Linear Algebra - Cheat Sheet (HS22)

Mattia Taiana

26. December 2022

1 Complex Numbers

General

$$z = \underbrace{x}_{\text{Re}} + i\underbrace{y}_{\text{Im}} = \underbrace{r \cdot (\cos(\varphi) + i \cdot \sin(\varphi)) = r \cdot e^{i\varphi}}_{\text{Polarform}}$$

$$\overline{z} = x - iy = r \cdot e^{i(2\pi - \varphi)}$$

$$|z| = r = \sqrt{x^2 + y^2} = \sqrt{z \cdot \overline{z}}$$

$$\varphi = \begin{cases} arctan(\frac{y}{x}), & \text{I } Q. \\ arctan(\frac{y}{x}) + \pi, & \text{II/III } Q. \\ arctan(\frac{y}{x}) + 2\pi, & \text{IV } Q. \end{cases}$$

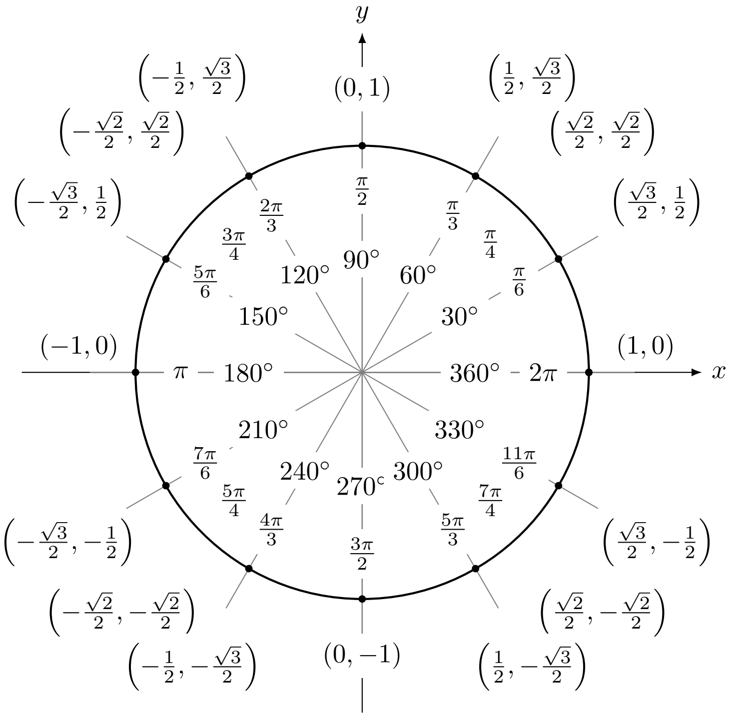
Operations

$$\begin{aligned} +/- :&\quad (x_1 + x_2) + (y_1 + y_2)i \\ z_1 \cdot z_2 :&\quad (x_1 + y_1 i)(x_2 + y_2 i) = r_1 \cdot r_2 \cdot e^{i(\varphi_1 + \varphi_2)} \\ \frac{z_1}{z_2} :&\quad \frac{z_1 \cdot \overline{z}_2}{|z_2|^2} = \frac{r_1}{r_2} \cdot e^{i(\varphi_1 - \varphi_2)} \\ z^n :&\quad r^n \cdot e^{i\varphi n} \\ \sqrt{a} :&\quad a = z^n \Leftrightarrow |a| \cdot e^{i\alpha} = r^n \cdot e^{i\varphi n} \begin{cases} r = \sqrt[n]{|a|} \\ \varphi = \frac{\alpha + 2k\pi}{n} \end{cases}_{k=0,\dots,n-1} \end{aligned}$$

Polynomials

$$\begin{aligned} \text{Grad 2:}&\quad z = \frac{b \pm \sqrt{b^2 - 4ac}}{2a} \\ \text{Sonderfall:}&\quad az^n + c = 0 \Leftrightarrow z = \sqrt[n]{-\frac{c}{a}} \end{aligned}$$

With polynomials with complex roots, the roots occur as a complex-conjugate pair.
Polynomials over $\mathbb{C}$ with an odd degree have at least one root in $\mathbb{R}$.



2 LSE

Gauss-Algorithm

runtime: $O(n^3)$
valid Ops: switch, multiply and add/subtract rows
goal: row echelon form

$$\begin{array}{cccc|c} x_1 & x_2 & \dots & x_n & \\ \hline a_{11} & a_{12} & \dots & a_{1n} & b_1 \\ 0 & a_{22} & \dots & a_{2n} & b_2 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & a_{rn} & b_r \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & b_m \end{array}$$

rank r

consistency conditions

LHS RHS

if RHS always 0: homogeneous LSE
consistency conditions: $b_{r+1} = \dots = b_m = 0$