project for MATH-403

December 2020

1 Task 4

From step 5 of the algorithm, we know that

$$\tilde{\mathcal{C}} = \tilde{U}_1^T \circ_1 \tilde{U}_2^T \circ_2 \tilde{U}_3^T \circ_3 \mathcal{X} \tag{1}$$

From step 6 of the algorithm, we know that

$$\tilde{\mathcal{C}} = V_1 \circ_1 V_2 \circ_2 V_3 \circ_3 \mathcal{C} \tag{2}$$

Thus we have

$$V_1 \circ_1 V_2 \circ_2 V_3 \circ_3 \mathcal{C} = \tilde{U}_1^T \circ_1 \tilde{U}_2^T \circ_2 \tilde{U}_3^T \circ_3 \mathcal{X} \tag{3}$$

Using the result from Exercise 8, problem 5, we get

$$\begin{aligned}
\mathcal{C} &= V_1^T \tilde{U}_1^T \circ_1 V_2^T \tilde{U}_2^T \circ_2 V_3^T \tilde{U}_3^T \circ_3 \mathcal{X} \\
&= U_1^T \circ_1 U_2^T \circ_2 U_3^T \circ_3 \mathcal{X}
\end{aligned} \tag{4}$$

Orthogonal projection $\tilde{\mathcal{X}} := (\pi_1 \circ \pi_2 \circ \pi_3) \mathcal{X}$ with $\pi_\mu \mathcal{X} := U_\mu U_\mu^T \circ_\mu \mathcal{X}$ \qquad (5)

$$\|\mathcal{X} - U_1 \circ_1 U_2 \circ_3 U_3 \circ_3 \mathcal{C}\|_F \tag{6}$$

$$\begin{aligned}
\|\mathcal{X} - \tilde{\mathcal{X}}\|^2 &= \|\mathcal{X} - (\pi_1 \circ \pi_2 \circ \pi_3) \mathcal{X}\|^2 \\
&= \|\mathcal{X} - \pi_1 \mathcal{X}\|^2 + \|\pi_1 \mathcal{X} - (\pi_1 \circ \pi_2) \mathcal{X}\|^2 + \cdots \\
&\quad \cdots + \|(\pi_1 \circ \pi_2) \mathcal{X} - (\pi_1 \circ \pi_2 \circ \pi_3) \mathcal{X}\|^2 \\
&\leq \left\|(I - U_1 U_1^T) X^{(1)}\right\|_F^2 + \left\|(I - U_2 U_2^T) X^{(2)}\right\|_F^2 + \left\|(I - U_3 U_3^T) X^{(3)}\right\|_F^2 \leq 3\epsilon^2
\end{aligned} \tag{7}$$