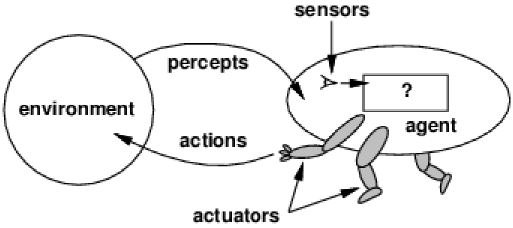
# Solving Problems by Searching

AIMA chapter 2 (partly), AIMA Sections 3.1–3.3

# Outline

- Rational agents
- Problem-solving agents
- Problem types
- Problem formulation
- Example problems
- General search algorithm

# Agents and environments

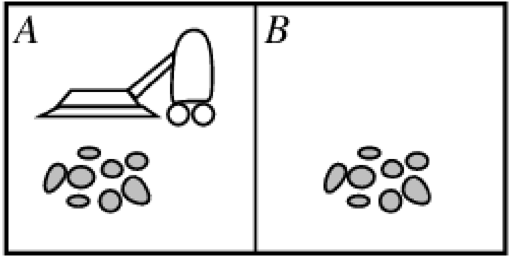



Agents include humans, robots, softbots, thermostats, etc.
The agent function maps from percept histories to actions:

$$f : \mathcal{P}^* \to \mathcal{A}$$

The agent program runs on the physical architecture to produce $f$

# Example: Vacuum-cleaner world




Perceptions: location and contents, e.g., [$A$, *Dirty*]
Actions: *Left*, *Right*, *Suck*, *NoOp*

# A vacuum-cleaner agent

| Percept sequence | Action |
| --- | --- |
| [$A$, *Clean*] | *Right* |
| [$A$, *Dirty*] | *Suck* |
| [$B$, *Clean*] | *Left* |
| [$B$, *Dirty*] | *Suck* |
| [$A$, *Clean*], [$A$, *Clean*] | *Right* |
| [$A$, *Clean*], [$A$, *Dirty*] | *Suck* |
| $\vdots$ | $\vdots$ |

What is the **right** function?

Can it be implemented in a **small** agent program?

# Agent Programs vs. Agent Functions

**Question**

If an agent has $|\mathcal{P}|$ possible perceptions, how many entries will the agent function have after $T$ time steps ?

# Agent Programs vs. Agent Functions

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If an agent has $|\mathcal{P}|$ possible perceptions, how many entries will the agent function have after $T$ time steps ?

## Sol

$\sum_{t=1}^{T} |\mathcal{P}|^t$

AI goal $\Rightarrow$ **Design small agent programs to represent huge agent functions**

# A possible agent program

**function** Reflex-Vacuum-Agent( [*location*,*status*]) **returns** an action

**if** *status* = *Dirty* **then return** *Suck*
**else if** *location* = *A* **then return** *Right*
**else if** *location* = *B* **then return** *Left*

# Rationality



Fixed performance measure evaluates the environment sequence

- one point per square cleaned up in time $T$?
- one point per clean square per time step, minus one per move?
- penalize for $> k$ dirty squares?

A rational agent chooses whichever action maximizes the expected value of the performance measure given the percept sequence to date

Rational $\neq$ omniscient

- percepts may not supply all relevant information

Rational $\neq$ clairvoyant

- action outcomes may not be as expected

Hence, rational $\neq$ successful

Rational $\implies$ exploration, learning, autonomy

# Multi-Robot Patrolling



## Exercise

Consider the following environment:

- Three rooms (A,B,C) and two robots ($r_1$,$r_2$)
- $r_1$ can patrol A and B, $r_2$ can patrol B and C
- $r_1$ starts from A and $r_2$ starts from C
- travel time between rooms is zero
- Performance measure: minimise sum of all rooms' average Idleness
- Average idleness = sum of time interval for which the room is not visited by any robot / total time interval

What would be a rational behavior for this environment ?

# PEAS

To design a rational agent, we must specify the task environment
Consider, e.g., the task of designing an automated taxi:
Performance measure??
Environment??
Actuators??
Sensors??

# PEAS

To design a rational agent, we must specify the task environment

Consider, e.g., the task of designing an automated taxi:

Performance measure?? safety, destination, profits, legality, comfort, . . .

Environment?? city streets/freeways, traffic, pedestrians, weather, . . .

Actuators?? steering, accelerator, brake, horn, speaker/display, . . .

Sensors?? video, accelerometers, gauges, engine sensors, keyboard, GPS, . . .

# Environment types



| | Crossword | Robo-selector | Poker | Taxi |
|---|---|---|---|---|
| Observable?? | | | | |
| Deterministic?? | | | | |
| Episodic?? | | | | |
| Static?? | | | | |
| Discrete?? | | | | |
| Single-agent?? | | | | |

**The environment type largely determines the agent design**

The real world is (of course) partially observable, stochastic, sequential, dynamic, continuous, multi-agent

# Environment types



| | Crossword | Robo-selector | Poker | Taxi |
|---|---|---|---|---|
| Observable?? | Yes | Partly | Partly | Partly |
| Deterministic?? | Yes | No | No | No |
| Episodic?? | No | Yes | No | No |
| Static?? | Yes | No | Yes | No |
| Discrete?? | Yes | No | Yes | No |
| Single-agent?? | Yes | Yes | No | No |

**The environment type largely determines the agent design**

The real world is (of course) partially observable, stochastic, sequential, dynamic, continuous, multi-agent

# Problem types



Deterministic, fully observable $\Longrightarrow$ single-state problem
  Agent knows exactly which state it will be in; solution is a sequence

Non-observable $\Longrightarrow$ conformant problem
  Agent may have no idea where it is; solution (if any) is a sequence

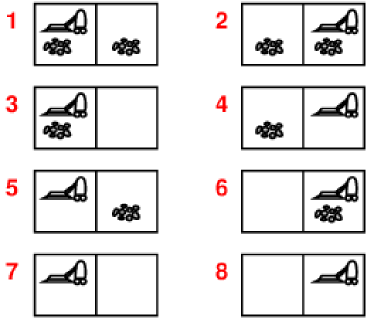
Nondeterministic and/or partially observable $\Longrightarrow$ contingency problem
  percepts provide **new** information about current state
  solution is a contingent plan or a policy
  often **interleave** search, execution

Unknown state space $\Longrightarrow$ exploration problem ("online")

# Example: vacuum world



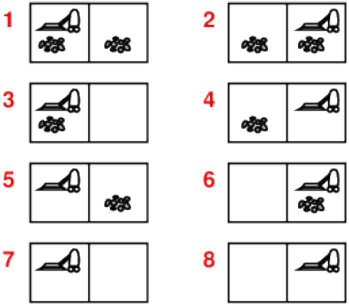
Single-state, start in #5. Solution??



# Example: vacuum world

Single-state, start in #5. Solution??
$[Right, Suck]$

Conformant
start in $\{1, 2, 3, 4, 5, 6, 7, 8\}$
e.g., *Right* goes to $\{2, 4, 6, 8\}$. Solution??



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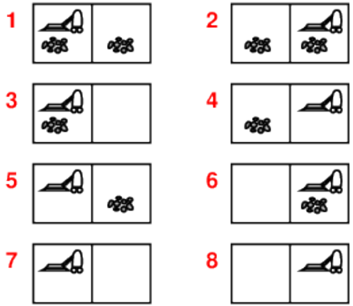
$[Right, Suck, Left, Suck]$

Contingency, start in #5
Murphy's Law: *Suck* can dirty a clean carpet
Local sensing: dirt, location only.
Solution??



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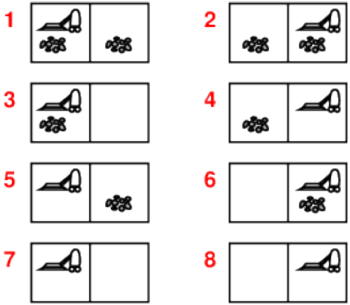
$[Right, Suck, Left, Suck]$

Contingency, start in #5
Murphy's Law: *Suck* can dirty a clean carpet
Local sensing: dirt, location only.
Solution??
$[Right,$ **if** $dirt$ **then** $Suck]$



# Problem-solving agents

Restricted form of general agent: **Goal based agents**

- formulate a goal and a problem given the current state
- search for a solution
- execute the solution **ignoring** perceptions

Note: this is offline problem solving; solution executed “eyes closed.”
Online problem solving involves acting without complete knowledge.

# Problem-solving agents

```
function Simple-Problem-Solving-Agent( percept) returns an action
    static: seq, an action sequence, initially empty
            state, some description of the current world state
            goal, a goal, initially null
            problem, a problem formulation

    state ← Update-State(state, percept)
    if seq is empty then
        goal ← Formulate-Goal(state)
        problem ← Formulate-Problem(state, goal)
        seq ← Search( problem)
    action ← First(seq)
    seq ← Rest(seq)
    return action
```

# An example: Traveling in Romania



**Example (Holidays in Romania)**

On holiday in Romania; currently in Arad.

Flight leaves tomorrow from Bucharest

Formulate goal:

be in Bucharest

Formulate problem:

states: various cities

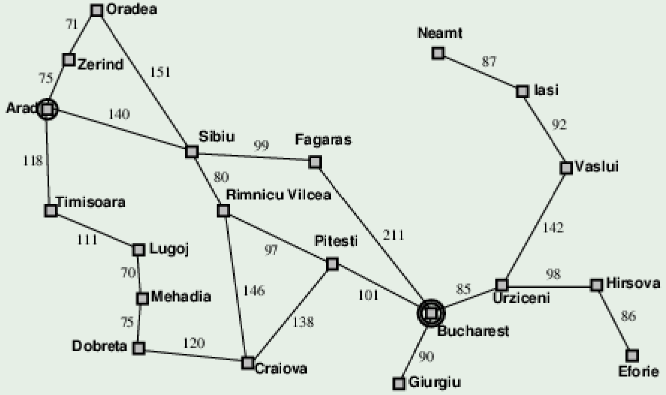
actions: drive between cities

Find solution:

sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest

# An example: Traveling in Romania

## Example (Holidays in Romania)



# Single-state problem formulation

A problem is defined by four items:

initial state e.g., "at Arad"

successor function $S(x)$ = set of action–state pairs

e.g., $S(A) = \{< Arad \rightarrow Zerind, Zerind >, \ldots\}$

goal test, can be

explicit, e.g., $x$ = "at Bucharest"

implicit, e.g., $NoDirt(x)$

path cost (additive)

e.g., sum of distances, number of actions executed, etc.

$c(x, a, y)$ is the step cost, assumed to be $\geq 0$

A solution is a sequence of actions
leading from the initial state to a goal state

# Selecting a state space



Real world is absurdly complex

$\Rightarrow$ state space must be **abstracted** for problem solving

(Abstract) state = set of real states

(Abstract) action = complex combination of real actions

e.g., "Arad $\rightarrow$ Zerind" represents a complex set of possible routes, detours, rest stops, etc.

For guaranteed realizability, **any** real state "in Arad" must get to some real state "in Zerind"

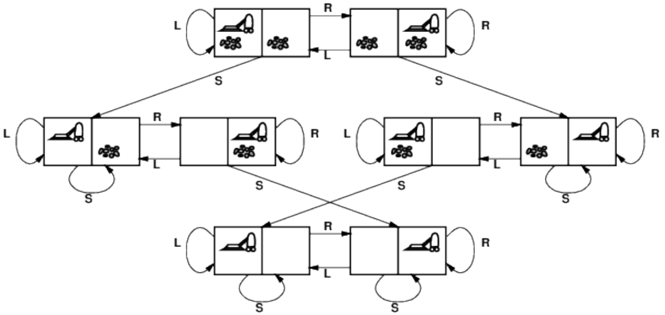
(Abstract) solution =

set of real paths that are solutions in the real world

Each abstract action should be "easier" than the original problem!

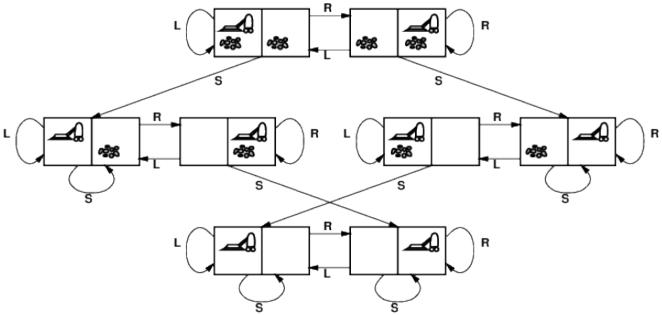
# Example: vacuum world state space graph






states??:
actions??:
goal test??:
path cost??:

# Example: vacuum world state space graph




states??: discrete dirt and robot locations (ignore dirt amounts)

actions??:

goal test??:

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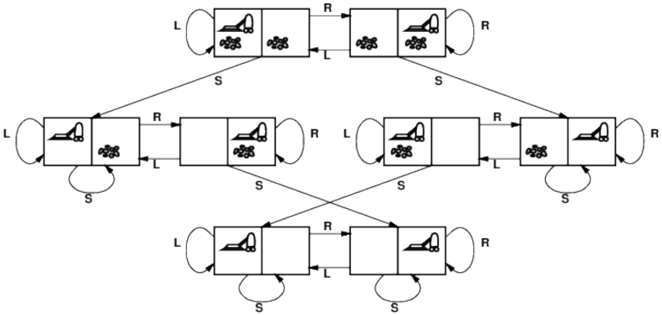


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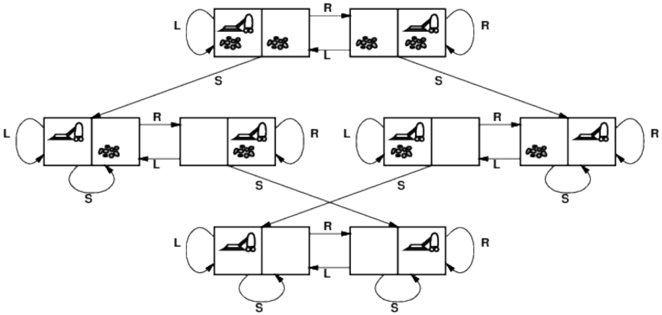


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path cost??:

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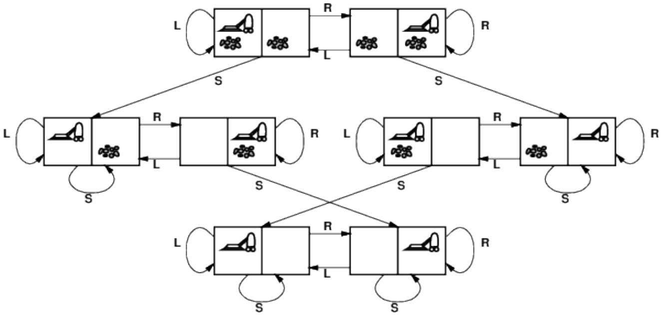


states??: discrete dirt and robot locations (ignore dirt amounts)

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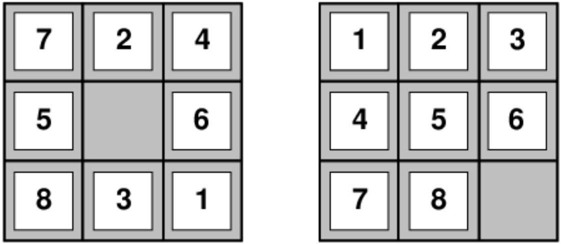
goal test??: no dirt

path cost??: 1 per action (0 for *NoOp*)

# Example: The 8-puzzle




Start State Goal State

states??:
actions??:
goal test??:
path cost??:

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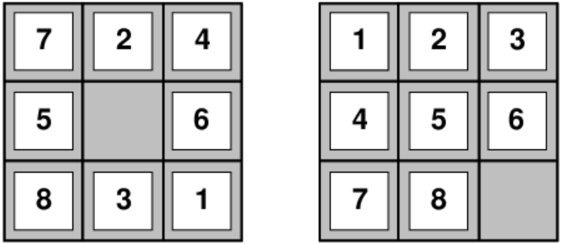


Start State Goal State

states??: integer locations of tiles (ignore intermediate positions)
actions??:
goal test??:
path cost??:

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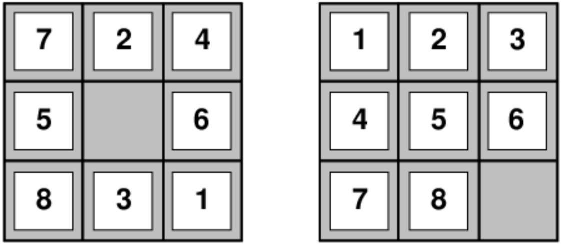


Start State Goal State

states??: integer locations of tiles (ignore intermediate positions)
actions??: move blank left, right, up, down
goal test??:
path cost??:

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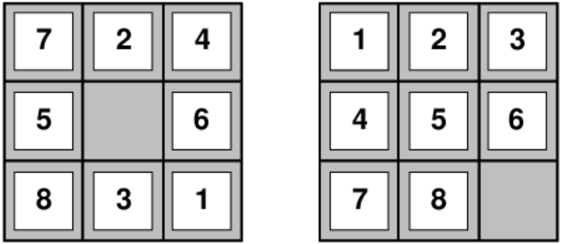


Start State    Goal State

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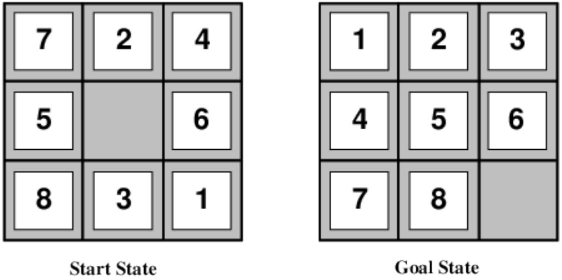


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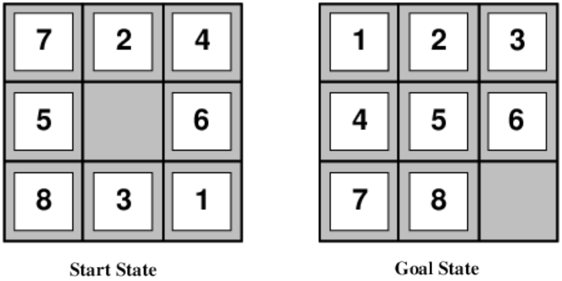


states??: integer locations of tiles (ignore intermediate positions)
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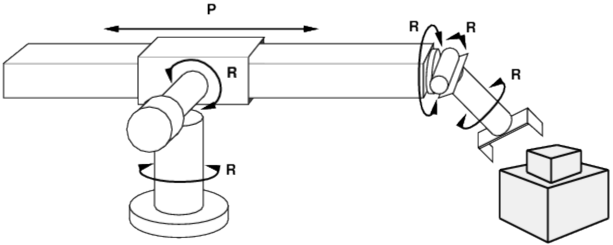
goal test??: given goal state
path cost??: 1 per move

[Note: optimal solution of *n*-Puzzle family is NP-hard]

# Example: robotic assembly






states??: real-valued coordinates of robot joint angles
parts of the object to be assembled

actions??: continuous motions of robot joints

goal test??: complete assembly **with no robot included!**

path cost??: time to execute

# Tree search algorithm

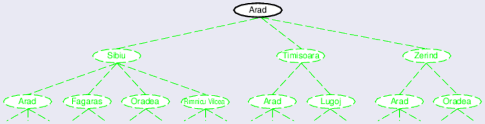


Basic idea:
offline, simulated exploration of state space
by generating successors of already-explored states
(a.k.a. expanding states)

```
function Tree-Search( problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if node contains a goal state then return the solution
        else add successor nodes to the search tree (expansion)
    end
```

# Tree search example





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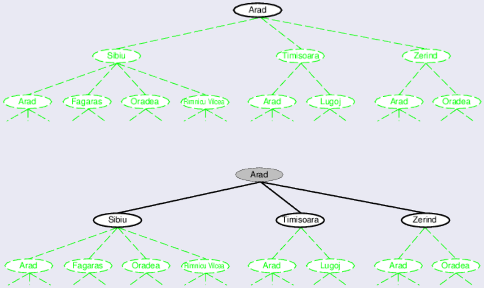

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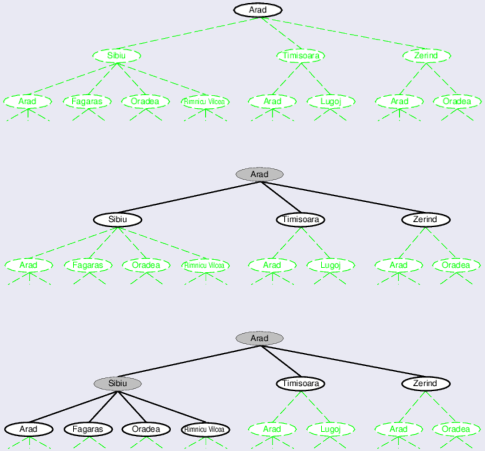

# Implementation: states vs. nodes



A state is a (representation of) a physical configuration

A node is a data structure constituting part of a search tree

  includes parent, action, children, depth, path cost (i.e., $g(x)$)

States do not have parents, actions,children, depth, or path cost!

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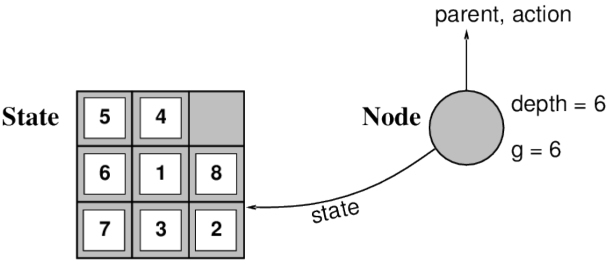



The Expand function creates new nodes, filling in the various fields and using the SuccessorFn of the problem to create the corresponding states.

# Implementation: general tree search



```
function Tree-Search( problem, frontier) returns a solution, or failure
  frontier ← Insert(Make-Node(problem.Initial-State))
  while not IsEmty(frontier) do
    node ← Pop(frontier)
    if problem.Goal-Test(node.State) then return node
    frontier ← InsertAll(Expand(node, problem))
  end loop
  return failure
```

# Implementation: expand nodes



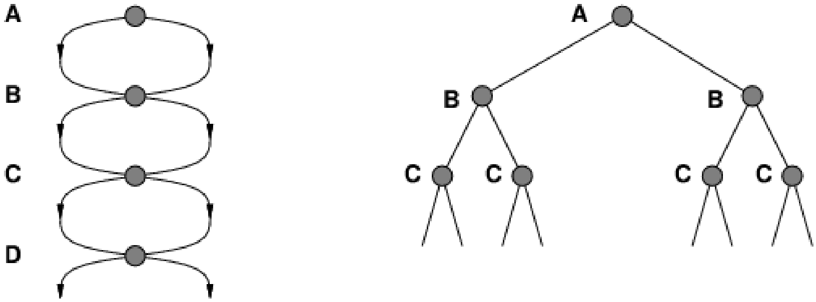
```
function Expand( node, problem) returns a set of nodes
    successors ← the empty set
    for each action, result in Successor-Fn(problem, node.State)
     do
        s ← a new Node
        s.Parent-Node ← node;
        s.Action ← action;
        s.State ← result
        s.Path-Cost ← node.Path-Cost +
                           Step-Cost(node.State, action, result)
        s.Depth ← node.Depth + 1
        add s to successors
    return successors
```

# Search strategies

- A strategy is defined by picking the **order of node expansion**
- Strategies are evaluated along the following dimensions:
  - completeness—does it always find a solution if one exists?
  - time complexity—number of nodes generated/expanded
  - space complexity—maximum number of nodes in memory
  - optimality—does it always find a least-cost solution?
- Time and space complexity are measured in terms of
  - $b$—maximum branching factor of the search tree
  - $d$—depth of the least-cost solution
  - $m$—maximum depth of the state space (may be $\infty$)

# Repeated states

Failure to detect repeated states can turn a linear problem into an exponential one!



# Graph search

```
function Graph-Search( problem, frontier) returns a solution, or failure
  explored ← an empty set
  frontier ← Insert(Make-Node(problem.Initial-State))
  while not IsEmty(frontier) do
    node ← Pop(frontier)
    if problem.Goal-Test(node.State) then return node
    if node.State is not in explored then
      add node.State to explored
      frontier ← InsertAll(Expand(node, problem))
    end if
  end loop
  return failure
```