



UNIVERSITY OF TRENTO

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Department of Industrial Engineering

Master Degree in Mechatronics Engineering

Accuracy influence in racing motorcycle dynamic  
models for minimum lap time solution

Supervisor:  
Prof. Francesco Biral.

Graduant:  
Mattia Piazza

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Thanks to me

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# Introduction

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# Chapter 1

## State of art on minimum lap time application in racing motorcycle

### 1.1 State of the art on Optimal Control Problems

Optimal control problem, also known as dynamic optimisation, are minimisation problem where the variables and parameters change with time. Dynamic systems are characterized by the states and often are controlled by a convenient choice of inputs (controls).

Dynamic optimisation aims to compute those controls and states for a dynamic system over a time interval to minimise one or more performance indexes. In other words, the input is chosen to optimize (minimize) an objective function while complying to constraint equations.

#### 1.1.1 Optimal Control Problems

Optimal control problems are challenging from the theoretical point of view and of practical interest. However due to dimensionality and complexity of system of equations the application in real problems and industrial environment is still not so widespread.

In general, OPC can be continuous or discrete, linear or non-linear, time-variant or time-invariant. However, in this thesis are addressed only optimal control problems that are continuous time-variant and highly non-linear. Those properties will be discussed in the following sections. [Ch.1.3]

In general, there are four main approaches to solve continuous-time OPC: state space approach, direct methods, indirect methods and differential dynamic programming.

## State-space approaches

State-space approaches follow the principle of optimality for which each subarc of an optimal trajectory must be optimal. In literature, those are referred to as Hamilton-Jacobi-Bellman (HJB) equation. However, the problem needs numerical methods to be solved, moreover, a solution can be found only for small dimension problems due to *course of dimensionality*. There is no practical application of this method to solve highly non-linear problem as a dynamic optimisation of a motorcycle model.

## Direct Method

Direct methods discretize the original optimal control problem into a nonlinear programming problem (NLP). In other words, the OPC is transformed in a discrete-time system that can be solved using numerical schemes and optimization techniques, namely Initial Value Solver (IVS) and Sequential Quadratic Programming (SQP) [1] The main advantage of direct methods is the possibility to use inequality constraints even in case of change in the constraints active set (activation/deactivation)[3]

Direct methods are easier to implement compared to the other three categories and this is one of the reasons why they are by far the most widespread. In fact, almost 90% of the available optimal control software rely on direct method. [9][10]

## Indirect Method

### Differential Dynamic Programming

#### 1.1.2 Minimum time Optimal Control Problem

#### 1.1.3 PINS

### 1.2 Theory of OCP

#### 1.2.1 Pontryagin

#### 1.2.2 TPBVP

### 1.3 Development in motorcycle dynamic

#### 1.3.1 History reference

#### 1.3.2 QSS problems

## Previous work



## Chapter 2

# Development of three motorcycle dynamic model using Pacejka tyre model

- 2.1 Motorcycle model with fixed suspension controlling slip
- 2.2 Motorcycle model with fixed suspension controlling torque
- 2.3 Motorcycle model with free suspension controlling torque

## Chapter 3

# Analysis of results

# Conclusions

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# Appendix