ACTION

$$S = \sum_{i,j} \bar{\psi}_i D_{ij} \psi_j + \bar{\chi}_i B_{ij} \chi_j + g \sum_i \bar{\psi}_i \psi_i \bar{\chi}_i \chi_j$$

Where D_{ij} and B_{ij} are invertible ~~antisymmetric~~ matrices $n \times n$

PATH INTEGRAL

$$\Rightarrow Z = \int \prod_i d\bar{\psi}_i \, d\psi_i \, d\bar{\chi}_i \, d\chi_i \, \exp(S) =$$

Using transformation:

$$\psi'_i := \sum_j D_{ij} \psi_j \qquad \chi'_i := \sum_j B_{ij} \chi_j$$

We get:

$$Z = \det D \cdot \det B \int \prod_i d\bar{\psi}_i \, d\psi'_i \, d\bar{\chi}_i \, d\chi'_i \, \exp(S') \quad (*)$$

where $S' = \sum_i \bar{\psi}_i \psi'_i + \bar{\chi}_i \chi'_i +$

$$+ g \sum_i \sum_j \sum_k \bar{\psi}_i D^{-1}_{ij} \psi'_j \cdot \bar{\chi}_i B^{-1}_{ik} \chi'_k =$$

~~The terms in the~~

The integrand $\exp(S') = 1 + S' + \frac{S'^2}{2} + \dots$

contains terms that are multiple of

$\prod_i d\bar{\psi}_i \, d\psi'_i \, d\bar{\chi}_i \, d\chi'_i$, These terms arise from powers of $(S')^m$

with $\frac{n}{2} \leq m \leq n$ and they give contributions to the coefficient of $\prod_i d\bar{\Psi}_i d\Psi_i' d\bar{X}_i dX_i'$ in powers of g, such that

$O(g^{n-m})$ originates from $(S')^m$

$$O(g^0) \longrightarrow 1 \cdot n!$$

$$O(g^1) \longrightarrow 0$$

$$O(g^2) \longrightarrow g^2 \cdot (n-2)! \sum_{j \neq k} (D^{-1})_{jj} (B^{-1})_{kk}$$

$$O(g^3) \longrightarrow 0$$

$$O(g^4) \longrightarrow g^4 (n-4)! \sum_{l,m,n,p} (D^{-1})_{ll} (D^{-1})_{mm} (B^{-1})_{nn} (B^{-1})_{pp}$$

etc.

Is there a closed formula for Z?