UNIVERSITY OF TRENTO

Department of Industrial Engineering

Master Degree in Mechatronics Engineering

Accuracy influence in racing motorcycle dynamic models for minimum lap time solution

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Introduction

The simulation of mechanical and mechatronical systems allow to test and validate the design in a safe and efficient environment withaut the need to build the physical object and measure the parameters. The simulation is based on digital tecnology with major benefis as cost and efficiency and the possibility of an easy reconfiguration and retesting of a system which is usally impossible or unfeasible for real model in terms of cost and time.[1]
A large number of vehicle model are avaiable in scientific litterature some are fairly complex and other are simple. Depending on the application and the time constraints a proper model shoul be chosen. Simple model are faster and therefore suitable for real time purposes while complex model are time consumin and are used in case where the model cannot be simplified or the goal of the study is to replicate in detail the behaviour of the analyzed system.
The work of this thesis aims to derive three different models of racing motorcycle with increasing complexity taking into account the force exchange between tyre and ground using Pacejka's magic formula [2]. The models will then be used to calculate controls and trajectory to achieve the minimum lap time of a specific circuit.
In particular the first model of the moto will rapresent a vehicle with fixed suspensions meaning that the rear swingarm and the steering fork have respectively a fixed angle and a fixed length. The motorcycle is controlled whith the steering torque and the longitudinal slip of front and rear wheel.
The second model has again fixed suspensions, however the vehicle is controlled with steering torue, braking torque at the front wheel and braking/traction torque at the rear.
The third model take into account the internal motion due to suspension deformation and the motorcycle is again controlled whith torques.
All the model are derived with the multibody approach and symbolic formulation. In order to achieve this goal the model is defined using Maple, a software well known for its capability in symbolic computation. Moreover, the equation of motion are obtained using MBSymba which is a custom free library for Maple avaiable online at http://www.multibody.net.
All three model are derived in a similar way as in the pubblication of Cossalter *et al*[3], without taking into account the lateral flexibility of the

front torque and the torsional flexibility of the swingarm. The tyre forces are derived using the Pacejka's magic formula [2] which is an empirical formula obtained from the assumption of similarity.
The minimum time trajectory and control is computed formulating a custom optimal control problem using XOptima package for Maple and then solving with PINS (acronym for *PINS Is Not a Solver*). The first is a library developed to transform the symbolic model of the vehicle (DAE), constraints, and target functions in C++ code that can be used by PINS. PINS is a software, free for academic purposes, developed at University of Trento by Prof. Bertolazzi, Prof. Biral and Prof. Bosetti that can solve optimal control problems (OCPs) with indirect method. As far as the author know, there are not other optimal control solvers that exploit Pontryagin maximum principle and calculus of variations to solve the problem with indirect method. [4]
The thesis is organized in the following way. In the first chapter there is a brief overview of the state of the art in optimal control, motorcycle dynamic model and minimum time application. In the second chapter there is a description of how the motorcycle models are derived and the optimal control problem. The third chapter confront the results of the three models. The thesis is ended with the conclusion, references and apendix.

Chapter 1

State of art on minimum lap time application in racing motorcycle

1.1 State of the art on Optimal Control Problems

Optimal control problem, also known as dynamic optimisation, are minimisation problem where the variables and parameters change with time. Dynamic systems are characterized by the states and often are controlled by a convenient choice of inputs (controls).
Dynamic optimisation aims to compute those controls and states for a dynamic system over a time interval to minimise one or more performance indexes. In other words, the input is chosen to optimize (minimize) an objective function while complying to constraint equations.

1.1.1 Optimal Control Problems

Optimal control problems are challenging from the theoretical point of view and of practical interest. However due to dimensionality and complexity of system of equations the application in real problems and industrial environment is still not so widespread.
In general, OPC can be continuous or discrete, linear or non-linear, time-variant or time-invariant. However, in this thesis are addressed only optimal control problems that are continuous time-variant and highly non-linear. Those properties will be discussed in the following sections. In general, there are four main approaches to solve continuous-time OPC: state space approach, direct methods, indirect methods and differential dynamic programming.

State-space approaches

State-space approaches follow the principle of optimality for which each subarc of an optimal trajectory must be optimal. In literature, those are referred to as Hamilton-Jacobi-Bellman (HJB) equation. However, the problem needs numerical methods to be solved, moreover, a solution can be found only for small dimension problems due to *course of dimensionality*. There is no practical application of this method to solve highly non-linear problem as a dynamic optimisation of a motorcycle model.

Direct Method

Direct methods discretize the original optimal control problem into a nonlinear programming problem (NLP). In other words, the OPC is transformed in a discrete-time system that can be solved using numerical schemes and optimization techniques, namely Initial Value Solver (IVS) and Sequential Quadratic Programming (SQP) [5] The main advantage of direct methods is the possibility to use inequality constraints even in case of change in the constraints active set (activation/deactivation)[6]

Direct methods are easier to implement compared to the other three categories and this is one of the reasons why they are by far the most widespread. In fact, almost 90% of the avaiable optimal control software rely on direct method. [7][8]

Indirect Method

Indirect methods exploits the necessary condition of optimality to derive a boundary value problem (BVP) in ordinary differential equations(ODE). Therefore the BVP can be solved numerically as a non linear problem. The indirect method allow to first optimize and then discretize meaning that the problem con be firstly written in continuous time and discretized later using different discretization techniques. The class of indirect methods exploits the well known calculus of variations and the Euler-Lagrange differential equations, and the so-called Pontryagin Maximum Principle.[4]

The numerical solution can be computed either by shooting techniques (single/multiple shooting) or by collocation. The major drawbacks of indirect methods are that the problem could be difficult to solve or unstable due to the nature of the underlying differential equations (nonlinearity and instability) and the changes in control structure (active constraints in specific arcs). Moreover, in some arcs, singularity arises therefore the DAE index increase leading to the necessity of specialized solution techniques. [6]

Differential Dynamic Programming

1.1.2 Minimum time Optimal Control Problem

1.1.3 PINS

1.2 Theory of OCP

1.2.1 Pontryagin

1.2.2 TPBVP

1.3 Development in motorcycle dynamic

1.3.1 History reference

1.3.2 QSS problems

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2.1 Motorcycle model with fixed suspension controlling slip

2.2 Motorcycle model with fixed suspension controlling torque

2.3 Motorcycle model with free suspension controlling torque

Chapter 3

Analysis of results

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Appendix