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Da un secolo, oltre.



Adaptive Importance Sampling for Rare Event Simulation in Fault Tree Analysis

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Introduction

Goal: Estimate failure probability of complex systems (fault trees) where failures are rare events.

Problems with standard Montecarlo

- requires millions of simulations;
- most samples yield no failures;
- computationally expensive;
- high variance for rare events;

Solution: Adaptive Importance Sampling

- Modify simulation parameters ($\lambda \rightarrow \alpha\lambda$, $\mu \rightarrow \beta\mu$) to observe more failures
- Correct bias with likelihood ratio
- **Challenge:** How to find optimal α , β ? GNN + MLP models.



System Architecture

Our approach:

- Generation of random fault trees (random number of components and ports);
- Self-supervised GNN predicts optimal α, β ranges for topology and period T;
- Self-supervised GNN predicts optimal samples number for IS and MC probability estimation;
- MLP + Cross-Entropy Method optimizes α, β within predicted range;
- CTMC Simulator: simulates with Importance Sampling.

Fault tree Generation

Random Topology Generator:

- 2 to 45 components (scalable);
- Gate types: AND, OR, KooN;
- Each gate type has a different probability to be chosen;
- Hierarchical bottom-up construction;
- $\lambda \in [10^{-5}, 10^{-4}]$, $\mu \in [0.1, 1.0]$;
- Used both in training and validation;

Topology determines Probability:

- Many OR gates → Higher P (multiple failure paths)
- Many AND gates → Lower P (all must fail)
- AND/OR ratio determines event rarity

IS Parameter Ranges Prediction

Key Technical Components

- **Graph Processing:** 3-Layer **GCNConv** stack extracts hierarchical features from gates (AND/OR) and components.
- **Temporal Context:** Injects Tnorm and $1/(1+Tnorm)$ into the latent space to enable time-adaptive biasing.
- **Global Feature Fusion:** Concatenates node embeddings with system-wide metrics (depth, Ncomp, avg λ/μ).

Execution Flow

- **Input:** Graph Data $[\lambda, \mu, \text{Type}, \text{Tnorm}]$
- **Logic:** Message Passing → Global Pooling → Feature Concatenation.
- **Output:** Four-dimensional vector $[\alpha_{\min}, \alpha_{\max}, \beta_{\min}, \beta_{\max}]$.

Adaptive Sample Size Optimization

Architecture Highlights

- **GNN Core:** A 2-layer **GCNConv** network that generates topological embeddings from node features ($\lambda, \mu, \text{Type}$).
- **Global Context Integration:** Fuses graph embeddings with system-wide metadata: component counts, gate distribution, and tree depth.
- **Logarithmic Regression:** Predicts $\log_{10}(N)$ using a **Sigmoid** activation scaled to a specific range.

Key Strategic Advantages

- **Adaptive Effort:** Automatically allocates more samples to rare-event scenarios (deep AND gates) and fewer to high-probability systems.
- **Exploration-Exploitation:** Gaussian sampling during training, ensuring the model discovers the minimum N required to achieve variance stability.

Self-supervised Training Loop

The system employs a **closed-loop feedback mechanism** where both Predictors (Range & Sample) are trained simultaneously without human-labeled data.

- **Step 1: Policy Prediction**
 - **RangePredictor** sets the "How" (biasing intensity α, β).
 - **SamplePredictor** sets the "How Much" (computational effort N).
- **Step 2: Simulation Execution**
 - The system runs parallel **CTMC simulations** using the predicted parameters on randomly generated Fault Trees.
- **Step 3: Statistical Feedback (The Reward)**
 - Performance is measured via the **Coefficient of Variation (CV)** and convergence speed.
 - A custom **Penalty Function** ensures physical consistency (e.g., preventing β from diverging).
- **Step 4: Gradient Update**
 - o The errors are backpropagated through the GNNs to refine future predictions.

Self-supervised Training Loop

The Challenge of Complexity

Directly training a GNN on systems with 30+ components is prone to instability due to the massive state space and extreme rarity of events.

The Incremental Fine-Tuning Approach

We implemented a **Curriculum Learning** strategy to progressively build the model's "topological intuition":

- **(Base):** Training on small systems (**2-10 components**) to learn basic AND/OR logic and fundamental biasing.
- **(Mid-Scale):** Fine-tuning the base model on medium systems (**10-20 components**) to adapt to deeper hierarchies.
- **(Large-Scale):** Final optimization on complex systems (**20-30+ components**) to master high-dimensional variance reduction.

Self-supervised Training Loop

Key Advantages

- **Faster Convergence:** The model starts with weights already optimized for simpler versions of the same problem.
- **Stability:** Prevents the "Gradient Explosion" or "Vanishing Rewards" typical of training on extremely rare events from scratch.
- **Knowledge Transfer:** Proves that the GNN learns generalized topological rules that scale across different system magnitudes.

Parameter Refinement – MLP + Cross-Entropy

Concept: Local Policy Optimization

While the GNN provides a global starting point, a dedicated **AlphaBetaMLP** performs fine-grained parallelized (16 workers) optimization for the specific Fault Tree and mission time T.

The CEM Workflow:

- **Stochastic Probing:** The MLP predicts the *mean* of a distribution; we sample N candidate configurations $[\alpha, \beta]_i$ using a Gaussian policy $N(\mu, \sigma)$.
- **Elite Selection (ρ -sampling):**
 - Runs N trajectories for each candidate.
 - Identifies the **Top $\rho\%$ (Elite)** candidates that yielded the highest variance reduction.
- **Softmax Weighting:** Elite samples are weighted using **Log-Sum-Exp** to stabilize training against extreme rare-event values (<10⁻⁹).

Parameter Refinement – MLP + Cross-Entropy

Optimization & Convergence

- **Policy Gradient Loss:** Updates the MLP to shift the sampling distribution toward the "Elite" parameters.
- **Entropy Bonus:** Penalizes premature convergence, forcing the model to explore the parameter space thoroughly.
- **Dynamic Refinement:** As T increases, the loop adaptively converges to the most efficient biasing strategy.



CTMC Simulation

Objective

To model the dynamic evolution of the Fault Tree and apply **Importance Sampling (IS)** at the trajectory level.

The system state evolves as a Continuous-Time Markov Chain, where components transition between "Working" and "Failed".

- **Failure Acceleration (α):** Failure rates are scaled by $\lambda_{IS} = \lambda \cdot \alpha$, making rare failures more frequent.
- **Repair Deceleration (β):** Repair rates are scaled by $\mu_{IS} = \mu \cdot \beta$, keeping the system in a failed state longer.



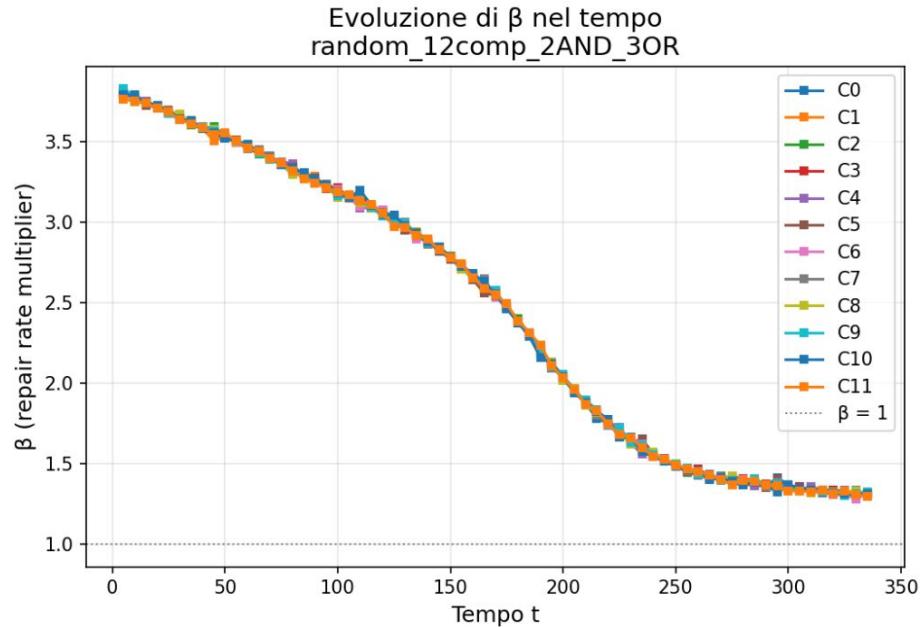
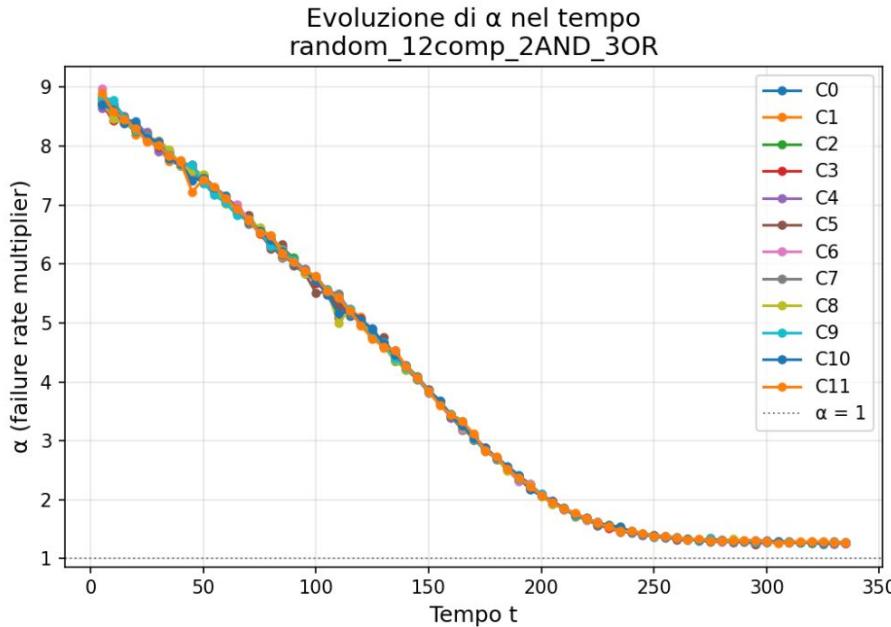
CTMC Simulation

Likelihood Ratio Accumulation

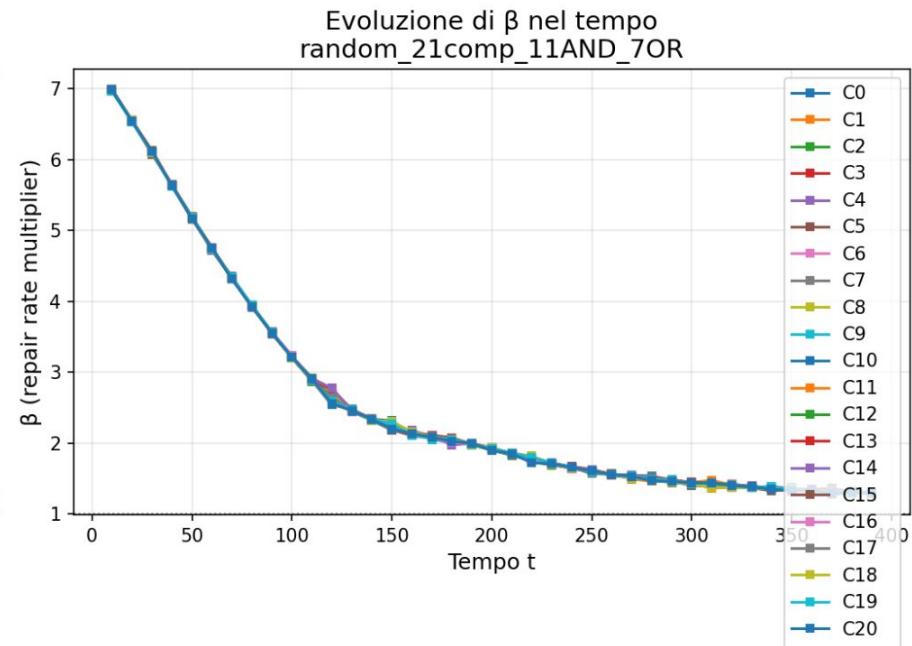
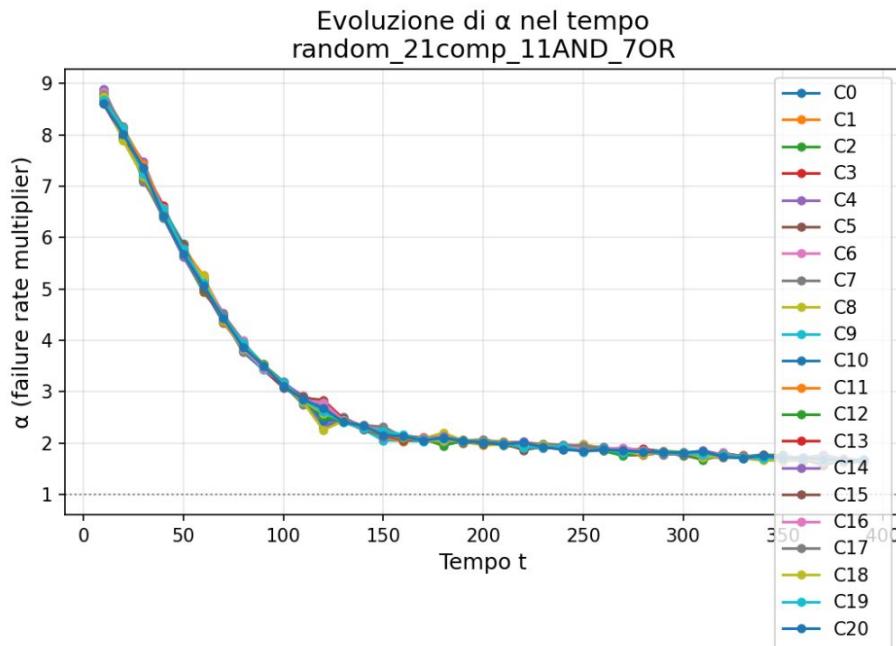
To ensure the final estimate is unbiased, we maintain a **Log-Weight (log_w)** for every trajectory, updated at each step:

1. **Holding Time Correction:** Adjusts for the biased time spent in each state:
 $(R_{IS} - R_{orig}) \cdot dt;$
2. **Transition Correction:** Adjusts for the biased choice of which component fails or is repaired next:
 $\log(\text{rate_orig}/\text{rate_IS});$

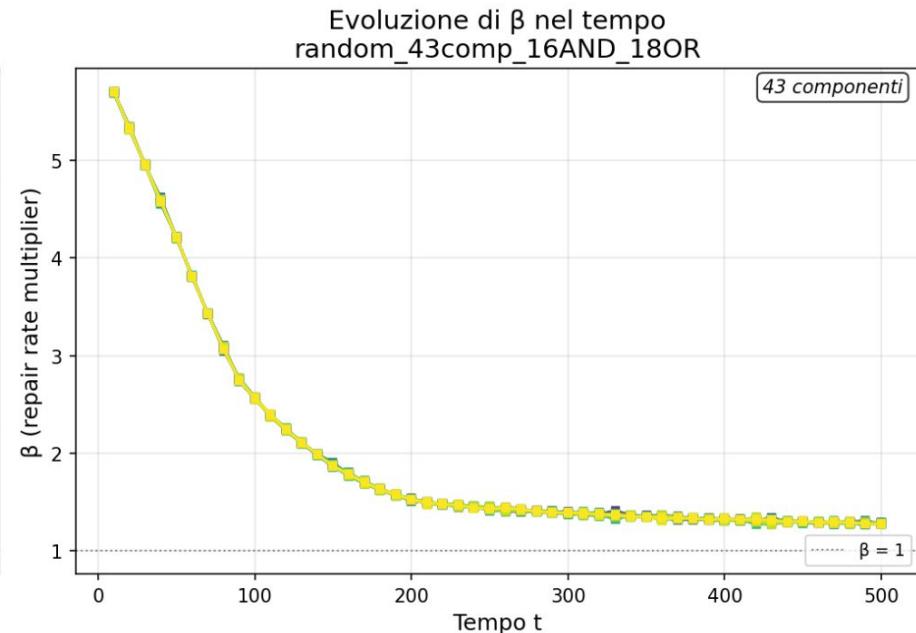
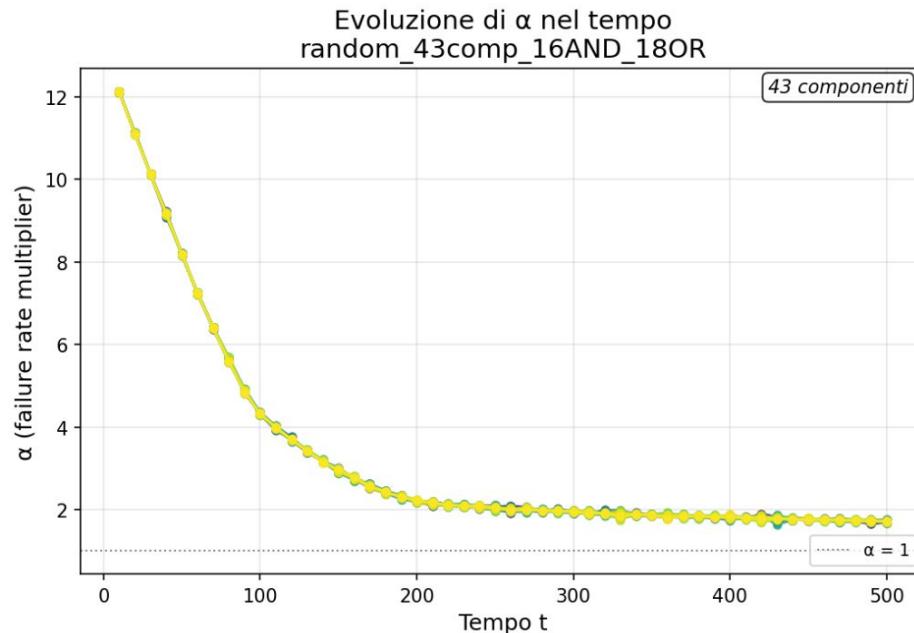
Results – Biasing Policy Dynamics



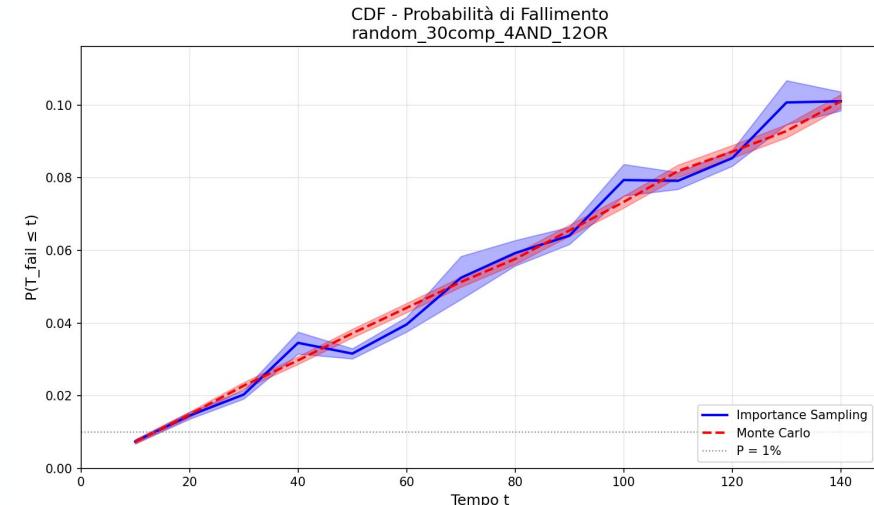
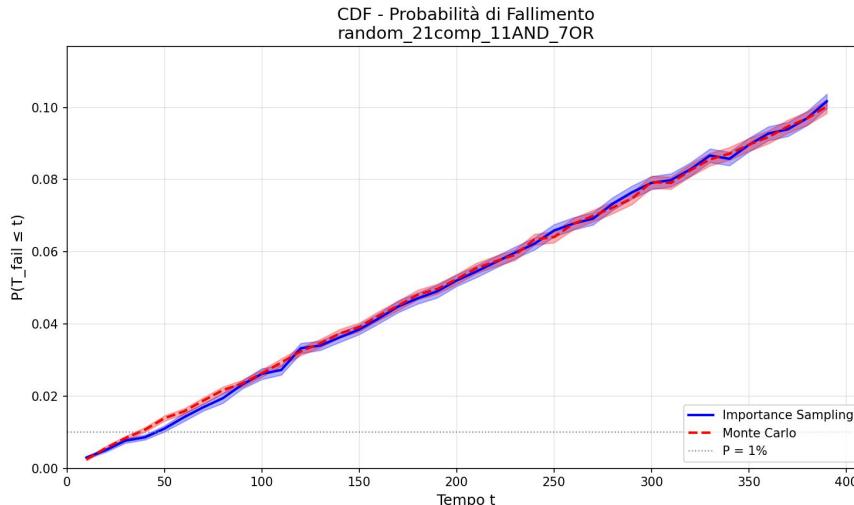
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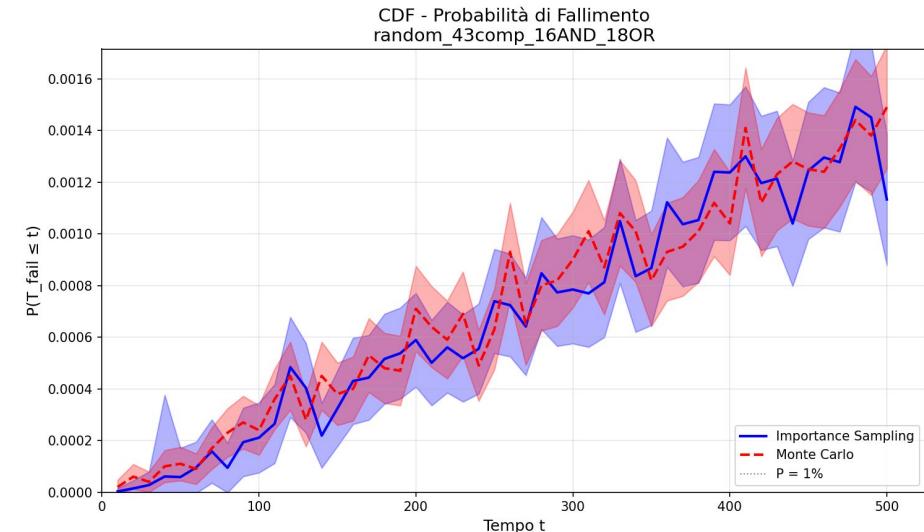
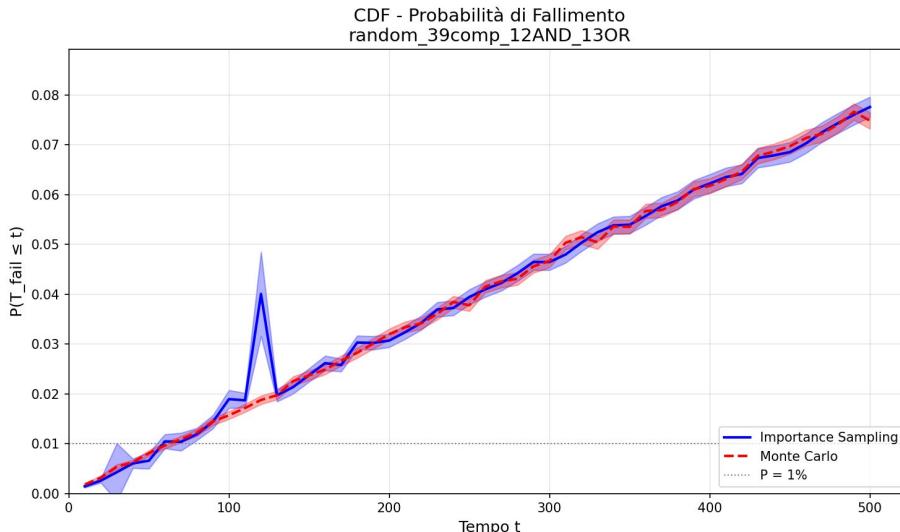
Results – Biasing Policy Dynamics



Results – CDF Estimation



Results – CDF Estimation



Challenges & Computational Bottlenecks

The MLP-CEM Bottleneck

Despite the high precision achieved, the local refinement phase (MLP + CEM) presents significant computational challenges:

- **Training Overhead:** While GNN inference is nearly instantaneous, training the MLP via CEM for a specific Fault Tree can take **several hours**, even for probabilities around 10^{-7} .
- **Simulation Intensity:** The "Stochastic Probing" requires thousands of CTMC trajectories per epoch to provide a stable gradient, making the optimization loop the primary bottleneck.
- **Cost-Benefit Trade-off:** Currently, the time saved in the final simulation is partially offset by the time required for parameter tuning on a single, specific system.
- **High variance for extremely rare events:** in order to keep computational costs affordable, the number of samples and trajectories per epoch has been kept quite small, leading to a non negligible variance.

Solutions and Future Work

- **Component-specific biasing:**
 - adjust biasing parameters with respect to the component criticality.
 - **Parallelisation of CTMC simulations.**
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- **Direct Parameter Prediction:** Instead of using the MLP-CEM loop for every new tree, the goal is to further train the **GNN** to predict the *refined* α and β directly.
 - **GAT & Transformers:** Moving from GCN to **Graph Attention Networks** to automatically weigh the importance of specific failure paths without manual ρ factor tuning.
 - **Large-Scale Stress Tests:** Extending the framework to industrial-grade systems with **100+ components** and more complex dynamic dependencies.
 - **Hybrid Engines:** Integration with **GPU-accelerated CTMC engines** to reduce the time per trajectory.