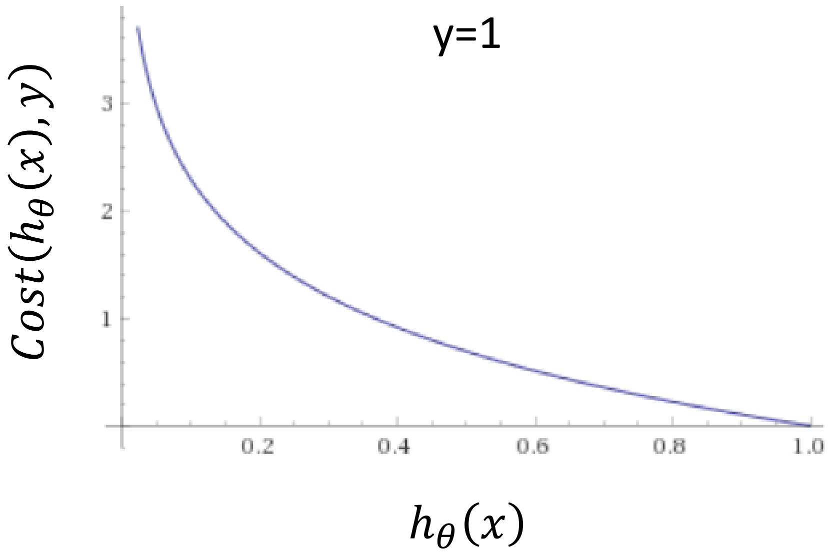

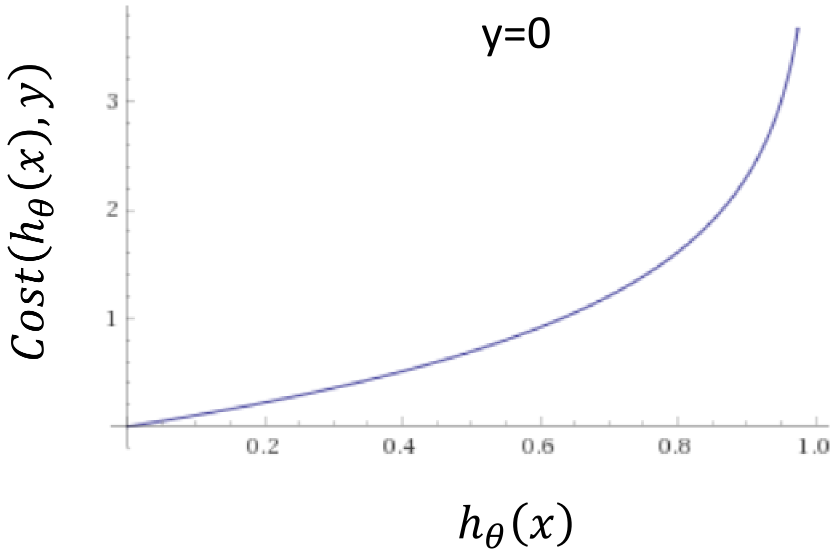

LOGISTIC REGRESSION
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$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m} Cost\big(h_\theta\big(x^{(i)}\big), y^{(i)}\big)$$

LOGISTIC REGRESSION

OBJECTIVE FUNCTION

$$Cost(h_\theta(x), y) = \begin{cases} -\log\big(h_\theta(x)\big), & if\ y = 1 \\ -\log\big(1 - h_\theta(x)\big), & if\ y = 0 \end{cases}$$

Rewritable as

$$Cost(h_\theta(x), y) = -\mathrm{y}.\log\big(h_\theta(x)\big) - (1 - y).\log\big(1 - h_\theta(x)\big)$$

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m} Cost\big(h_\theta\big(x^{(i)}\big), y^{(i)}\big) = -\frac{1}{m}\left[\sum_{i=1}^{m} y^{(i)} \log\Big(h_\theta\big(x^{(i)}\big)\Big) + \big(1 - y^{(i)}\big).\log\Big(1 - h_\theta\big(x^{(i)}\big)\Big)\right]$$

LOGISTIC REGRESSION

WEIGHTS UPDATE

$$Cost(h_\theta(x), y) = \begin{cases} -\log(h_\theta(x)), & if\ y = 1 \\ -\log(1 - h_\theta(x)), & if\ y = 0 \end{cases}$$

Rewritable as

$$Cost(h_\theta(x), y) = -\mathrm{y}.\log(h_\theta(x)) - (1 - y).\log(1 - h_\theta(x))$$

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m} Cost\left(h_\theta\left(x^{(i)}\right), y^{(i)}\right) = -\frac{1}{m}\left[\sum_{i=1}^{m} y^{(i)} \log\left(h_\theta\left(x^{(i)}\right)\right) + \left(1 - y^{(i)}\right).\log\left(1 - h_\theta\left(x^{(i)}\right)\right)\right]$$

To apply gradient descent, we just need to calculate the derivative of the new cost function, and plug it into the weights updating formula

$$\theta_j \coloneqq \theta_j \ - \ \alpha \frac{\partial}{\partial \theta_j} J(\theta)$$

LOGISTIC REGRESSION

WEIGHTS UPDATE

$$Cost(h_\theta(x), y) = \begin{cases} -\log(h_\theta(x)), & if\ y = 1 \\ -\log(1 - h_\theta(x)), & if\ y = 0 \end{cases}$$

Rewritable as

$$Cost(h_\theta(x), y) = -\mathrm{y}.\log(h_\theta(x)) - (1 - y).\log(1 - h_\theta(x))$$

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m} Cost(h_\theta(x^{(i)}), y^{(i)}) = -\frac{1}{m}\left[\sum_{i=1}^{m} y^{(i)} \log\left(h_\theta(x^{(i)})\right) + (1 - y^{(i)}).\log\left(1 - h_\theta(x^{(i)})\right)\right]$$

To apply gradient descent, we just need to calculate the derivative of the new cost function, and plug it into the weights updating formula

$$\theta_j \coloneqq \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta) = \theta_j - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}).\, x_j^{(i)}$$

$$Cost(h_\theta(x), y) = \begin{cases} -\log\left(h_\theta(x)\right), & if\ y = 1 \\ -\log\left(1 - h_\theta(x)\right), & if\ y = 0 \end{cases}$$

$\boxed{\text{Rewritable as } Cost(h_\theta(x), y) = -\mathrm{y}.\log\left(h_\theta(x)\right) - (1-y).\log\left(1 - h_\theta(x)\right)}$

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m} Cost\left(h_\theta\left(x^{(i)}\right), y^{(i)}\right) = -\frac{1}{m}\left[\sum_{i=1}^{m} y^{(i)} \log\left(h_\theta\left(x^{(i)}\right)\right) + \left(1 - y^{(i)}\right).\log\left(1 - h_\theta\left(x^{(i)}\right)\right)\right]$$

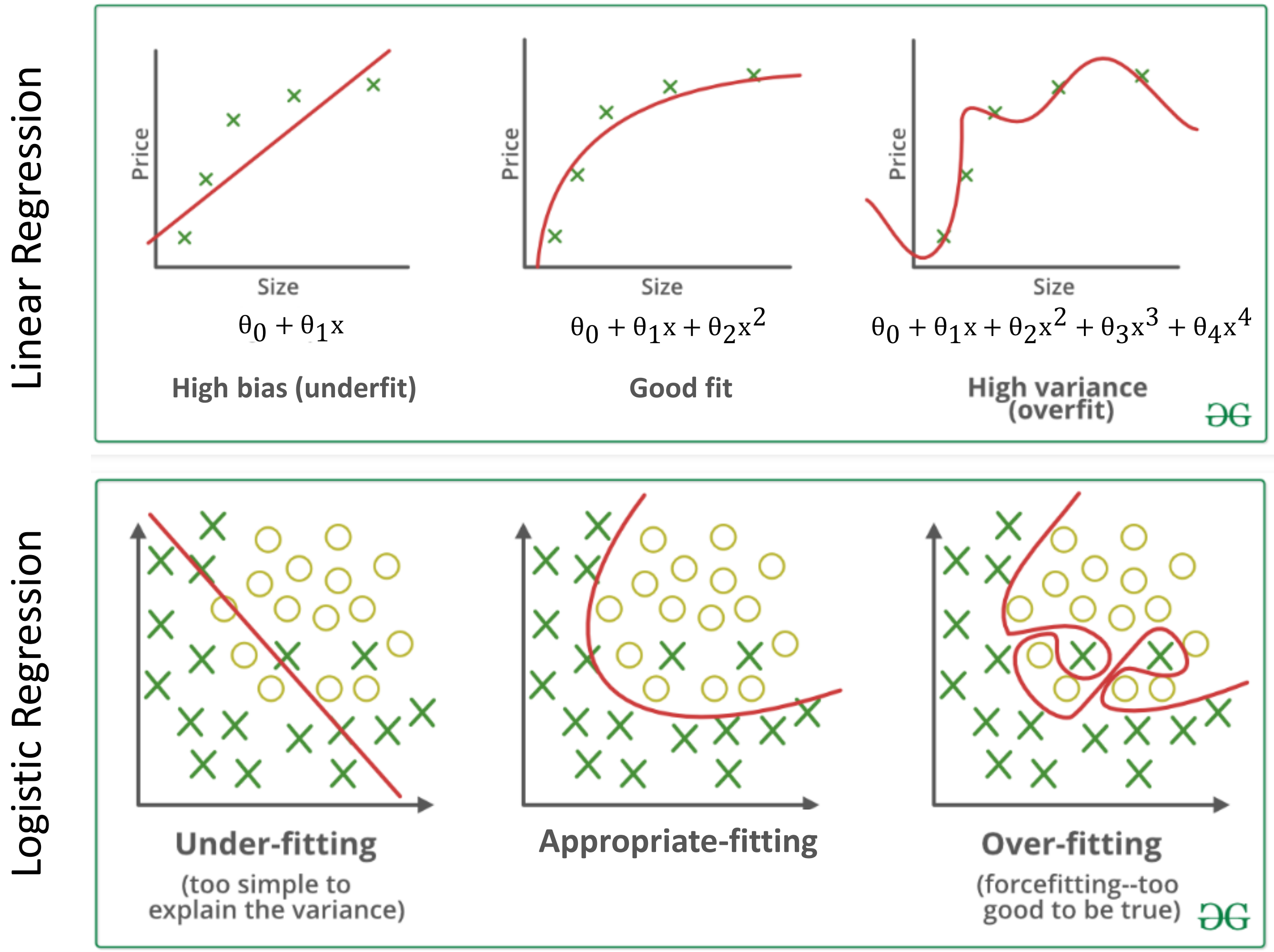
To apply gradient descent, we just need to calculate the derivative of the new cost function, and plug it into the weights updating formula

$$\theta_j \coloneqq \theta_j \ - \ \alpha \frac{\partial}{\partial \theta_j} J(\theta) = \ \boxed{\theta_j \ - \ \alpha \frac{1}{m} \textstyle\sum_{i=1}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right).x_j^{(i)}}$$

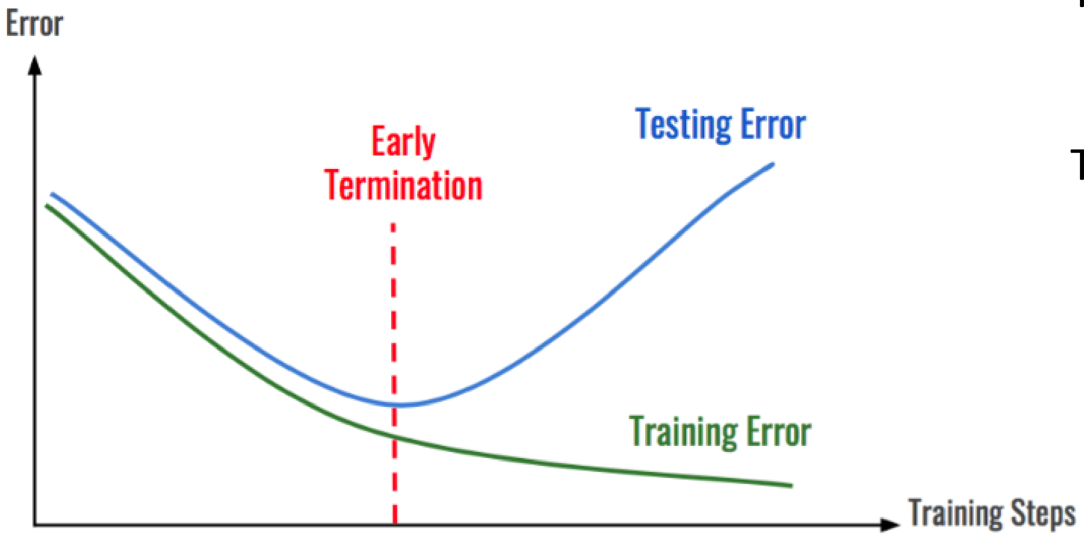
Same as for linear regression!
Only $h_\theta(x)$ is different!

Assessing Fitness

UNDERFIT/OVERFIT



UNDERFIT/OVERFIT



When the **testing error** starts to **increase**, it's time to stop!

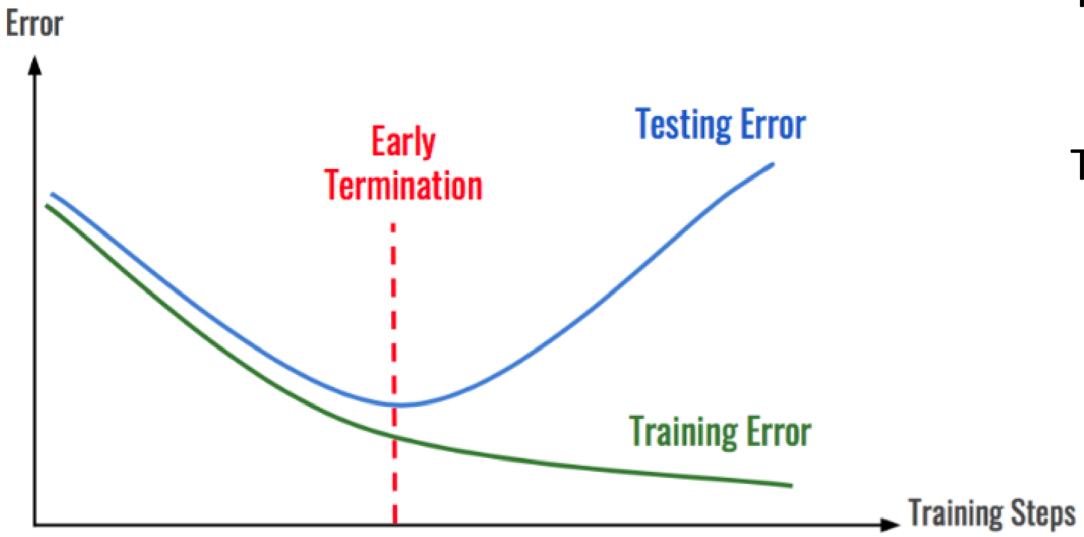
Too few iterations => model can still be underfit

- There is still potential to improve on the validation set

Too many iteration => model can be overfit

- Training error continues to decrease, as model memorizes samples
- Testing error increases has model loses capability of generalization

UNDERFIT/OVERFIT



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Too few iterations => model can still be underfit

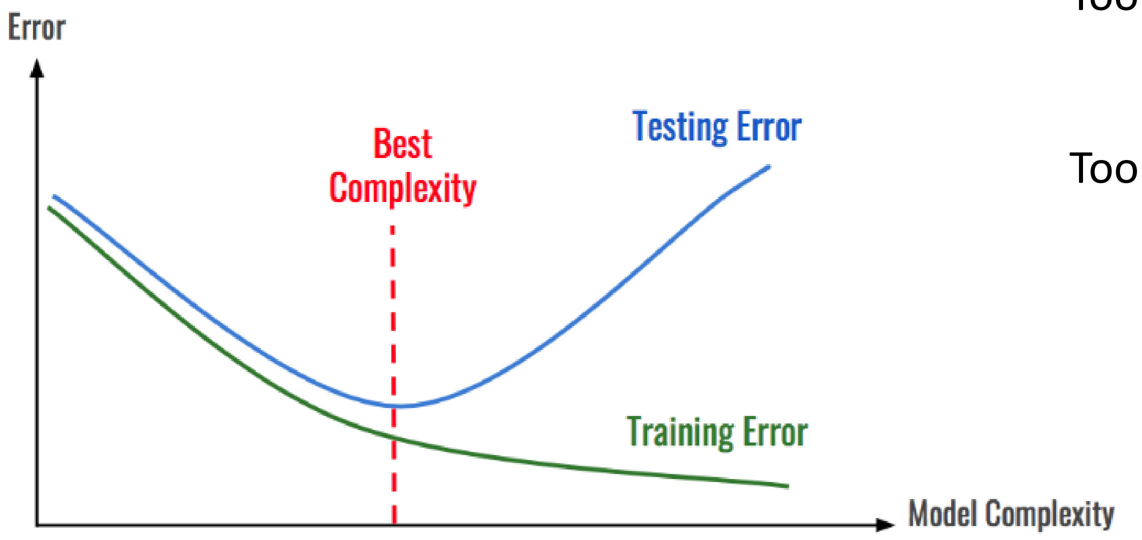
- There is still potential to improve on the validation set

Too many iterations => model can overfit

- Training error continues to decrease, as model memorizes samples
- Testing error increases has model loses capability of generalization

Good number of iterations depends on hyperparameters and model complexity!

UNDERFIT/OVERFIT



On the left, the model is too simple. On the right it overfits.

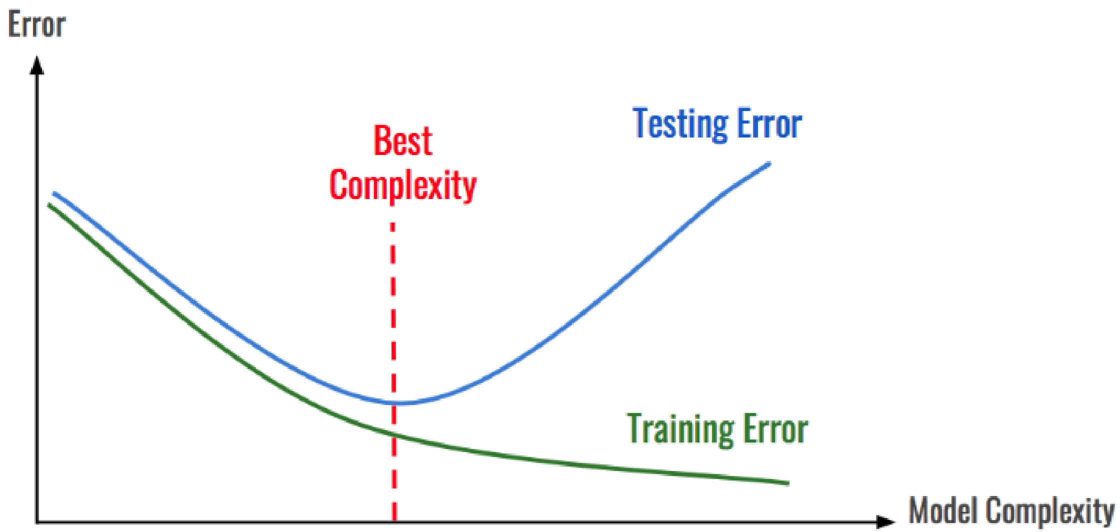
Too simple a model => model can still be underfit

- There is still potential to improve on the validation set

Too complex of a model => model can overfit

- Training error continues to decrease, as model memorizes samples
- Testing error increases has model loses capability of generalization

UNDERFIT/OVERFIT



On the left, the model is too simple. On the right it overfits.

simple a model => model can still be underfit

There is still potential to improve on the validation set

complex of a model => model can overfit

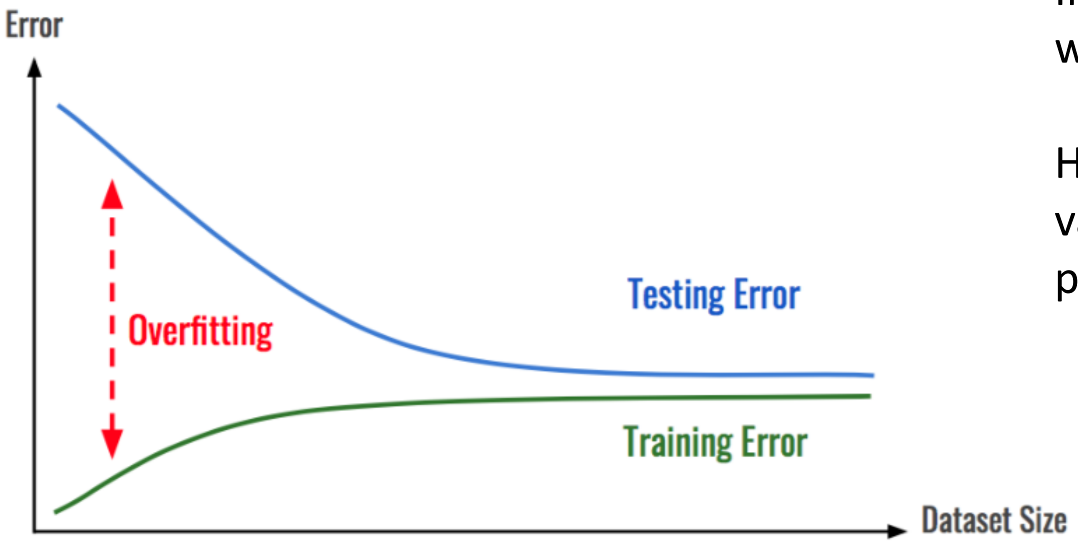
Training error continues to decrease, as model memorizes samples

Testing error increases has model loses capability of generalization

Choice of model complexity depends greatly on complexity of the data.

Typically one must try different architectures/complexities and adapt the model complexity in order to avoid underfit/overfit

UNDERFIT/OVERFIT



The **more data** you get, the **less** likely the model is to **overfit.**

Increasing the dataset size tends to reduce overfitting as there will exist more data to learn on and conditioning the learning

However, if increase in dataset also increases greatly the variance of patterns existing in the data, we can easily end in a problem that is to complex for our current model complexity.

UNDERFIT/OVERFIT

To address overfitting:

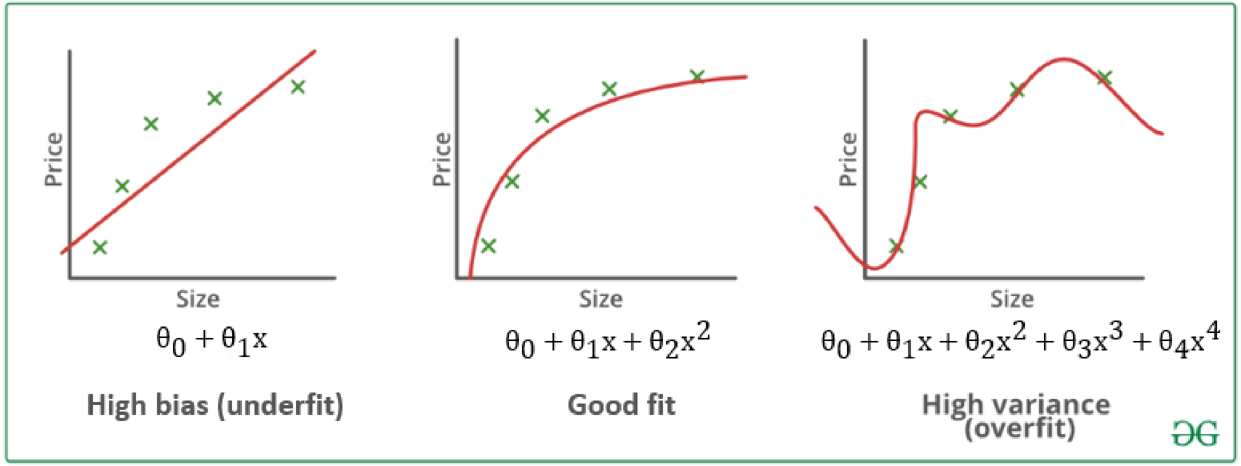
- Get more data
- Reduce the number of features
- Regularization
- Reduce the number of iterations and/or learning rate

To address underfit:

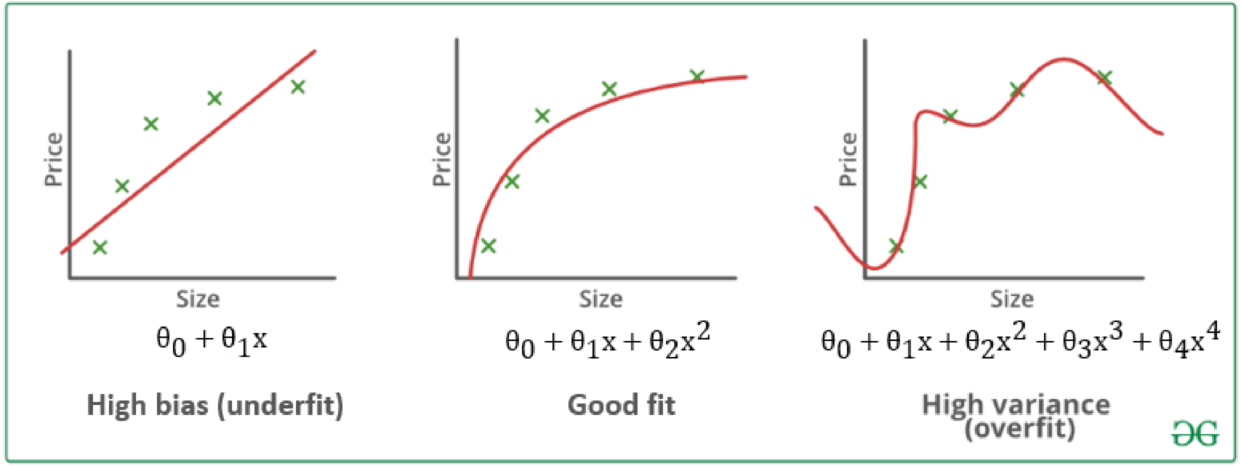
- Increase model complexity
- Get/engineer more features
- Decrease strength of regularization
- Increase the number of iterations (can be time-expensive)

Regularization

REGULARIZATION
INTUITION



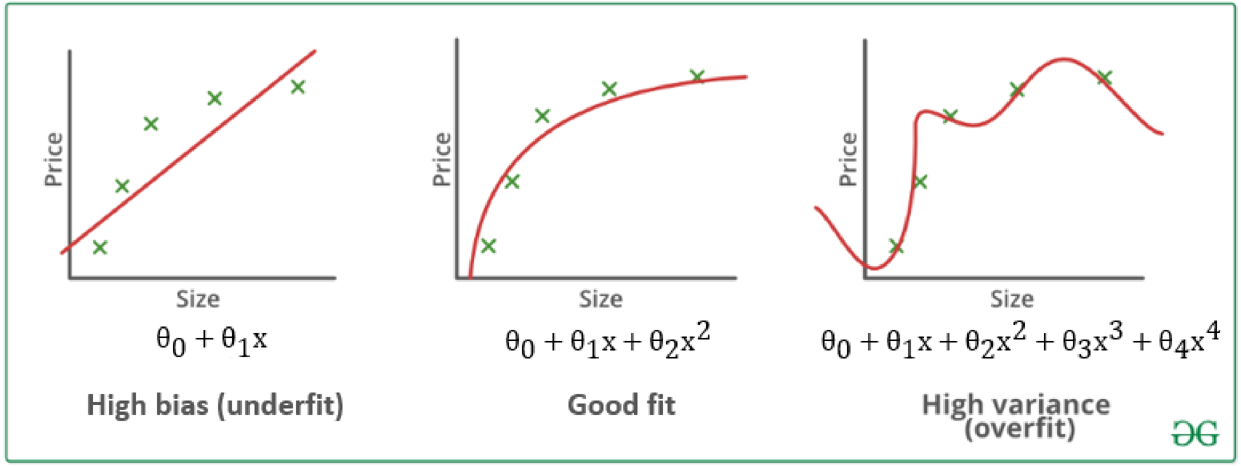
REGULARIZATION
INTUITION



$$h_\theta(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4\, x^4 \qquad ==> \qquad h_\theta(x) = \theta_0 + \theta_1 x + \theta_2 x^2$$

REGULARIZATION

INTUITION



$$h_\theta(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4\, x^4 \qquad ==> \qquad h_\theta(x) = \theta_0 + \theta_1 x + \theta_2 x^2$$

Drive θ_3 and θ_4 to be ≈ 0

REGULARIZATION

INTUITION

$$\min_{\theta} \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x_i) - y_i)^2$$

REGULARIZATION
INTUITION

$$\min_{\theta} \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x_i) - y_i)^2 \;\; + bigM_1.\theta_3 + bigM_2.\theta_4$$

REGULARIZATION
INTUITION

$$\min_{\theta} \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x_i) - y_i)^2 \ + \underbrace{bigM_1 . \theta_3}_{\theta_3 \approx 0} + \underbrace{bigM_2 . \theta_4}_{\theta_4 \approx 0}$$

REGULARIZATION

GENERALIZATION

$$\min_{\theta} \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x_i) - y_i)^2 \; + \lambda \sum_{j=1}^{n} \theta_j^2$$

REGULARIZATION

GENERALIZATION

$$\min_{\theta} \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x_i) - y_i)^2 + \lambda \sum_{j=1}^{n} \theta_j^2$$

Regularization parameter

By tuning λ it is possible to control the strength of the regularization and thus control the underfitting/overfitting

Notice that θ_0 is not regularized, and so, if λ is very high and all $\theta_1 \dots \theta_n$ are very small, then $h_\theta(x_i) \approx \theta_0$ (model underfit)

On the other hand, if $\lambda \approx 0$ we go back to the original situation and can have overfit

Lasso (l1) regularization: $\sum_{j=1}^{n} \left| \theta_j \right|$

Ridge (l2) regularization: $\sum_{j=1}^{n} \theta_j^2$

Ridge coefficients can get very small, but never exactly zero

On the other hand, Lasso coefficients can be zero, and consequently perform one kind of feature selection

However, Ridge tends to be computationally less intensive than Lasso

Updated gradients (with L2):

$$\begin{cases} \theta_0 \coloneqq \theta_0 - \alpha \left[\dfrac{1}{m} \displaystyle\sum_{i=1}^{m} (h_\theta\left(x^{(i)}\right) - y^{(i)}).\, x_0^{(i)} \right], \text{for } j = 0 \\ \theta_j \coloneqq \theta_j - \alpha \left[\dfrac{1}{m} \displaystyle\sum_{i=1}^{m} (h_\theta\left(x^{(i)}\right) - y^{(i)}).\, x_j^{(i)} + \dfrac{\lambda}{m}\theta_j \right], \text{for } j \neq 0 \end{cases}$$

Update is the same for linear and logistic regression. Only $(h_\theta\left(x^{(i)}\right)$ is different

REGULARIZATION

$$\min_{\theta} \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x_i) - y_i)^2 \; + \lambda \sum_{j=1}^{n} \theta_j^2$$

How to choose lambda, such that we don't underfit/overfit?

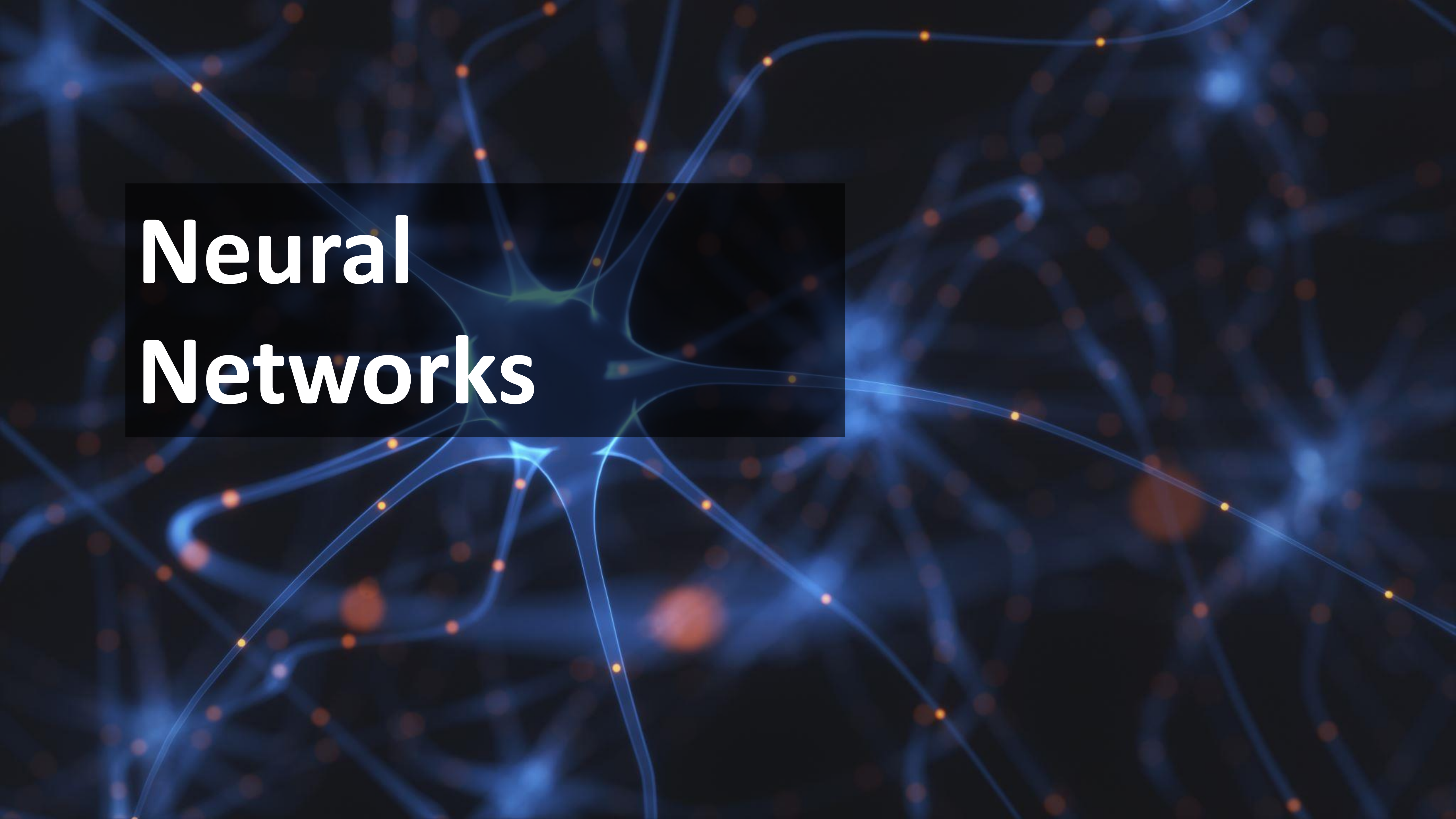
REGULARIZATION

$$\min_{\theta} \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x_i) - y_i)^2 \; + \lambda \sum_{j=1}^{n} \theta_j^2$$

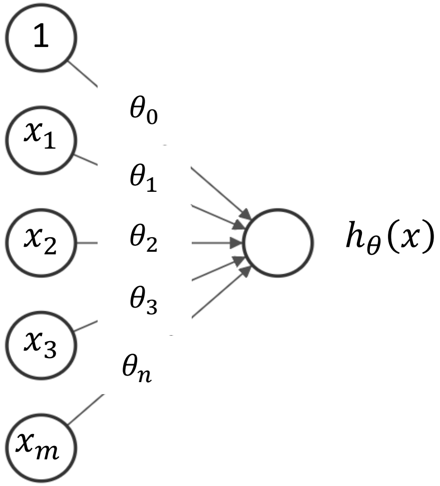
How to choose lambda, such that we don't underfit/overfit?

- Perform a grid search on different order of magnitudes with reduced, different, datasets (coarse search)
- Plot the error vs Model Complexity or vs Lambda
- Around the selected order of magnitude perform a finer search with the full dataset

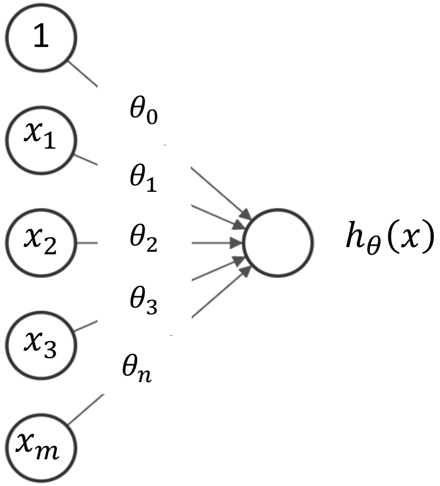
Neural Networks

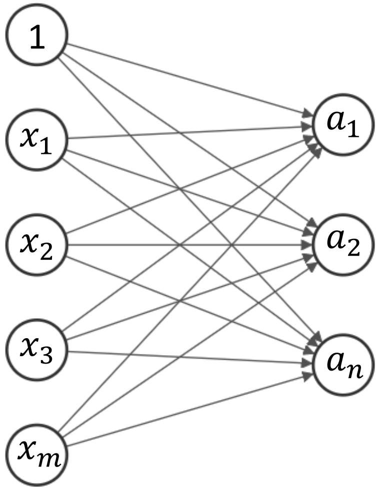


SHALLOW NETWORK
INTUITION



SHALLOW NETWORK
INTUITION

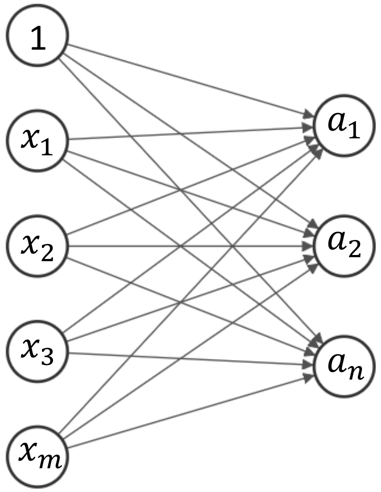




SHALLOW NETWORK
INTUITION

$$a_n = g(\theta_n^T x) = g(\theta_{n0} + \theta_{n1}x_1 + \theta_{n2}x_2 + \theta_{n3}x_3 + \theta_{n4}x_4)$$

$$g(z) = \frac{1}{1+e^{-z}}$$



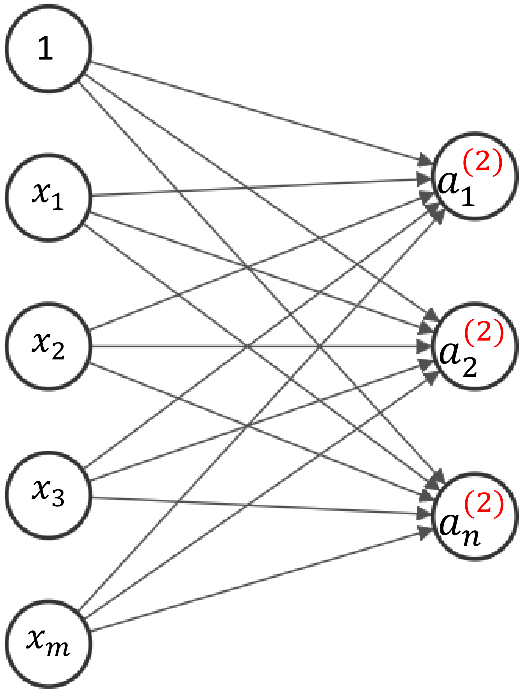
SHALLOW NETWORK
INTUITION

$$a_n = g(\theta_n^T x) = g(\theta_{n0} + \theta_{n1}x_1 + \theta_{n2}x_2 + \theta_{n3}x_3 + \theta_{n4}x_4)$$

$$g(z) = \frac{1}{1 + e^{-z}}$$

$$a_n^{(2)} = g(\Theta_{n0}^{(1)} + \Theta_{n1}^{(1)}x_1 + \Theta_{n2}^{(1)}x_2 + \cdots + \Theta_{nm}^{(1)}x_m)$$

m – number of input features
n – number of neurons in current layer



SHALLOW NETWORK

INTUITION

$$a_n = g(\theta_n^T x) = g(\theta_{n0} + \theta_{n1}x_1 + \theta_{n2}x_2 + \theta_{n3}x_3 + \theta_{n4}x_4)$$

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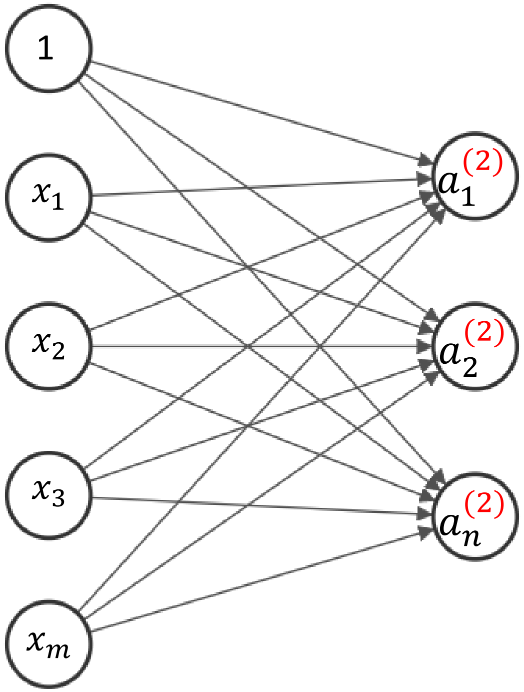


m – number of input features
n – number of neurons in current layer

Θ_{nm} is a matrix!

If $m = 4$ then $\dim(\Theta_{nm}) = (3, 5)$

m is 0 based due to the bias unit!

SHALLOW NETWORK

GENERALIZATION

$$a_n^{(2)} = g(\Theta_{n0}^{(1)} + \Theta_{n1}^{(1)} x_1 + \Theta_{n2}^{(1)} x_2 + \cdots + \Theta_{nm}^{(1)} x_m)$$

n – number of neurons in next layer
m – number of neurons in current layer

$$a^{(1)} = x$$

SHALLOW NETWORK
GENERALIZATION

$$a_n^{(2)} = g(\Theta_{n0}^{(1)} + \Theta_{n1}^{(1)} x_1 + \Theta_{n2}^{(1)} x_2 + \cdots + \Theta_{nm}^{(1)} x_m)$$

$$a^{(1)} = x$$

n – number of neurons in next layer
m – number of neurons in current layer
l – number of current layer

$$a_n^{(l+1)} = g(\Theta_{n0}^{(l)} + \Theta_{n1}^{(l)} a_1^{(l)} + \Theta_{n2}^{(l)} a_2^{(l)} + \cdots + \Theta_{nm}^{(l)} a_m^{(l)})$$

SHALLOW NETWORK

GENERALIZATION

$$a_n^{(2)} = g(\Theta_{n0}^{(1)} + \Theta_{n1}^{(1)} x_1 + \Theta_{n2}^{(1)} x_2 + \cdots + \Theta_{nm}^{(1)} x_m)$$

$$a^{(1)} = x$$

n – number of neurons in next layer
m – number of neurons in current layer
l – number of current layer

$$a_n^{(l+1)} = g(\Theta_{n0}^{(l)} + \Theta_{n1}^{(l)} a_1^{(l)} + \Theta_{n2}^{(l)} a_2^{(l)} + \cdots + \Theta_{nm}^{(l)} a_m^{(l)})$$

l – number of current layer
$s_l \equiv m$
$s_{l+1} \equiv n$

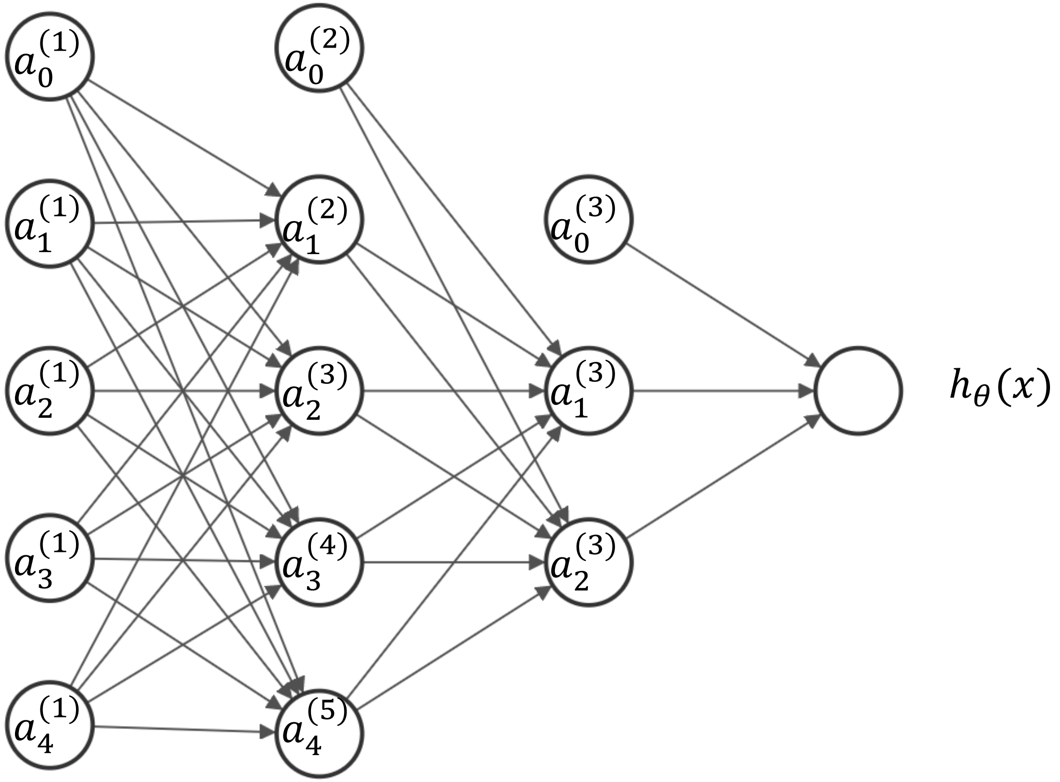
$$a_{s_{l+1}}^{(l+1)} = g(\Theta_{s_{l+1},0}^{(l)} + \Theta_{s_{l+1},1}^{(l)} a_1^{(l)} + \Theta_{s_{l+1},2}^{(l)} a_2^{(l)} + \cdots + \Theta_{s_{l+1},s_l}^{(l)} a_{s_l}^{(l)})$$

DEEP NETWORKS

GENERALIZATION

$$a_{s_{l+1}}^{(l+1)} = g(\Theta_{s_{l+1},0}^{(l)}\underbrace{a_0^{(l)}}_{=1} + \Theta_{s_{l+1},1}^{(l)}a_1^{(l)} + \Theta_{s_{l+1},2}^{(l)}a_2^{(l)} + \cdots + \Theta_{s_{l+1},s_l}^{(l)}a_{s_l}^{(l)})$$

s_{l+1} – number of neurons in next layer
s_l – number of neurons in current layer
l – number of current layer

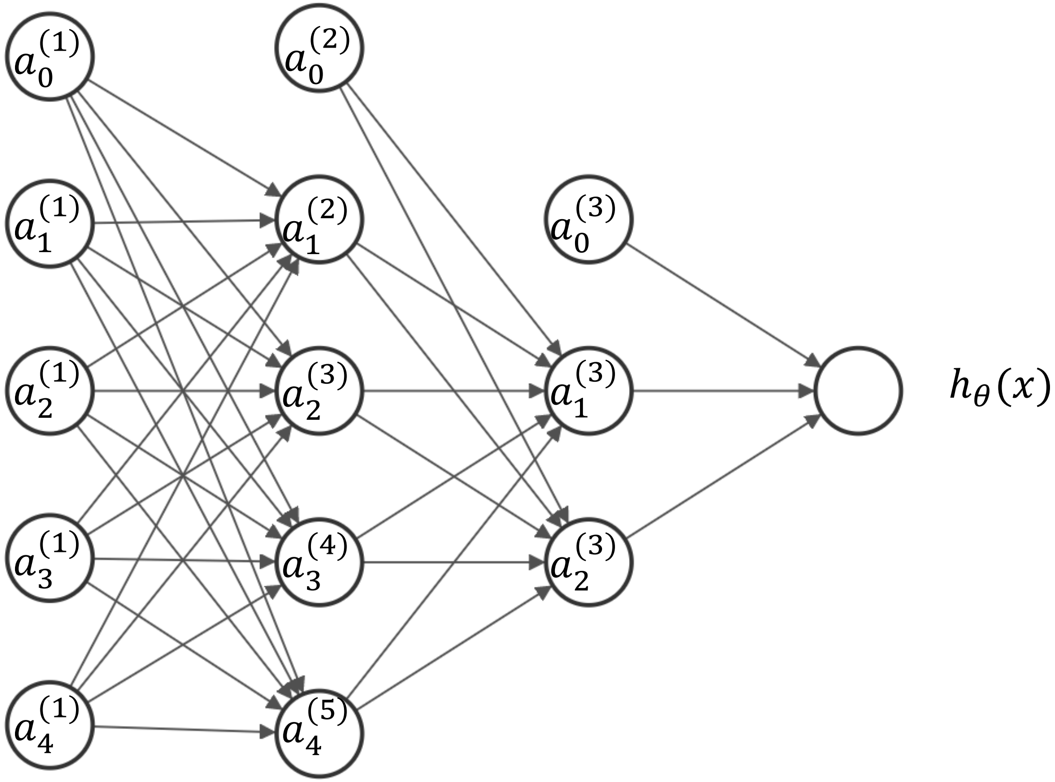


DEEP NETWORKS

GENERALIZATION

$$a_{s_{l+1}}^{(l+1)} = g(\Theta_{s_{l+1},0}^{(l)} \underbrace{a_0^{(l)}}_{=1} + \Theta_{s_{l+1},1}^{(l)} a_1^{(l)} + \Theta_{s_{l+1},2}^{(l)} a_2^{(l)} + \cdots + \Theta_{s_{l+1},s_l}^{(l)} a_{s_l}^{(l)})$$

s_{l+1} – number of neurons in next layer
s_l – number of neurons in current layer
l – number of current layer



$$a^{(1)} = x$$

$$a^{(2)} = g(\Theta^{(1)}.a^{(1)})$$

$$a^{(3)} = g(\Theta^{(2)}.a^{(2)})$$

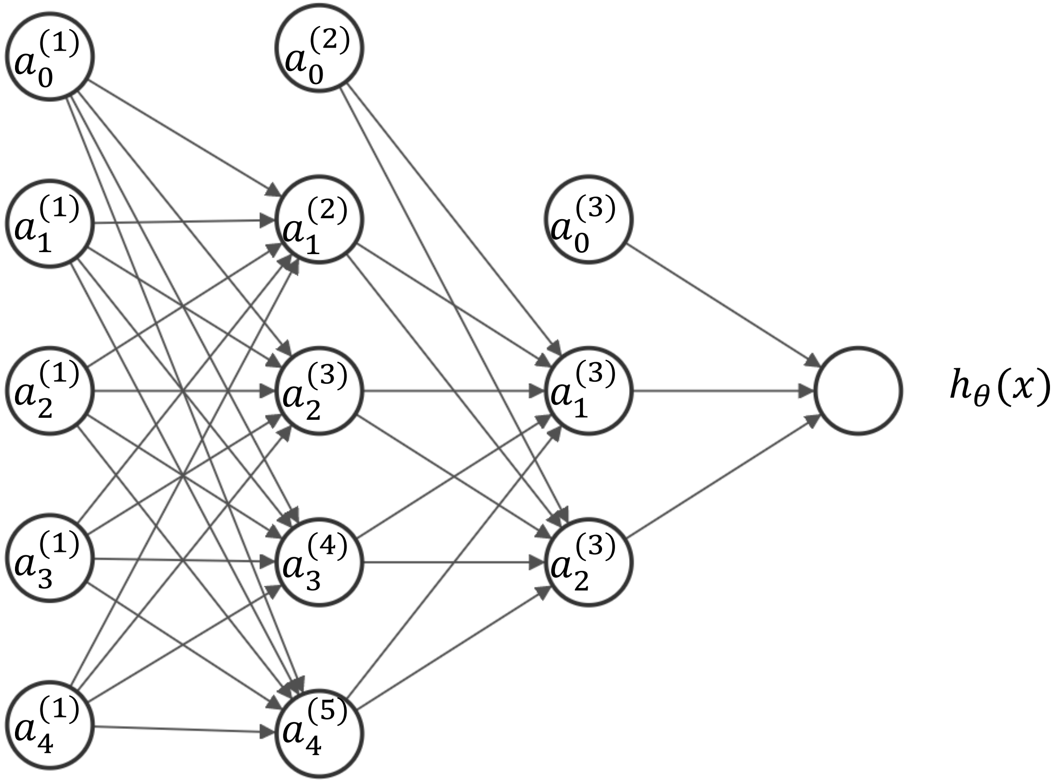
$$h_\theta(x) = g(\Theta^{(3)}.a^{(3)})$$

DEEP NETWORKS

GENERALIZATION

$$a_{s_{l+1}}^{(l+1)} = g(\Theta_{s_{l+1},0}^{(l)}\underbrace{a_0^{(l)}}_{=1} + \Theta_{s_{l+1},1}^{(l)}a_1^{(l)} + \Theta_{s_{l+1},2}^{(l)}a_2^{(l)} + \cdots + \Theta_{s_{l+1},s_l}^{(l)}a_{s_l}^{(l)})$$

s_{l+1} – number of neurons in next layer
s_l – number of neurons in current layer
l – number of current layer



$$a^{(1)} = x$$
...
$$a^{(2)} = g(\Theta^{(1)}.a^{(1)})$$
...
$$a^{(3)} = g(\Theta^{(2)}.a^{(2)})$$
...
$$h_\theta(x) = g(\Theta^{(3)}.a^{(3)})$$

Add $a_0^{(1)} = x_0 = 1$

Add $a_0^{(2)} = 1$

Add $a_0^{(3)} = 1$

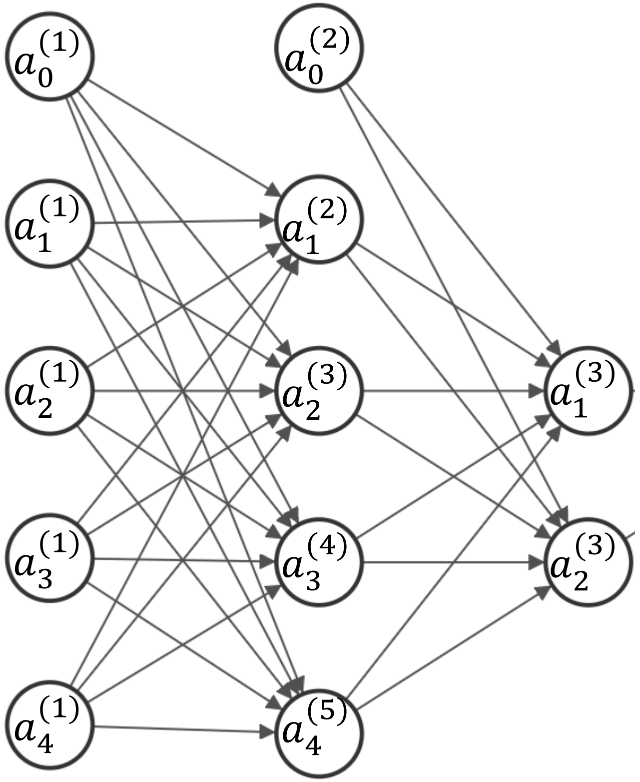
DEEP NETWORKS

GENERALIZATION

MULTICLASSIFICATION

$$a_{s_{l+1}}^{(l+1)} = g(\Theta_{s_{l+1},0}^{(l)}\overbrace{a_0^{(l)}}^{=1} + \Theta_{s_{l+1},1}^{(l)}a_1^{(l)} + \Theta_{s_{l+1},2}^{(l)}a_2^{(l)} + \cdots + \Theta_{s_{l+1},s_l}^{(l)}a_{s_l}^{(l)})$$

s_{l+1} – number of neurons in next layer
s_l – number of neurons in current layer
l – number of current layer



$h_\theta(x)$

$$a^{(1)} = x$$

Add $a_0^{(1)} = x_0 = 1$

…

$$a^{(2)} = g(\Theta^{(1)}.a^{(1)})$$

Add $a_0^{(2)} = 1$

…

$$h_\theta(x) = a^{(3)} = g(\Theta^{(2)}.a^{(2)})$$

Logistic Regression

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m} Cost\left(h_\theta\left(x^{(i)}\right), y^{(i)}\right) = -\frac{1}{m}\left[\sum_{i=1}^{m} y^{(i)} \log\left(h_\theta\left(x^{(i)}\right)\right) + \left(1 - y^{(i)}\right).\log\left(1 - h_\theta\left(x^{(i)}\right)\right)\right] + \frac{\lambda}{2m}\sum_{j=1}^{n}\theta^2$$

Neural Network

$$J(\theta) = -\frac{1}{m}\left[\sum_{i=1}^{m}\sum_{k=1}^{K} y_k^{(i)} \log\left(h_\theta\left(x^{(i)}\right)\right)_k + \left(1 - y_k^{(i)}\right).\log\left(1 - h_\theta\left(x^{(i)}\right)\right)_k\right] + \frac{\lambda}{2m}\sum_{l=1}^{L-1}\sum_{i=1}^{s_l}\sum_{j=1}^{s_{l+1}}\left(\Theta_{ji}^{(l)}\right)^2$$

Logistic Regression

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m} Cost\left(h_\theta\left(x^{(i)}\right), y^{(i)}\right) = -\frac{1}{m}\left[\sum_{i=1}^{m} y^{(i)} \log\left(h_\theta\left(x^{(i)}\right)\right) + \left(1 - y^{(i)}\right).\log\left(1 - h_\theta\left(x^{(i)}\right)\right)\right] + \frac{\lambda}{2m}\sum_{j=1}^{n} \theta^2$$

Neural Network

$$J(\theta) = -\frac{1}{m}\left[\sum_{i=1}^{m}\sum_{k=1}^{K} y_k^{(i)} \log\left(h_\theta\left(x^{(i)}\right)\right)_k + \left(1 - y_k^{(i)}\right).\log\left(1 - h_\theta\left(x^{(i)}\right)\right)_k\right] + \frac{\lambda}{2m}\sum_{l=1}^{L-1}\sum_{i=1}^{s_l}\sum_{j=1}^{s_{l+1}} (\Theta_{ji}^{(l)})^2$$

For K outputs

Logistic Regression

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m} Cost\left(h_\theta\left(x^{(i)}\right), y^{(i)}\right) = -\frac{1}{m}\left[\sum_{i=1}^{m} y^{(i)} \log\left(h_\theta\left(x^{(i)}\right)\right) + \left(1 - y^{(i)}\right).\log\left(1 - h_\theta\left(x^{(i)}\right)\right)\right] + \frac{\lambda}{2m}\sum_{j=1}^{n}\theta^2$$

Neural Network

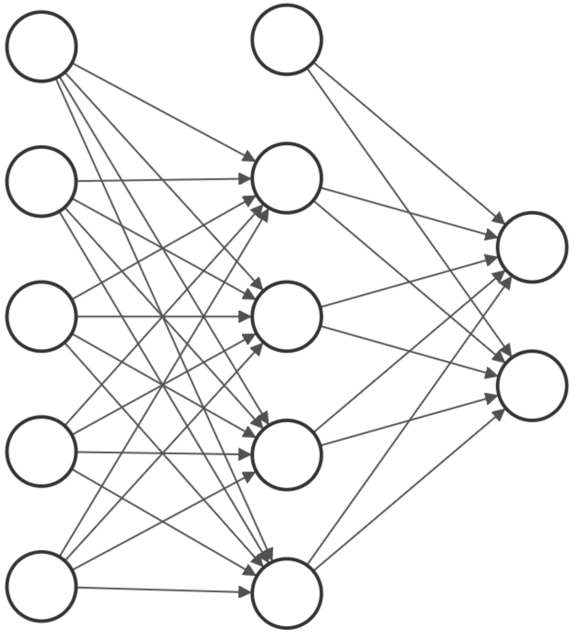
$$J(\theta) = -\frac{1}{m}\left[\sum_{i=1}^{m}\sum_{k=1}^{K} y_k^{(i)} \log\left(h_\theta\left(x^{(i)}\right)\right)_k + \left(1 - y_k^{(i)}\right).\log\left(1 - h_\theta\left(x^{(i)}\right)\right)_k\right] + \frac{\lambda}{2m}\sum_{l=1}^{L-1}\sum_{i=1}^{s_l}\sum_{j=1}^{s_{l+1}}\left(\Theta_{ji}^{(l)}\right)^2$$

For K outputs

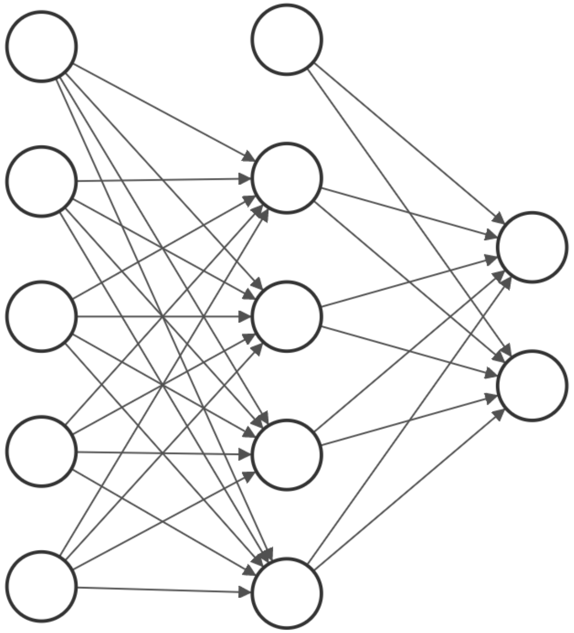
Sum of regularization term for all weights, on all layers

DEEP NETWORKS
BACKPROPAGATION

$$\delta_j^{(l)} = \text{"}error\text{"}\ of\ node\ j\ in\ layer\ l$$



DEEP NETWORKS
BACKPROPAGATION

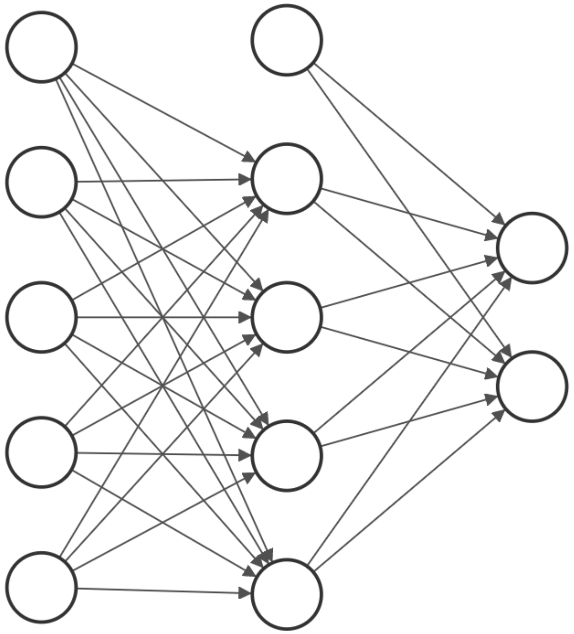


$$\delta_j^{(l)} = \text{"}error\text{"}\ of\ node\ j\ in\ layer\ l$$

$$\delta_j^{(3)} = a_j^{(3)} - y_j$$

DEEP NETWORKS

BACKPROPAGATION



$\delta_j^{(l)} = \text{"}error\text{"}\ of\ node\ j\ in\ layer\ l$

$\delta_j^{(3)} = a_j^{(3)} - y_j$

If we use the chain rule to manipulate the derivative of the cost function it is possible to obtain the following expression to compute $\delta_j^{(l-1)}$:

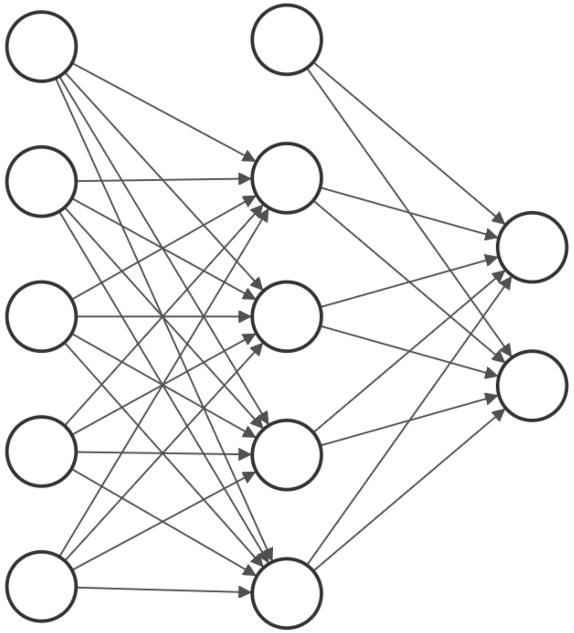
$$\delta_j^{(l-1)} = (\Theta^{(l-1)})^T \delta_j^{(l)} .* \, g'\left(z^{(l-1)}\right)$$

with

$$z^{(l-1)} = \Theta^{(l-2)} . a^{(l-2)}$$

Computed on the forward pass

DEEP NETWORKS
BACKPROPAGATION



$$\delta_j^{(l)} = \text{"}error\text{"}\ of\ node\ j\ in\ layer\ l$$

$$\delta_j^{(3)} = a_j^{(3)} - y_j$$

$$\delta_j^{(l-1)} = (\Theta^{(l-1)})^T \delta_j^{(l)} .* \, g'\big(z^{(l-1)}\big)$$

$$\boxed{z^{(l-1)} = \Theta^{(l-2)}.a^{(l-2)}}$$

$$\frac{\partial}{\partial \Theta_{ij}^{(l)}} J(\Theta) = a_j^{(l)} \delta_i^{(l+1)} = D_{ij}^{(l)}$$

How to perform updates over all the training examples ?

For each epoch (or alternatively batch):

- Initialize an error matrix $\Delta_{ij}^{(l)} = 0$, (for all i, j, l)
- For each example in epoch/batch
  - Compute prediction
  - Compute error ($\delta^{(L)} = a^{(L)} - y$)
  - Compute all $\delta_j^{(l)}, 1 \leq l \leq L-1$
  - Update $\Delta_{ij}^{(l)}$ as $\Delta_{ij}^{(l)} := \Delta_{ij}^{(l)} + a_j^{(l)} \delta_i^{(l+1)}$
- Average $\Delta_{ij}^{(l)}$ over number of examples used: $D_{ij}^{(l)} := \frac{1}{m} \Delta_{ij}^{(l)}$
- If using regularization, add the regularization term:

$$D_{ij}^{(l)} := \frac{1}{m} \Delta_{ij}^{(l)} + \lambda \Theta_{ij}^{(l)} \text{ if } j \neq 0$$

$$D_{ij}^{(l)} := \frac{1}{m} \Delta_{ij}^{(l)} \text{ if } j = 0$$

Update weights on the entire network with:

$$\Theta_{ij}^{(l)} := \Theta_{ij}^{(l)} - \alpha \frac{\partial}{\partial \Theta_{ij}^{(l)}} J(\Theta) := \Theta_{ij}^{(l)} - \alpha D_{ij}^{(l)}$$

$$\delta_j^{(3)} = a_j^{(3)} - y_j$$

$$\delta_j^{(l-1)} = (\Theta^{(l-1)})^T \delta_j^{(l)} .* g'\left(z^{(l-1)}\right)$$

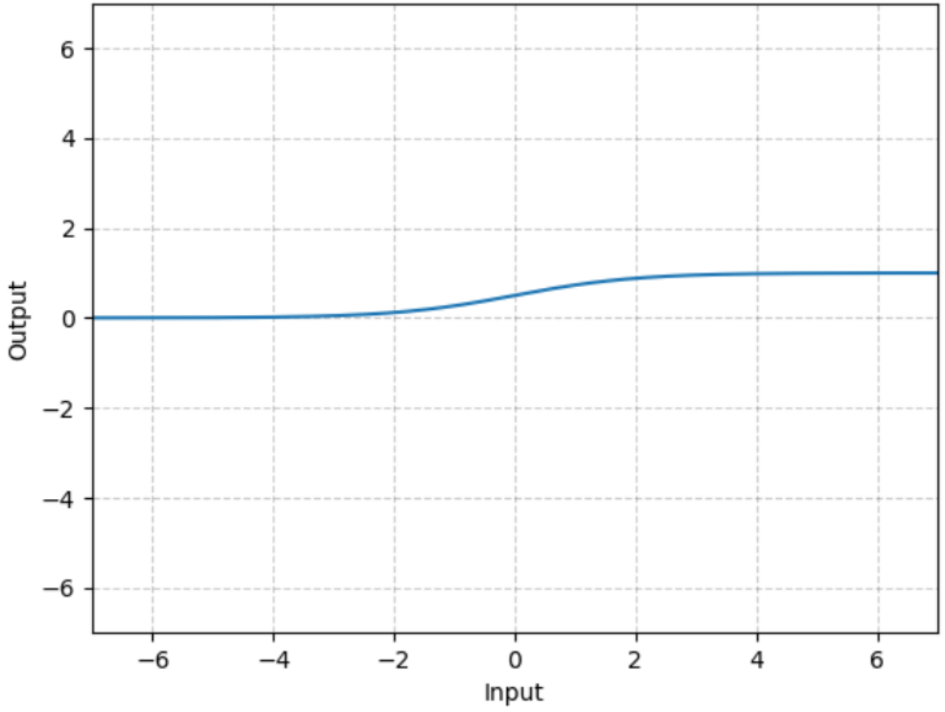
$$\boxed{z^{(l-1)} = \Theta^{(l-2)} . a^{(l-2)}}$$

$$\frac{\partial}{\partial \Theta_{ij}^{(l)}} J(\Theta) = a_j^{(l)} \delta_i^{(l+1)} = D_{ij}^{(l)}$$

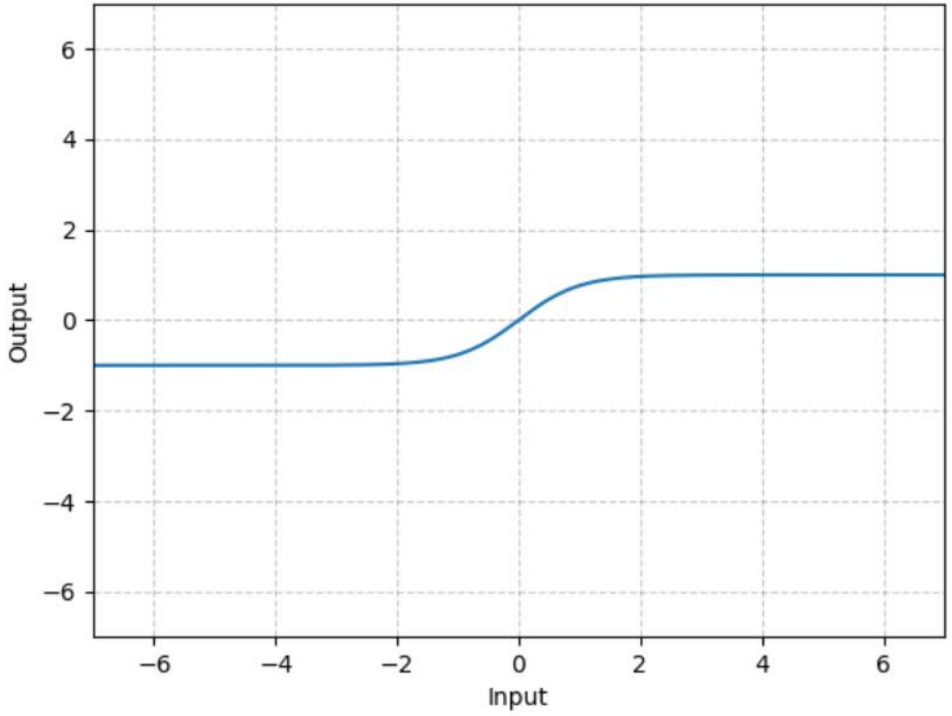
For a single example! If several examples are to be used to compute one update, the value must be averaged, as at each example the errors are added cumulatively

ACTIVATION FUNCTIONS

Sigmoid

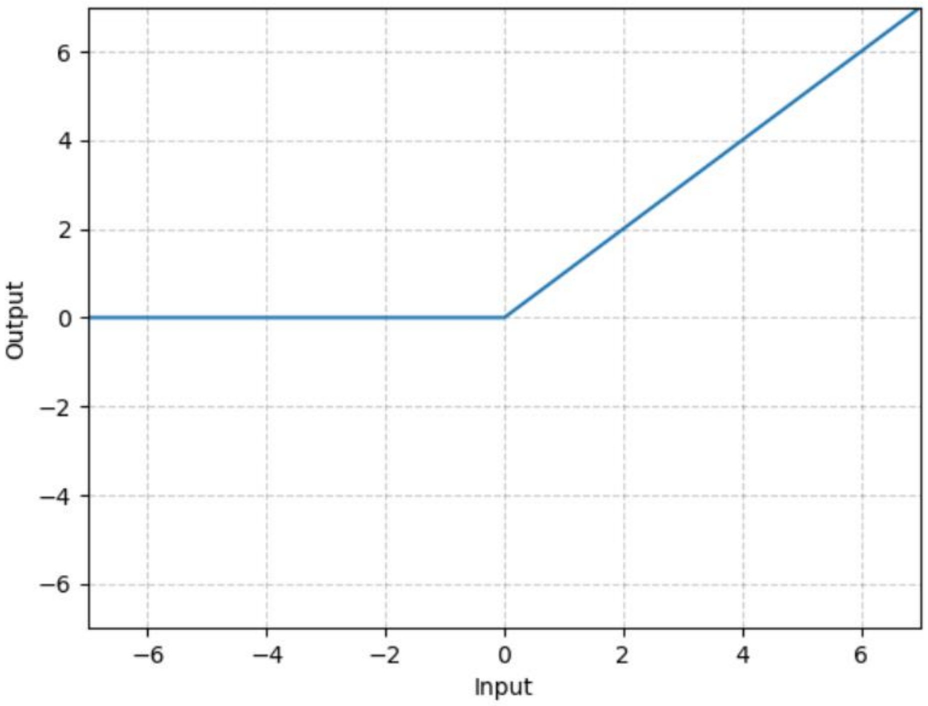


Tanh

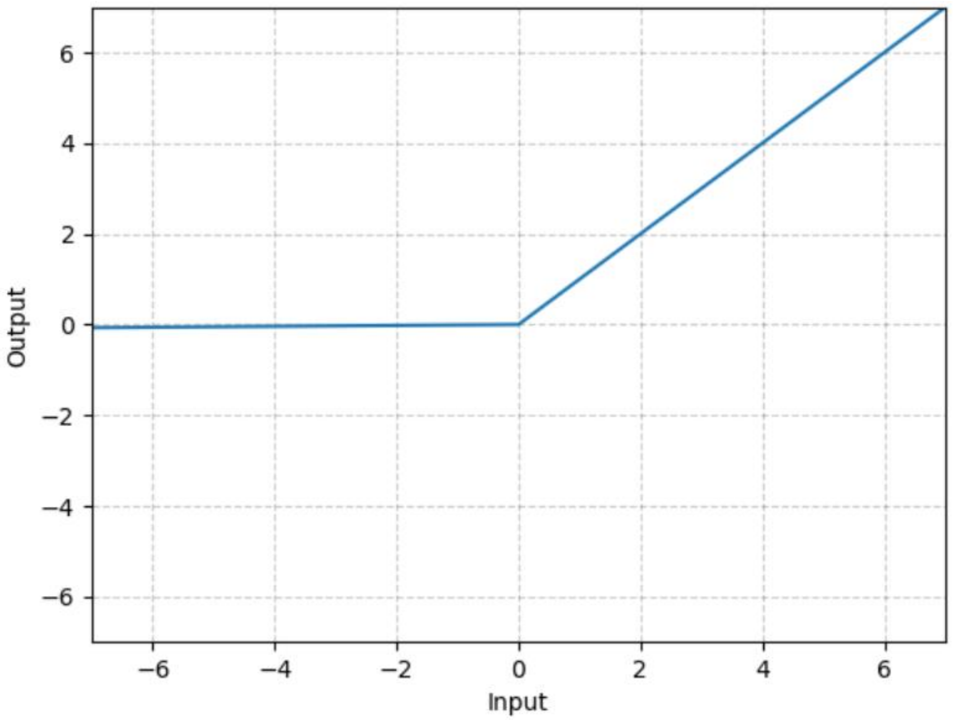


ACTIVATION FUNCTIONS

ReLU



LeakyReLU



https://pytorch.org/docs/stable/nn.html

INPUT NORMALIZATION/STANDARDIZATION

- Inputs should be on same scale (order of magnitude)
- When magnitudes are different, weights values can also be very different
- Consequently, updates on weights based on the derivatives can rapidly make some weights explode or switch signal or practically don’t update at all
- It can also generate exploding or vanishing gradients, resulting that the last layers can still learn with the errors, but the error backpropagation don’t reach the initial layers
- This can make the network to not converge (or even diverge), despite the learning rate chosen

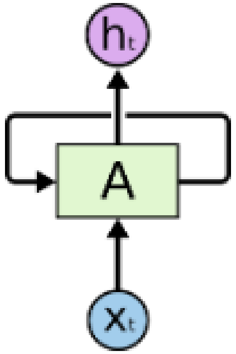
REGULARIZATION TECHNIQUES ON NEURAL NETWORKS

- L1 and L2 regularization is possible
- Additionally, it is possible to introduce dropout or batch normalization layers between hidden layers
- Dropout layers turn off a set number of neurons in the previous layer (given as a ratio). The neurons are chosen randomly.
- Batch normalization layers perform normalization of the outputs of the previous layer, resetting the outputs of that layer to a range within 0 to 1. This is especially effective when using activation functions with a linear behavior.
- Tuning the batch size can also have a regularization effect. Usually higher batch sizes tend to oblige the network to perform more generalist updates

Other Architectures

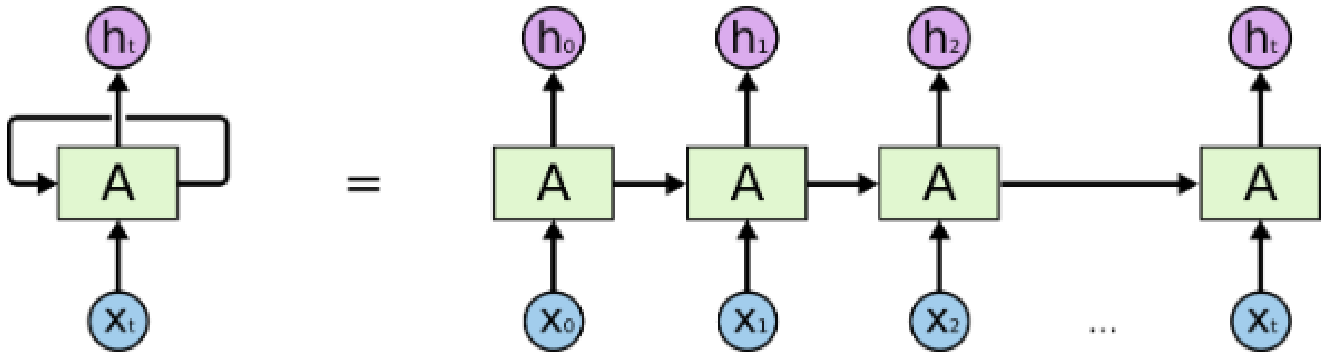
OTHER ARCHITECTURES

RECURRENT NEURAL NETWORK (RNN)

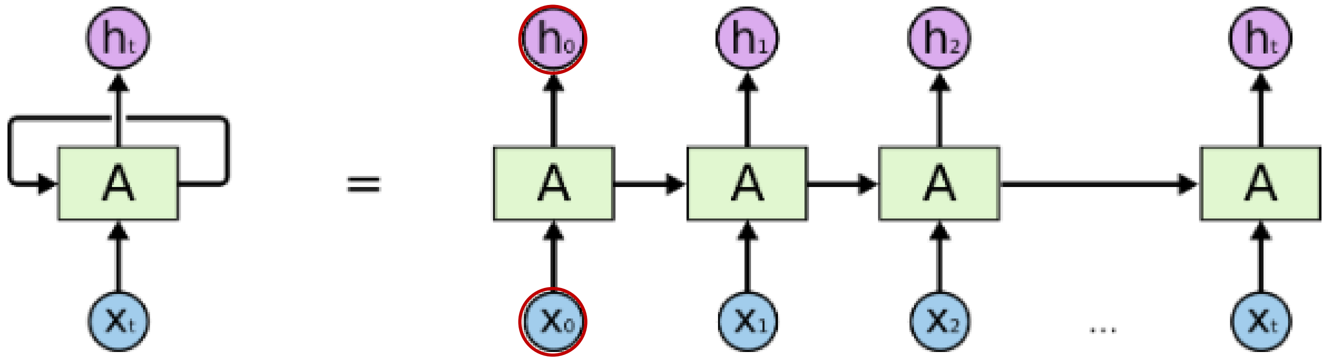


OTHER ARCHITECTURES

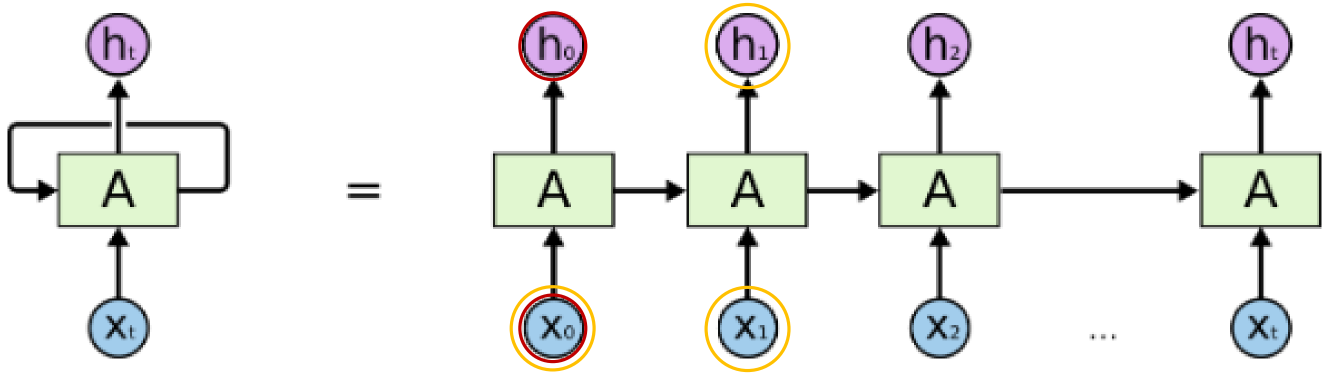
RECURRENT NEURAL NETWORK (RNN)



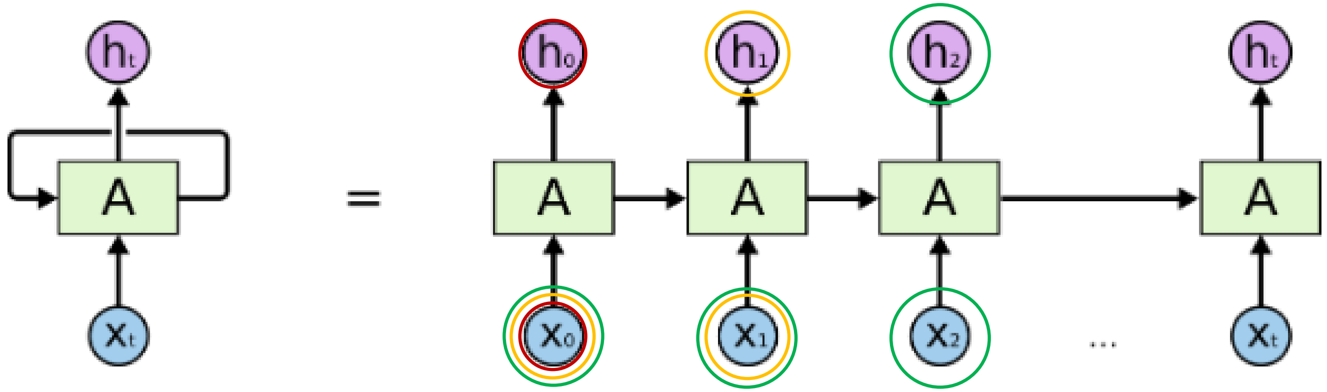
OTHER ARCHITECTURES
RECURRENT NEURAL NETWORK (RNN)



OTHER ARCHITECTURES
RECURRENT NEURAL NETWORK (RNN)

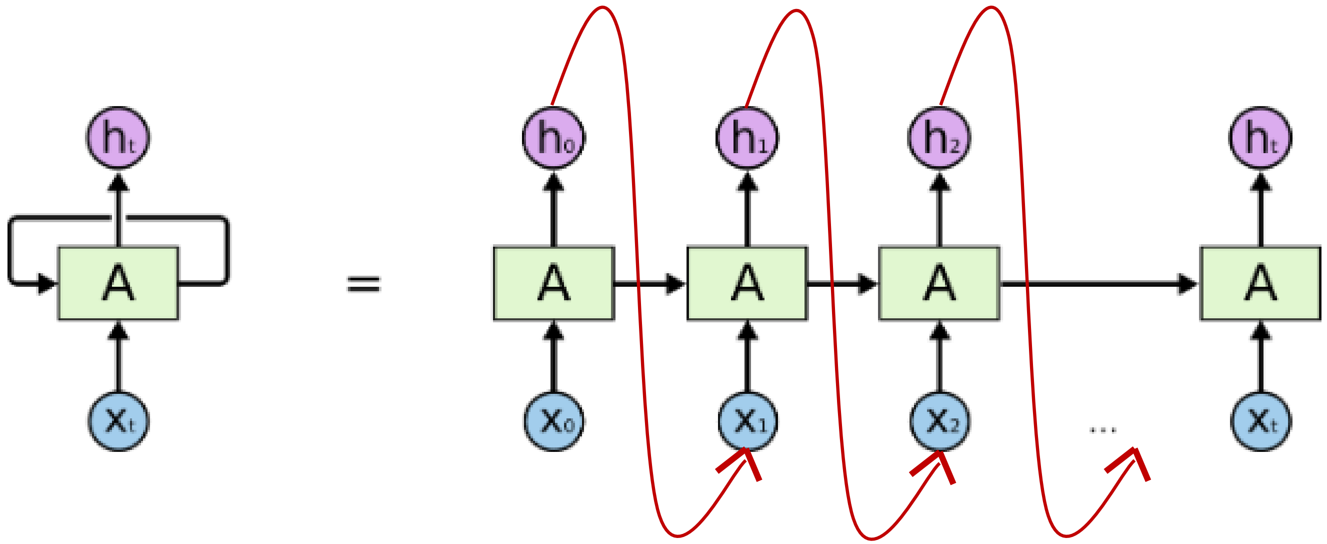


OTHER ARCHITECTURES
RECURRENT NEURAL NETWORK (RNN)



OTHER ARCHITECTURES

RECURRENT NEURAL NETWORK (RNN)



OTHER ARCHITECTURES
CONVOLUTIONAL NEURAL NETWORKS (CNN)

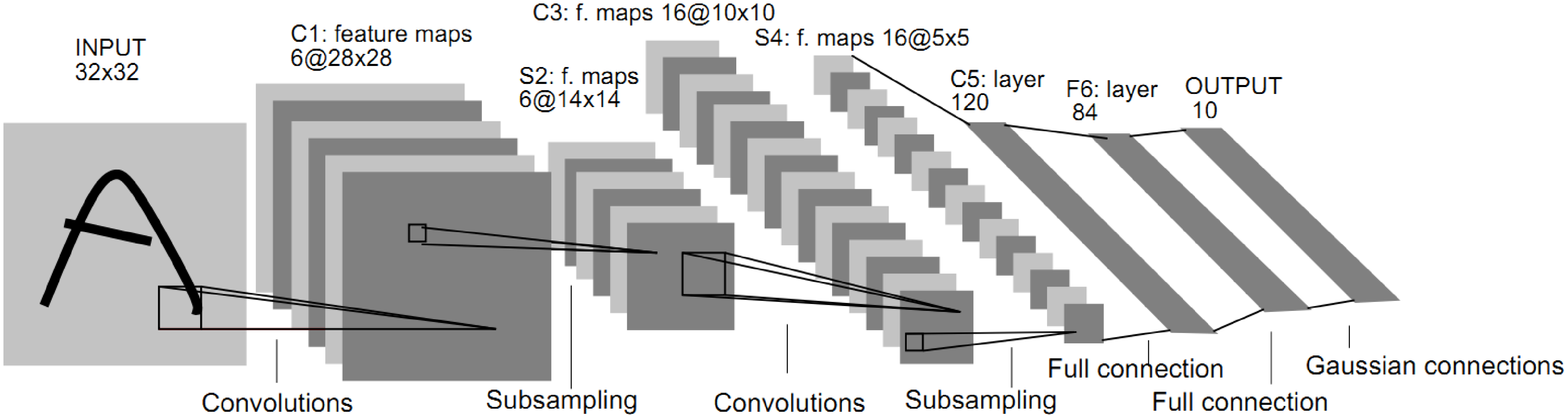
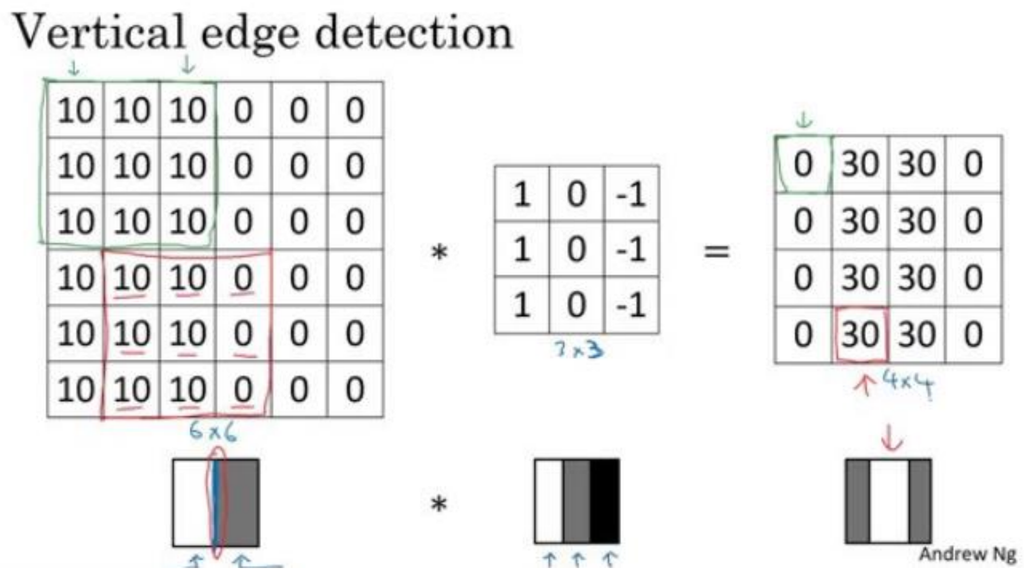
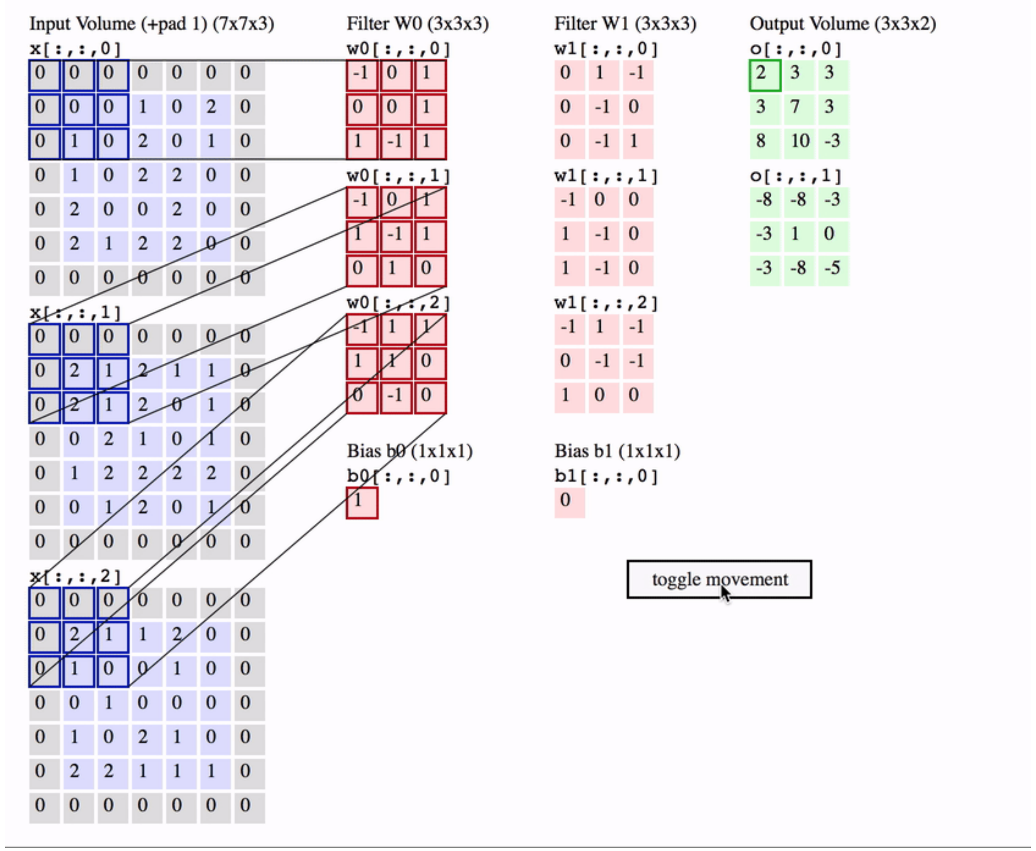


Fig. 2. Architecture of LeNet-5, a Convolutional Neural Network, here for digits recognition. Each plane is a feature map, i.e. a set of units whose weights are constrained to be identical.

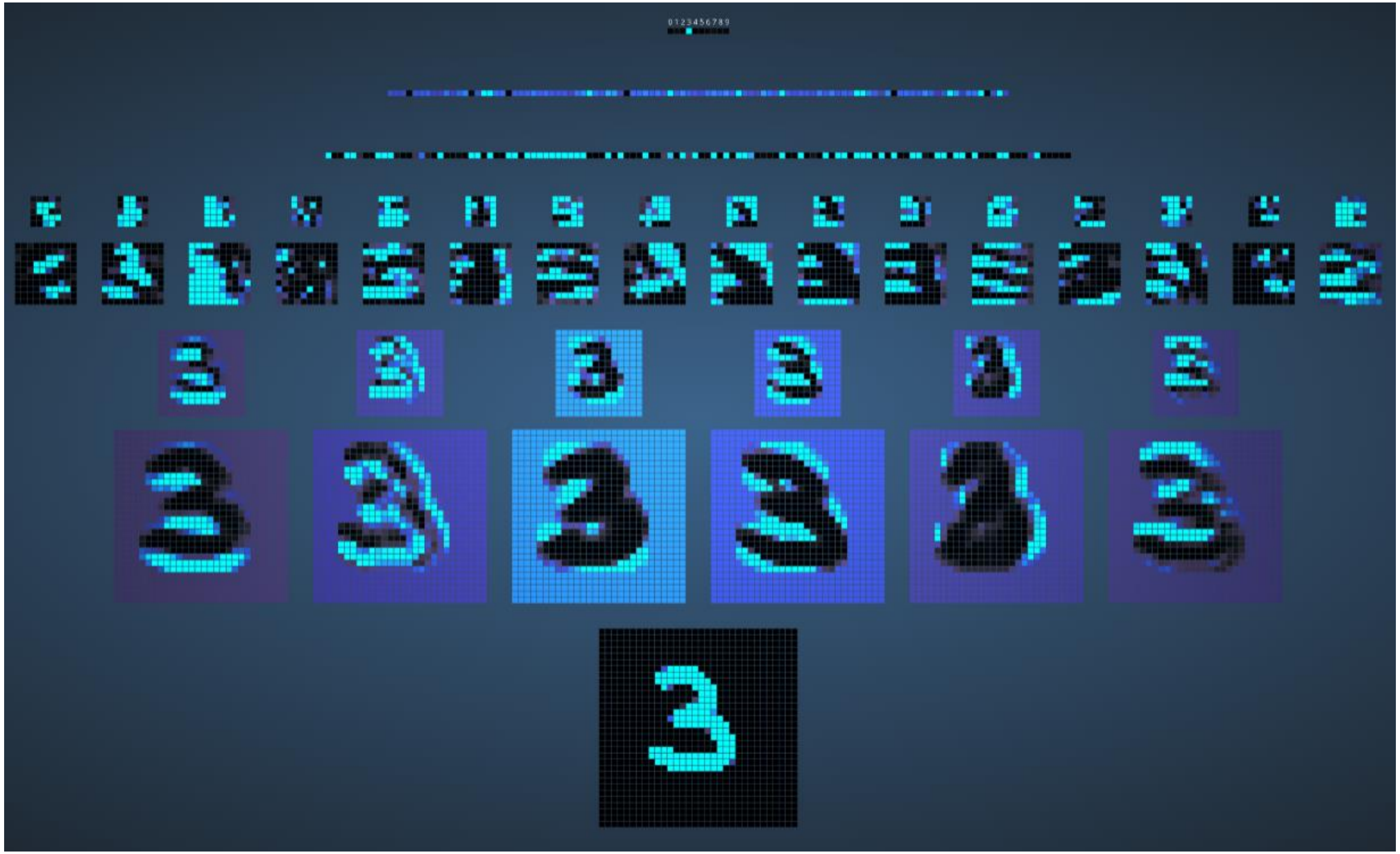
OTHER ARCHITECTURES

CONVOLUTIONAL NEURAL NETWORKS (CNN)

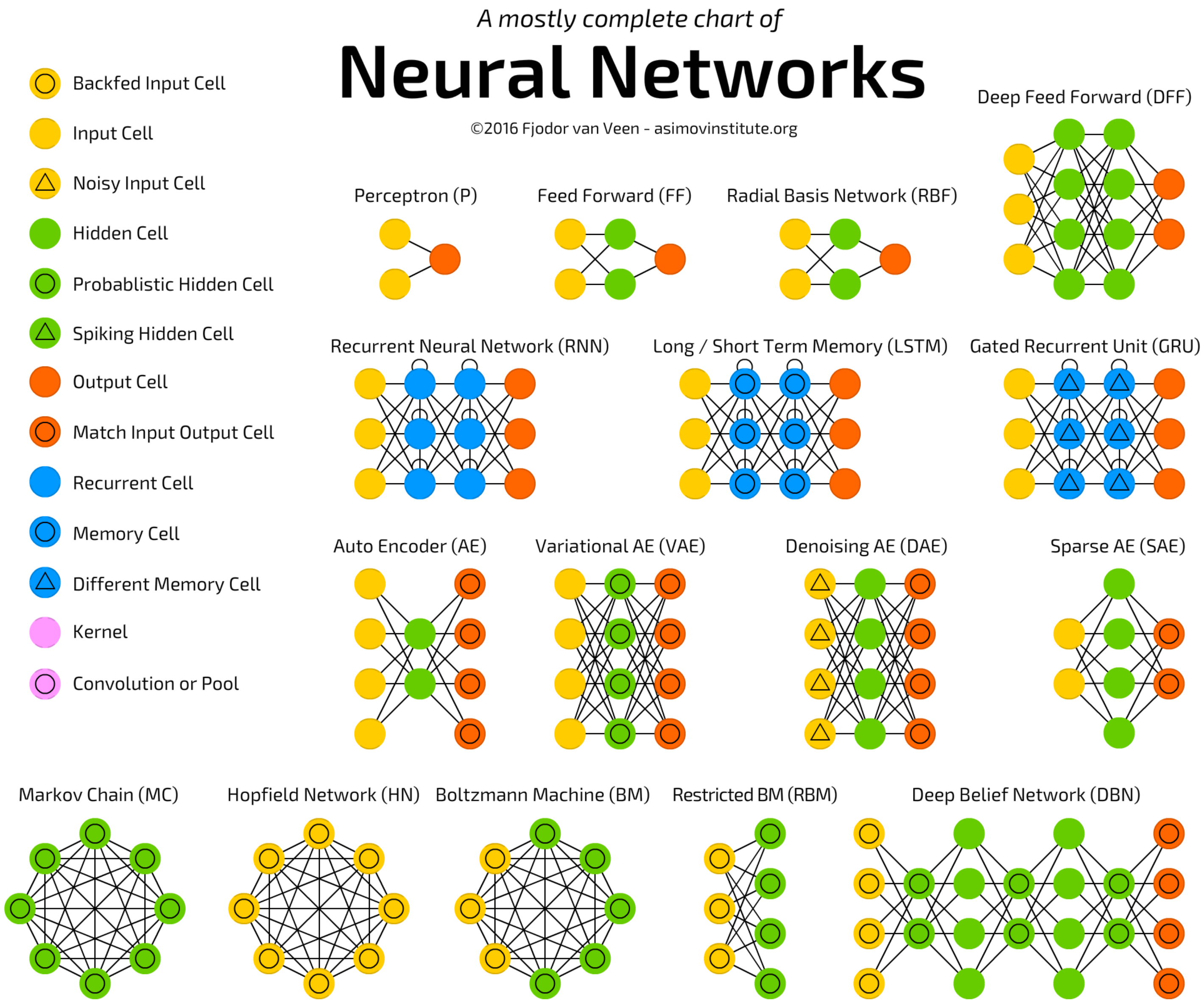




OTHER ARCHITECTURES
CONVOLUTIONAL NEURAL NETWORKS (CNN)



https://www.cs.ryerson.ca/~aharley/vis/conv/flat.html



A mostly complete chart of
Neural Networks
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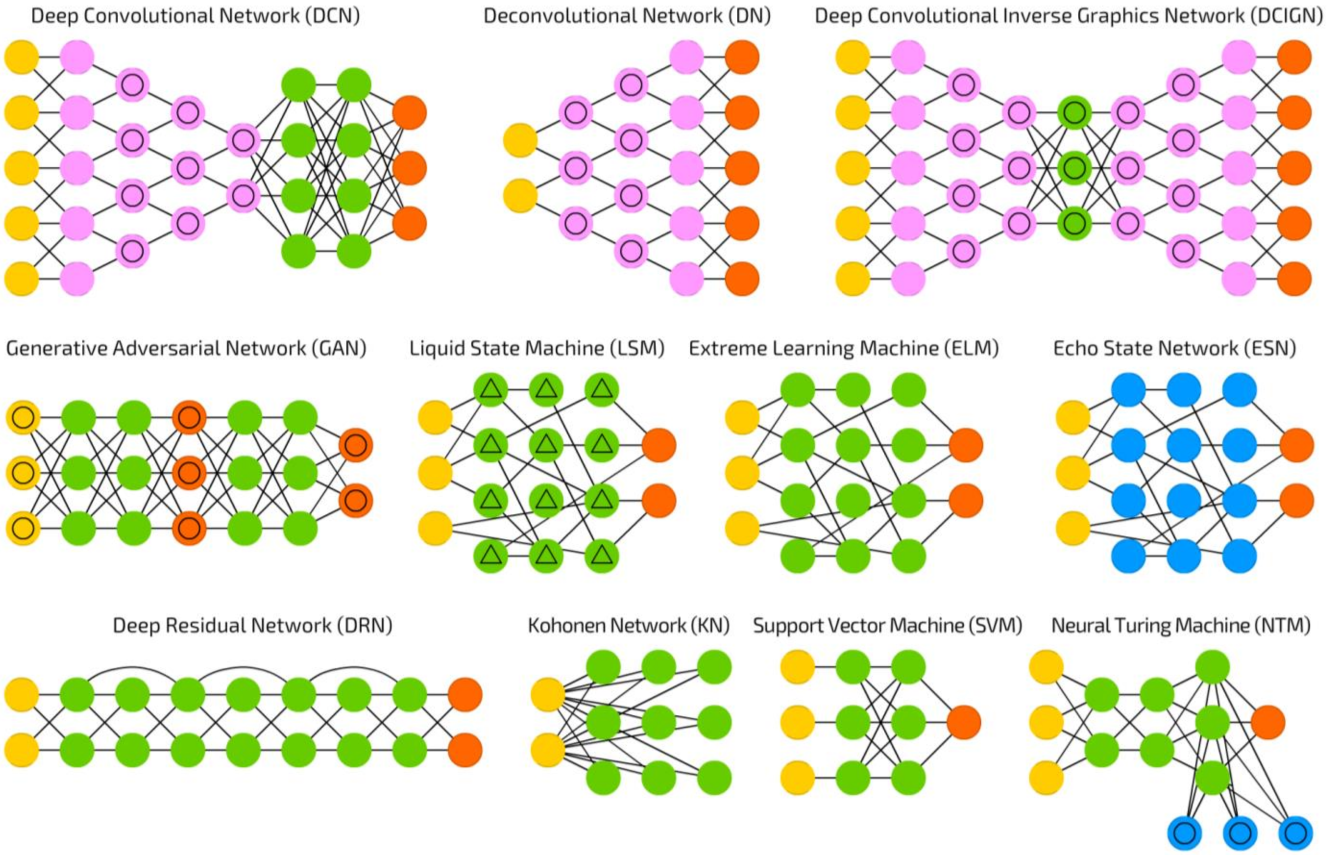
Backfed Input Cell
Input Cell
Noisy Input Cell
Hidden Cell
Probablistic Hidden Cell
Spiking Hidden Cell
Output Cell
Match Input Output Cell
Recurrent Cell
Memory Cell
Different Memory Cell
Kernel
Convolution or Pool
Perceptron (P)
Feed Forward (FF)
Radial Basis Network (RBF)
Deep Feed Forward (DFF)
Recurrent Neural Network (RNN)
Long / Short Term Memory (LSTM)
Gated Recurrent Unit (GRU)
Auto Encoder (AE)
Variational AE (VAE)
Denoising AE (DAE)
Sparse AE (SAE)
Markov Chain (MC)
Hopfield Network (HN)
Boltzmann Machine (BM)
Restricted BM (RBM)
Deep Belief Network (DBN)



https://towardsdatascience.com/the-mostly-complete-chart-of-neural-networks-explained-3fb6f2367464