# Artificial Intelligence

## Informed Search strategies (AIMA sections 3.5 – 3.6.3 [3.5.5 and 3.5.6 excluded])

# Summary

- Greedy Best-First search
- A$^*$ search
- Heuristics

# Review: Tree search

```
function Tree-Search( problem, frontier) returns a solution, or failure
  frontier ← Insert(Make-Node(problem.Initial-State))
  while not IsEmty(frontier) do
    node ← Pop(frontier)
    if problem.Goal-Test(node.State) then return node
    frontier ← InsertAll(Expand(node, problem))
  end loop
  return failure
```

A strategy is defined by picking the **order of node expansion**

# Best-First search



Idea: use an evaluation function for each node – estimate of “desirability”

$\Rightarrow$ Expand most desirable unexpanded node

Implementation: *frontier* is a queue sorted in decreasing order of desirability

Special cases:

- greedy best-first search
- A$^*$ search

# Romania with straight-line distances to Bucharest

| | | | |
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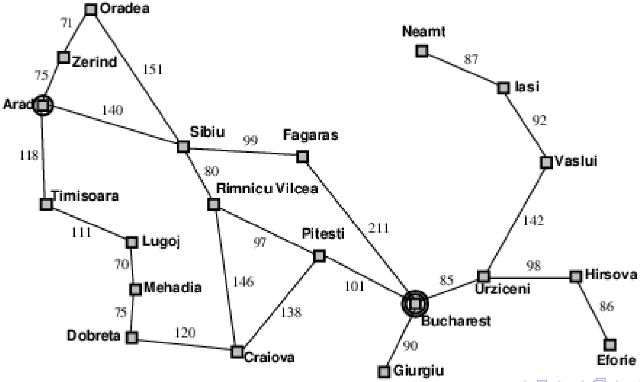

# Greedy search



Evaluation function $h(n)$ (**h**euristic)

$=$ estimate of cost from $n$ to the closest goal

E.g., $h_{\mathrm{SLD}}(n) =$ straight-line distance from $n$ to Bucharest

Greedy search expands the node that **appears** to be closest to goal

# Greedy search example

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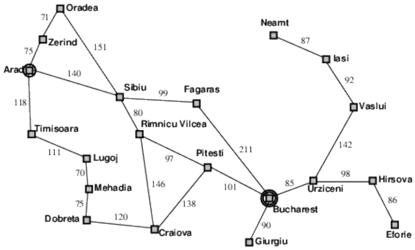


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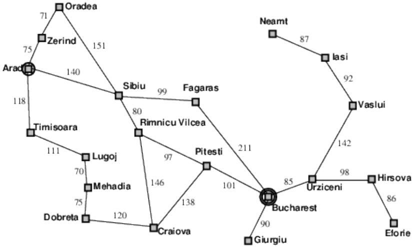


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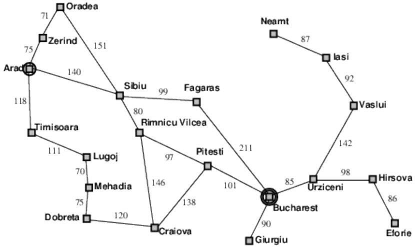


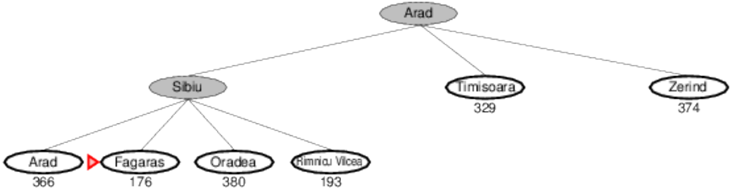

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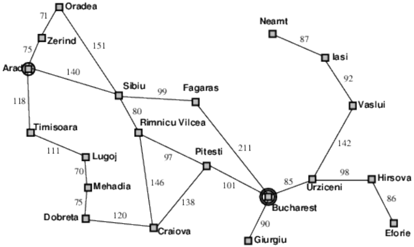


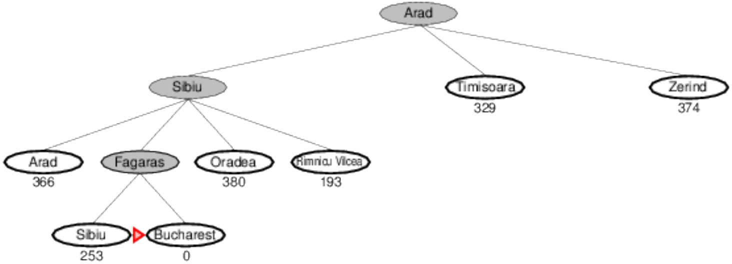

# Properties of greedy search



Complete??

# Properties of greedy search

Complete?? No–can get stuck in loops, e.g.,

Start: Iasi, Goal: Fagaras

Iasi $\rightarrow$ Neamt $\rightarrow$ Iasi $\rightarrow$ Neamt $\rightarrow \cdots$

Complete in finite space with repeated-state checking

Time??

# Properties of greedy search



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Start: Iasi, Goal: Fagaras

Iasi $\rightarrow$ Neamt $\rightarrow$ Iasi $\rightarrow$ Neamt $\rightarrow \cdots$

Complete in finite space with repeated-state checking

Time?? $O(b^m)$, but a good heuristic can give dramatic improvement

Space??

# Properties of greedy search

Complete?? No–can get stuck in loops, e.g.,
Start: Iasi, Goal: Fagaras
Iasi $\to$ Neamt $\to$ Iasi $\to$ Neamt $\to \cdots$
Complete in finite space with repeated-state checking
Time?? $O(b^m)$, but a good heuristic can give dramatic improvement
Space?? $O(b^m)$—keeps all nodes in memory
Optimal??

# Properties of greedy search



Complete?? No–can get stuck in loops, e.g.,
  Start: Iasi, Goal: Fagaras
  Iasi $\rightarrow$ Neamt $\rightarrow$ Iasi $\rightarrow$ Neamt $\rightarrow \cdots$
Complete in finite space with repeated-state checking
Time?? $O(b^m)$, but a good heuristic can give dramatic improvement
Space?? $O(b^m)$—keeps all nodes in memory
Optimal?? No

# $A^*$ search

Idea: avoid expanding paths that are already expensive

Evaluation function $f(n) = g(n) + h(n)$

- $g(n)$ = cost so far to reach $n$
- $h(n)$ = estimated cost to goal from $n$
- $f(n)$ = estimated total cost of path through $n$ to goal

◇ $A^*$ search uses an admissible heuristic
i.e., $h(n) \leq h^*(n)$ where $h^*(n)$ is the **true** cost from $n$.
(Also require $h(n) \geq 0$, so $h(G) = 0$ for any goal $G$.)

◇ E.g., $h_{\mathrm{SLD}}(n)$ never overestimates the actual road distance

◇ Theorem: $A^*$ search is optimal

# A* search example



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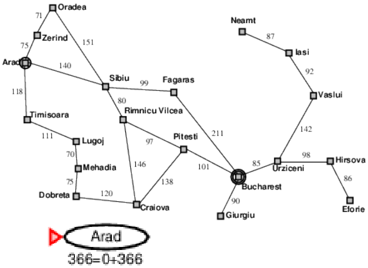

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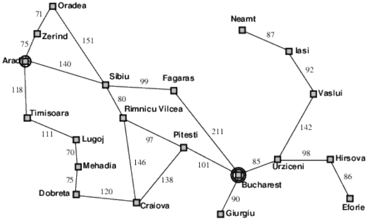


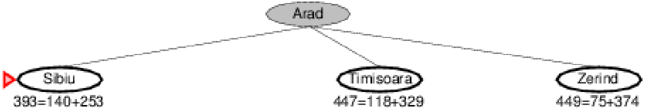

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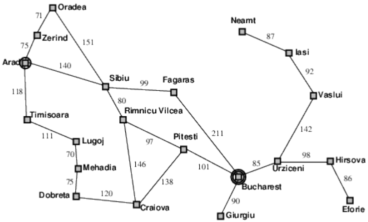


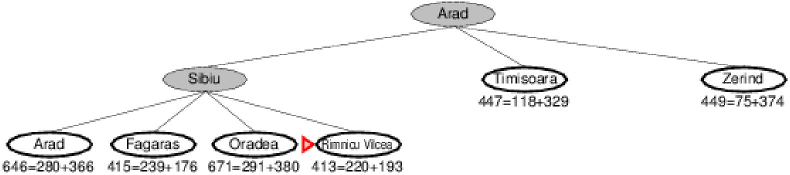

# A* search example

Artificial Intelligence

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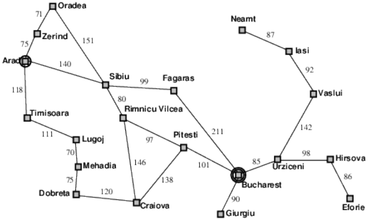


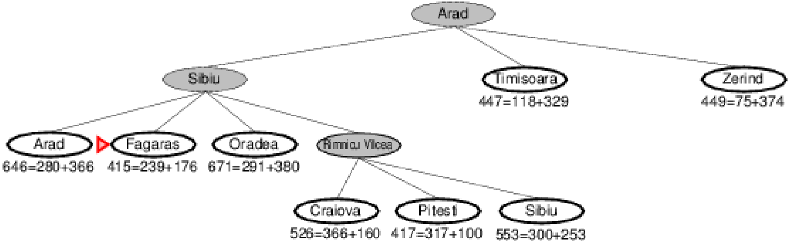

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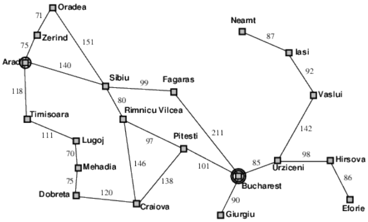


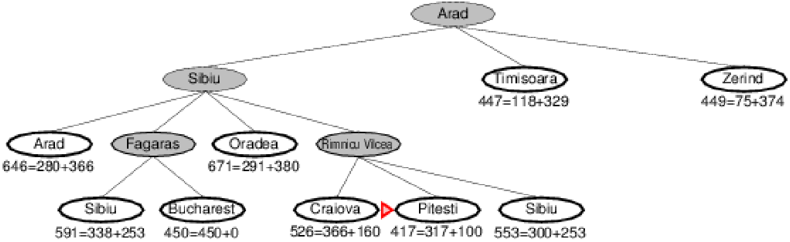

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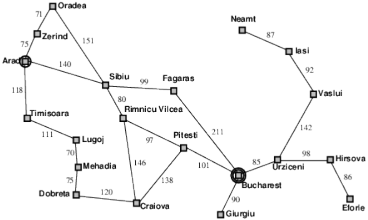
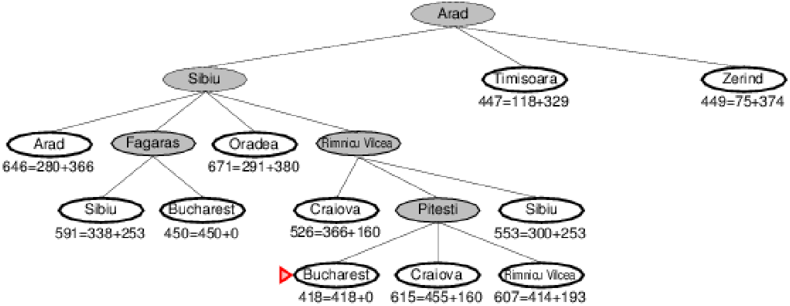


# Properties of A*



Complete??

# Properties of A$^*$



Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time??

# Properties of A*



Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$
Time?? Exponential in [relative error in $h$ $\times$ length of soln.]
Space??

# Properties of A$^*$



Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$
Time?? Exponential in [relative error in $h \times$ length of soln.]
Space?? Keeps all nodes in memory
Optimal??

# Properties of A*



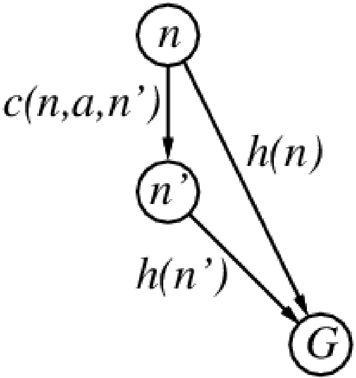
Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$
Time?? Exponential in [relative error in $h$ $\times$ length of soln.]
Space?? Keeps all nodes in memory
Optimal?? Yes, **but** it requires assumptions on heuristics (admissibility, consistency) and search strategy (tree or graph search)

# Consistent heuristic definition

A heuristic is consistent if

$$h(n) \leq c(n, a, n') + h(n')$$

- We can show that, if $h$ is consistent $f(n)$ is nondecreasing along any path.
- A* expands nodes in order of increasing $f$, hence it will find the least cost solution.



# Admissible vs Consistent Heuristic

**consistency** $\rightarrow$ **admissible**
Can be proved by induction on the path to goal
**admissible** $\not\rightarrow$ **consistency**
Find a counter example...

Tree-Search + admissible Heuristic $\rightarrow$ optimality of $A^*$

Graph-Search + admissible Heuristic $\not\rightarrow$ optimality of $A^*$

- Can discard the optimal path to a repeated node

Graph-Search + **consistent** Heuristic $\rightarrow$ optimality of $A^*$

# Admissible heuristics

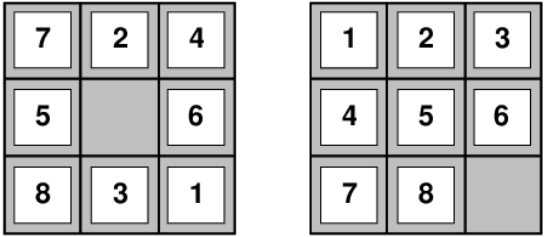


E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles

$h_2(n)$ = total Manhattan distance

(i.e., no. of squares from desired location of each tile)




Start State Goal State

$h_1(S) =$??

$h_2(S) =$??

# Admissible heuristics



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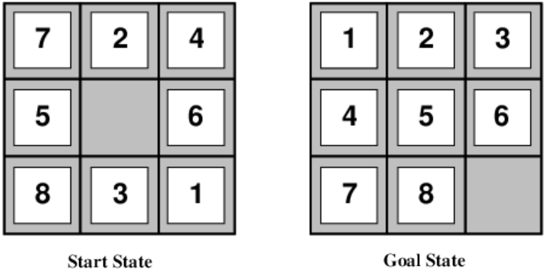


$h_1(S) =$?? 6

$h_2(S) =$??

# Admissible heuristics

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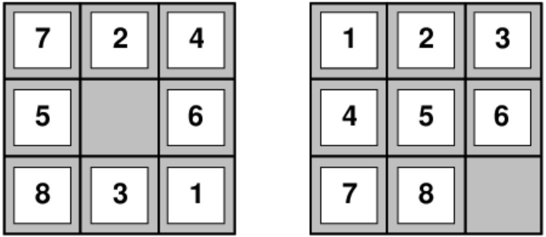



Start State Goal State

$h_1(S)$ =?? 6

$h_2(S)$ =?? 4+0+3+3+1+0+2+1 = 14

# Dominance



If $h_2(n) \geq h_1(n)$ for all $n$ (both admissible) then $h_2$ dominates $h_1$ and is better for search

Typical search costs:

| | |
|---|---|
| $d = 14$ | IDS = 3,473,941 nodes |
| | $A^*(h_1)$ = 539 nodes |
| | $A^*(h_2)$ = 113 nodes |
| $d = 24$ | IDS ≈ 54,000,000,000 nodes |
| | $A^*(h_1)$ = 39,135 nodes |
| | $A^*(h_2)$ = 1,641 nodes |

Given any admissible heuristics $h_a$, $h_b$,

$$h(n) = \max(h_a(n), h_b(n))$$

is also admissible and dominates $h_a$, $h_b$

# Relaxed problems

Admissible heuristics can be derived from the **exact** solution cost of a **relaxed** version of the problem

If the rules of the 8-puzzle are relaxed so that a tile can move **anywhere**, then $h_1(n)$ gives the shortest solution

If the rules are relaxed so that a tile can move to **any adjacent square**, then $h_2(n)$ gives the shortest solution

**Key point**: the optimal solution cost of a relaxed problem is no greater than the optimal solution cost of the real problem

## Summary

- ◊ Heuristic functions estimate costs of shortest paths
- ◊ Good heuristics can dramatically reduce search cost
- ◊ Greedy best-first search expands lowest $h$
  - – incomplete and not always optimal
- ◊ $\mathrm{A}^*$ search expands lowest $g + h$
  - – complete and optimal
  - – also optimally efficient (up to tie-breaks, for forward search)

Admissible heuristics can be derived from exact solution of relaxed problems

# Exercise: Going from Lugoj to Bucharest



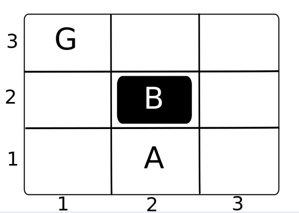
## From Lugoj to Bucharest

◇ Trace the operation of A* search applied to the problem of going from Lugoj to Bucharest using the straight-line distance heuristic.

◇ Trace the operation of greedy best-first search applied to the problem of going from Lugoj to Bucharest using the straight-line distance heuristic.

# Exercise: Navigation

## Navigation with obstacles




The figure shows an artificial environment where an agent A is positioned in the square (1, 2)[a], the goal G is in (3, 1), and there is a block B in (2, 2). The agent can not pass through blocks and can move in the four directions (Up, Down, Left, Right).

[a] where the position is (row,column)

# Exercise: Navigation II



## Navigation with obstacles II

Formalize the problem of reaching G as a state problem

- Describe the state space, the initial and final state.
- Describe the operators.
- Find an admissible heuristics for $A^*$.
- Assume the operators have cost 1, draw the tree generated by $A^*$.

# Exercise: confusing problems for greedy best-first search



## confusing problems for greedy best-first search

When going from Iasi to Fagaras the straight-line distance heuristic result in poor performance for greedy best-first search. But from Fagaras to Iasi it is perfect.

Are there problems for which the heuristic produces sub-optimal paths in both directions ?