

Assignment 3: Functional Dependencies and Normalization

Task 1

FD1: $\{A\} \rightarrow \{B,C\}$ FD2: $\{C\} \rightarrow \{A,D\}$ FD3: $\{D,E\} \rightarrow \{F\}$

a) $\{C\} \rightarrow \{B\}$

1. FD4a: $C \rightarrow A$ (Decomposition with FD2)
2. FD5a: $C \rightarrow BC$ (Transitivity with FD1)
3. FD6a: $C \rightarrow B$ (Decomposition with FD5a)

b) $\{A,E\} \rightarrow \{F\}$

1. FD4b: $A \rightarrow C$ (Decomposition with FD1)
2. FD5b: $A \rightarrow AD$ (Transitivity with FD2)
3. FD6b: $A \rightarrow D$ (Decomposition with FD5b)
4. FD7b: $AE \rightarrow F$ (Pseudo-transitivity with FD6b and FD3)

Task 2

$R(A, B, C, D, E, F)$ and FD1: $\{A\} \rightarrow \{B,C\}$ FD2: $\{C\} \rightarrow \{A,D\}$ FD3: $\{D,E\} \rightarrow \{F\}$

a) $X = \{A\}$

1. FD4: $A \rightarrow B$ (Decomposition with FD1)
2. FD5: $A \rightarrow C$ (Decomposition with FD1)
3. FD6: $A \rightarrow D$ (Transitivity FD2, FD5 and decomposition)

Therefore $X^+ = \{A, B, C, D\}$

b) $X = \{C, E\}$

1. FD4: $CE \rightarrow ADE$ (Augmentation with FD2 and E)
2. FD5: $CE \rightarrow DE$ (Decomposition)
3. FD6: $CE \rightarrow F$ (Transitivity between FD5 and FD3)
4. FD7: $CE \rightarrow BC$ (Decomposition FD4, Transitivity FD1, FD4)

Therefore, $X^+ = \{A, B, C, D, E, F\}$

Task 3

$R(A, B, C, D, E, F)$, FD1: $\{A,B\} \rightarrow \{C,D,E,F\}$ FD2: $\{E\} \rightarrow \{F\}$ FD3: $\{D\} \rightarrow \{B\}$

a) **Determine the candidate key(s) for R.**

$X=\{A,B\}$ is a superkey and a candidate key since $X^+=\{A,B,C,D,E,F\}$ and no subset of X achieves that.

$X = \{A, D\}$ is also a superkey and a candidate key since $X^+=\{A,B,C,D,E,F\}$ and no subset of X achieves that. FD4: $\{AD\} \rightarrow \{BCEF\}$

- Assume we have $X = \{AD\}$ and X^+ initially is $\{AD\}$, then: $\{D\} \rightarrow \{B\}$ (FD3), now $X^+=\{A,B,D\}$, and $\{AB\} \rightarrow \{CDEF\}$ (FD1) now $X^+=\{A,B,C,D,E,F\}$.

b) Note that R is not in BCNF. Which FD(s) violate the BCNF condition?

FD2 and FD3 are not superkeys and may therefore leave room for redundancy, hence R is not in BCNF.

c) Decompose R into a set of BCNF relations, and describe the process step by step (don't forget to determine the FDs and the candidate key(s) for all of the relation schemas along the way)

1. From FD2 we create two new relations R1 and R2 from R:
 - R1(E,F)
 - R2(A,B,C,D,E)
2. Now we have that FD2: $\{E\} \rightarrow \{F\}$ for R1, so $\{E\}$ is a candidate key. We also have that FD5: $\{AB\} \rightarrow \{CDE\}$, and FD4: $\{AD\} \rightarrow \{BCE\}$ hence $\{AB\}$, $\{AD\}$ are candidate keys for R2 and we also have FD3: $\{D\} \rightarrow \{B\}$ where $\{D\}$ is not a candidate key or a superkey. Now we create R3, R4 from FD3 and R2.
 - R3(D,B)
 - R4(A,C,D,E)
3. Now we have that FD2: $\{E\} \rightarrow \{F\}$ where $\{E\}$ is a candidate key for R1, FD3: $\{D\} \rightarrow \{B\}$ where $\{D\}$ is a candidate key for R3 and FD6: $\{AD\} \rightarrow \{CE\}$ where $\{AD\}$ is a candidate key for R4.
4. Now we have:
 - a. R1(E,F),
 - b. R3(D,B),
 - c. R4(A,C,D,E)

Task 4

$R(A, B, C, D, E)$, FD1: $\{A,B,C\} \rightarrow \{D,E\}$ FD2: $\{B,C,D\} \rightarrow \{A,E\}$ FD3: $\{C\} \rightarrow \{D\}$

a) Show that R is not in BCNF

We can see that both FD1 and FD2 are superkeys since the closure is $X^+=\{A,B,C,D,E\}$ for both $X=\{A,B,C\}$ and $X=\{B,C,D\}$.

We look at a subset of FD1, let $X = \{B,C\}$ where $X^+=\{B,C\}$. From FD3 we get $X^+=\{B,C,D\}$, from FD2 we get $X^+=\{A,B,C,D,E\}$ where $\{B,C\}$ is a candidate key.

So, $\{B,C\}$ is a candidate key, $\{A,B,C\}$, $\{B,C,D\}$ are superkeys and $\{C\}$ is neither.

With the Armstrong rules we can then derive a new functional dependency ($\{B,C\} \rightarrow \{A,D,E\}$)

- FD5: $BC \rightarrow BD$ (FD3 and augmentation with B)
- FD6: $BC \rightarrow BCD$ (FD5 and augmentation with C ($C \cup C=C$))

- FD7: $BC \rightarrow AE$ (transitivity with FD6 & FD2)
- FD8: $BC \rightarrow D$ (FD5 decomposition)
- **FD9: $BC \rightarrow AED$ (FD7,FD8 union)**

FD3 violates the BCNF condition for R since $\{C\}$ is not a superkey.

b) Decompose R into a set of BCNF relations (describe the process step by step)

We now have that $\{B,C\}$ is a candidate key, we can also see that $\{A,B,C\}$ and $\{B,C,D\}$ are superkeys for R. We note that FD3 violates the BCNF condition, therefore we decompose R over FD3 into R1 and R2.

R1(C,D)

R2(A,B,C,E)

We now have FD3: $\{C\} \rightarrow \{D\}$ for R1 where $\{C\}$ is a candidate key for R1 and FD7: $\{BC\} \rightarrow \{AE\}$ for R2 where $\{BC\}$ is a candidate key for R2. R1 and R2 are now in BCNF.