UNIVERSITY OF TRENTO

Department of Industrial Engineering

Master Degree in Mechatronics Engineering

Influence of slip versus torque control formulation on minimum time manoeuvres of a motorcycle

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Introduction

The simulation of mechanical and mechatronics systems allow to test and validate the design in a safe and efficient environment without the need to build the physical object and measure the parameters. The simulation is based on digital technology with major benefits as cost and efficiency and the possibility of an easy reconfiguration and retesting of a system which is usually impossible or infeasible for real model in terms of cost and time.**maria1997introduction**

A large number of vehicle model are available in scientific literature some are fairly complex and other are simple. Depending on the application and the time constraints a proper model should be chosen. Simple models are faster and therefore suitable for real-time purposes while complex models are time-consuming and are used in the case where the model cannot be simplified or the goal of the study is to replicate in detail the behaviour of the analysed system.

The work of this thesis aims to derive three different models of racing motorcycle with increasing complexity taking into account the force exchange between tyre and ground using Pacejka's magic formula **pacejka2006tyre**. The models will then be used to calculate controls and trajectory to achieve the minimum lap time of a specific circuit.

In particular, the first model of the motorcycle will represent a vehicle with fixed suspensions meaning that the rear swingarm and the steering fork have respectively a fixed angle and a fixed length. The motorcycle is controlled with the steering torque and the longitudinal slip of front and rear wheel.

The second model has again fixed suspensions, however, the vehicle is controlled with steering torque, braking torque at the front wheel and braking/traction torque at the rear.

The third model takes into account the internal motion due to suspension deformation and the motorcycle is again controlled with torques.

All the model are derived with the multi-body approach and symbolic formulation. In order to achieve this goal, the model is defined using Maple, a software well known for its capability in symbolic computation. Moreover, the equations of motion are obtained using MBSymba which is a custom free library for Maple available online at http://www.multibody.net.

All three models are derived in a similar way as in the publication of

Cossalter *et al***cossalter2007influence**, without taking into account the lateral flexibility of the front torque and the torsional flexibility of the swingarm. The tyre forces are derived using the Pacejka's magic formula **pacejka2006tyre** which is an empirical formula obtained from the assumption of similarity.
The minimum time trajectory and control is computed formulating a custom optimal control problem using XOptima package for Maple and then solving with PINS (acronym for *PINS Is Not a Solver*). The first is a library developed to transform the symbolic model of the vehicle (DAE), constraints, and target functions in C++ code that can be used by PINS. PINS is a software, free for academic purposes, developed at University of Trento by Prof. Bertolazzi, Prof. Biral and Prof. Bosetti that can solve optimal control problems (OCPs) with indirect method. As far as the author knows, there are not other optimal control solvers that exploit Pontryagin maximum principle and calculus of variations to solve the problem with the indirect method. **bertolazzi2006symbolic**
The thesis is organized in the following way. In the first chapter, there is a brief overview of the state of the art in optimal control, motorcycle dynamic model and minimum time application. In the second chapter, there is a description of how the motorcycle models are derived and the optimal control problem. The third chapter confronts the results of the three models. The thesis is ended with the conclusion, references and appendix.

Chapter 1

State of art on minimum lap time application in racing motorcycle

1.1 State of the art on Optimal Control Problems

Optimal control problem, also known as dynamic optimisation, are minimisation problem where the variables and parameters change with time. Dynamic systems are characterized by the states and often are controlled by a convenient choice of inputs (controls).
Dynamic optimisation aims to compute those controls and states for a dynamic system over a time interval to minimise one or more performance indexes. In other words, the input is chosen to optimize (minimize) an objective function while complying to constraint equations.

1.1.1 Optimal Control Problems

Optimal control problems are challenging from the theoretical point of view and of practical interest. However due to dimensionality and complexity of system of equations the application in real problems and industrial environment is still not so widespread.
In general, OPC can be continuous or discrete, linear or non-linear, time-variant or time-invariant. However, in this thesis are addressed only optimal control problems that are continuous time-variant and highly non-linear. Those properties will be discussed in the following sections. In general, there are four main approaches to solve continuous-time OPC: state space approach, direct methods, indirect methods and differential dynamic programming.

State-space approaches

State-space approaches follow the principle of optimality for which each subarc of an optimal trajectory must be optimal. In literature, those are referred to as Hamilton-Jacobi-Bellman (HJB) equation. However, the problem needs numerical methods to be solved, moreover, a solution can be found only for small dimension problems due to *course of dimensionality*. There is no practical application of this method to solve highly non-linear problem as a dynamic optimisation of a motorcycle model.

Direct Method

Direct methods discretize the original optimal control problem into a non-linear programming problem (NLP). In other words, the OPC is transformed in a discrete-time system that can be solved using numerical schemes and optimization techniques, namely Initial Value Solver (IVS) and Sequential Quadratic Programming (SQP) **bertolazzi2005symbolic**
The main advantage of direct methods is the possibility to use inequality constraints even in case of change in the constraints active set (activation/deactivation)**biral2016notes**
Direct methods are easier to implement compared to the other three categories and this is one of the reasons why they are by far the most widespread. In fact, almost 90% of the avaiable optimal control software rely on direct method. **rao2009surveyrodrigues2014optimal**

Indirect Method

Indirect methods exploits the necessary condition of optimality to derive a boundary value problem (BVP) in ordinary differential equations(ODE). Therefore the BVP can be solved numerically as a non linear problem. The indirect method allow to first optimize and then discretize meaning that the problem con be firstly written in continuous time and discretized later using different discretization techniques. The class of indirect methods exploits the well known calculus of variations and the Euler-Lagrange differential equations, and the so-called Pontryagin Maximum Principle.**bertolazzi2006symbolic**
The numerical solution can be computed either by shooting techniques (single/multiple shooting) or by collocation. The major drawbacks of indirect methods are that the problem could be difficult to solve or unstable due to the nature of the underlying differential equations (nonlinearity and instability) and the changes in control structure (active constraints in specific arcs). Moreover, in some arcs, singularity arises therefore the DAE index increase leading to the necessity of specialized solution techniques. **biral2016notes**

Differential Dynamic Programming

1.1.2 Minimum time Optimal Control Problem

1.1.3 PINS

1.2 Theory of OCP

1.2.1 Pontryagin

1.2.2 TPBVP

1.3 Development in motorcycle dynamic

1.3.1 History reference

1.3.2 QSS problems

Previous work

Chapter 2

Motorcycle dynamic model with Pacejka's tyre model

2.1 Motorcycle model with fixed suspension controlling slip

There are multiple way to derive describe the behaviour of the motorcycle and to derive the equation of motion. Some use the lagrangian approach to use a minimum set of coordinates**pacejka2006tyresharp2004advancesleonelli2019optimal**. Other derive the equation of motion using Newton-Euler equations.
In this thesis the dynamic model of the motorcycle has been derived using a multi-body approach and assuming the ISO convention for the orientation of the z-axis (upward). To this end multiple reference frames, points, bodies, forces and torques are defined in the following sections. The model was derived symbolically using Maple and MBSymba that deals with rotation and transformation matrices using homogeneous coordinates. In particular, working on the model, the author chose to derive the kinematics using a combination of global and recursive approach in order to use a minimum set of coordinates while containing the size of the derived equations.

2.2 Reference frames

A common choice is to start by defining a reference frame in movement with respect to the ground, or fixed, one. This reference frame have in general three linear and three angular velocities. However, for the purpose of this thesis, we consider only planar roads. This means that we need to define the movement of a frame in a plane, therefore only three degrees of freedom are needed (three velocities). The moving reference frame ia addressed as RF_1 and has velocities $u(t)$, $v(t)$ and $\Omega(t)$. This set of velocities, also called

quasi-coordinates, are suitable to be used later in the definition of curvilinear coordinate. RF_1 has the x-axis aligned with the direction of motion of the motorcycle.
The frame RF_ϕ is the reference frame attached to a plane rotated of an angle $\phi(t)$ commonly addressed as rolling angle around the moving x-axis and it is obtained recursively multiplying RF_1 for a rotation matrix.

$$RF_\phi = RF_1 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\left(\phi\left(t\right)\right) & -\sin\left(\phi\left(t\right)\right) & 0 \\ 0 & \sin\left(\phi\left(t\right)\right) & \cos\left(\phi\left(t\right)\right) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.1}$$

Then a reference frame attached to the joint between the swingarm and the rear frame is defined with a translation in the vertical direction of the rolled frame of a certain height $h(t)$ and a rotation around the rolled y-axis of an angle $\theta(t)$ plus the caster angle ϵ. The new frame will be from here addressed as RF_{Rear}

$$RF_{Rear} = RF_\phi \begin{bmatrix} \cos\left(\theta\left(t\right)+\epsilon\right) & 0 & -\sin\left(\theta\left(t\right)+\epsilon\right) & 0 \\ 0 & 1 & 0 & 0 \\ \sin\left(\theta\left(t\right)+\epsilon\right) & 0 & \cos\left(\theta\left(t\right)+\epsilon\right) & h\left(t\right) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.2}$$

From RF_{Rear} frame one can define the reference frame attached to the swingarm and the one attached to the steering assembly. The first is obtained with a rotation around the y-direction of the previous reference frame of a relative angle $eta(t)$ and a translation of the length of the swingarm. The advantage of choosing this relative angle is that there are already define relationship between this rotation an the force of the suspension. The new reference frame is addressed as RF_η and will coincide with the centre of the rear wheel.

$$RF_\eta = RF_{Rear} \begin{bmatrix} \cos\left(\eta\left(t\right)\right) & 0 & \sin\left(\eta\left(t\right)\right) & -\cos\left(\eta\left(t\right)\right)L_{swa} \\ 0 & 1 & 0 & 0 \\ -\sin\left(\eta\left(t\right)\right) & 0 & \cos\left(\eta\left(t\right)\right) & \sin\left(\eta\left(t\right)\right)L_{swa} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.3}$$

The second reference frame, the one attached to the steering assembly has already the z-axis with the same direction of the rotation axis of the steer. Therefore, RF_δ can be obtained with a translation of a fixed quantity in the x direction and a rotation of an angle $\delta(t)$ around the rotation axis z. This steering angle is small for racing motorcycle and it is always smaller than 10 degrees therefore can be linearised from here since the equation of motion are derived using Newton-Euler approach instead of Lagrange.

$$RF_{\delta} = RF_{Rear} \begin{bmatrix} 1 & -\delta(t) & 0 & L_b \\ \delta(t) & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.4}$$

The reference frame attached to the centre of the front wheel is defined as RF_{susp} and it is obtained with a translation in the negative vertical direction of a fixed quantity s_{fs} plus a time-varying $s_f(t)$ which is the deformation of the suspension and a translation in the x direction of x_{off}, an offset always present in the suspension fork.

$$RF_{susp} = RF_{\delta} \begin{bmatrix} 1 & 0 & 0 & x_{\mathit{off}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -s_{\mathit{fs}} + s_f(t) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.5}$$

In order to have a simplified model one can introduce two new reference frames one for the front and one for the rear wheel starting from RF_1. RF_{FW} is defined with a translation of the components $x_f(t)$, $y_f(t)$ and $z_f(t)$ and a rotation of an angle $\delta_f(t)$ around the vertical direction and then one around the new longitudinal direction of angle $\phi_f(t)$.

2.3 Motorcycle model with fixed suspension controlling torque

2.4 Motorcycle model with free suspension controlling torque

Chapter 3

Chapters/Analysis of results

Conclusions

Appendix