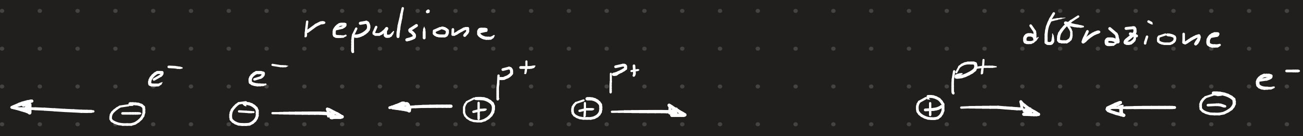
Elettrostatica: fenomeno in cui non c'è movimento



I fenomeni elettrostatici sono descrivibili come iterazioni tra protoni ed elettroni.

Legge di Coulomb

$$\vec{F}_{e_{q_2}} = \boxed{\frac{1}{4\pi\varepsilon_0}} \frac{q_1 \cdot q_2}{r^2} \vec{u}$$

$$\frac{1}{4\pi\varepsilon_0} = 9 \cdot 10^9 \frac{N \cdot m^2}{C^2}$$

Se q_1 e q_2 hanno lo stesso segno $\vec{F}$ e $\vec{u}$ sono concordi, se invece hanno segno opposto $\vec{F}$ e $\vec{u}$ sono di verso opposto.

La legge di Coulomb, in formula, è simile alla formula di attrazione gravitazionale, con la differenza che la seconda è solo attrattiva.

Valore carica di un elettrone/protone:

$$|e^-| = |p^+| \cong 1{,}6 \cdot 10^{-19}\ C$$

Esercizio

Si consideri un atomo di idrogeno semplice, calcolare la forza elettrostatica e la forza gravitazionale.

$r = 0{,}53 \cdot 10^{-10}\ m$

$m_e = 9{,}109 \cdot 10^{-31}\ Kg$

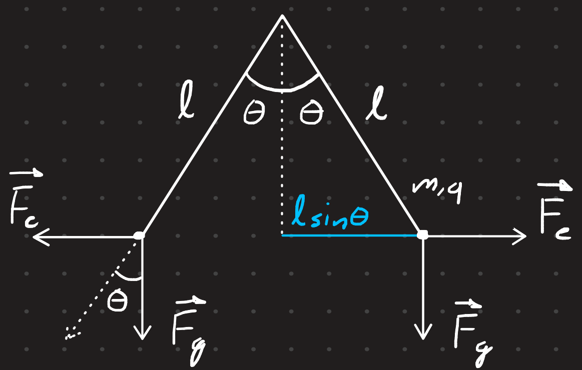
$m_p = 1{,}67 \cdot 10^{-27}\ Kg$

$\gamma = 6{,}67 \cdot 10^{-11}\ \frac{N \cdot m^2}{Kg}$

$$\frac{F_e}{F_g} = \frac{\frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{\cancel{r^2}}}{\gamma \cdot \frac{m_e \cdot m_p}{\cancel{r^2}}} \simeq 2{,}3 \cdot 10^{39}$$

Forza elettrostatica più forte della forza gravitazionale.

Esercizio



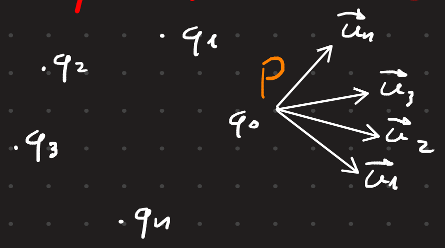
$q(\theta) = ?$

$$\tan\theta = \frac{F_e}{F_g} = \frac{\frac{1}{4\pi\varepsilon_0} \frac{q^2}{(2l\sin\theta)^2}}{m \cdot g}$$

$$q = 2l\sin\theta \sqrt{4\pi\varepsilon_0 \cdot mg \cdot tg\theta} \simeq \theta^{3/2}$$

per angoli piccoli

Campo elettrostatico



$$\vec{F}_{q_0} = \sum_{K=1}^{N} \vec{F}_K = \sum_{K=1}^{N} \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_K \cdot q_0}{r_K^2} \vec{u}_K$$

$$= q_0 \cdot \underbrace{\frac{1}{4\pi\varepsilon_0} \sum_{K=1}^{N} \frac{q_K}{r_K^2} \vec{u}_K}_{\vec{E}_P}$$

$$\vec{E}_{q_0} = \lim_{q \to 0} \frac{\vec{F}_{q_0}}{q_0}$$

Se ho solo una carica

$$\vec{E}_P = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \vec{u}$$

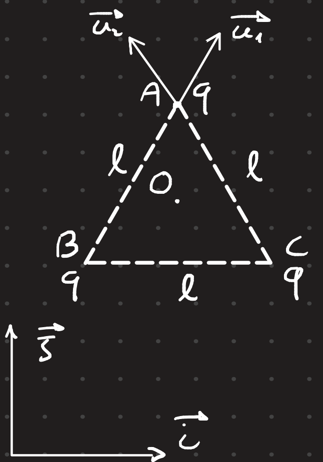
Il segno della carica mi dà il verso del campo

$$[E] = N/C = V/m$$

→ volt su metro

Il campo elettrostatico è definito in tutti i punti dello spazio, tranne nelle cariche che lo generano dove vale infinito.

Esercizio



Calcolare il campo elettrico e forza in A, e campo elettrico nel punto O.

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{\ell^2} \vec{u}_1 \qquad \vec{u}_1 = \frac{1}{2}\vec{i} + \frac{\sqrt{3}}{2}\vec{j}$$

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{\ell^2} \vec{u}_2 \qquad \vec{u}_2 = -\frac{1}{2}\vec{i} + \frac{\sqrt{3}}{2}\vec{j}$$

$$\vec{E}_A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{\ell^2}(\vec{u}_1 + \vec{u}_2) = \frac{1}{4\pi\varepsilon_0} \frac{q}{\ell^2} \sqrt{3}\ \vec{j}$$

↓ componente verticale

$$\vec{F}_A = q\,\vec{E}_A$$

Nel punto O il campo è nullo per la simmetria del problema.

$$\vec{E}_{0,1} = \frac{1}{4\pi\varepsilon_0} \frac{q}{\ell^2/\sqrt{3}} \left(\frac{\sqrt{3}}{2} i + \frac{1}{2} j\right)$$

$$\vec{E}_{0,3} = \frac{1}{4\pi\varepsilon_0} \frac{q}{\ell^2/\sqrt{3}} \left(-\frac{\sqrt{3}}{2} i + \frac{1}{2} j\right)$$

$$\vec{E}_{0,2} = \frac{1}{4\pi\varepsilon_0} \frac{q}{\ell^2/\sqrt{3}} (-j)$$

$$\vec{E}_0 = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = 0$$

$.P$

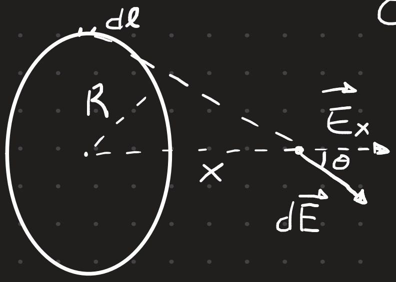
Oggetto caricato elettrostaticamente.
Quanto vale il campo elettrostatico generato dall'oggetto?

$$\rho \equiv \frac{dq}{d\tau} \quad \text{densità di carica}$$

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\vec{u} = \frac{1}{4\pi\varepsilon_0}\frac{\rho\, d\tau}{r^2}\vec{u}$$

$$\boxed{\vec{E}_P = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho\, d\tau}{r^2}\vec{u}}$$

Esercizio anello



Q con densità uniforme

$$\boxed{\lambda = \frac{dq}{dl}} = \frac{Q}{2\pi R} \qquad Q = \lambda \cdot 2\pi R$$

densità di carica lineare

$$dE_x = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\lambda\, dl}{x^2+R^2}\cdot\underbrace{\frac{x}{\sqrt{x^2+R^2}}}_{\cos\theta}$$

$$E_P = \int dE_x = \frac{1}{4\pi\varepsilon_0}\frac{\lambda x}{(x^2+R^2)^{3/2}}\int dl = 2\pi R$$

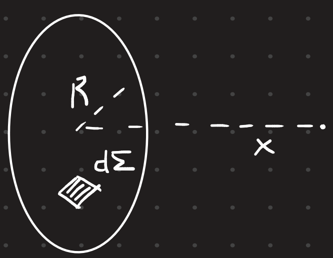
$$\vec{E}_P = \frac{1}{4\pi\varepsilon_0}\frac{Q\,x}{(x^2+R^2)^{3/2}}\vec{i} \underset{x>>R}{\simeq} \boxed{\frac{1}{4\pi\varepsilon_0}\frac{Q}{x^2}} \qquad \frac{x}{|x|}$$

questo termine mi dà solo il verso

legge di Coulomb

$$\vec{E}_{anello} = \frac{1}{4\pi\varepsilon_0}\frac{Q\,x}{(x^2+R^2)^{3/2}}\vec{i}$$

Esercizio disco

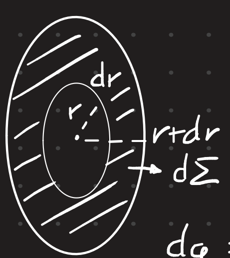


$$\boxed{\sigma = \frac{dq}{d\Sigma}} = \frac{Q}{\pi R^2}$$

densità superficiale

$$\vec{E}_P = \frac{1}{4\pi\varepsilon_0}\cdot\int_\Sigma \frac{\sigma}{r^2}\vec{u}\, d\Sigma$$

Divido la mia superficie in anelli infinitesimi.



$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq\cdot x}{(x^2+r^2)^{3/2}} = \frac{1}{4\pi\varepsilon_0}\frac{\sigma 2\pi r x\, dr}{(x^2+r^2)^{3/2}}$$

$$d\Sigma = \pi(r+dr)^2 - \pi r^2 = 2r\pi\, dr$$

$$dq = \sigma\cdot d\Sigma$$

$$E = \frac{\sigma \cancel{\pi} x}{4\cancel{\pi}\varepsilon_0} \int_0^R \frac{2r\,dr}{(x^2+r^2)^{3/2}} = \frac{\sigma x}{4\varepsilon_0}\left[\frac{-2}{\sqrt{x^2+r^2}}\right]_0^R$$

$$= \frac{\sigma x}{2\varepsilon_0}\left(\frac{1}{|x|} - \frac{1}{\sqrt{x^2+R^2}}\right)$$

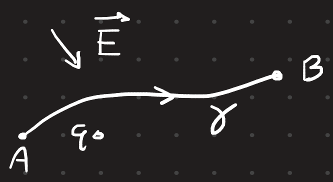
Cosa succede se il piano è infinito?

$$E = \frac{\sigma x}{2\varepsilon_0}\left(\frac{1}{|x|} - \frac{1}{\sqrt{x^2+R^2}}\right) \underset{R>>0}{\simeq} \frac{\sigma}{2\varepsilon_0}\cdot\frac{x}{|x|} \longrightarrow \text{campo elettrostatico costante}$$

Linee di campo

→ le linee non si incontrano mai, eccetto nelle cariche

- Nella zona in cui le linee sono più dense il campo è più forte



$$L_{AB} = \int_\gamma \overbrace{\vec F_{q_0}}^{q_0\cdot\vec E}\cdot d\vec s = q_0 \int_A^B \underbrace{\vec E\, d\vec s}_{\text{tensione campo elettrico}}$$

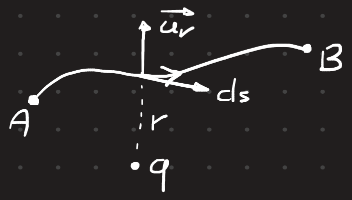
Un campo è conservativo quando l'integrale non dipende dal percorso, ma dal punto iniziale a quello finale.

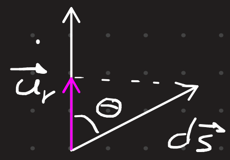
$$L_{AB} = -\Delta U = U_A - U_B = q_0\,\underbrace{(V_A - V_B)}_{\text{differenza di potenziale}}$$

$$V_A - V_B = \int_A^B \vec E\cdot d\vec s \iff \boxed{V_B - V_A = -\int_A^B \vec E\cdot d\vec s}$$

$$C_\gamma(\vec E) = \oint_\gamma \vec E\cdot d\vec s = 0$$ (percorso chiuso)

Dimostriamo che il campo elettrostatico è conservativo:





$d\vec{s}$ è lo spostamento infinitesimo

$$\vec{u}_r \cdot d\vec{s} = ds \cdot \cos\theta = dr$$

$$\int_A^B \vec{E} \cdot d\vec{s} = \int_A^B \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \vec{u}_r \cdot d\vec{s} = \int_A^B \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} dr =$$

$$= \left[\frac{1}{4\pi\varepsilon_0} \frac{-q}{r} \right]_A^B = \frac{1}{4\pi\varepsilon_0} \frac{q}{r_A} - \frac{1}{4\pi\varepsilon_0} \frac{q}{r_B} = V_A - V_B$$

Il campo Coulombiano è un campo conservativo: non dipende dal percorso, ma solo del punto finale ed iniziale.

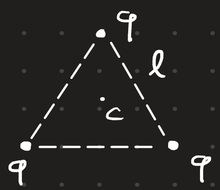
$$\boxed{V(r) = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}} + K$$

formula del potenziale Coulombiano

$$[V] = \frac{1\,J}{1\,C} = 1 \text{ Volt}$$

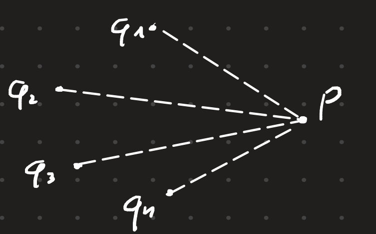
$V(\infty) = 0$ (è una scelta per eliminare K)

Esercizio



$$\overline{qC} = r = l\frac{\sqrt{3}}{3}$$

$$V = \left(\frac{1}{4\pi\varepsilon_0} \frac{q}{l\frac{\sqrt{3}}{3}} \right) \cdot 3$$



$$V_P = \sum \frac{1}{4\pi\varepsilon_0} \frac{q_k}{r_k}$$

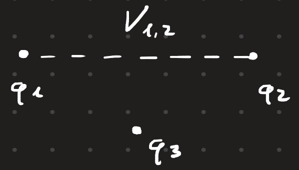
densità di carica

$$V_P = \int d\tau \, \frac{1}{4\pi\varepsilon_0} \frac{\rho}{r}$$

Energia potenziale di un sistema di cariche

$$L_{ext} = -\int_{\infty}^{V_{1,2}} \vec{F} \cdot d\vec{s} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 \cdot q_2}{r_{1,2}}$$

il lavoro è opposto al campo elettrico



$$U_{TOT} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 \cdot q_2}{r_{1,2}} + \frac{q_1 \cdot q_3}{r_{1,3}} + \frac{q_2 \cdot q_3}{r_{2,3}} \right)$$

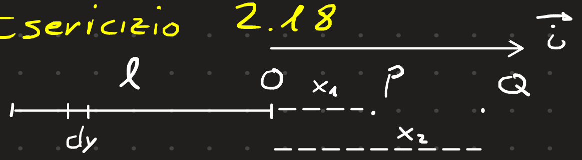
$$= \frac{1}{4\pi\varepsilon_0} \sum_{i,j} \frac{1}{2} \frac{q_i \cdot q_j}{r_{ij}}$$

energia spesa per arrivare ad una determinata configurazione di cariche

$$L_{AB} = \int_A^B \vec{F}\, d\vec{s} = \frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2 = -q_0 (V_B - V_A)$$

$$\frac{1}{2} m v_B^2 + q V_B = \frac{1}{2} m v_A^2 + q V_A$$

Esercizio 2.18



$\ell = 20\,cm$
$q = 6 \cdot 10^{-8}\,C = 60\,nC$
$x_1 = 15\,cm$
$x_2 = 20\,cm$

1° modo

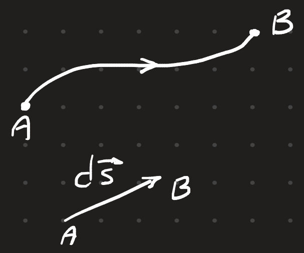
formula campo lineare

$$L_{ext} = \int_P^Q \left(-\vec{F}_e\right) d\vec{s} \qquad \vec{F}_e = q_0 \frac{1}{4\pi\varepsilon_0} \int_{-\ell}^{0} \frac{\lambda\, dy}{(x-y)^2}\, \vec{i}$$

2° Metodo

$$L_{ext} = +\Delta U = U_Q - U_P = q_0 (V_Q - V_P)$$

$$V(x) = \frac{1}{4\pi\varepsilon_0} \int_{-\ell}^{0} \frac{\lambda\, dy}{(x-y)} = \frac{\lambda}{4\pi\varepsilon_0} \Big[-\ln(x-y) \Big]_{-\ell}^{0} = \frac{\lambda}{4\pi\varepsilon_0} \ln\left(\frac{x+\ell}{x}\right)$$



$$\overbrace{V_B - V_A}^{dV} = -\int_A^B \vec{E}\cdot d\vec{s}$$

$$\vec{E} = E_x\vec{i} + E_y\vec{j} + E_z\vec{k}$$

$$d\vec{s} = dx\,\vec{i} + dy\,\vec{j} + dz\,\vec{k}$$

$$dV = -\vec{E}\cdot d\vec{s} =$$

$$= -\vec{E}_x d\vec{x} - \vec{E}_y d\vec{y} - \vec{E}_z d\vec{z}$$

$V_B - V_A$

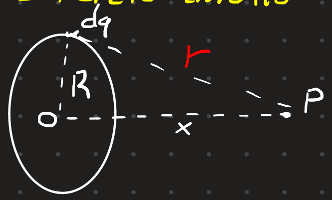
$$dV = V_{(\vec{x}+d\vec{s})} - V_{(\vec{x})} = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz \qquad \forall\, d\vec{s}$$

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_x = -\frac{\partial V}{\partial z}$$

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) = \text{NABLA}$$

$$\vec{\nabla}V = \left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}\right) = \text{gradiente di } V \implies \vec{E} = \vec{\nabla}V$$

Esercizio anello



$$V_p = ?$$

$$\lambda = \frac{dq}{d\ell} = \frac{Q}{2\pi R} = \text{cost}$$

$$V_P = \frac{1}{4\pi\varepsilon_0}\int \frac{dQ}{r} = \frac{1}{4\pi\varepsilon_0}\int\frac{\lambda\, d\ell}{\sqrt{x^2+R^2}} =$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{\sqrt{x^2+R^2}}\,2\pi R = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{x^2+R^2}}$$

$$E_z - E_y = 0$$

$$E_x = -\frac{\partial V}{\partial x} = \cancel{+}\frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{(x^2+R^2)^3}}\left(\cancel{-}\frac{1}{\cancel{2}}\right)\cancel{2}x = \frac{1}{4\pi\varepsilon_0}\frac{Q\cdot x}{(x^2+R^2)^{3/2}}$$

Esercizio disco

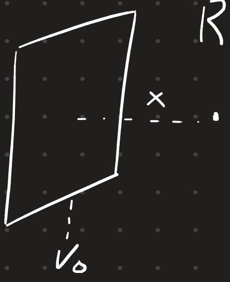
$$dq = \sigma\, d\Sigma = \sigma\, 2\pi r\, dr$$

$$dV_{(dq)} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\sqrt{x^2+r^2}}$$

$$V_P = \int\frac{1}{4\pi\varepsilon_0}\frac{dq}{\sqrt{x^2+r^2}} = \int_0^R \frac{1}{4\pi\varepsilon_0}\frac{\sigma 2\pi r\, dr}{\sqrt{x^2+r^2}} = \frac{\sigma}{4\varepsilon_0}\left[2\sqrt{x^2+r^2}\right]_0^R =$$

$$= \frac{\sigma}{2\varepsilon_0}\left[\sqrt{x^2+R^2} - |x|\right]$$

$$E_x = -\frac{\partial V}{\partial x} = \frac{\sigma}{2\varepsilon_0}\left[-\frac{x}{\sqrt{x^2+R^2}} + \frac{|x|}{x}\right]$$



$R >> x$ $\qquad \vec{E} = \frac{\sigma}{2\varepsilon_0}\vec{u} = \frac{\sigma}{2\varepsilon_0}\frac{|x|}{x}\vec{u} = \sigma\, 2\pi r\, dr$

$$V = V_0 \cdot - \frac{\sigma}{2\varepsilon_0}|x|$$

$$\vec{E} = -\frac{\partial V}{\partial x} = \frac{\sigma}{2\varepsilon_0}\frac{|x|}{x} = E_x$$

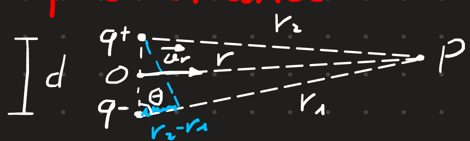
Superfici equipotenziali

$V(x,y,z) = V_0$

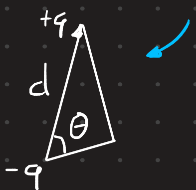
$$dV = 0 = -\vec{E}\cdot d\vec{s} \qquad \vec{E} \perp d\vec{s}$$

- Come le linee di campo anche queste non si intersecano mai, per ogni punto passa una sola superficie equipotenziale.

Dipolo elettrico



$$V_p = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{-q}{r_2} = \frac{q}{4\pi\varepsilon_0}\left(\frac{r_2 - r_1}{r_1\cdot r_2}\right)$$



Se $r >> d$ $\qquad r_1 \cdot r_2 \simeq r^2 \qquad r_2 - r_1 \simeq d\cos\theta$

$$V_p \simeq \frac{q}{4\pi\varepsilon_0}\frac{d\cos\theta}{r^2} = \boxed{\frac{\vec{p}\cdot\vec{u}_r}{4\pi\varepsilon_0\cdot r^2}}$$

$\vec{p} = q\vec{d} \equiv$ momento di dipolo

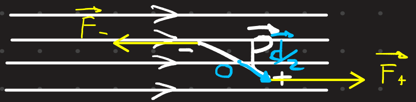
$\vec{p} = \sum_k \vec{r}_k q_k \qquad Q = \sum_k q_k$

$$V_p = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

$$E_r = -\frac{\partial V}{\partial r} = \frac{1}{4\pi\varepsilon_0}\frac{2p\cos\theta}{r^3}$$

$$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} = \frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3}$$

$$\boxed{\vec{E} = \frac{p}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\,\vec{u}_r + \sin\theta\,\vec{u}_\theta\right)}$$



$\vec{E}$ uniforme

$\vec{F} = q\vec{E}$ (il segno della forza dipende dalla carica + o -)

$$\vec{M} = \frac{\vec{d}}{2} \times \vec{F}_+ + \left(-\frac{\vec{d}}{2}\right) \times \vec{F}_- = \frac{\vec{d}}{2} \times q\vec{E} + \left(-\frac{\vec{d}}{2}\right) \times \left(-q\vec{E}\right) = q\vec{d} \times \vec{E}$$

momento di dipolo

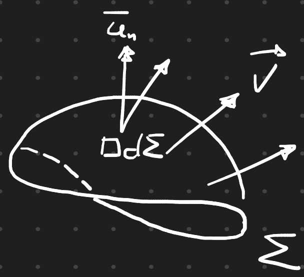
$$\boxed{\vec{M} = \vec{p} \times \vec{E}}$$

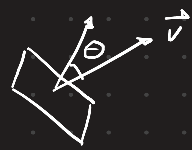
$$U = U_+ + U_- = qV_A - qV_B = q(V_A - V_B) \simeq -q\vec{E} \cdot \vec{d}$$

$$\boxed{U = -\vec{p} \cdot \vec{E}} \qquad U_{min} = -|p||E| \qquad U_{max} = +|p||E|$$

• Def. di flusso

$$\Phi_\Sigma(\vec{v}) = \int_\Sigma \vec{v} \cdot \vec{u}_n \, d\Sigma$$





$$\Phi_\Sigma(\vec{v}) = v \cdot \Sigma \cdot \cos\theta$$

Teorema di Gauss

$$\Phi_\Sigma(\vec{E}) = \oint_\Sigma \vec{E} \cdot \vec{u}_n \cdot d\Sigma = \sum \frac{q_{interne}}{\varepsilon_0}$$

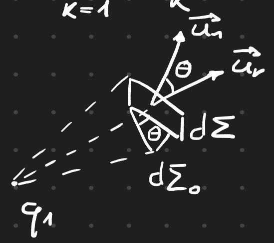
sommatoria

(campo * normale * superficie)

Σ è chiusa

Dimostrazione

$$\vec{E} = \sum_{k=1}^{N} \vec{E}_k \qquad \vec{E}_k \text{ è generato da } q_k$$



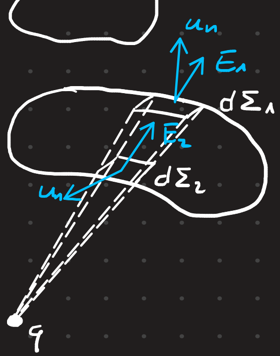
$$\vec{E}_1 \cdot u_n \, d\Sigma = \frac{1}{4\pi\varepsilon_0} \frac{q_1}{r^2} \overbrace{\vec{u}_r \cdot \vec{u}_n}^{\cos\theta} d\Sigma$$

$$= \frac{1}{4\pi\varepsilon_0} q_1 \frac{d\Sigma_0}{r^2} = \frac{1}{4\pi\varepsilon_0} q_1 \, d\Omega$$

angolo solido

$$d\theta = \frac{dl}{r}$$

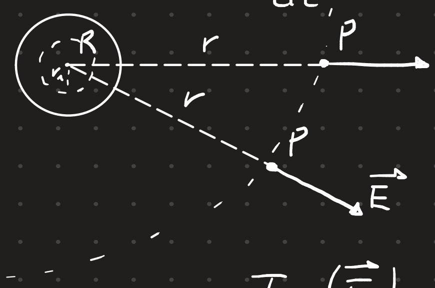
$$\oint_\Sigma \vec{E} \cdot \vec{u}_n \, d\Sigma = \frac{q_1}{4\pi\varepsilon_0} \oint_\Sigma d\Omega = \frac{q_1}{\cancel{4\pi}\varepsilon_0} \cancel{4\pi} = \frac{q_1}{\varepsilon_0}$$



$$\Phi(\vec{E}) = \Phi_{\Sigma_1}(\vec{E}) + \Phi_{\Sigma_2}(\vec{E}) = \frac{q_1}{4\pi\varepsilon_0}\Omega_1 - \frac{q_1}{4\pi\varepsilon_0}\Omega_1 = 0$$

Esercizio

$\cdot\ \rho = \frac{dq}{d\tau} = cost$ densità di carica costante nella sfera



$\vec{E} = E(r)\,\vec{u}_r$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho\, d\tau}{r^2}\,\vec{u}_r$$

$$\Phi_{S_r}(\vec{E}) = E(r)\,4\pi r^2 = \frac{\rho\frac{4}{3}\pi R^3}{\varepsilon_0} = \frac{q_{int}}{\varepsilon_0} \qquad r \geq R$$

$$Q = \rho\frac{4}{3}\pi$$

$$E(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \qquad \text{campo esterno}$$

$$\Phi_{S_r}(\vec{E}) = E(r)\,4\pi r^2 = \frac{\rho\frac{4}{3}\pi r_1^3}{\varepsilon_0} = \frac{Q r_1^3}{\varepsilon_0 R^3}$$

$$E(r) = \begin{cases} \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}r & r < R \\ \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} & r \geq R \end{cases}$$

$\rho = cost$

$$E(r) = \begin{cases} \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}r & r < R & \text{interno} \\ \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} & r \geq R & \text{esterno} \end{cases}$$

$$\boxed{V(r) = -\int E(r)\,dr}$$

$$V(r) = \begin{cases} -\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}\frac{r^2}{2} + C_1 & r < R & \text{interno} \\ \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} + C_2 & r \geq R & \text{esterno} \end{cases}$$

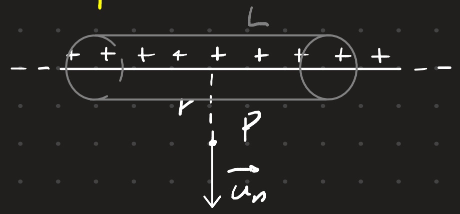
Come trovo C_1 e C_2?

$V(\infty) = 0 = C_2$

$\frac{Q}{R^3}\frac{R^2}{2}$

$$V(R_+) = V(R_-) \iff \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} = \frac{-1}{8\pi\varepsilon_0}\frac{Q}{R} + C_1 \implies C_1 = \frac{3}{8\pi\varepsilon_0}\frac{Q}{R}$$

Esempi di utilizzo del Th. di Gauss



$$\lambda = cost = \frac{dq}{d\ell}$$

$$dq = \lambda \, d\ell$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda \, d\ell}{r^2}\,\vec{u}_r$$

$$\phi = \oint \vec{E}\cdot\vec{u}_n \, d\Sigma$$

$$\vec{E} = E_{(r)}\,\vec{u}_r$$

$$\phi(\vec{E}) = E_{(r)}\,2\pi r \cancel{L} \overset{Gauss}{=} \frac{\lambda \cancel{L}}{\varepsilon_0}$$

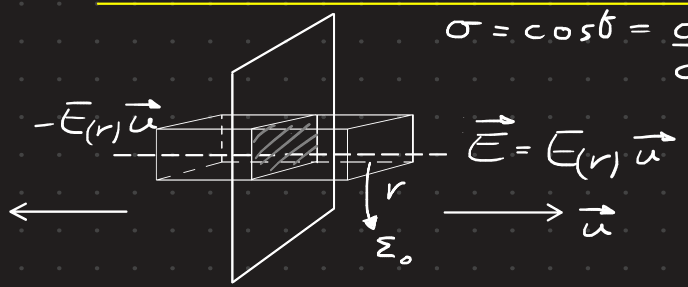
cariche interne

Nelle superfici il campo elettrico è perpendicolare alla normale: il flusso fa 0.

$$E_{(r)} = \frac{1}{2\pi\varepsilon_0}\,\frac{\lambda}{r}$$

$$\ln c_1 = \ln r_0$$

$$V_{(r)} = -\int E_{(r)}\,dr = -\frac{1}{2\pi\varepsilon_0}(\ln r + c_1) = -\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r}{r_0}$$



$$\sigma = cost = \frac{dq}{d\Sigma}$$

$$\vec{E} = E_{(r)}\,\vec{u}$$

$$\phi(\vec{E}) = 0 + 2\cancel{\Sigma_0}\vec{E}_{(r)} \overset{Gauss}{=} \frac{\sigma\cancel{\Sigma_0}}{\varepsilon_0} \implies E_{(r)} = \frac{\sigma}{2\varepsilon_0}$$

Due proprietà fondamentali dell'elettrostatica viste fino ad ora

$$C_\gamma(\vec{E}) = \oint_\gamma \vec{E}\,d\vec{s} = 0$$

- Campo conservativo → posso derivare il potenziale

$$\phi_\Sigma(\vec{E}) = \oint_\Sigma \vec{E}\,\vec{u}_n\,d\Sigma = \frac{q_{int}}{\varepsilon_0}$$

- Posso ricavare il campo

Voglio riscrivere queste due equazioni usando delle derivate

Th. della Divergenza

superficie — volume

$$\oint_\Sigma \vec{v}\cdot\vec{u}_n\,d\Sigma = \int_\tau \underbrace{\vec{\nabla}\cdot}_{divergenza}\vec{v}\,d\tau$$

superficie che è bordo di un volume

$$\Sigma = \partial\tau$$

$$\vec{\nabla}\cdot\vec{v} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

Assomiglia al Th. fondamentale del calcolo integrale

$$f_{(b)} - f_{(a)} = \int_a^b f'(x)\,dx$$

$$\oint_\Sigma \vec{E}\,\vec{u}_n\,d\Sigma = \frac{q_{int}}{\varepsilon_0}$$

$$\int_\tau \vec{\nabla}\cdot\vec{E}\,d\tau = \int_\tau \frac{\rho}{\varepsilon_0}\,d\tau$$ (esprimo la carica come l'integrale della densità di carica)

Quindi posso riscrivere il teorema di Gauss come

$$\Phi_\Sigma(\vec{E}) = \oint_\Sigma \vec{E}\,\vec{u}_n\,d\Sigma = \frac{q_{int}}{\varepsilon_0} \iff \vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

Teorema del rotore

$$\underbrace{\oint_\gamma \vec{v}\cdot d\vec{s}}_{\text{linea}} = \underbrace{\int_\Sigma \big(\underbrace{\vec{\nabla}\times\vec{v}}_{\text{rotore di } v}\big)\vec{u}_n\,d\Sigma}_{\text{superficie}}$$

abbiamo una linea che è bordo di una superficie

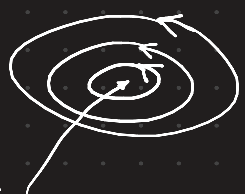
$$\oint_\gamma \vec{E}\,d\vec{s} = 0 = \int_\Sigma \big(\vec{\nabla}\times\vec{v}\big)\vec{u}_n\,d\Sigma \qquad \forall\Sigma / \gamma = \partial\Sigma$$

$$\Longrightarrow \vec{\nabla}\times\vec{E} = 0$$

Posso riscrivere la circuitazione come

$$C_\gamma(\vec{E}) = \oint_\gamma \vec{E}\,d\vec{s} = 0 \iff \vec{\nabla}\times\vec{E} = 0$$

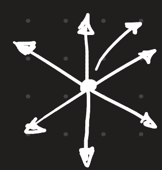
Spiegazione geometrica



$\vec{\nabla}\times\vec{v} \neq 0$

rotore non nullo:
punti in cui le linee di campo girano intorno.
Nel campo elettrico le linee di campo non si uniscon
perciò è sempre 0

$\vec{\nabla}\cdot\vec{v} \neq 0$



divergenza non nulla:
punti in cui entrano o escono le linee di campo

$$C_\gamma(\vec{E}) = \oint_\gamma \vec{E}\, d\vec{s} = 0 \iff \vec{\nabla} \times \vec{E} = 0 \iff \vec{E} = -\vec{\nabla} V$$

$$\Phi_\Sigma(\vec{E}) = \oint_\Sigma \vec{E} \cdot \vec{u}\, d\Sigma = \frac{q_{int}}{\varepsilon_0} \iff \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \iff \vec{\nabla} \cdot (\vec{\nabla} V) = -\frac{\rho}{\varepsilon_0}$$

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho}{\varepsilon_0}$$

eq. di Poisson $\longrightarrow \Delta V = -\frac{\rho}{\varepsilon_0}$

$$\Delta V = -\frac{\rho}{\varepsilon_0} \implies V = \frac{1}{4\pi\varepsilon_0} \int_\tau \frac{\rho\, d\tau}{r}$$

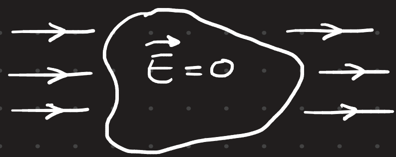
Conduttori

Proprietà in condizioni elettrostatiche

fenomeno di induzione

1 - $\vec{E}_{interno} = 0$

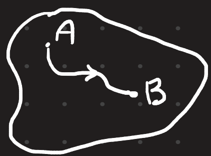
Le cariche sul conduttore si modificano per creare un campo uguale e opposto al campo esterno



2 - La carica in eccesso si deposita sulla superficie (Dim. con Th. di Gauss)

Siccome il campo interno è 0, qualsiasi per qualsiasi superficie interna che possiamo prendere il flusso è 0.

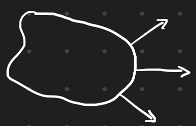
3 - Tutto il conduttore è allo stesso potenziale



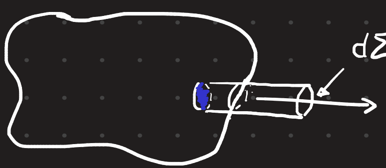
$$V_A - V_B = -\int_B^A \underbrace{\vec{E}}_{\text{il campo è nullo}} \cdot d\vec{s} = 0$$

4 - Teorema di Coulomb

$$\vec{E}_{sup} = \frac{\sigma}{\varepsilon_0} \vec{u}_n$$



Dimostrazione



$$\sigma = \frac{dq}{d\Sigma} \implies dq = \sigma\, d\Sigma$$

$$\Phi(\vec{E}) = \underset{\text{laterale}}{0} + \underset{\text{base interna}}{0} + E\,\cancel{d\Sigma} = \frac{\sigma\,\cancel{d\Sigma}}{\varepsilon_0}$$

Esempio 4.1

Come si dispongono le cariche sulle due sfere?

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{R_1} \qquad V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{R_2}$$

$$\begin{cases} \dfrac{q_1}{R_1} = \dfrac{q_2}{R_2} \\ q_1 + q_2 = q \end{cases}$$

Esercizio 3.20

$R = 10\,cm$
$q = 8 \cdot 10^{-3}\,C$
$\rho(r) = br$

- Calcolare b
- Campo elettrostatico $E(r)$
- Differenza di potenziale tra il centro O e la superficie della sfera

$$q = \int_\tau \rho\, d\tau = \int_0^R br \cdot 4\pi r^2 dr =$$

$$= \int_0^R 4b\pi r^3 dr = \pi b R^4$$

$$\Longrightarrow b = \frac{q}{\pi R^4}$$

$V = \frac{4}{3}\pi r^3$
$dV = 4\pi r^2 dr$

la carica è l'integrale della densità sul volume!!!

$$\vec{E} = E(r)\,\vec{u}_n$$

$$\Phi_S(E) = \vec{E}(r)\,4\pi r^2 = \frac{q_{int}(r)}{\varepsilon_0}$$

$r \geq R \qquad q_{int}(r) = q$

$r < R \qquad q_{int}(r) = \int_0^r 4\pi b y^2 dy = \pi b r^4 = q\frac{r^4}{R^4}$

$$\vec{E}(r) = \begin{cases} \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2} & r \geq R \\ \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{R^4}r^2 & r < R \end{cases}$$

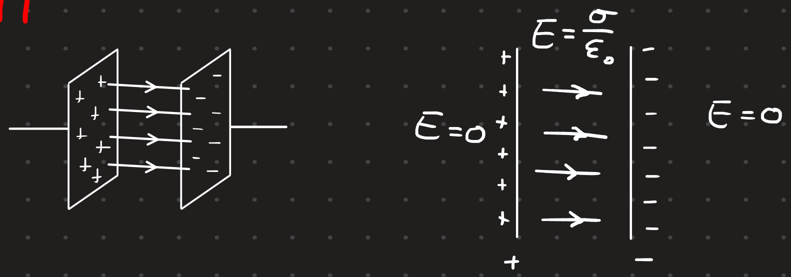
$$V_0 - V_R = -\int_R^0 \vec{E}(r)\,dr = \int_0^R \frac{1}{4\pi\varepsilon_0}\frac{q}{R^4}r^2\,dr = \frac{1}{12\pi\varepsilon_0}\frac{q}{R}$$

• Conduttori cavi

Per il Th. di Gauss la carica totale interna è 0.

$$C_\gamma(\vec{E}) = \int_A^B \vec{E}\, d\vec{s} + \cancel{\int_B^A \vec{E}\, d\vec{s}}$$

• CONDESATORI



$$C = \text{capacità} = \left|\frac{Q}{\Delta V}\right| \qquad [C] = \frac{1C}{1V} = 1F \ \text{(Farad)}$$

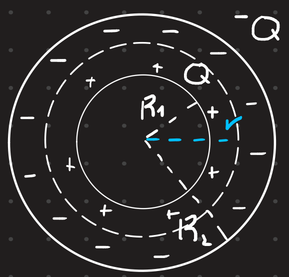
Esercizio

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 \Sigma}$$

$$\Delta V = \int_A^B \vec{E}\, d\vec{s} = \frac{Q}{\varepsilon_0 \Sigma}\int ds = \frac{Q}{\varepsilon_0\Sigma} d$$

$$C = \frac{Q}{\Delta V} = \frac{\cancel{Q}}{\frac{\cancel{Q}}{\varepsilon_0\Sigma} d} = \varepsilon_0 \frac{\Sigma}{d}$$

Esercizio



$$E(r)\, 4\pi r^2 = \frac{Q}{\varepsilon_0} \qquad E(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

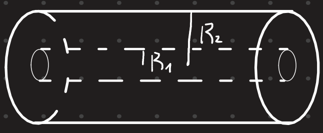
$$\Delta V = \left| \int_{R_1}^{R_2} E(r)\, dr \right| = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$R_2 - R_1 = d \ll R_{1,2}$$

$$R_1 \sim R_2 \sim R$$

$$C = \frac{Q}{\Delta V} = \frac{\cancel{Q}}{\frac{\cancel{Q}}{4\pi\varepsilon_0}\frac{R_2-R_1}{R_1 R_2}} = \varepsilon_0 \frac{4\pi R_2 R_1}{R_2 - R_1} \underset{d \ll R_2 - R_1}{\simeq} \varepsilon_0 \frac{4\pi R^2}{d}$$

Esercizio



$$E(r)\, 2\pi r L = \frac{Q}{\varepsilon_0}$$

$$E(r) = \frac{1}{2\pi\varepsilon_0} \frac{Q/L}{r}$$

$d \ll R_1, R_2$

$R_2 = R_1 + d$

$$\ln \frac{R_1+d}{R_1} = \ln\left(1 + \frac{d}{R_1}\right) \simeq \frac{d}{R_1}$$

$$\Delta V = \int E\,dr = \frac{1}{2\pi\varepsilon_0} \frac{Q}{L} \ln \frac{R_2}{R_1}$$

$$C = \frac{Q}{\Delta V} = \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{R_2}{R_1}\right)} \simeq \varepsilon_0 \frac{2\pi L}{d/R_1} = \varepsilon_0 \frac{2\pi R_1 L}{d}$$

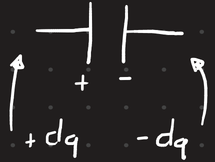
Serie

$$\frac{1}{C_T} = \frac{1}{C_1} + \dots + \frac{1}{C_N}$$

Parallelo

$$C_N = C_1 + \dots + C_N$$

Energia condensatori



$$dL_{ext} = +dq\,\Delta V = +dq\,\frac{q}{C}$$

$$L_{TOT} = \int_0^Q \frac{q}{C}\,dq = \frac{1}{2}\frac{Q^2}{C} = U_e$$

energia elettrostatica accumulata sul condensatore

$$U_e = \frac{1}{2} Q\,\Delta V = \frac{1}{2} C\,\Delta V^2$$

Per il condensatore piano la capacità vale

$$C = \varepsilon_0 \frac{\Sigma}{d}$$

$$U_e = \frac{1}{2} C \Delta V^2 = \frac{1}{2}\varepsilon_0 \frac{\Sigma}{d} \Delta V^2 = \frac{1}{2}\varepsilon_0 \frac{\Sigma}{\cancel{d}} E^2 d^{\cancel{2}} = \frac{1}{2}\varepsilon_0 E^2 \overbrace{\Sigma d}^{\tau \text{ volume}}$$

$\Delta V = E \cdot d$ solo per condensatore piano o quando E è costante

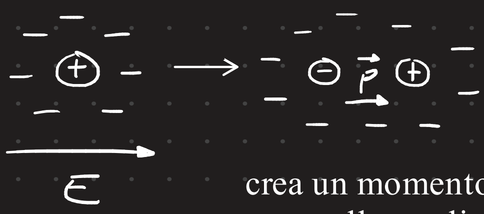
$$u_e = \frac{dU_e}{d\tau} = \frac{1}{2}\varepsilon_0 E^2$$ (densità di energia elettrostatica)

$$U_e = \int_\tau \frac{1}{2}\varepsilon_0 E^2\,d\tau$$

energia accumulata nella regione di spazio grazie al campo elettrico

Campo in materiali isolanti: dielettrici

- Polarizzazione per spostamento

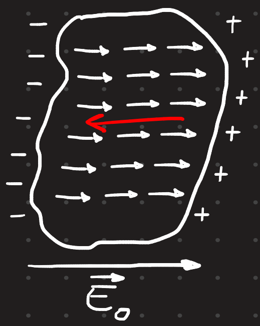


$\langle \vec{p} \rangle \neq 0$

crea un momento di dipolo
non nullo negli atomi

- Polarizzazione per orientamento

$\langle \vec{p} \rangle \neq 0$



carica di
polarizzazione
q_P

genera un campo elettrico che si sommerà
al campo esterno (di verso opposto)

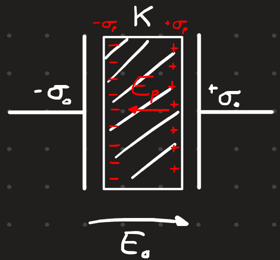
$\vec{P}$ = vettore di polarizzazione $= \dfrac{d\vec{p}}{d\tau}$

$$\sigma_p = \vec{P} \cdot \vec{u}_n$$

densità superficiale
di carica di polarizzazione

$$\vec{P} = \varepsilon_0 \frac{K-1}{K} \vec{E}_0$$

K = costante dielettrica > 1



$$E_0 = \frac{\sigma_0}{\varepsilon_0}$$

$$P = \boxed{\frac{K-1}{K} \sigma_0 = \sigma_p}$$

$$E_p = \frac{\sigma_p}{\varepsilon_0} \longrightarrow E_0 = \frac{\sigma_0}{\varepsilon_0}$$

$$\longleftarrow E_p = \frac{\sigma_p}{\varepsilon_0}$$

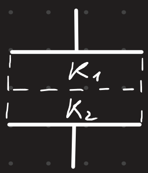
$K\varepsilon_0 = \varepsilon$

$$E = E_0 - E_p = \frac{\sigma_0}{\varepsilon_0} - \frac{K-1}{K}\frac{\sigma_0}{\varepsilon_0} = \frac{\sigma_0}{K\varepsilon_0} = \frac{E_0}{K}$$

campo elettrico nel dielettrico:
campo elettrico nel vuoto diviso
la costante dielettrica

$$\Delta V = \frac{\Delta V_0}{K} \qquad C = K \varepsilon_0 \frac{\Sigma}{d}$$

Esercizio



$\Sigma = 600 \, cm^2$ $\qquad \Delta V = 2 \, kV$

$d = 5 \, mm$

$K_1 = 5 \quad K_2 = 2$

1) $C = ?$ 2) $q = ?$ 3) $U_e = ?$

$$C_T = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{K_1 \varepsilon_0 \frac{\Sigma}{d/2}} + \frac{1}{K_2 \varepsilon_0 \frac{\Sigma}{d/2}}$$

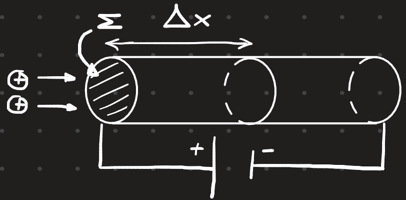
$$\Delta V = q_T \cdot C_T \longrightarrow q_T = \frac{\Delta V}{C_T}$$

$$U_e = \frac{1}{2} q \Delta V$$

Elettrodinamica

$V_1 > V_2$

$\mathcal{E} = \Delta V$ (il generatore mantiene costante la differenza di potenziale)



$i = \frac{dq}{dt} = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t}$ = CORRENTE ELETTRICA

$[i] = \frac{1C}{1s} = 1A$

$\langle \vec{v} \rangle \neq 0$ $\quad \langle \vec{v} \rangle = \vec{v_d}$ = velocità di deriva

in Δt ogni carica percorre $\Delta x = \vec{v_d} \cdot \Delta t$

$\Delta q = n\, q^+ \Delta \tau =$
$= n\, q^+ d\Sigma \cos\theta\, \Delta x =$
$n\, q^+ d\Sigma \cos\theta\, \vec{v_d} \Delta t$

$\frac{\Delta q}{\Delta t} = di = n\, q^+ v_d\, d\Sigma \cos\theta = \vec{J} \cdot \vec{u_n} \cdot d\Sigma$

$\vec{J} = n\, q^+ \vec{v_d}$ = densità di corrente $\quad [J] = \frac{1A}{1m^2}$

$i = \int_\Sigma \vec{J}\, \vec{u_n}\, d\Sigma = \Phi_\Sigma(\vec{J})$

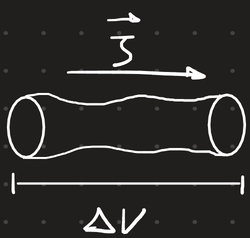
$\vec{J} = n_+ q^+ \vec{v_+} - n_- |q^-| \vec{v_-}$

Legge di Ohm

$\vec{J} = \sigma \vec{E} = \frac{1}{\rho} \vec{E}$

$\sigma =$ conducibilità propria di ogni materiale $= \frac{1}{\rho}$

ρ = resistività



$\Delta V = \int_A^B \vec{E}\, d\vec{s} = \int_A^B \rho \vec{J}\, d\vec{s} = \int_A^B \rho J\, ds = \int_A^B \rho \frac{i}{\Sigma}\, ds =$

$= i \int_A^B \frac{\rho}{\Sigma}\, ds$

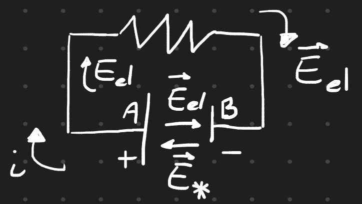
$R = \int_A^B \frac{\rho}{\Sigma}\, ds = \frac{\rho\, l}{\Sigma}$ (se ρ, Σ costanti)

$[\rho] = \Omega \cdot m$

$\Delta V = R \cdot i$

$[R] = \frac{1V}{1A} = 1\Omega$

Legge di Ohm generalizzata

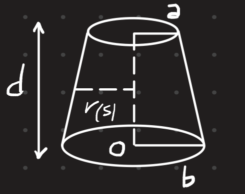


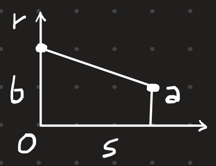
E_* = campo elettromotore

$\vec{E}_* = -\vec{E}_{el}$ (all'interno del generatore)

$$C_{(\vec{E})} = \oint_\gamma \vec{E}\, d\vec{s} = \int_A^B E_{el}\, d\vec{s} + \int_B^A 0 \cdot d\vec{s} = R\, i$$

Esercizio 5.6





ρ data

$R = ?$

$$R = \int_0^d \frac{\rho\, ds}{\Sigma} = \rho \int_0^d \frac{ds}{\Sigma(s)}$$

$$r - b = \frac{a-b}{d-0}(s-0) \qquad r = b + \frac{a-b}{d} s$$

$$\Sigma = \pi r^2 = \pi\left(b + \frac{a-b}{d} s\right)^2$$

$$R = \rho \int_0^d \frac{ds}{\pi\left(b + \frac{a-b}{d} s\right)^2} = \frac{\rho}{\pi}\left[-\frac{1}{b + \frac{a-b}{d}} \times \frac{d}{a-b}\right]_0^d$$

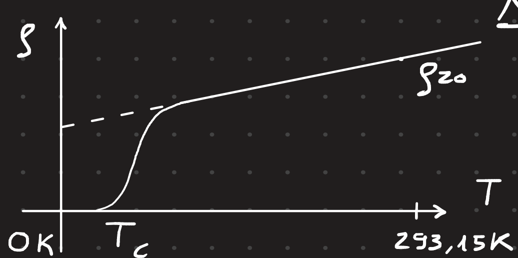
$$= \frac{\rho d}{\pi a b}$$

Effetti termici sulla resistenza

$$\rho(T) = \rho_{20}(1 + \alpha \Delta T)$$

ρ_{20} = resistività a 20°C

$\Delta T = T - 20°C$



T_c = temperatura critica

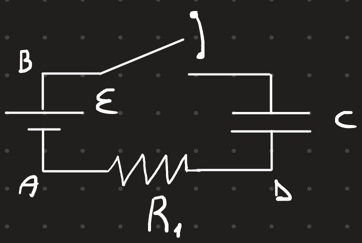
$$dL = dq \cdot \Delta V$$

$$P = \frac{dL}{dq} = \underbrace{\boxed{\frac{dq}{dt}}}_{i} \Delta V = i\, \Delta V = R \cdot i^2 \Longrightarrow \text{effetto Joule}$$

$$E_J = \int P\, dt = \int_0^{T_1} R\, i^2\, dt$$

Carica e scarica di un condensatore (circuito RC)

Carica



$$V_B - V_A = (V_B - V_D) + (V_D - V_A)$$

$$\varepsilon = \frac{q(t)}{C} + R\, i_{(t)}$$

$$\varepsilon = \frac{q(t)}{C} + R \frac{dq(t)}{dt}$$

$$\varepsilon - \frac{q}{C} = R\frac{dq}{dt} \Longrightarrow dt = \frac{R}{\varepsilon - \frac{q}{C}}\, dq = \frac{R}{\frac{\varepsilon C - q}{C}}\, dq = \frac{CR}{\varepsilon C - q}\, dq$$

$$\Longrightarrow \frac{dt}{RC} = \frac{dq}{\varepsilon C - q}$$

$$\int_0^t \frac{dt}{RC} = \int_0^q \frac{dq}{\varepsilon C - q}$$

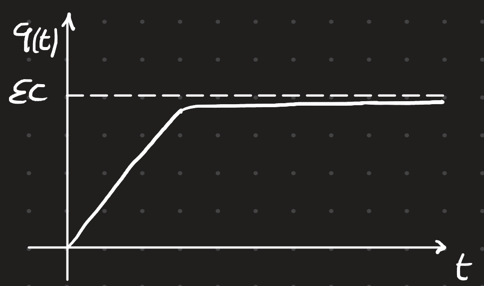
al tempo t = 0 la carica è 0 siccome il circuito è aperto

$$\frac{t}{RC} = \Big[-\ln(\varepsilon C - q)\Big]_0^q = -\ln(\varepsilon C - q) + \ln(\varepsilon C) = -\ln\frac{\varepsilon C - q(t)}{\varepsilon C}$$

$$-\frac{t}{RC} = \ln\left(1 - \frac{q(t)}{\varepsilon C}\right) \qquad e^{-t/RC} = 1 - \frac{q}{\varepsilon C}$$

$$q_{(t)} = \varepsilon C\left[1 - e^{-t/\tau}\right] \qquad \tau = RC \quad [\Omega \cdot F = s]$$

costante di tempo



$t = 4,5\tau$

$e^{-t/\tau} \ll 1$

$$\varepsilon = \frac{q}{C} + Ri$$

$$P_{gen} = \varepsilon \cdot i(t) = \underbrace{\frac{1}{C}\, q(t)\, i(t)}_{\text{condensatore}} + \underbrace{R\, i_{(t)}^2}_{\text{resistenza}}$$

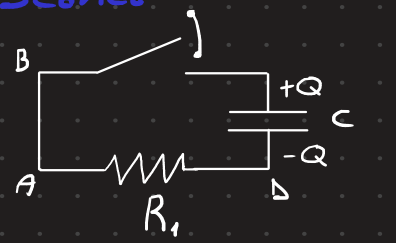
$$i(t) = \frac{dq}{dt} = \frac{\varepsilon}{R} e^{-t/\tau}$$

$$L_{gen} = \int_0^{+\infty} P_{gen}(t)\, dt = \int_0^{+\infty} \frac{1}{C}\, q\, \frac{dq}{\cancel{dt}}\, \cancel{dt} + \int_0^{+\infty} R i^2\, dt =$$

$$\frac{1}{2}\frac{q^2}{C} = \frac{1}{2} C \varepsilon^2 \equiv U_e$$

$$\int_0^{+\infty} R i^2 dt = \frac{\varepsilon^2}{R} \int_0^{+\infty} e^{-\frac{2t}{\tau}} dt = \frac{\varepsilon^2}{R}\left[-\frac{\tau}{2} e^{-\frac{2t}{\tau}}\right] = \frac{\varepsilon^2}{R}\frac{\tau}{2} = \frac{1}{2} C \varepsilon^2$$

Scarica



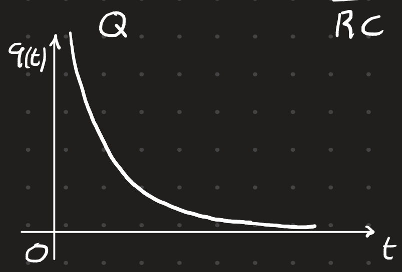
$t=0 \rightarrow Q =$ carica in C

$$\overset{\varepsilon}{\underset{\parallel}{}} 0 = \frac{q(t)}{C} + R\,\frac{dq(t)}{dt}$$

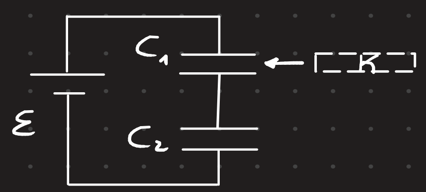
$$-\frac{q}{C} = R\frac{dq}{dt} \Rightarrow -\frac{dt}{RC} = \frac{dq}{q}$$

$$-\int_0^t \frac{dt}{RC} = \int_0^q \frac{dq}{q}$$

$$-\frac{t}{RC} = \ln\frac{q}{Q} \qquad q(t) = Q e^{-t/\tau}$$



Esercizio 4.10



$C_1 = 500$ pF
$C_2 = 100$ pF
$\varepsilon = 400$ V
$K = 4$

a) Δq energia dal generatore
b) variazione ΔV_1 (ai capi di C_1)
c) energia fornita dal generatore

ⓐ $\varepsilon = \frac{q}{C_T} \longrightarrow q = \varepsilon C_T$ q prima dielettrico

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 C_2}{C_1 + C_2}$$

$$C_T' = \frac{K C_1 C_2}{K C_1 + C_2}$$

$$\Delta q = C_T'\varepsilon - C_T\varepsilon = (C_T' - C_T)\varepsilon$$

ⓑ

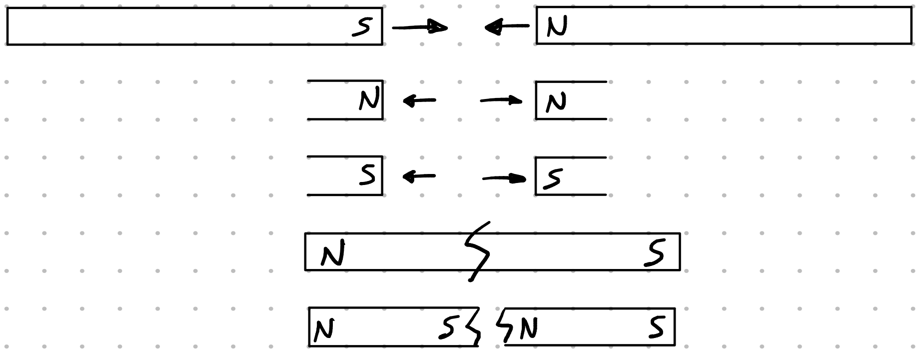
$$V_1 = \frac{Q}{C_1} = \frac{C_T}{C_1}\varepsilon \qquad \Delta V_1 = V_1' - V_1$$

$$V_1' = \frac{Q'}{C_1'} = \frac{C_T'\varepsilon}{K C_1}$$

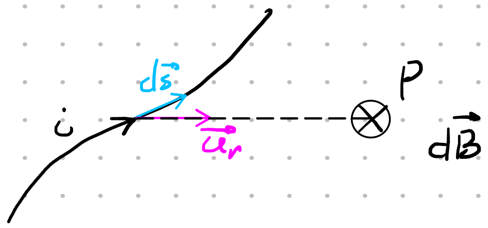
ⓒ

$$E_{gen} = \Delta U_e = \frac{1}{2}C_T'\varepsilon^2 - \frac{1}{2}C_T\varepsilon^2 = \frac{1}{2}(C_T' - C_T)\varepsilon^2$$

Magnetismo



I^a legge di Laplace



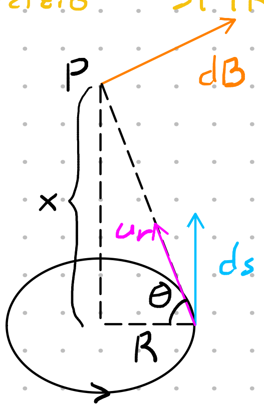
B = campo magnetico

$[B]$ = Tesla (T)

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,|d\vec{s}\times\vec{u}_r|}{r^2}}$$

$$\frac{\mu_0}{4\pi} = 10^{-7}\ \frac{T\cdot m}{A}$$

Esercizio SPIRA CIRCOLARE



ds è entrante nel foglio

$$dB_z = \frac{\mu_0}{4\pi}\frac{i\,|d\vec{s}\times\vec{u}_r|}{r^2}\cos\theta$$

$$= \frac{\mu_0}{4\pi}\, i\, \frac{d\vec{s}}{R^2+x^2}\, \frac{R}{\sqrt{x^2+R^2}}$$

$$\vec{B}_z = \int dB_z = \frac{\mu_0}{4\pi}\, i\, \frac{R}{(x^2+R^2)^{3/2}}\int d\vec{s} =$$

$2\pi R$

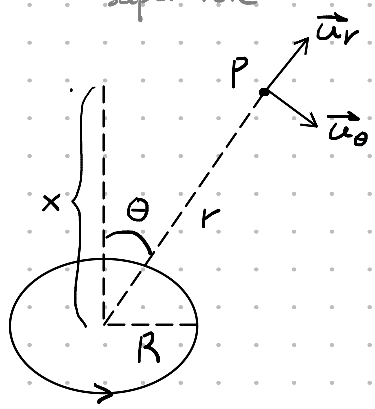
$$= \frac{\mu_0}{4\pi}\frac{i\,2\pi R}{(x^2+R^2)^{3/2}}$$

$m = i\Sigma = iR^2\pi \equiv$ momento di dipolo $\qquad \vec{m} = i\Sigma\,\vec{u}_n$

normale alla superficie

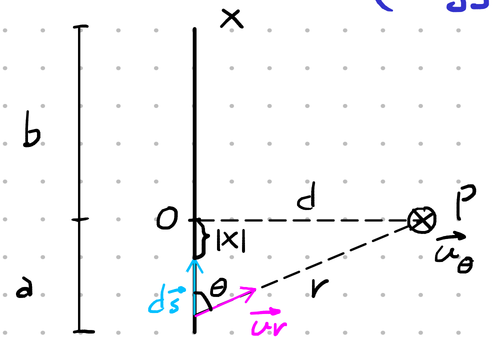
$$B_z = \frac{\mu_0}{2\pi}\frac{m}{(x^2+R^2)^{3/2}} \simeq r^3$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{m}{r^3}\left(2\cos\theta\,\vec{u}_r + \sin\theta\,\vec{u}_r\right) \qquad r \gg R$$



per qualsiasi punto P

Filo infinito (Legge di Biot-Savart) $\vec{B} = \frac{\mu_0 i}{2\pi d}\vec{u}_\theta$



a, b, i, d dati

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i|d\vec{s}\times\vec{u}_r|}{r^2}$$

$$= \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{d^2/\sin^2\theta}\vec{u}_\theta \qquad r = \frac{d}{\sin\theta}$$

$ds = dx \qquad x_\theta = -\frac{d}{tg\theta} \qquad dx = \frac{d}{\sin^2\theta}d\theta$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,\cancel{d}/\cancel{\sin^2\theta}\,\sin\theta}{\frac{d^{\cancel{2}}}{\cancel{\sin^2\theta}}}d\theta\,\vec{u}_\theta$$

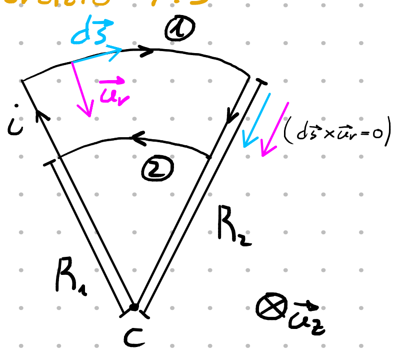
$$\cos\theta_0 = \frac{a}{\sqrt{a^2+d^2}} \qquad \cos\theta_1 = \frac{b}{\sqrt{b^2+d^2}}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{i\vec{u}_\theta}{d}\int_{\theta_0}^{\theta_1}\sin\theta\,d\theta = \frac{\mu_0}{4\pi}\frac{i}{d}\vec{u}_\theta(\cos\theta_0 - \cos\theta_1)$$

filo infinito $\quad \theta_0 \to 0 \quad \theta_1 \to \pi$

$$\boxed{\vec{B} = \frac{\mu_0 i}{2\pi d}\vec{u}_\theta}$$

Esercizio 7.9



$\vec{B}_c = ?$

$\vec{m} = ?$

$$m = i\theta\left(\frac{R_2^2 - R_1^2}{2}\right)$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i|d\vec{s}\times\vec{u}_r|}{r^2} \qquad \int ds = \theta R_2$$

$$d\vec{B}_1 = \frac{\mu_0}{4\pi}\frac{i\,ds}{R_2^2}\vec{u}_z \implies \vec{B}_1 = \frac{\mu_0}{4\pi}\frac{i\theta}{R_2}\vec{u}_z$$

$$\vec{B}_2 = -\frac{\mu_0}{4\pi}\frac{i\theta}{R_1}\vec{u}_z$$

$$B_c = \frac{\mu_0 i\theta}{4\pi}\left(\frac{1}{R_2} - \frac{1}{R_1}\right)\vec{u}_z$$

Riassunto legge di Laplace

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\,|d\vec{s} \times \vec{u}_r|}{r^2} \Longrightarrow \frac{\mu_0}{4\pi} \frac{dq\,\vec{v} \times \vec{u}_r}{r^2}$$

$$i = \frac{dq}{dt} \qquad i\,d\vec{s} = \frac{dq}{dt} \cdot d\vec{s} = dq \cdot \vec{v}$$

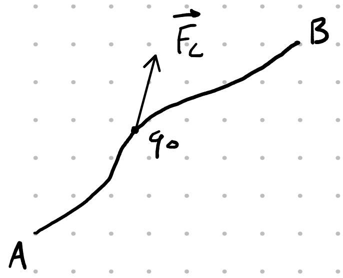
$\vec{v}$ → velocità di deriva cariche

$$B = \frac{\mu_0}{4\pi} \frac{q\,\vec{v} \times \vec{u}_r}{r^2} \qquad |\vec{v}| \ll c \ (\text{velocità luce})$$

$$\vec{F}_e = q\vec{E}$$ → carica in presenza di un campo esterno

carica q in
un campo esterno

Forza di Lorentz



$$\boxed{\vec{F}_L = q_0 \left(\vec{v} \times \vec{B}\right)} \qquad \vec{F}_L = q_0\left(\cancel{\vec{E}} + \vec{v} \times \vec{B}\right) \quad \vec{E} = 0$$

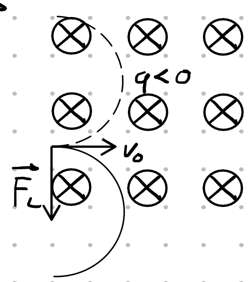
$$L_{AB} = \int_A^B \vec{F}_L \, d\vec{s} = \int_A^B q_0 \overbrace{\left(\vec{v} \times \vec{B}\right) \vec{v}}^{0} \, dt = 0$$

$$\Delta E_K = \frac{1}{2}\left(\vec{v}_B - \vec{v}_A\right) m$$

$$\Longrightarrow |\vec{v}_A| = |\vec{v}_B|$$

Campo magnetico uniforme

$V_0 \perp \vec{B}$



$\vec{B}$

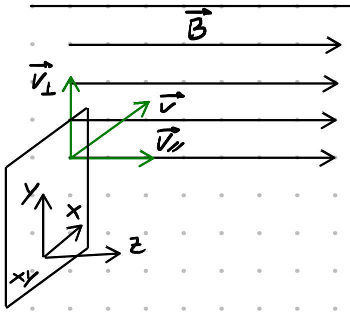
$$|\vec{F}_L| = q\,V_0 \times \vec{B}$$

↑ verso il basso

$$F_c = m\frac{V_0^2}{R} = |F_L| = q V_0 B$$

forza centrifuga →

$$R = \frac{m V_0}{qB} \qquad \omega = \frac{V_0}{r} = \frac{qB}{m} \qquad T = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

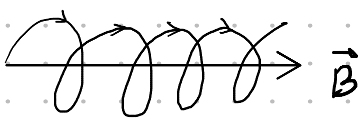


$$\vec{v} \not\perp \vec{B} \qquad \vec{v} = \vec{v}_{//} + \vec{v}_\perp$$

$$\vec{F}_L = q\,\vec{v} \times \vec{B} = q\,\vec{v}_\perp \times \vec{B} + \cancel{q\,\vec{v}_{//} \times \vec{B}} = 0 \quad \text{paralleli}$$

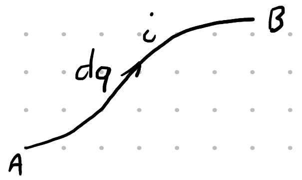
$$= q\,\vec{v} \times \vec{B}$$

$$\vec{E}_z = 0 \qquad z(t) = z_0 + v_{//} t$$ moto rettilineo uniforme lungo z



moto circolare lungo xy

II^a legge di Laplace



$$d\vec{F} = dq\,\vec{v}_d \times \vec{B} \qquad dq\,\vec{v}_d = i\,d\vec{s}$$

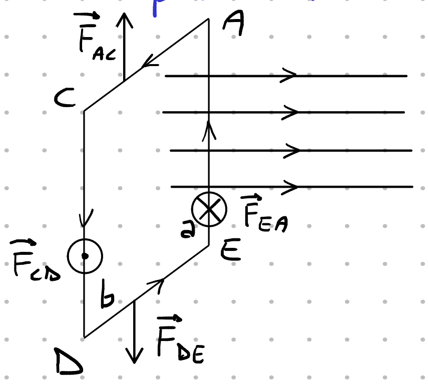
$$\boxed{d\vec{F} = i\,d\vec{s} \times \vec{B}}$$

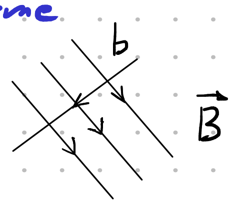
Forza subita dal filo percorso dalla corrente in presenza di un campo magnetico

$$\vec{F} = \int_A^B i\,d\vec{s} \times \vec{B}$$ con $\vec{B}$ costante e uniforme

$$\Longrightarrow i\left(\int_A^B d\vec{s}\right) \times \vec{B} = i\,\vec{AB} \times \vec{B}$$

Circuito piano in un campo uniforme

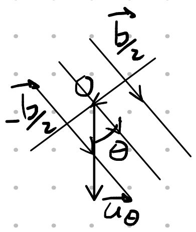




$$\left.\begin{array}{l}\vec{F}_{AC} = i\,\vec{AC} \times \vec{B} \\ \vec{F}_{DE} = i\,\vec{DE} \times \vec{B}\end{array}\right\}$$ si annullano

$$\vec{F}_{EA} = i\,\vec{EA} \times \vec{B}$$ entrante

$$\vec{F}_{CD} = i\,\vec{CD} \times \vec{B} = -\vec{F}_{EA}$$ uscente



Il centro di massa è 0
le due forze generano un momento

$$\vec{M} = \frac{\vec{b}}{2} \times \vec{F}_{AE} - \frac{\vec{b}}{2} \times \vec{F}_{CD} = \vec{b} \times \vec{F}_{AE}$$

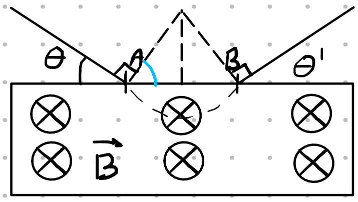
$$|\vec{M}| = b \cdot \vec{F}_{AE} \sin\theta = i\,a\,b\,B \sin\theta$$

$$\vec{m} = i\,a\,b\,\vec{u}_n = i\,\Sigma\,\vec{u}_n$$

momento di dipolo magnetico

$$\vec{M} = \vec{m} \times \vec{B}$$

Esercizio 6.1



$$d = 2R\cos\left(\boxed{\frac{\pi}{2} - \theta}\right)$$

$$R = \frac{mv_0}{qB}$$

Riassunto

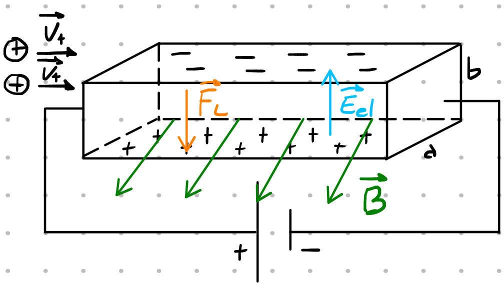
$$\vec{F}_L = q(\vec{E} + v \times \vec{B})$$

$$d\vec{F} = i\,d\vec{s} \times \vec{B}$$

$$\vec{m} = i\,\Sigma\,\vec{u}_n \qquad \vec{M} = \vec{m} \times \vec{B} \qquad U_m = \vec{m}\,\vec{E}$$ magnetico

$$\vec{M}_e = \vec{p} \times \vec{E} \qquad U_e = -\vec{p}\,\vec{E}$$ elettrico

Effetto Hall



B perpendicolare al momento delle cariche

$$\vec{F}_L = q \cdot \vec{v} \times \vec{B}$$

La faccia superiore si carica negativamente, quella inferiore positivamente. Così facendo si forma un campo elettrostatico verso l'alto.

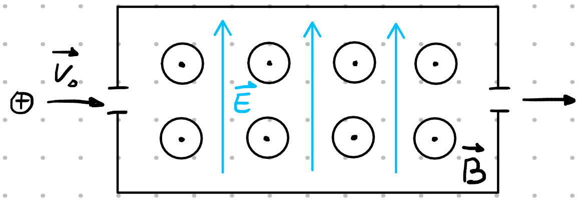
$$\vec{E}_H = \frac{\vec{F}_L}{q_+} = \vec{v} \times \vec{B} \qquad \text{campo di Hall}$$

$\Longrightarrow$ situazione a regime: $\vec{E}_H + \vec{E}_{el} = 0$

$$|\vec{E}_{el}| = |\vec{E}_H| = v \cdot \vec{B} \cdot \underbrace{\cancel{\sin 90°}}_{=1} = \frac{J}{n q_+} B = \frac{i}{n a b q_+} \vec{B}$$

$$\Delta V_H = |E_H|\, b = \frac{iB}{n a q_+}$$

Selettore di velocità



$$\vec{F}_L = q(E + v_0 \times \vec{B}) = 0$$

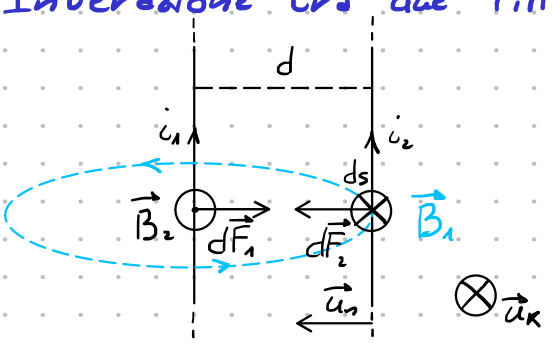
(la forza è 0 per avere un moto rettilineo uniforme)

$$E = -v_0 \times \vec{B}$$

$$E = v_0 B \underbrace{\cancel{\sin\theta}}_{=1} = v_0 B \Longrightarrow v_0 = \frac{E}{B}$$

$$1\,T = \frac{1\,V}{m\, m/s} = \frac{Vs}{m^2}$$

Interazione tra due fili percorsi da corrente



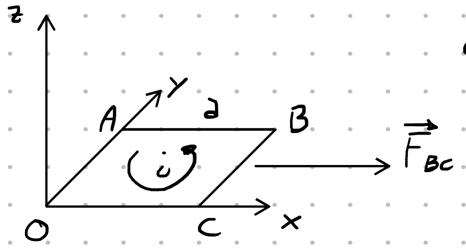
$$\vec{B}_1 = \frac{\mu_0}{2\pi d} i_1 \vec{u}_K$$

$$d\vec{F}_2 = i_2 d\vec{s} \times \vec{B}_1 = \frac{\mu_0 i_1 i_2}{2\pi d} ds\, (\vec{u}_n)$$

$$\frac{dF}{ds} = \frac{\mu_0 i_1 i_2}{2\pi d}$$

Esercizio 6.23

$a = 20\,cm$
$i = 5A$
$\vec{B} = \lambda x \vec{u}_z$
λ costante
$\vec{F} = ?$



$d\vec{F} = i\, d\vec{s} \times d\vec{B}$

II^a legge di Laplace

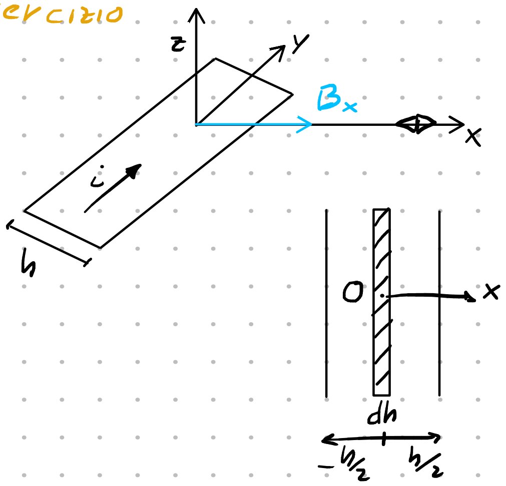
$\vec{F}_{AO} = 0$ per $x=0$ il campo è nullo

$\vec{F}_{BC} = i \cdot a \vec{u}_y \times \lambda a \vec{u}_z = \lambda i a^2 \vec{u}_x$

$$\vec{F}_{OC} = \int_0^a i\, d\vec{s} \times (\lambda x \vec{u}_z) = \lambda i \int_0^a x\, dx (-\vec{u}_y)$$
$$= \lambda i \frac{a^2}{2} (-\vec{u}_y)$$

$\vec{F}_{AB} = -\vec{F}_{OC} \implies \vec{F}_{AB} + \vec{F}_{OC} = 0$

Esercizio



$\frac{di}{i} = \frac{dh}{h} \implies di = \frac{dh}{h} \cdot i$

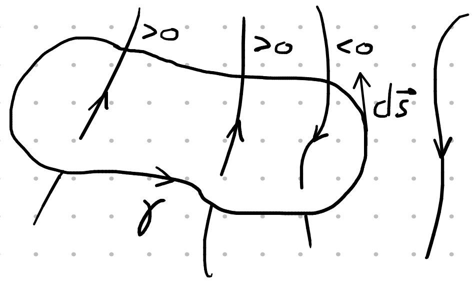
$$B = \int_{-h/2}^{+h/2} \frac{\mu_0}{2\pi} \frac{di}{(x-y)} = \int_{-h/2}^{+h/2} \frac{\mu_0}{2\pi} \frac{i}{(x-y)h}\, dh = \dots$$

distanza filo/punto

$\vec{M} = \vec{m} \times \vec{B} = 0$ $\quad \vec{m} \perp \vec{B}$

Legge di Ampere

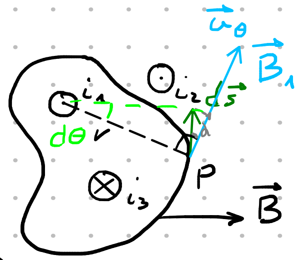
$$C_\gamma(\vec{B}) = \oint_\gamma \vec{B}\cdot d\vec{s} = \boxed{\mu_0 \sum_K i_K^{(conc)}}$$



Dimostrazione

$$\vec{B} = \sum_K \vec{B}_K$$

$$C_\gamma(\vec{B}) = \sum_K \oint_\gamma \vec{B}_K \cdot d\vec{s}$$



Considero solo un filo

$$K=1) \quad \vec{B}_1 = \frac{\mu_0 i_1}{2\pi r}\vec{u}_\theta \iff \vec{B}_1\cdot d\vec{s} = \frac{\mu_0 i_1}{2\pi r} d\vec{s}\,\vec{u}_\theta = \frac{\mu_0 i_1}{2\pi r}\underbrace{ds\cdot\cos\lambda}_{\pm dl} =$$

$$= \pm\frac{\mu_0 i_1}{2\pi}\frac{dl}{r} = \pm\frac{\mu_0 i_1}{2\pi} d\theta$$

definizione di angolo in radianti (arco/raggio)

$$\vec{B}\cdot d\vec{s} = \pm\frac{\mu_0 i}{2\pi} d\theta = \frac{\mu_0 (\pm) i}{2\pi} d\theta$$

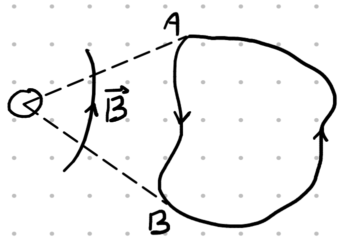
$$\oint \vec{B}\cdot d\vec{s} = \frac{\mu_0 (\pm) i}{2\pi}\oint d\theta$$

→ i concatenato $\quad \oint d\theta = 2\pi$

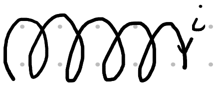
→ i non concatenato

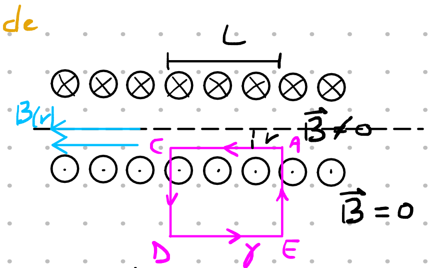
$$(\pm) i \oint d\theta = -i\int_A^B d\theta + i\int_B^A d\theta$$

$$= -i\theta + i\theta = \emptyset$$



Esempio solenoide





$$C_\gamma(\vec{B}) = \oint_A^C \vec{B}\cdot d\vec{s} + \cancel{\oint_C^D \vec{B}\cdot d\vec{s}} + \cancel{\oint_D^E \vec{B}\cdot d\vec{s}} + \cancel{\oint_E^A \vec{B}\cdot d\vec{s}} = B_{(r)}\oint_A^C d\vec{s} = B_{(r)} L$$

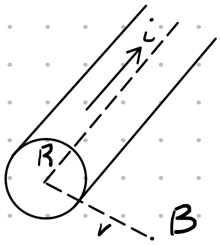
$B \perp ds \qquad \overset{\shortparallel}{0} \qquad B\perp ds$

$$C_\gamma(\vec{B}) \overset{Ampere}{=} \mu_0 \sum_K i_K^{conc} = \mu_0 N i$$

numero di "avvolgimenti" nel tratto L

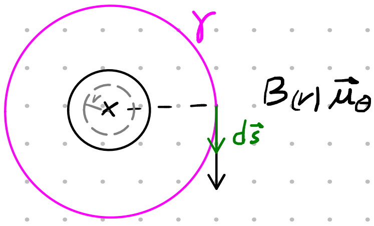
$$B_{(r)} L = \mu_0 N i \rightarrow \boxed{B = \mu_0 n i} \qquad n = \frac{N}{L}$$

Esercizio 7.2



$B(r) = ?$

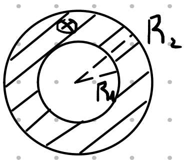
J uniforme



$$C_\gamma(\vec{B}) = \oint_\gamma \vec{B}(r)\,d\vec{s} = B(r)\oint_\gamma d\vec{s} = B(r)\cdot 2\pi r$$

$$C_\gamma(\vec{B}) \overset{\text{Ampere}}{=} \mu_0 i_c = \begin{cases} \mu_0 i & r \geq R \\ \mu_0 J \pi r^2 & r < R \end{cases} \qquad J = \frac{i}{\pi R^2}$$

$$B(i) = \begin{cases} \dfrac{\mu_0 i}{2\pi r} & r \geq R \\ \dfrac{\mu_0 i}{2\pi R^2}\, r & r < R \end{cases}$$



$$J = \frac{i}{\pi(R_2^2 - R_1^2)} \qquad C(B) = B\,2\pi r = \begin{cases} \mu_0 i & r \geq R_2 \\ J\pi(r^2 - R_1^2) & R_1 < r < R_2 \\ 0 & r \leq R_1 \end{cases}$$

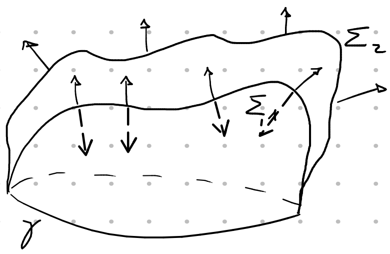
Ripasso leggi di Maxwell

- Campo elettrostatico

$$C_\gamma(\vec{E}) = 0 \qquad C_\gamma(\vec{B}) = \mu_0 i^{conc} \qquad \vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$$

$$\Phi_\Sigma(\vec{E}) = \frac{q_{int}}{\varepsilon_0} \qquad \Phi_\Sigma(\vec{B}) = 0$$

$\partial X =$ bordo di X



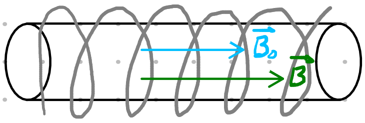
$$\partial\Sigma_1 = \partial\Sigma_2 = \gamma$$

$$\Sigma = \Sigma_1 \cup \Sigma_2$$

$$\Phi_\Sigma(\vec{B}) = 0 = \Phi_{\Sigma_2}(\vec{B}) + \Phi_{\tilde{\Sigma}_1}(\vec{B})$$

$$= \Phi_{\Sigma_2}(\vec{B}) - \Phi_{\Sigma_1}(\vec{B})$$

$$\Phi_{\Sigma_1}(\vec{B}) = \Phi_{\Sigma_2}(\vec{B}) = \Phi_\gamma(\vec{B})$$



$$\vec{B}_0 = \mu_0 n i \vec{u}_n \qquad \vec{H} = \frac{\vec{B}_0}{\mu_0} = \text{campo magnetizzante}$$

$$\vec{B} = k_m \vec{B}_0 = \qquad \vec{E} = \frac{\vec{E}_0}{k_e}$$

$$= \vec{B}_0 + (k_m - 1)\vec{B}_0 =$$

$$= \mu_0 \vec{H} + \underbrace{\mu_0 (k_m - 1)\vec{H}}_{\vec{M} = \text{magnetizzazione}} = \mu_0 (\vec{H} + \vec{M})$$

- $k_m < 1$ diamagentici
- $k_m > 1$ paramagnetici
- $k_m >> 1$ ferromagnetici

$$\vec{M} = (k_m - 1)\vec{H} = \chi_m \vec{H} \qquad \vec{H} = \frac{\vec{B}_0}{\mu_0}$$

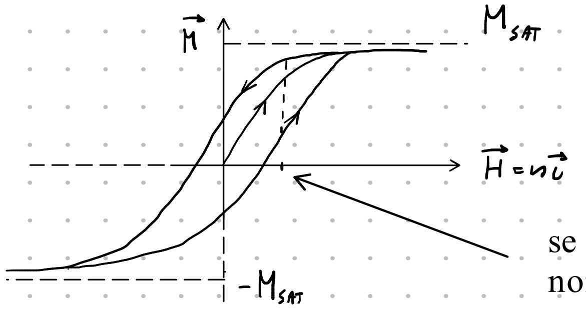
$$\vec{B} = \mu_0 (\vec{H} + \vec{M})$$

↳ solenoide

i_m = corrente di magnetizzazione

$$B = \mu_0 n i + \mu_0 n \underset{i_m}{\chi_m i} = k_m \mu_0 n i \qquad \mu_0 \longrightarrow k_m \mu_0 = \mu$$

Ciclo di isteresi



$$\vec{M} = \chi_m \vec{H}$$

(In parole povere, processo di magnetizzazione dei materiali)

se conosco solo il campo magnetizzante, non conosco anche la magnetizzazione

Ancora Maxwell (forma differenziale)

$$C_\gamma(\vec{E}) = 0 \iff \vec{\nabla} \times \vec{E} = 0$$

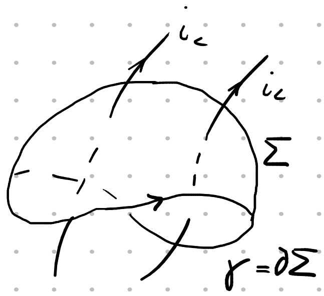
$$\Phi(\vec{E}) = \frac{q_{int}}{\varepsilon_0} \iff \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$C_\gamma(\vec{B}) = \mu_0 i_{conc} \iff \vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

$$\Phi_\Sigma(\vec{B}) = 0 \iff \vec{\nabla} \cdot \vec{B} = 0$$

$$C_\gamma(\vec{B}) = \int_\Sigma (\vec{\nabla} \times \vec{B}) \, \vec{u}_n \, d\Sigma$$

$$\mu_0 i_{conc} = \mu_0 \int_\Sigma \underbrace{\vec{J} \cdot \vec{u}_n \, d\Sigma}_{\Phi_\Sigma(\vec{J})}$$



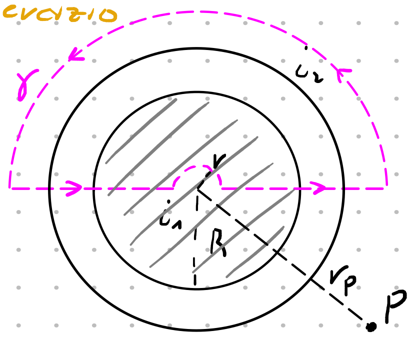
$$\int i_{nc}$$

$$\Phi_\Sigma(\vec{B}) = \int_\tau \vec{\nabla} \cdot \vec{B} \, d\tau$$

TH DI GAUSS $\Downarrow$ 0 $\qquad \vec{\nabla} \cdot \vec{B} = 0$

$$\Sigma = \partial \tau$$

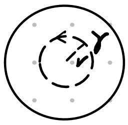
Esercizio



$R = 2cm$
$r = R/4$
$B_r = 20\mu T$
$C_\gamma = 0$
$r_p = 10cm$

$i_1 = ?$
$B_p = ?$

①



$$C_\gamma(\vec{B}) = B(r)\,2\pi r \overset{Ampere}{=} \mu_0 \frac{i_1}{\pi R^2}\pi r^2$$

$$i_1 = 2\pi \cancel{r} B(r) \frac{R^2}{\mu_0 r^{\cancel{2}}}$$

②

$$B_p = \frac{\mu_0(i_1+i_2)}{2\pi r_p} = \frac{\mu_0 \frac{1}{16} i_1}{2\pi r_p}$$

$$i_{conc} = \frac{i_1}{\cancel{\pi}R^2}\left(\frac{\cancel{\pi}R^2}{2} - \frac{\cancel{\pi}r^2}{2}\right) = i_1\left(\frac{1}{2} - \frac{r^2}{2R^2}\right) =$$

$$C_\gamma = \mu_0\left(\frac{i_2}{2} + i_1\left(\frac{1}{2} - \frac{r^2}{2R^2}\right)\right) = 0$$

$$i_2 = -i_1\left(1 - \frac{r^2}{R^2}\right) = i_1 \frac{15}{16}$$

Maxwell

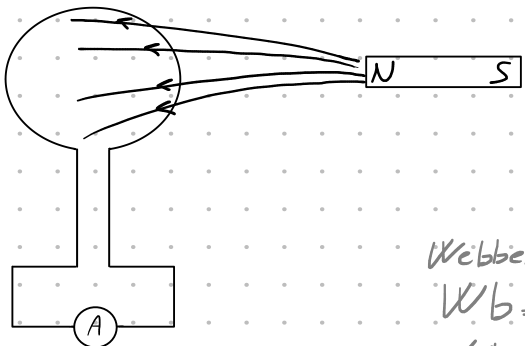
$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0} \qquad \vec{\nabla}\times\vec{E} = 0$$

$$\vec{\nabla}\cdot\vec{B} = 0 \qquad \vec{\nabla}\times\vec{B} = \mu_0\vec{J}$$

Legge di Faraday-Lentz

Circuito collegato ad un amperometro

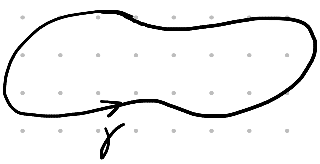
Muovendo la calamità, o un altro circuito viene creata una forza elettromotrice indotta nel circuito



Webber

$Wb = T \cdot m^2 = [\Phi(\vec{B})]$

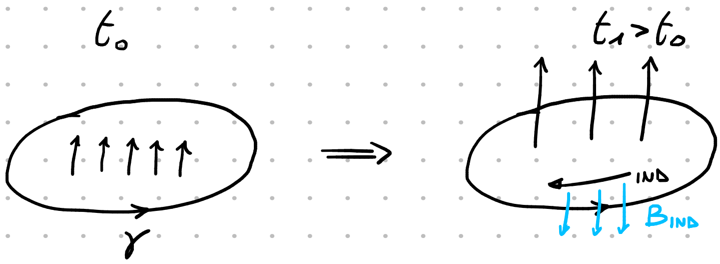
$\frac{Wb}{s} = V$



$$C_\gamma(\vec{E}) = -\frac{d\Phi_\gamma(\vec{B})}{dt}$$

$$\Phi_\gamma(\vec{B}) = \int_\Sigma \vec{B}\cdot\vec{u}_n\cdot d\Sigma$$

$$\boxed{i_{IND} = -\frac{1}{R}\frac{d\Phi_\gamma(\vec{B})}{dt}}$$

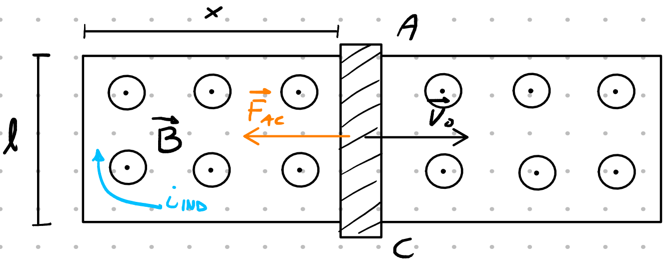


B aumenta

1) $\circlearrowleft > 0 \Rightarrow \phi$ aumenta $\Rightarrow -\dfrac{d\phi}{dt} < 0 \Rightarrow i_{IND} < 0$

2) $\circlearrowleft < 0 \Rightarrow \phi$ diminuisce $\Rightarrow -\dfrac{d\phi}{dt} > 0 \Rightarrow i_{IND} > 0$

Generatore fem costante



Campo costante, varia la superficie!

$\vec{v}_0 = cost$

$$\phi(\vec{B}) = B\Sigma = B\,l\,x(t)$$

$$f_{em} = C_\gamma(\vec{E}) = -\frac{d\phi}{dt} = -Bl\frac{dx}{dt}(t) = -Blv_0$$

$$i_{IND} = -\frac{Blv_0}{R}$$

$$\boxed{\vec{F}_{AC} = i_{IND}\,\vec{AC} \times \vec{B}}$$

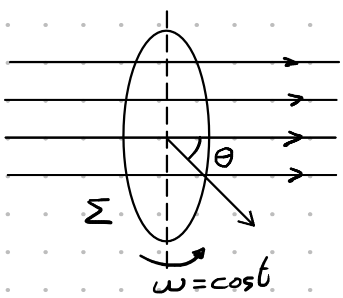
Lavoro per mantenere la bacchetta a velocità costante

$\vec{F}_{ext} = -\vec{F}_{AC}$ $\qquad \vec{F}_{ext} = i\,l\,B$ $\qquad l \perp B \quad l \times B \Rightarrow l \cdot B \cdot \sin\theta$ (1)

$$P_{ext} = F_{ext} \cdot v_0 = i_{IND}\,l\,B\,v_0 = \frac{(Blv_0)^2}{R}$$

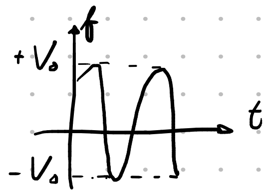
$$P_J = R\,i_{IND}^2 = \cancel{R}\frac{(Blv_0)^2}{\cancel{R}} = P_{ext}$$

Fem alternata



$$\phi(B) = \vec{B} \cdot \vec{u}_n \cdot \Sigma = NB\Sigma\cos\theta(t)$$

$$f_{em} = -\frac{d\phi}{dt} = +NB\Sigma\sin\theta(t) \cdot \underbrace{\frac{d\theta}{dt}}_{\omega} = \underbrace{NB\Sigma\omega}_{V_0}\sin(\omega t)$$



$$i(t) = \frac{V_0}{R}\sin(\omega t) = i_0\sin(\omega t)$$

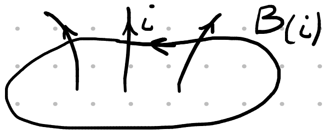
$2\pi\nu = \omega$

$\nu = 50\ Hz$

$$\langle P_J \rangle = \frac{1}{T}\int dt\,R\,i(t)^2 = \frac{1}{2}R\,i^2 = R\,i_{eff}^2 \qquad i_{eff} = \frac{i_0}{\sqrt{2}} \qquad V_{eff} = \frac{V_0}{\sqrt{2}}$$

$$C_\gamma = -\frac{d\Phi_\gamma(\vec{B})}{dt}$$

Coefficiente di auto induzione (L)



I^a legge di Laplace

autoflusso

$$\Phi_\Sigma(\vec{B}) = \int_\Sigma \vec{B}\cdot\vec{u}_n \cdot d\Sigma = \int_\Sigma \left(\oint_\gamma \frac{\mu_0}{4\pi}\frac{i\, d\vec{s}\times\vec{u}_r}{r^2}\right)\cdot \vec{u}_n \cdot d\Sigma$$

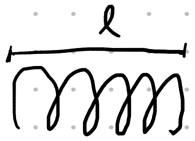
dipende dalla geometria del circuito

$$= i_0 \cdot L$$

$$L = \frac{\Phi_\gamma(\vec{B})}{i_0} \qquad [L] = \frac{T\, m^2}{A} \equiv H \text{ (Henry)}$$

$$i_{IND}(t) = -\frac{1}{R}\frac{d\Phi_\gamma(\vec{B})}{dt} = -\frac{1}{R}\frac{d}{dt}\left(L\, i_0(t)\right) = -\frac{L}{R}\frac{di(t)}{dt}$$

$$\boxed{i_{IND}(t) = -\frac{L}{R}\frac{di(t)}{dt}} \qquad \boxed{f_{em}(t) = -L\frac{di}{dt}}$$

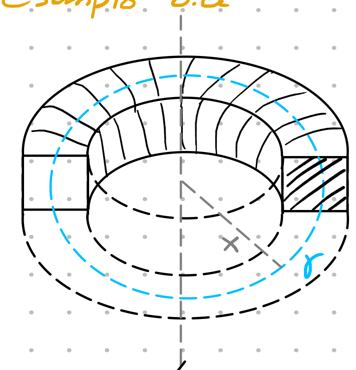


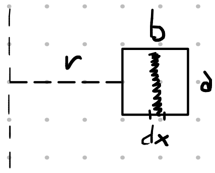
N spire

$$\Phi(\vec{B}) = N\cdot B\cdot\Sigma = N\,\mu_0 \frac{N}{l}\, i\,\Sigma$$

$$L = \frac{\Phi(\vec{B})}{i} = \mu_0\frac{N^2}{l}\Sigma \qquad C = \varepsilon_0\frac{\Sigma}{d}$$

Esempio 8.4





N spire

$$C_\gamma(\vec{B}) = \oint_\gamma \vec{B}\, d\vec{s} = B(x)\, 2\pi x \overset{\text{Ampere}}{=} \mu_0 N i_0$$

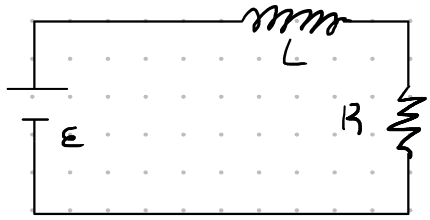
$$B(x) = \frac{\mu_0 N i_0}{2\pi x}$$

campo magnetico non costante

$$\Phi(B) = N\int_r^{r+b} B(x)\, a\, dx = \frac{\mu_0 N^2 i a}{2\pi}\int_r^{r+b}\frac{dx}{x} = \frac{\mu_0 N^2 i a}{2\pi}\ln\left(\frac{r+b}{r}\right)$$

$$L = \frac{\mu_0 N^2 a}{2\pi}\ln\left(\frac{r+b}{r}\right)$$

Circuito RL



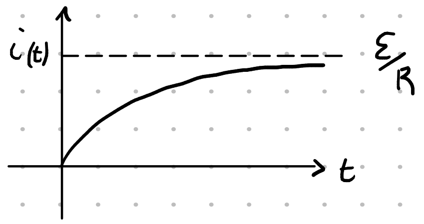
$$V = R \cdot i$$

$$\varepsilon - L\frac{di(t)}{dt} = R \cdot i(t)$$

$$\varepsilon - Ri = L\frac{di(t)}{dt} \Rightarrow \int_0^t dt = \int_0^i L\frac{di}{\varepsilon - Ri} \Rightarrow t = -\frac{L}{R}\ln\frac{\varepsilon - Ri}{\varepsilon}$$

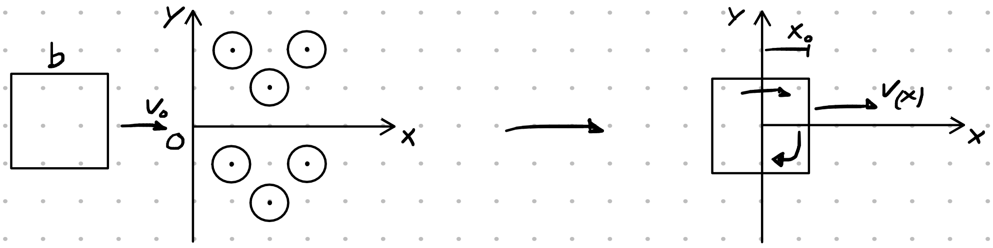
$$-\frac{R}{L}t = \ln\frac{\varepsilon - Ri}{\varepsilon} \Rightarrow \frac{\varepsilon - Ri}{\varepsilon} = e^{-\frac{R}{L}t} \Rightarrow \varepsilon\left[1 - e^{-\frac{R}{L}t}\right] = Ri$$

$$i(t) = \frac{\varepsilon}{R}\left(1 - e^{-\frac{t}{R/L}}\right) \qquad \tau_{LR} = \frac{L}{R} \qquad \tau_{RC} = RC$$



Esercizio

$b = 20\,cm$
$m = 4g$
$R = 25\,\Omega$
$v_0 = 0{,}04\,m/s$
$B = 0{,}5T$



a) $v(x) = ?$
b) $v_1 = ?$ spira completamente entrata
c) $W_{diss} = ?$

Per calcolare la velocità devo prima calcolare la forza. Per la forza mi serve la corrente indotta. Quindi calcolo il flusso

$$\Phi(B) = B \cdot \overbrace{b \cdot x}^{\text{superficie}}$$

$$i_{IND} = -\frac{1}{R}\frac{d\Phi(B)}{dt} = -\frac{Bb}{R}\frac{dx}{dt} = -\frac{Bb}{R}v(x)$$

$$F = ma$$

$$\vec{F}_{att} = i_{IND}\,\vec{b} \times \vec{B} \qquad \vec{F}_{att} = |i_{IND}|bB = \frac{B^2b^2}{R}v(x) = -m\frac{dv}{dt}$$

$$\frac{B^2b^2}{R}\frac{dx}{\cancel{dt}} = -m\frac{dv}{\cancel{dt}} \qquad \frac{dv}{dx} = -\frac{B^2b^2}{Rm} = a$$

$$\int_{v_0}^{v} dv = -\frac{B^2b^2}{Rm}\int_0^x dx \Rightarrow v_1 = v_0 - \frac{B^2b^2}{Rm}x = v(b) > 0$$

$$W_{diss} = \frac{1}{2}mv_0^2 - \frac{1}{2}mv_1^2$$