

Elettrostatica: fenomeno in cui non c'è movimento



I fenomeni elettrostatici sono descrivibili come iterazioni tra protoni ed elettroni.

Legge di Coulomb



$$\vec{F}_c = \frac{1}{4\pi\epsilon_0} \frac{q_1 \cdot q_2}{r^2} \vec{u}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \cdot 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}$$

Se q_1 e q_2 hanno lo stesso segno \vec{F} e \vec{u} sono concordi, se invece hanno segno opposto \vec{F} e \vec{u} sono di verso opposto.

La legge di Coulomb, in formula, è simile alla formula di attrazione gravitazionale, con la differenza che la seconda è solo attrattiva.

Valore carica di un elettrone/protone:

$$|e^-| = |p^+| \approx 1,6 \cdot 10^{-19} \text{ C}$$

Esercizio

Si consideri un atomo di idrogeno semplice, calcolare la forza elettrostatica e la forza gravitazionale.

$$r = 0,53 \cdot 10^{-10} \text{ m}$$

$$m_e = 9,109 \cdot 10^{-31} \text{ Kg}$$

$$m_p = 1,67 \cdot 10^{-27} \text{ Kg}$$

$$\gamma = 6,67 \cdot 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{Kg}}$$

$$\frac{F_e}{F_g} = \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{r^2} \simeq 2,3 \cdot 10^{39}$$

Forza elettrostatica più forte della forza gravitazionale.

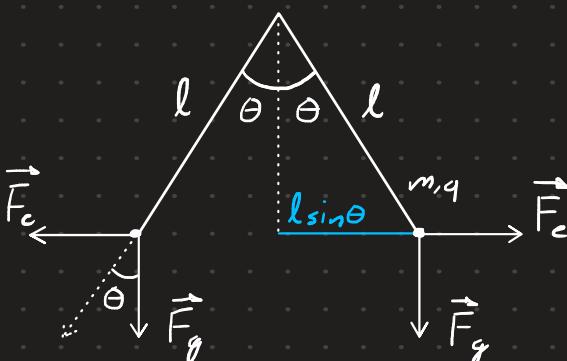
Esercizio

$$q(\theta) = ?$$

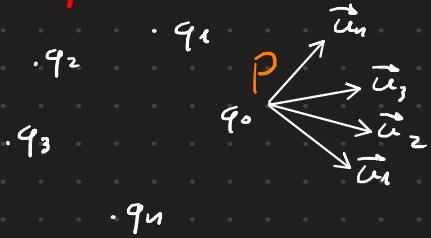
$$\tan \theta = \frac{F_e}{F_g} = \frac{1}{4\pi\epsilon_0} \frac{q^2}{(2l \sin \theta)^2}$$

$$q = 2l \sin \theta \sqrt{4\pi\epsilon_0 \cdot mg \cdot \tan \theta} \simeq \Theta^{3/2}$$

per angoli piccoli



Campo elettrostatico



$$\begin{aligned}\vec{F}_{q_0} &= \sum_{K=1}^N \vec{F}_K = \sum_{K=1}^N \frac{1}{4\pi\epsilon_0} \cdot \frac{q_K \cdot q_0}{r_K^2} \vec{u}_K \\ &= q_0 \cdot \underbrace{\frac{1}{4\pi\epsilon_0} \sum_{K=1}^N \frac{q_K}{r_K^2}}_{\vec{E}_P} \vec{u}_K\end{aligned}$$

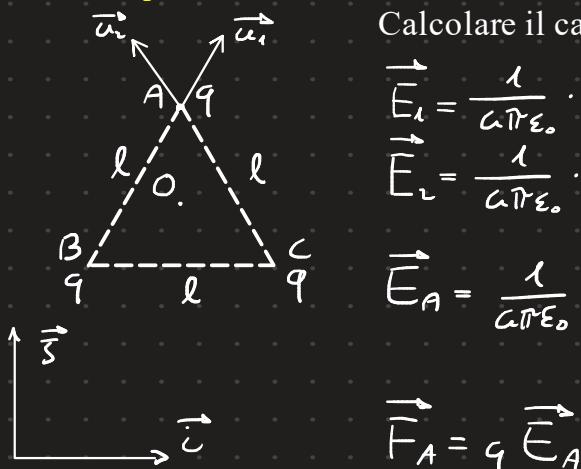
$$\vec{E}_{q_0} = \lim_{q \rightarrow 0} \frac{\vec{F}_q}{q_0}$$

Se ho solo una carica

$$\begin{aligned}P &\quad \vec{u} \\ q &\quad \vec{E}_P = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \vec{u} \\ &\quad \text{volt su metro} \\ [E] &= N/C = V/m\end{aligned}$$

Il segno della carica mi dà il verso del campo

Esercizio



Calcolare il campo elettrico e forza in A, e campo elettrico nel punto O.

$$\begin{aligned}\vec{E}_1 &= \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{l^2} \vec{u}_1 \\ \vec{E}_2 &= \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{l^2} \vec{u}_2 \\ \vec{u}_1 &= \frac{1}{2} \vec{i} + \frac{\sqrt{3}}{2} \vec{j} \\ \vec{u}_2 &= -\frac{1}{2} \vec{i} + \frac{\sqrt{3}}{2} \vec{j}\end{aligned}$$

$$\vec{E}_A = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{l^2} (\vec{u}_1 + \vec{u}_2) = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{l^2} \sqrt{3} \vec{j}$$

$$\vec{F}_A = q \vec{E}_A$$

componente verticale

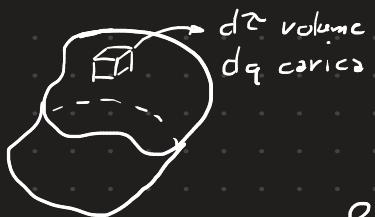
Nel punto O il campo è nullo per la simmetria del problema.

$$\vec{E}_{O,1} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{l/\sqrt{3}} \left(\frac{\sqrt{3}}{2} \vec{i} + \frac{1}{2} \vec{j} \right)$$

$$\vec{E}_{O,3} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{l/\sqrt{3}} \left(-\frac{\sqrt{3}}{2} \vec{i} + \frac{1}{2} \vec{j} \right)$$

$$\vec{E}_{O,2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{l/\sqrt{3}} (-\vec{i})$$

$$\vec{E}_O = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = 0$$



P

Oggetto caricato elettrostaticamente.

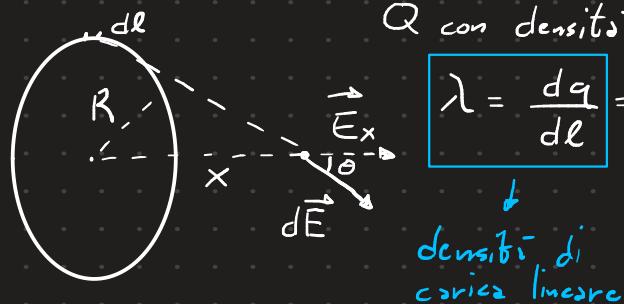
Quanto vale il campo elettrostatico generato dall'oggetto?

$$\rho \equiv \frac{dq}{dV} \quad \text{densità di carica}$$

$$d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} \vec{u} = \frac{1}{4\pi\epsilon_0} \frac{\rho dV}{r^2} \vec{u}$$

$$\vec{E}_P = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho dV}{r^2} \vec{u}$$

Esercizio anello



Q con densità uniforme

$$\lambda = \frac{dq}{dl} = \frac{Q}{2\pi R}$$

$$Q = \lambda \cdot 2\pi R$$

$$dE_x = \frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda dl}{x^2 + R^2} \cdot \underbrace{\frac{x}{\sqrt{x^2 + R^2}}}_{\cos\theta}$$

$$E_P = \int dE_x = \frac{1}{4\pi\epsilon_0} \frac{\lambda x}{(x^2 + R^2)^{3/2}} \int dl =$$

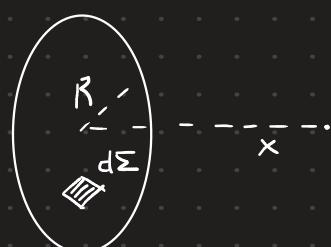
$$\vec{E}_P = \frac{1}{4\pi\epsilon_0} \frac{Q x}{(x^2 + R^2)^{3/2}} \vec{u} \underset{x \gg R}{\approx} \frac{1}{4\pi\epsilon_0} \frac{Q}{x^2} \frac{x}{|x|}$$

$$\vec{E}_{anello} = \frac{1}{4\pi\epsilon_0} \frac{Q x}{(x^2 + R^2)^{3/2}} \vec{u}$$

legge di Coulomb

questo termine mi dà solo il verso

Esercizio disco



$$\sigma = \frac{dq}{d\Sigma} = \frac{Q}{\pi R^2}$$

densità superficiale

$$\vec{E}_P = \frac{1}{4\pi\epsilon_0} \cdot \int_{\Sigma} \frac{\sigma}{r^2} \vec{u} d\Sigma$$

Divido la mia superficie in anelli infinitesimi.



$$dE = \frac{1}{4\pi\epsilon_0} \frac{dq \cdot x}{(x^2 + r^2)^{3/2}} = \frac{1}{4\pi\epsilon_0} \frac{\sigma \cdot 2\pi r \times dr}{(x^2 + r^2)^{3/2}}$$

$$dq = \sigma \cdot d\Sigma$$

$$d\Sigma = \pi (r + dr)^2 - \pi r^2 = 2r\pi dr$$

$$E = \frac{\sigma x}{2\epsilon_0} \int_0^R \frac{2r dr}{(x^2 + r^2)^{3/2}} = \frac{\sigma x}{2\epsilon_0} \left[-\frac{z}{\sqrt{x^2 + r^2}} \right]_0^R$$

$$= \frac{\sigma x}{2\epsilon_0} \left(\frac{1}{|x|} - \frac{1}{\sqrt{x^2 + R^2}} \right)$$

Cosa succede se il piano è infinito?

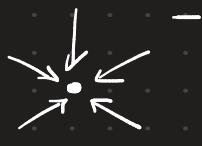
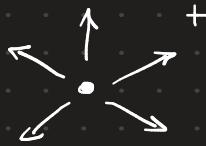
$$E = \frac{\sigma x}{2\epsilon_0} \left(\frac{1}{|x|} - \frac{1}{\sqrt{x^2 + R^2}} \right) \underset{R \gg 0}{\approx} \frac{\sigma}{2\epsilon_0} \cdot \frac{x}{|x|}$$

campo elettrostatico costante

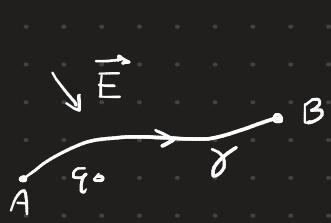
Linee di campo



le linee non si incontrano mai, eccetto nelle cariche



- Nella zona in cui le linee sono più dense il campo è più forte



$$\mathcal{L}_{AB} = \int_{\gamma} \vec{F}_{q_0} \cdot d\vec{s} = q_0 \int_A^B \vec{E} \cdot d\vec{s}$$

tensione campo elettrico

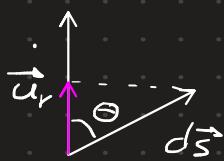
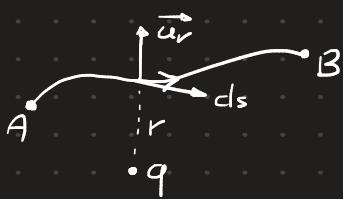
Un campo è conservativo quando l'integrale non dipende dal percorso, ma dal punto iniziale a quello finale.

$$\mathcal{L}_{AB} = -\Delta V = V_A - V_B = q_0 (V_A - V_B) \quad \text{differenza di potenziale}$$

$$V_A - V_B = \int_A^B \vec{E} \cdot d\vec{s} \iff \boxed{V_B - V_A = - \int_A^B \vec{E} \cdot d\vec{s}}$$

$$\mathcal{L}_\gamma(\vec{E}) = \oint_\gamma \vec{E} \cdot d\vec{s} = 0 \quad (\text{percorso chiuso})$$

Dimostriamo che il campo elettrostatico è conservativo:



$d\vec{s}$ è lo spostamento infinitesimo

$$\vec{a}_r \cdot d\vec{s} = ds \cdot \cos\theta = dr$$

$$\int_A^B \vec{E} \cdot d\vec{s} = \int_A^B \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \vec{a}_r \cdot d\vec{s} = \int_A^B \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} dr =$$

$$= \left[\frac{1}{4\pi\epsilon_0} \frac{-q}{r} \right]_A^B = \frac{1}{4\pi\epsilon_0} \frac{q}{r_B} - \frac{1}{4\pi\epsilon_0} \frac{q}{r_A} = V_A - V_B$$

Il campo Coulombiano è un campo conservativo: non dipende dal percorso, ma solo del punto finale ed iniziale.

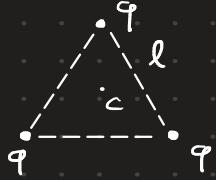
$V(r) = \frac{1}{4\pi\epsilon_0} \frac{q}{r} + K$

formula del potenziale Coulombiano

$V_{(\infty)} = 0 \quad (\text{è una scelta per eliminare } K)$

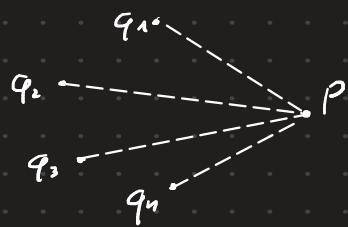
$[V] = \frac{1 \text{ J}}{1 \text{ C}} = 1 \text{ Volt}$

Esercizio



$\overline{qc} = r = l \frac{\sqrt{3}}{3}$

$V = \left(\frac{1}{4\pi\epsilon_0} \frac{q}{l\frac{\sqrt{3}}{3}} \right) \cdot 3$



$V_P = \sum \frac{1}{4\pi\epsilon_0} \frac{q_K}{r_K}$



$V_P = \int dV \frac{1}{4\pi\epsilon_0} \frac{S}{r}$
densità di carica

Energia potenziale di un sistema di cariche

$$U_{ext} = - \int_{\infty}^{V_{1,2}} \vec{F} \cdot d\vec{s} = \frac{1}{4\pi\epsilon_0} \frac{q_1 \cdot q_2}{r_{1,2}}$$

il lavoro è opposto al campo elettrico

$$U_{TOT} = \frac{1}{4\pi\epsilon_0} \left(\frac{q_1 \cdot q_2}{r_{1,2}} + \frac{q_1 \cdot q_3}{r_{1,3}} + \frac{q_2 \cdot q_3}{r_{2,3}} \right)$$

$$= \frac{1}{4\pi\epsilon_0} \sum_{i,j} \frac{1}{2} \frac{q_i \cdot q_j}{r_{ij}}$$

energia spesa per arrivare ad una determinata configurazione di cariche

$$U_{AB} = \int_A^B \vec{F} \cdot d\vec{s} = \frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2 = -q_0 (V_B - V_A)$$

$$\frac{1}{2} m v_B^2 + q V_B = \frac{1}{2} m v_A^2 + q V_A$$

Esercizio 2.18



$$\begin{aligned} l &= 20 \text{ cm} \\ q &= 6 \cdot 10^{-8} \text{ C} = 60 \text{ nC} \\ x_1 &= 15 \text{ cm} \\ x_2 &= 20 \text{ cm} \end{aligned}$$

1° modo

$$U_{ext} = \int_P^Q (-\vec{F}_c) \cdot d\vec{s} \quad \vec{F}_c = q_0 \frac{1}{4\pi\epsilon_0} \int_{-l}^0 \frac{\lambda dy}{(x-y)^2} \vec{i}$$

formula campo lineare

2° Metodo.

$$U_{ext} = + \Delta U = U_Q - U_P = q_0 (V_Q - V_P)$$

$$V(x) = \frac{1}{4\pi\epsilon_0} \int_{-l}^0 \frac{\lambda dy}{(x-y)} = \frac{\lambda}{4\pi\epsilon_0} \left[-\ln(x-y) \right]_{-l}^0 = \frac{\lambda}{4\pi\epsilon_0} \ln \left(\frac{x+l}{x} \right)$$

$$V_B - V_A = - \int_A^B \vec{E} \cdot d\vec{s}$$

$$dV = - \vec{E} \cdot d\vec{s} = - E_x dx - E_y dy - E_z dz$$

$$dV = V_{(\vec{x} + d\vec{s})} - V_{(\vec{x})} = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz$$

$$E_x = - \frac{\partial V}{\partial x}, \quad E_y = - \frac{\partial V}{\partial y}, \quad E_z = - \frac{\partial V}{\partial z}$$

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) = \text{NABLA}$$

$$\vec{\nabla} V = \left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z} \right) = \text{gradiente di } V \Rightarrow \vec{E} = \vec{\nabla} V$$

Esercizio anello



$$V_P = ?$$

$$\lambda = \frac{dq}{dl} = \frac{Q}{2\pi R} = \text{cost}$$

$$V_P = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r} = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda dl}{\sqrt{x^2 + R^2}} =$$

$$= \frac{1}{4\pi\epsilon_0} \frac{\lambda}{\sqrt{x^2 + R^2}} [2\pi R] = \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{x^2 + R^2}}$$

$$E_z - E_y = 0$$

$$E_x = - \frac{\partial V}{\partial x} = - \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{(x^2 + R^2)^3}} \left(-\frac{1}{2} \right) \cancel{x} = \frac{1}{4\pi\epsilon_0} \frac{Q \cdot x}{(x^2 + R^2)^{3/2}}$$

Esercizio disco



$$dq = \sigma d\sum = \sigma 2\pi r dr$$

$$dV(dq) = \frac{1}{4\pi\epsilon_0} \frac{dq}{\sqrt{x^2 + r^2}}$$

$$V_P = \int_0^R \frac{1}{4\pi\epsilon_0} \frac{dq}{\sqrt{x^2 + r^2}} = \int_0^R \frac{1}{4\pi\epsilon_0} \frac{\sigma 2\pi r dr}{\sqrt{x^2 + r^2}} = \frac{\sigma}{4\pi\epsilon_0} \left[2\sqrt{x^2 + r^2} \right]_0^R =$$

$$= \frac{\sigma}{2\epsilon_0} \left[\sqrt{x^2 + R^2} - |x| \right]$$

$$E_x = - \frac{\partial V}{\partial x} = \frac{\sigma}{2\epsilon_0} \left[-\frac{x}{\sqrt{x^2 + R^2}} + \frac{|x|}{x} \right]$$

$R \gg x$

$$\vec{E} = \frac{\sigma}{2\epsilon_0} \vec{u} = \frac{\sigma}{2\epsilon_0} \frac{|x|}{x} \vec{u} = \sigma 2\pi r dr$$

$$V = V_0 - \frac{\sigma}{2\epsilon_0} |x|$$

$$\vec{E} = -\frac{\partial V}{\partial x} = \frac{\sigma}{2\epsilon_0} \frac{|x|}{x} = E_x$$

Superfici equipotenziali

$$V(x, y, z) = V_0$$



$$dV = 0 = -\vec{E} \cdot d\vec{s} \quad \vec{E} \perp d\vec{s}$$

- Come le linee di campo anche queste non si intersecano mai, per ogni punto passa una sola superficie equipotenziale.



Dipolo elettrico

$\int d$

$$V_P = \frac{1}{4\pi\epsilon_0} \frac{q}{r_1} + \frac{1}{4\pi\epsilon_0} \frac{-q}{r_2} = \frac{q}{4\pi\epsilon_0} \left(\frac{r_2 - r_1}{r_1 \cdot r_2} \right)$$

Se $r \gg d$ $r_1 \cdot r_2 \approx r^2$ $r_2 - r_1 \approx d \cos \theta$

$$V_P \approx \frac{q}{4\pi\epsilon_0} \frac{d \cos \theta}{r^2} = \boxed{\frac{\vec{P} \cdot \vec{u}_r}{4\pi\epsilon_0 \cdot r^2}}$$

$\vec{P} = q\vec{d} \equiv$ momento di dipolo

$$\vec{P} = \sum_k \vec{r}_k q_k \quad Q = \sum_k q_k$$

$$V_P = \frac{1}{4\pi\epsilon_0} \frac{P \cos \theta}{r^2}$$

$$\vec{E}_r = -\frac{\partial V}{\partial r} = \frac{1}{4\pi\epsilon_0} \frac{2P \cos \theta}{r^3}$$

$$\vec{E}_\theta = -\frac{1}{r} \frac{\partial V}{\partial \theta} = \frac{1}{4\pi\epsilon_0} \frac{P \sin \theta}{r^3}$$

$$\boxed{\vec{E} = \frac{P}{4\pi\epsilon_0 r^3} (2 \cos \theta \vec{u}_r + \sin \theta \vec{u}_\theta)}$$



\vec{E} uniforme

$$\vec{F} = q \vec{E} \quad (\text{il segno della forza dipende dalla carica + o -})$$

$$\vec{M} = \frac{\vec{d}}{2} \times \vec{F}_+ + \left(-\frac{\vec{d}}{2}\right) \times \vec{F}_- = \frac{\vec{d}}{2} \times q \vec{E} + \left(-\frac{\vec{d}}{2}\right) \times (-q \vec{E}) = q \frac{\vec{d}}{2} \times \vec{E}$$

$\vec{M} = \vec{P} \times \vec{E}$

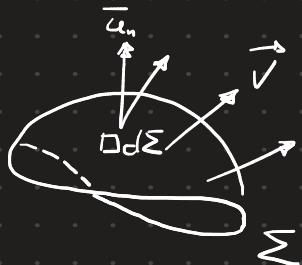
momento di dipolo.

$$U = U_+ + U_- = qV_A - qV_B = q(V_A - V_B) \approx -q\vec{E} \cdot \vec{d}$$

$$U = -|\vec{p}| |\vec{E}| \quad U_{\min} = -|\vec{p}| |\vec{E}| \quad U_{\max} = +|\vec{p}| |\vec{E}|$$

Def. di Flusso

$$\Phi_{\Sigma}(\vec{v}) = \int_{\Sigma} \vec{v} \cdot \vec{n} d\Sigma$$



$$\Phi_{\Sigma}(\vec{v}) = v \cdot \Sigma \cdot \cos \theta$$

Teorema di Gauss

$$\Phi_{\Sigma}(\vec{E}) = \oint_{\Sigma} \vec{E} \cdot \vec{n} d\Sigma = \sum \frac{q_{\text{interna}}}{\epsilon_0}$$

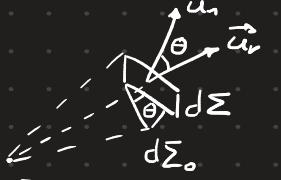
Sommatario

(campo * normale * superficie)

Σ è chiusa

Dimostrazione

$$\vec{E} = \sum_{k=1}^N \vec{E}_k \quad \vec{E}_k \text{ è generato da } q_k$$



$$\begin{aligned} \vec{E}_k \cdot \vec{n} d\Sigma &= \frac{1}{4\pi\epsilon_0} \frac{q_1}{r^2} \vec{u}_r \cdot \vec{n} d\Sigma \\ &= \frac{1}{4\pi\epsilon_0} q_1 \frac{d\Sigma}{r^2} = \frac{1}{4\pi\epsilon_0} q_1 d\Omega \end{aligned}$$

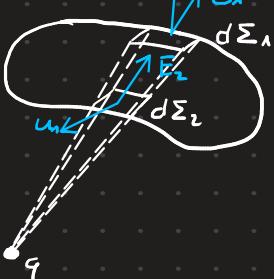
cos theta

angolo solido

$$d\Omega = \frac{d\ell}{r}$$



$$\oint_{\Sigma} \vec{E} \cdot \vec{n} d\Sigma = \frac{q_1}{4\pi\epsilon_0} \oint_{\Sigma} d\Omega = \frac{q_1}{4\pi\epsilon_0} 4\pi = \frac{q_1}{\epsilon_0}$$

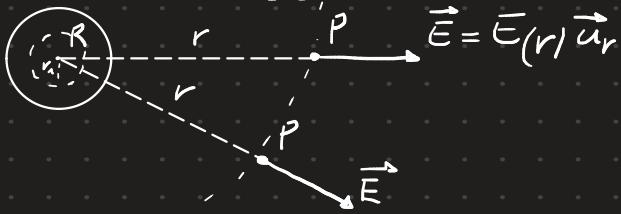


$$\Phi(\vec{E}) = \Phi_{\Sigma_1}(\vec{E}) + \Phi_{\Sigma_2}(\vec{E}) = \frac{q_1}{4\pi\epsilon_0} \Omega_1 - \frac{q_1}{4\pi\epsilon_0} \Omega_2 = 0$$

Esercizio

$$\rho = \frac{dq}{d\tau} = \text{cost}$$

densità di carica costante nella sfera



$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{\rho d\tau}{r^2} \vec{u}_r$$

$$\Phi_{Sr}(\vec{E}) = E(r) 4\pi r^2 = \frac{\rho \frac{4}{3}\pi R^3}{\epsilon_0} = \frac{Q_{int}}{\epsilon_0}$$

$$r \geq R$$

$$E(r) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \quad \text{campo esterno}$$

$$Q = \rho \frac{4}{3}\pi R^3$$

$$\Phi_{Sr}(\vec{E}) = E(r) 4\pi r^2 = \frac{\rho \frac{4}{3}\pi r_n^3}{\epsilon_0} = \frac{Q r_n^3}{\epsilon_0 R^3}$$

$$E(r) = \begin{cases} \frac{1}{4\pi\epsilon_0} \frac{Q}{R^3} r & r < R \\ \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} & r \geq R \end{cases}$$



$$\rho = \text{cost}$$

$$E(r) = \begin{cases} \frac{1}{4\pi\epsilon_0} \frac{Q}{R^3} r & r < R \\ \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} & r \geq R \end{cases} \quad \begin{matrix} \text{interno} \\ \text{esterno} \end{matrix}$$

$$V(r) = - \int E(r) dr$$

$$V(r) = \begin{cases} -\frac{1}{4\pi\epsilon_0} \frac{Q}{R^3} \frac{r^2}{2} + C_1 & r < R \\ \frac{1}{4\pi\epsilon_0} \frac{Q}{r} + C_2 & r \geq R \end{cases} \quad \begin{matrix} \text{interno} \\ \text{esterno} \end{matrix}$$

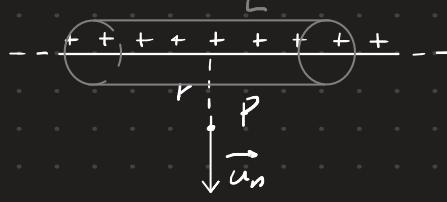
Come trovo C_1 e C_2 ?

$$V(\infty) = 0 = C_2$$

$$V(R_+) = V(R_-) \iff \frac{1}{4\pi\epsilon_0} \frac{Q}{R} = \frac{-1}{8\pi\epsilon_0} \frac{Q}{R} + C_1 \implies C_1 = \frac{3}{8\pi\epsilon_0} \frac{Q}{R}$$

$$\frac{Q}{R^3} \frac{R^2}{2}$$

Esempi di utilizzo del Th. di Gauss



$$\lambda = \text{cost} = \frac{dq}{dl}$$

$$dq = \lambda dl$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda dl}{r^2} \hat{u}_r$$

$$\phi = \oint \vec{E} \cdot \hat{u}_n d\Sigma$$

$$\vec{E} = E(r) \hat{u}_r$$

$$\phi(\vec{E}) = E(r) 2\pi r L = \frac{\lambda L}{\epsilon_0} \rightarrow \text{cariche interne}$$

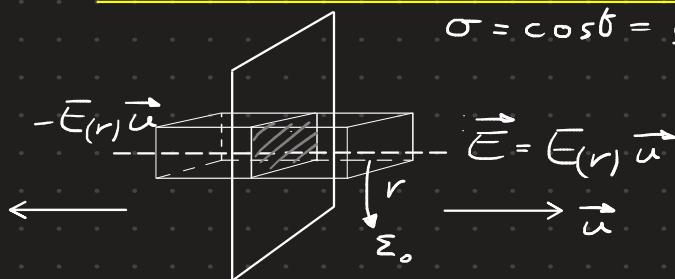
Nelle superfici il campo elettrico è perpendicolare alla normale: il flusso fa 0.

$$E(r) = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r}$$

$$V(r) = - \int E(r) dr = - \frac{1}{2\pi\epsilon_0} (\ln r + C_1) = - \frac{\lambda}{2\pi\epsilon_0} \ln \frac{r}{r_0}$$

$$\ln C_1 = \ln r_0$$

$$\sigma = \cos\theta = \frac{dq}{d\Sigma}$$



$$\phi(\vec{E}) = 0 + 2 \sum_{\Sigma} \vec{E}(r) \stackrel{\text{Gauss}}{=} \frac{\sigma \Sigma}{\epsilon_0} \Rightarrow E(r) = \frac{\sigma}{2\epsilon_0}$$

Due proprietà fondamentali dell'elettrostatica viste fino ad ora

$$C_r(\vec{E}) = \oint_r \vec{E} d\vec{s} = 0$$

• Campo conservativo \rightarrow posso derivare il potenziale

$$\phi_{\Sigma}(\vec{E}) = \oint_{\Sigma} \vec{E} \cdot \hat{u}_n d\Sigma = \frac{q_{\text{int}}}{\epsilon_0}$$

• Posso ricavare il campo

Voglio riscrivere queste due equazioni usando delle derivate

Th. della Divergenza

$$\oint_{\Sigma} \vec{v} \cdot \hat{u}_n d\Sigma = \int_{\tau} \vec{\nabla} \cdot \vec{v} d\tau$$

superficie

volume

divergenza



$$\Sigma = \partial \tau$$

superficie che è bordo di un volume

$$\vec{\nabla} \cdot \vec{v} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

Assomiglia al Th. fondamentale del calcolo integrale

$$f(b) - f(a) = \int_a^b f'(x) dx$$

$$\oint_{\Sigma} \vec{E} \cdot \vec{n} d\Sigma = \frac{q_{int}}{\epsilon_0}$$

$$\int_{\mathcal{C}} \vec{\nabla} \cdot \vec{E} d\gamma = \int_{\Sigma} \frac{\rho}{\epsilon_0} d\Sigma \quad (\text{esprimo la carica come l'integrale della densità di carica})$$

Quindi posso riscrivere il teorema di Gauss come

$$\phi_{\Sigma}(\vec{E}) = \oint_{\Sigma} \vec{E} \cdot \vec{n} d\Sigma = \frac{q_{int}}{\epsilon_0} \iff \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

Teorema del rotore

$$\oint_{\gamma} \vec{v} \cdot d\vec{s} = \iint_{\Sigma} (\vec{\nabla} \times \vec{v}) \cdot \vec{n} d\Sigma$$

linea superficie
 Σ $\gamma = \partial \Sigma$
 rotore di v



abbiamo una linea che è bordo di una superficie

$$\oint_{\gamma} \vec{E} \cdot d\vec{s} = 0 = \iint_{\Sigma} (\vec{\nabla} \times \vec{E}) \cdot \vec{n} d\Sigma \quad \forall \Sigma / \gamma = \partial \Sigma$$

$$\Rightarrow \vec{\nabla} \times \vec{E} = 0$$

Posso riscrivere la circuitazione come

$$C_{\gamma}(\vec{E}) = \oint_{\gamma} \vec{E} \cdot d\vec{s} = 0 \iff \vec{\nabla} \times \vec{E} = 0$$

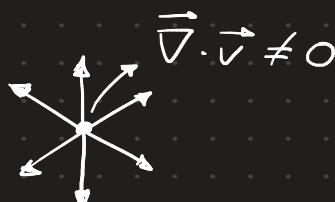
Spiegazione geometrica



rotore non nullo:

punti in cui le linee di campo girano intorno.

Nel campo elettrico le linee di campo non si uniscono perciò è sempre 0



divergenza non nulla:

punti in cui entrano o escono le linee di campo

$$C_y(\vec{E}) = \oint_{\gamma} \vec{E} \cdot d\vec{s} = 0 \iff \vec{\nabla} \times \vec{E} = 0 \iff \vec{E} = -\vec{\nabla} V$$

$$\Phi_{\Sigma}(\vec{E}) = \oint_{\Sigma} \vec{E} \cdot \vec{n} d\Sigma = \frac{q_{int}}{\epsilon_0} \iff \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \iff \vec{\nabla} \cdot (\vec{\nabla} V) = -\frac{\rho}{\epsilon_0}$$

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho}{\epsilon_0}$$

$$\text{eq. cl. Poisson} \rightarrow \Delta V = -\frac{\rho}{\epsilon_0}$$

$$\Delta V = -\frac{\rho}{\epsilon_0} \implies V = \frac{1}{4\pi\epsilon_0} \int_{\Sigma} \frac{\rho d\Sigma}{r}$$

Conduttori:

Proprietà in condizioni elettrostatiche

$$1 - \vec{E}_{\text{interno}} = 0$$



Le cariche sul conduttore si modificano per creare un campo uguale e opposto al campo esterno



2 - La carica in eccesso si deposita sulla superficie (Dim. con Th. di Gauss)

Siccome il campo interno è 0, qualsiasi per qualsiasi superficie interna che possiamo prendere il flusso è 0.

3 - Tutto il conduttore è allo stesso potenziale

$$V_A - V_B = - \int_A^B \vec{E} \cdot d\vec{s} = 0$$

B → il campo è nullo



4 - Teorema di Coulomb

$$\vec{E}_{\text{sup}} = \frac{\sigma}{\epsilon_0} \vec{n}$$

Dimostrazione



$$\sigma = \frac{dq}{d\Sigma} \Rightarrow dq = \sigma d\Sigma$$



$$\Phi(\vec{E}) = 0 + 0 + \vec{E} d\Sigma = \frac{\sigma d\Sigma}{\epsilon_0}$$

laterale base
interna

Esempio A.1



Come si dispongono le cariche sulle due sfere?

$$V_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1}{R_1} \quad V_2 = \frac{1}{4\pi\epsilon_0} \frac{q_2}{R_2}$$

$$\begin{cases} \frac{q_1}{R_1} = \frac{q_2}{R_2} \\ q_1 + q_2 = q \end{cases}$$

Esercizio 3.20

$$\begin{aligned} R &= 10 \text{ cm} \\ q &= 8 \cdot 10^{-5} \text{ C} \\ \rho(r) &= br \end{aligned}$$



- Calcolare b
- Campo elettrostatico $\vec{E}(r)$
- Differenza di potenziale tra il centro O e la superficie della sfera

$$\begin{aligned} q &= \int_V \rho \, dV = \int_0^R br \cdot 4\pi r^2 dr = \\ &= \int_0^R ab \pi r^3 dr = \pi b R^4 \\ \Rightarrow b &= \frac{q}{\pi R^4} \end{aligned}$$

$$\begin{aligned} V &= \frac{4}{3} \pi r^3 \\ dV &= 4\pi r^2 dr \end{aligned}$$

la carica è l'integrale della densità sul volume!!!

$$\vec{E} = E(r) \hat{a}_r$$

$$\Phi_s(E) = \vec{E}(r) 4\pi r^2 = \frac{q_{int}(r)}{\epsilon_0}$$

$$r \geq R \quad q_{int}(r) = q_r$$

$$r < R \quad q_{int}(r) = \int_0^r 4\pi y b y^2 dy = \pi b r^4 = q \frac{r^4}{R^4}$$

$$\vec{E}(r) = \begin{cases} \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{a}_r & r \geq R \\ \frac{1}{4\pi\epsilon_0} \frac{q}{R^4} r^2 \hat{a}_r & r < R \end{cases}$$

$$V_0 - V_R = - \int_R^0 \vec{E}(r) dr = \int_0^R \frac{1}{4\pi\epsilon_0} \frac{q}{R^4} r^2 dr = \frac{1}{12\pi\epsilon_0} \frac{q}{R}$$

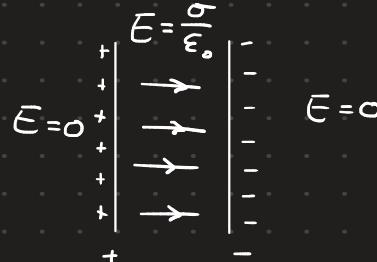
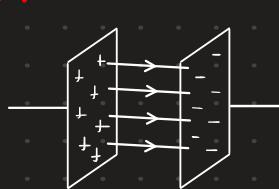
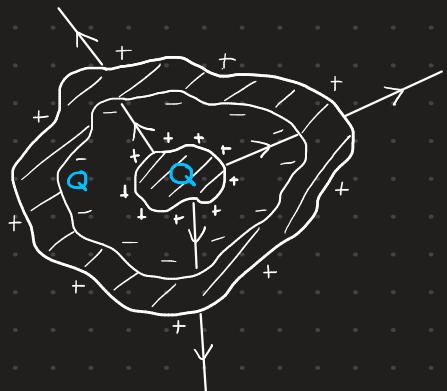
• Conduttori cavi



Per il Th. di Gauss la carica totale interna è 0.

$$C_V(\vec{E}) = \int_A^B \vec{E} \cdot d\vec{s} + \int_B^A \vec{E} \cdot d\vec{s}$$

• CONDESATORI



$$C = \text{capacità} = \left| \frac{Q}{\Delta V} \right|$$

$$[C] = \frac{1C}{1V} = 1F \quad (\text{Fard})$$

Esercizio

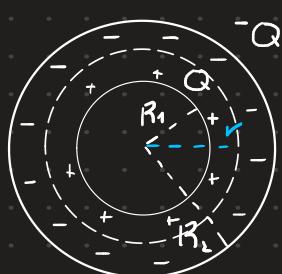


$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 \Sigma}$$

$$\Delta V = \int_A^B \vec{E} \cdot d\vec{s} = \frac{Q}{\epsilon_0 \Sigma} \int ds = \frac{Q}{\epsilon_0 \Sigma} d$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{\frac{Q}{\epsilon_0 \Sigma} d} = \epsilon_0 \frac{\Sigma}{d}$$

Esercizio



$$E(r) \propto \pi r^2 = \frac{Q}{\epsilon_0}$$

$$E(r) = \frac{1}{\epsilon_0 \pi} \frac{Q}{r^2}$$

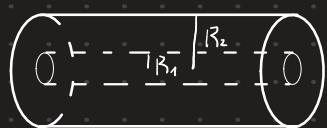
$$\Delta V = \left| \int_{R_1}^{R_2} E(r) dr \right| = \frac{Q}{\epsilon_0 \pi} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{\frac{Q}{\epsilon_0 \pi} \frac{R_2 - R_1}{R_1 R_2}} = \epsilon_0 \frac{\epsilon_0 \pi R_2 R_1}{R_2 - R_1} \underset{d \ll R_2 - R_1}{\approx} \epsilon_0 \frac{\epsilon_0 \pi R^2}{d}$$

$$R_2 - R_1 = d \ll R_{1,2}$$

$$R_1 \sim R_2 \sim R$$

Esercizio



$$E(r) = \frac{Q}{2\pi r L} = \frac{Q}{\epsilon_0}$$

$d \ll R_1, R_2$

$$E(r) = \frac{1}{2\pi\epsilon_0} \frac{Q}{r}$$

$$R_2 = R_1 + d$$

$$\Delta V = \int E dr = \frac{1}{2\pi\epsilon_0} \frac{Q}{L} \ln \frac{R_2}{R_1}$$

$$\ln \frac{R_2+d}{R_1} = \ln \left(1 + \frac{d}{R_1}\right) \approx \frac{d}{R_1}$$

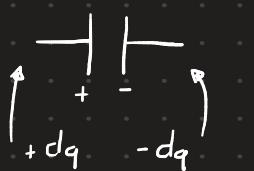
$$C = \frac{Q}{\Delta V} = \frac{2\pi\epsilon_0 L}{\ln\left(\frac{R_2}{R_1}\right)} \approx \epsilon_0 \frac{2\pi L}{d/R_1} = \epsilon_0 \frac{2\pi R_1 L}{d}$$

Serie $\frac{Q}{C_1} \parallel \frac{Q}{C_2} \parallel \dots \parallel \frac{Q}{C_N}$

$$\frac{1}{C_T} = \frac{1}{C_1} + \dots + \frac{1}{C_N}$$

Parallelo $\Delta V = \frac{1}{C_1 + C_2 + \dots + C_N} = \Delta V \quad C_T = C_1 + \dots + C_N$

Energia condensatori



$$dU_{ext} = +dq \Delta V = +dq \frac{q}{C}$$

$$U_{TOT} = \int_0^Q \frac{q}{C} dq = \frac{1}{2} \frac{Q^2}{C} = U_e$$

energia elettrostatica
accumulata sul
condensatore

$$U_e = \frac{1}{2} Q \Delta V = \frac{1}{2} C \Delta V^2$$

Per il condensatore piano la capacità vale

$$C = \epsilon_0 \sum_d \tau \text{ volume}$$

$$U_e = \frac{1}{2} C \Delta V^2 = \frac{1}{2} \epsilon_0 \sum_d \Delta V^2 = \frac{1}{2} \epsilon_0 \sum_d \frac{\epsilon^2 d^2}{\cancel{d}} = \frac{1}{2} \epsilon_0 \overbrace{\epsilon^2}^{\tau} \overbrace{\sum d}^{V}$$

$\Delta V = E \cdot d$ solo per condensatore piano o quando E è costante

$$u_e = \frac{dU_e}{dz} = \frac{1}{2} \epsilon_0 E^2 \quad (\text{densità di energia elettrostatica})$$



$$U_e = \int_z \frac{1}{2} \epsilon_0 E^2 dz$$

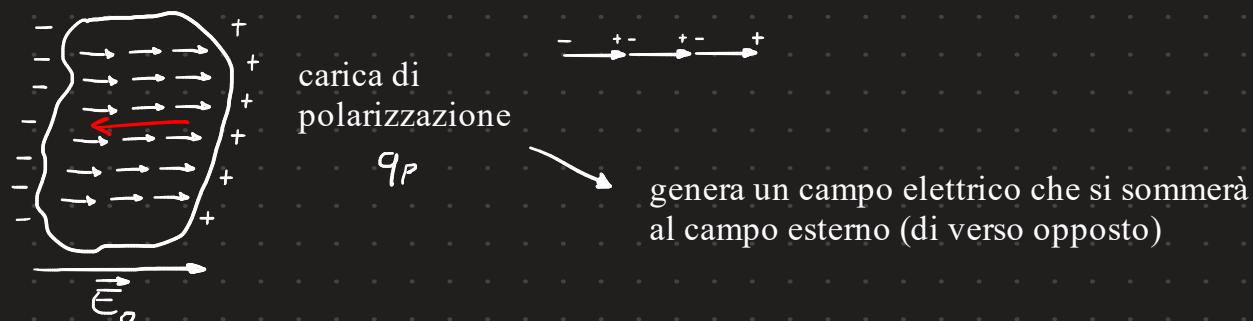
energia accumulata nella regione di spazio grazie al campo elettrico

Campo in materiali isolanti: dielettrici

- Polarizzazione per spostamento



- Polarizzazione per orientamento



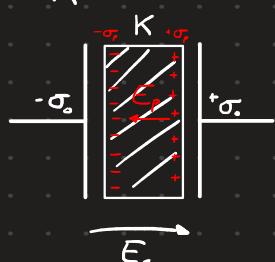
$$\vec{P} = \text{vettore di polarizzazione} = \frac{d\vec{p}}{d\tau}$$

$$\sigma_p = \vec{P} \cdot \vec{u}_n$$

densità superficiale
di carica di polarizzazione



$$\vec{P} = \epsilon_0 \frac{K-1}{K} \vec{E}_0 \quad K = \text{costante dielettrica} > 1$$



$$E_0 = \frac{\sigma_0}{\epsilon_0}$$

$$P = \frac{K-1}{K} \sigma_0 = \sigma_p$$

$$E_p = \frac{\sigma_p}{\epsilon_0}$$

$$E_0 = \frac{\sigma_0}{\epsilon_0}$$

$$K\epsilon_0 = \epsilon$$

$$E_p = \frac{\sigma_p}{\epsilon_0}$$

$$E = E_0 - E_p = \frac{\sigma_0}{\epsilon_0} - \frac{K-1}{K} \frac{\sigma_0}{\epsilon_0} = \frac{\sigma_0}{K\epsilon_0} = \frac{E_0}{K}$$

campo elettrico nel dielettrico:
campo elettrico nel vuoto diviso
la costante dielettrica

$$\Delta V = \frac{\Delta V_0}{K}$$

$$C = K \epsilon_0 \frac{\sum}{d}$$

Esercizio



$$\sum = 600 \text{ cm}^2$$

$$\Delta V = z KV$$

$$d = 5 \text{ mm}$$

$$K_1 = 5 \quad K_2 = 2$$

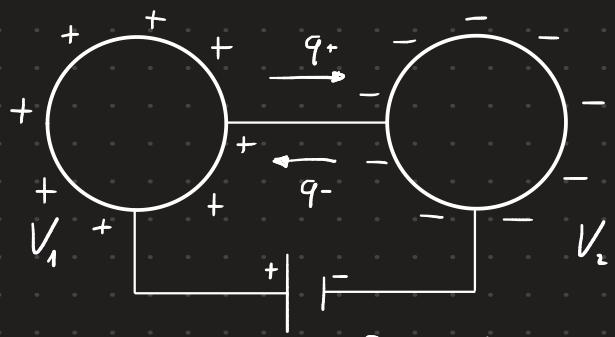
$$1) C = ? \quad 2) q = ? \quad 3) U_e = ?$$

$$C_T = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{K_1 \epsilon_0 \frac{\sum}{d/2}} + \frac{1}{K_2 \epsilon_0 \frac{\sum}{d/2}}$$

$$\Delta V = q_T \cdot C_T \rightarrow q_T = \frac{\Delta V}{C_T}$$

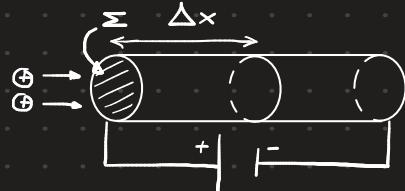
$$U_e = \frac{1}{2} q \Delta V$$

Elettrodinamica



$$V_1 > V_2$$

$\mathcal{E} = \Delta V$ (il generatore mantiene costante la differenza di potenziale)



$$i = \frac{dq}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\Delta q}{\Delta t} = \text{CORRENTE ELETTRICA}$$

$$[i] = \frac{1C}{1s} = 1A$$

$$\langle \vec{v} \rangle \neq 0 \quad \langle \vec{v} \rangle = \vec{v}_d = \text{velocità di deriva}$$

$$\text{in } \Delta t \text{ ogni carica percorre } \Delta x = \vec{v}_d \cdot \Delta t \quad \Delta q = n q^+ \Delta \Sigma = \\ \frac{\Delta q}{\Delta t} = di = n q^+ v_d d\Sigma \cos \theta = \vec{j} \cdot \vec{u}_n \cdot d\Sigma \quad n q^+ d\Sigma \cos \theta \vec{v}_d \Delta t$$

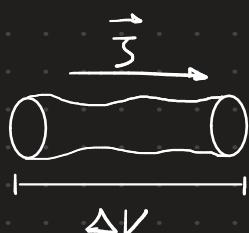
$$\vec{j} = n q^+ \vec{v}_d = \text{densità di corrente} \quad [j] = \frac{1A}{1m^2}$$

$$i = \int_{\Sigma} \vec{j} \cdot \vec{u}_n d\Sigma = \oint_{\Sigma} (\vec{j})$$

$$\vec{j} = n_+ q^+ \vec{v}_+ - n_- |q^-| \vec{v}_-$$

Legge di Ohm

$$\vec{j} = \sigma \vec{E} = \frac{1}{\rho} \vec{E}$$



$$\sigma = \text{conducibilità propria di ogni materiale} = \frac{1}{\rho}$$

$\rho = \text{resistività}$

$$\Delta V = \int_A^B \vec{E} \cdot d\vec{s} = \int_A^B \rho \vec{j} \cdot d\vec{s} = \int_A^B \rho j ds = \int_A^B \rho \frac{i}{\Sigma} ds = \\ = i \int_A^B \frac{\rho}{\Sigma} ds$$

$$R = \int_A^B \frac{\rho}{\Sigma} ds = \frac{\rho l}{\Sigma}$$

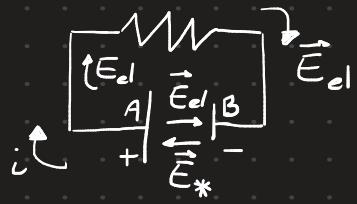
se ρ, Σ costanti

$$[\rho] = \Omega \cdot m$$

$$\Delta V = R \cdot i$$

$$[R] = \frac{1V}{1A} = 1\Omega$$

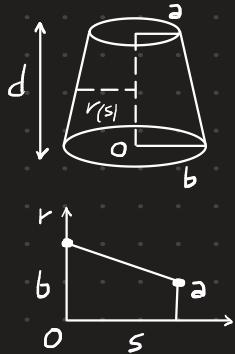
Legge di Ohm generalizzata



$\vec{E}_* = \text{campo elettromotore}$
 $\vec{E}_* = -\vec{E}_{el}$ (all'interno del generatore)

$$C(\vec{E}) = \oint_S \vec{E} \cdot d\vec{s} = \int_A^B \vec{E}_{el} \cdot d\vec{s} + \int_B^A \vec{O} \cdot d\vec{s} = R_i i$$

Esercizio 5.6



ρ data

$R = ?$

$$R = \sqrt{\frac{\rho \sum ds}{\sum (s)}} = \rho \sqrt{\frac{\sum ds}{\sum (s)}}$$

$$r - b = \frac{a - b}{d - 0} (s - 0) \quad r = b + \frac{a - b}{d} s$$

$$\sum = \pi r^2 = \pi \left(b + \frac{a - b}{d} s \right)^2$$

$$R = \rho \sqrt{\int_0^d \frac{ds}{\pi \left(b + \frac{a - b}{d} s \right)^2}} = \frac{\rho}{\pi} \left[- \frac{1}{b + \frac{a - b}{d}} \times \frac{d}{a - b} \right]_0^d$$

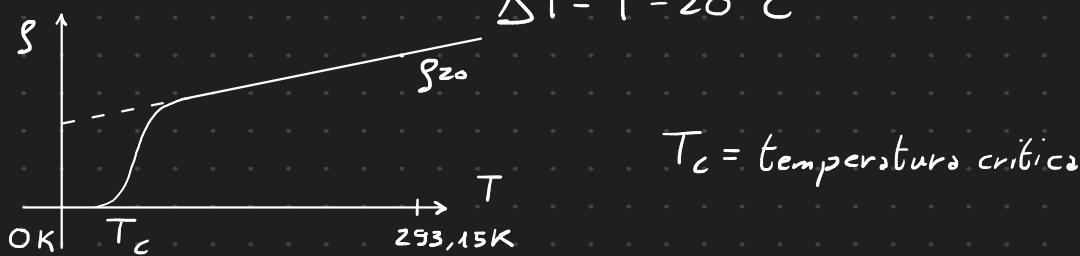
$$= \frac{\rho d}{\pi a b}$$

Effetti termici sulla resistenza

$$\rho(T) = \rho_{20} (1 + \alpha \Delta T)$$

ρ_{20} = resistività a $20^\circ C$

$$\Delta T = T - 20^\circ C$$



T_c = temperatura critica

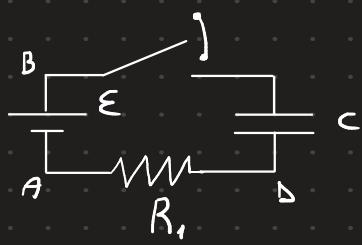
$$dL = d_q \cdot \Delta V \cdot i$$

$$P = \frac{dL}{dq} = \left[\frac{d_q}{dt} \right] \Delta V = i \Delta V = R \cdot i^2 \Rightarrow \text{effetto Joule}$$

$$E_J = \int P dt = \int_0^{T_1} R i^2 dt$$

Carica e scarica di un condensatore (circuito RC)

Carica



$$V_B - V_A = (V_B - V_D) + (V_D - V_A)$$

$$\mathcal{E} = \frac{q(t)}{C} + R_i i(t)$$

$$\mathcal{E} = \frac{q(t)}{C} + R \frac{dq(t)}{dt}$$

$$\mathcal{E} - \frac{q}{C} = R \frac{dq}{dt} \Rightarrow dt = \frac{R}{\mathcal{E} - q} dq = \frac{R}{\frac{\mathcal{E}C - q}{C}} dq = \frac{CR}{\mathcal{E}C - q} dq$$

$$\Rightarrow \frac{dt}{RC} = \frac{dq}{\mathcal{E}C - q}$$

$$\int_0^t \frac{dt}{RC} = \int_0^q \frac{dq}{\mathcal{E}C - q}$$

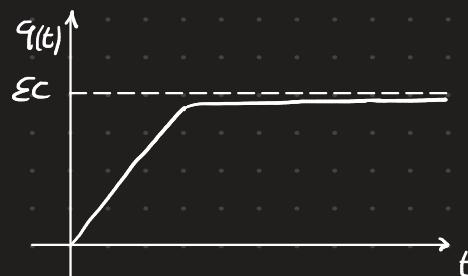
al tempo $t=0$ la carica è 0 siccome il circuito è aperto

$$\frac{t}{RC} = \left[-\ln(\mathcal{E}C - q) \right]_0^q = -\ln(\mathcal{E}C - q) + \ln(\mathcal{E}C) = -\ln \frac{\mathcal{E}C - q(t)}{\mathcal{E}C}$$

$$-\frac{t}{RC} = \ln \left(1 - \frac{q(t)}{\mathcal{E}C} \right) \quad e^{-\frac{t}{RC}} = 1 - \frac{q}{\mathcal{E}C}$$

$$q(t) = \mathcal{E}C \left[1 - e^{-\frac{t}{RC}} \right] \quad \tau = RC \quad [\Omega \cdot F = s]$$

costante di tempo



$$t = 4,5 \tau$$

$$e^{-\frac{t}{\tau}} \ll 1$$

$$\mathcal{E} = \frac{q}{C} + Ri$$

condensatore resistenza

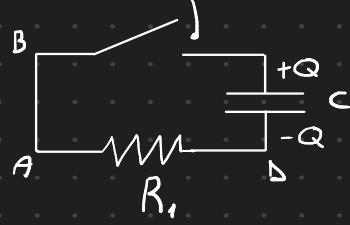
$$P_{gen} = \mathcal{E} \cdot i(t) = \frac{1}{C} q(t) i(t) + R i(t)^2$$

$$L_{gen} = \int_0^{+\infty} P_{gen}(t) dt = \int_0^{+\infty} \frac{1}{C} q \frac{dq}{dt} dt + \int_0^{+\infty} R i^2 dt =$$

$$\frac{1}{2} \frac{q^2}{C} = \frac{1}{2} C \mathcal{E}^2 = U_e$$

$$\int_0^{+\infty} R i^2 dt = \frac{\mathcal{E}^2}{R} \int_0^{+\infty} e^{-\frac{2t}{\tau}} dt = \frac{\mathcal{E}^2}{R} \left[-\frac{\tau}{2} e^{-\frac{2t}{\tau}} \right] = \frac{\mathcal{E}^2}{R} \frac{\tau}{2} = \frac{1}{2} C \mathcal{E}^2$$

Scarica



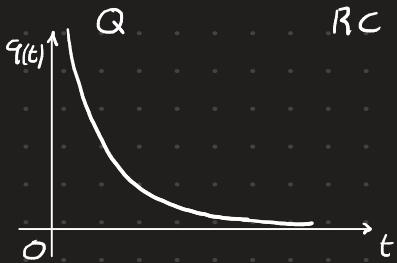
$$t=0 \rightarrow Q = \text{carica in } C$$

$$\frac{\epsilon}{C} = \frac{Q(t)}{C} + R \frac{dq(t)}{dt}$$

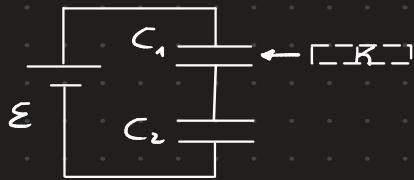
$$-\frac{q}{C} = R \frac{dq}{dt} \Rightarrow -\frac{dt}{RC} = \frac{dt}{q}$$

$$-\int_0^t \frac{dt}{RC} = \int_0^q \frac{dq}{q}$$

$$-\frac{t}{RC} = \ln \frac{q}{Q} \quad q(t) = Q e^{-\frac{t}{RC}}$$



Esercizio 4.10



$$\begin{aligned} C_1 &= 500 \text{ pF} \\ C_2 &= 100 \text{ pF} \\ \epsilon &= 600 \text{ V} \\ K &= \zeta \end{aligned}$$

a) Δq energia dal generatore
 b) variazione ΔV_t (ai capi di C_1)
 c) energia fornita dal generatore

$$\textcircled{a} \quad \epsilon = \frac{q}{C_T} \rightarrow q = \epsilon C_T \quad q \text{ prima dielettrico}$$

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 C_2}{C_1 + C_2}$$

$$C_T' = \frac{K C_1 C_2}{K C_1 + C_2}$$

$$\Delta q = C_T' \epsilon - C_T \epsilon = (C_T' - C_T) \epsilon$$

\textcircled{b}

$$V_t = \frac{Q}{C_1} = \frac{C_T}{C_1} \epsilon$$

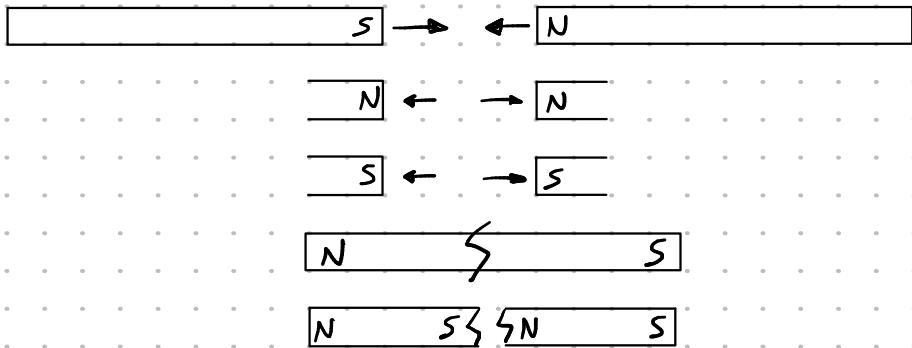
$$\Delta V_t = V_t' - V_t$$

$$V_t' = \frac{Q'}{C_1'} = \frac{C_T' \epsilon}{K C_1}$$

\textcircled{c}

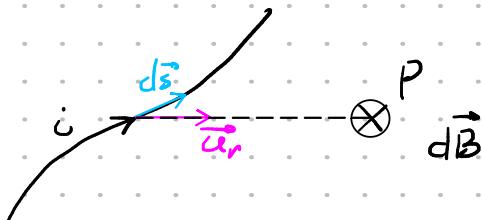
$$E_{\text{gen}} = \Delta U_e = \frac{1}{2} C_T' \epsilon^2 - \frac{1}{2} C_T \epsilon^2 = \frac{1}{2} (C_T' - C_T) \epsilon^2$$

Magnetismo



I^a legge di Laplace

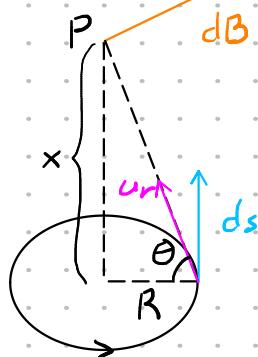
B = campo magnetico
 $[B] = \text{Tesla (T)}$



$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i |d\vec{s} \times \vec{ur}|}{r^2}$$

$$\frac{\mu_0}{4\pi} = 10^{-7} \frac{\text{T} \cdot \text{m}}{\text{A}}$$

Esercizio SPIRA CIRCOLARE



$$dB_z = \frac{\mu_0}{4\pi} \frac{i |d\vec{s} \times \vec{ur}|}{r^2} \cos \theta$$

$$= \frac{\mu_0}{4\pi} i \frac{ds}{R^2 + x^2} \frac{R}{\sqrt{x^2 + R^2}}$$

$$\vec{B}_z = \int dB_z = \frac{\mu_0}{4\pi} i \frac{R}{(x^2 + R^2)^{3/2}} \int d\vec{s} =$$

$$= \frac{\mu_0}{4\pi} i \frac{2\pi R^2}{(x^2 + R^2)^{3/2}}$$

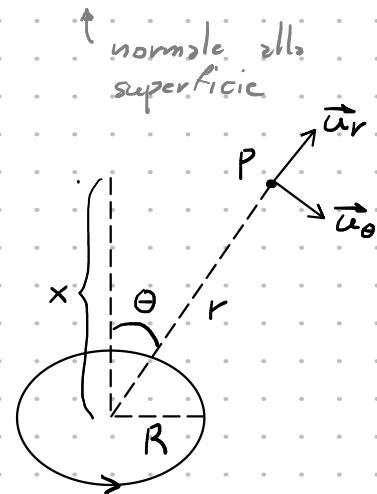
ds è entrante nel foglio

$$m = i \sum = i R^2 \pi \equiv \text{momento di dipolo} \quad \vec{m} = i \sum \vec{a}_n$$

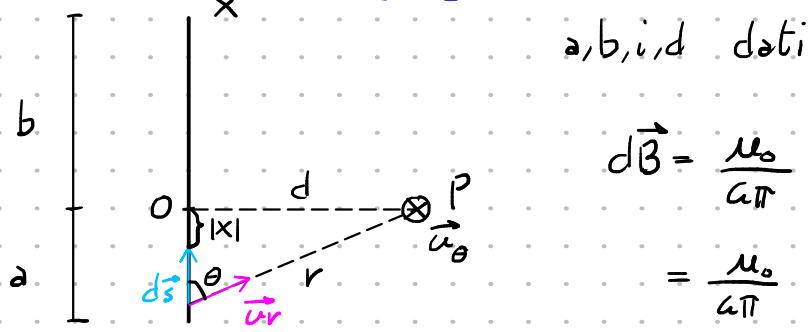
$$B_z = \frac{\mu_0}{2\pi} \frac{m}{(x^2 + R^2)^{3/2}} \approx r^3$$

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{m}{r^3} (2 \cos \theta \vec{u}_r + \sin \theta \vec{u}_\theta) \quad r \gg R$$

per qualsiasi punto P



Filo infinito (Legge di Biot-Savart) $\vec{B} = \frac{\mu_0 i}{2\pi d} \vec{u}_\theta$



$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i |d\vec{s} \times \vec{u}_r|}{r^2}$$

$$= \frac{\mu_0}{4\pi} \frac{i d\vec{s} \sin\theta}{d^2 / \sin^2\theta} \vec{u}_\theta \quad r = \frac{d}{\sin\theta}$$

$$ds = dx \quad x_\theta = -\frac{d}{\tan\theta} \quad dx = \frac{d}{\sin^2\theta} d\theta$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i \cancel{d/\sin\theta} \sin\theta}{\cancel{d^2/\sin^2\theta}} d\theta \vec{u}_\theta$$

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{i \vec{u}_\theta}{d} \int_{\theta_0}^{\theta_1} \sin\theta d\theta = \frac{\mu_0}{4\pi} \frac{i}{d} \vec{u}_\theta (\cos\theta_0 - \cos\theta_1)$$

$$\cos\theta_0 = \frac{d}{\sqrt{a^2 + d^2}}$$

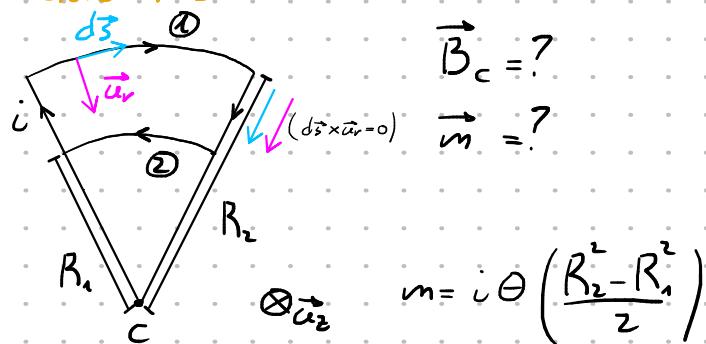
$$\cos\theta_1 = \frac{b}{\sqrt{b^2 + d^2}}$$

filo infinito

$$\begin{matrix} \theta_0 \rightarrow 0 \\ \theta_1 \rightarrow \pi \end{matrix}$$

$$\boxed{\vec{B} = \frac{\mu_0 i}{2\pi d} \vec{u}_\theta}$$

Esercizio 7.5



$$\vec{B}_c = ?$$

$$\vec{m} = ?$$

$$m = i \Theta \left(\frac{R_2^2 - R_1^2}{2} \right)$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i |d\vec{s} \times \vec{u}_r|}{r^2} \quad \int ds = \Theta R_2$$

$$d\vec{B}_1 = \frac{\mu_0}{4\pi} \frac{i ds}{R_2^2} \vec{u}_z \quad \Rightarrow \quad \vec{B}_1 = \frac{\mu_0}{4\pi} \frac{i \Theta}{R_2} \vec{u}_z$$

$$\vec{B}_2 = -\frac{\mu_0}{4\pi} \frac{i \Theta}{R_1} \vec{u}_z$$

$$\vec{B}_c = \frac{\mu_0 i \Theta}{4\pi} \left(\frac{1}{R_2} - \frac{1}{R_1} \right) \vec{u}_z$$

Riassunto legge di Laplace

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i |d\vec{s} \times \vec{v}_r|}{r^2} \Rightarrow \frac{\mu_0}{4\pi r} \frac{dq \cdot \vec{v} \times \vec{v}_r}{r^2}$$

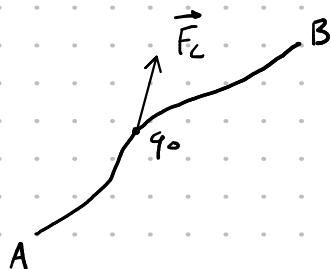
$$i = \frac{dq}{dt} \quad i d\vec{s} = \frac{dq}{dt} d\vec{s} = dq \cdot \vec{v} \quad \rightarrow \text{velocità di deriva cariche}$$

$$B = \frac{\mu_0}{4\pi} \frac{q \cdot \vec{v} \times \vec{v}_r}{r^2} \quad |\vec{v}| \ll c \quad (\text{velocità luce})$$

$$\vec{F}_e = q \vec{E} \quad \rightarrow \quad \text{carica in presenza di un campo esterno}$$

carica q in
un campo esterno

Forza di Lorentz



$$\vec{F}_L = q_0 (\vec{v} \times \vec{B})$$

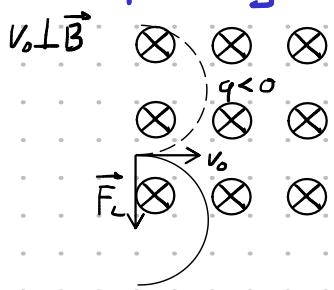
$$\vec{F}_L = q_0 (\cancel{\vec{E}} + \vec{v} \times \vec{B}) \quad \vec{E} = 0$$

$$L_{AB} = \int_A^B \vec{F}_L \cdot d\vec{s} = \int_A^B q_0 (\vec{v} \times \vec{B}) \vec{v} dt = 0$$

$$\Delta E_K = \frac{1}{2} (\vec{v}_B - \vec{v}_A) m$$

$$\Rightarrow |\vec{v}_A| = |\vec{v}_B|$$

Campo magnetico uniforme

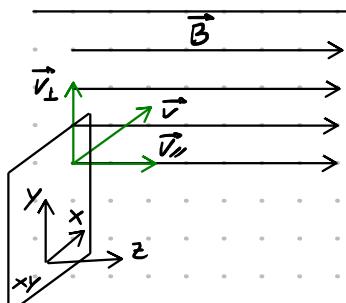


$$|\vec{F}_L| = q v_0 \times \vec{B}$$

verso il basso

$$F_c = m \frac{v_0^2}{R} = |\vec{F}_L| = q v_0 B$$

$$R = \frac{mv_0}{qB} \quad \omega = \frac{v_0}{R} = \frac{qB}{m} \quad T = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

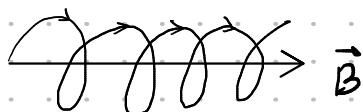


$$\vec{v} \times \vec{B} \quad \vec{v} = \vec{v}_{\parallel} + \vec{v}_{\perp}$$

$$\vec{F}_L = q \vec{v} \times \vec{B} = q \vec{v}_{\perp} \times \vec{B} + q \vec{v}_{\parallel} \times \vec{B} \quad \text{paralleli}$$

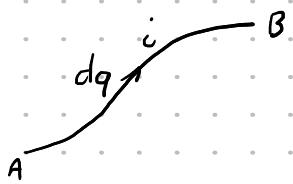
$$= q \vec{v}_{\perp} \times \vec{B}$$

$$\vec{E}_z = 0 \quad z(t) = z_0 + v_{\parallel} t \quad \text{moto rettilineo uniforme lungo z}$$



moto circolare lungo xy

II^a legge di Laplace



$$d\vec{F} = dq \vec{v}_d \times \vec{B}$$

$$d\vec{F} = i d\vec{s} \times \vec{B}$$

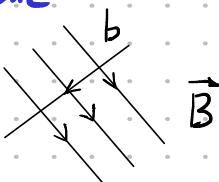
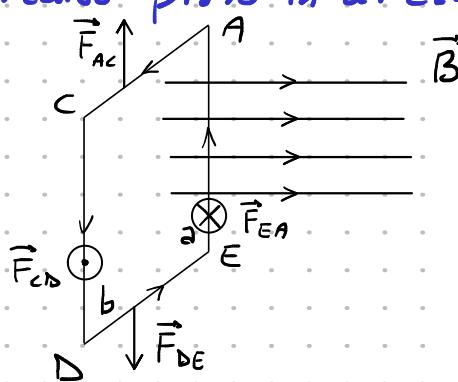
$$dq \vec{v}_d = i d\vec{s}$$

Forza subita dal filo percorso dalla corrente in presenza di un campo magnetico

$$\vec{F} = \int_A^B i d\vec{s} \times \vec{B} \quad \text{con } \vec{B} \text{ costante e uniforme}$$

$$\Rightarrow i \left(\int_A^B d\vec{s} \right) \times \vec{B} = i \vec{AB} \times \vec{B}$$

Circuito piano in un campo uniforme.



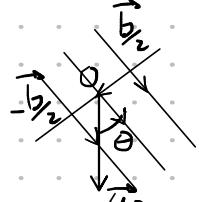
$$\left. \begin{array}{l} \vec{F}_{AC} = i \vec{AC} \times \vec{B} \\ \vec{F}_{DE} = i \vec{DE} \times \vec{B} \end{array} \right\} \text{si annullano.}$$

$$\vec{F}_{EA} = i \vec{EA} \times \vec{B}$$

$$\vec{F}_{CD} = i \vec{CD} \times \vec{B} = -\vec{F}_{EA}$$

entrante

uscente



Il centro di massa è 0
le due forze generano un momento

$$\vec{M} = \frac{\vec{b}}{2} \times \vec{F}_{AE} - \frac{\vec{b}}{2} \times \vec{F}_{CD} = \vec{b} \times \vec{F}_{AE}$$

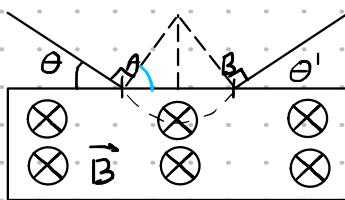
$$|\vec{M}| = b \cdot \vec{F}_{AE} \sin \theta = i ab B \sin \theta$$

$$\vec{M} = \vec{m} \times \vec{B}$$

$$\vec{m} = iab \vec{u}_n = i \sum \vec{u}_n$$

momento di dipolo magnetico

Esercizio 6.1



$$d = 2R \cos\left(\frac{\pi}{z} - \theta\right)$$

$$R = \frac{mv_0}{qB}$$

Riassunto

$$\vec{F}_C = q(E + v \times \vec{B})$$

$$d\vec{F} = i d\vec{s} \times \vec{B}$$

$$\vec{m} = i \sum \vec{u}_n$$

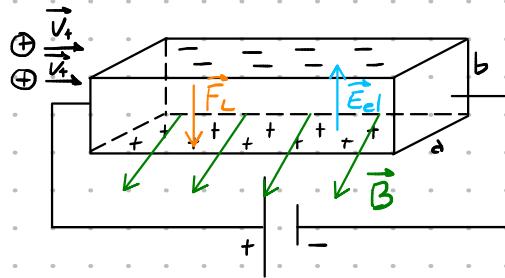
$$\vec{M} = \vec{m} \times \vec{B}$$

$$U_m = \vec{m} \cdot \vec{E} \quad \text{magnetico}$$

$$\vec{M}_e = \vec{p} \times \vec{E}$$

$$U_e = -\vec{p} \cdot \vec{E} \quad \text{elettrico}$$

Effetto Hall



B perpendicolare al momento delle cariche

$$\vec{F}_L = q \cdot \vec{v} \times \vec{B}$$

La faccia superiore si carica negativamente, quella inferiore positivamente. Così facendo si forma un campo elettrico verso l'alto.

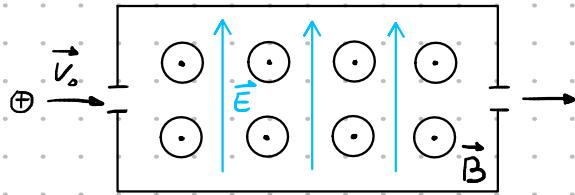
$$\vec{E}_H = \frac{\vec{F}_L}{q_+} = \vec{v} \times \vec{B} \quad \text{campo di Hall}$$

\Rightarrow situazione a regime: $\vec{E}_H + \vec{E}_{el} = 0$

$$|\vec{E}_{el}| = |\vec{E}_H| = v \cdot B \cdot \sin 30^\circ = \frac{I}{nq_+} B = \frac{i}{nabq_+} B$$

$$\Delta V_H = |\vec{E}_H| b = \frac{iB}{nabq_+}$$

Selettore di velocità



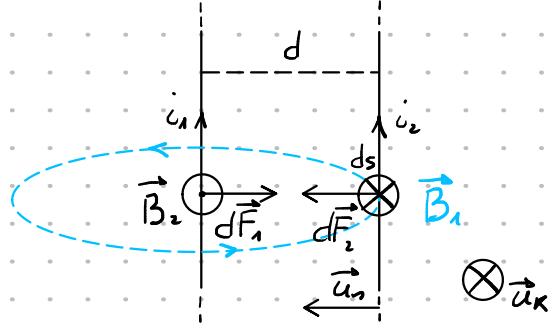
$$\vec{F}_L = q(E + v_o \times \vec{B}) = 0 \quad (\text{la forza è 0 per avere un moto rettilineo uniforme})$$

$$E = -v_o \times \vec{B}$$

$$E = v_o B \sin \theta = v_o B \Rightarrow v_o = \frac{E}{B}$$

$$1T = \frac{1V}{m \cdot m/s} = \frac{Vs}{m^2}$$

Interazione tra due fili percorsi da corrente



$$\vec{B}_1 = \frac{\mu_0}{2\pi d} i_1 \vec{a}_n$$

$$d\vec{F}_2 = i_2 d\vec{s} \times \vec{B}_1 = \frac{\mu_0 i_1 i_2}{2\pi d} ds (\vec{a}_n)$$

$$\frac{dF}{ds} = \frac{\mu_0 i_1 i_2}{2\pi d}$$

Esercizio 6.23

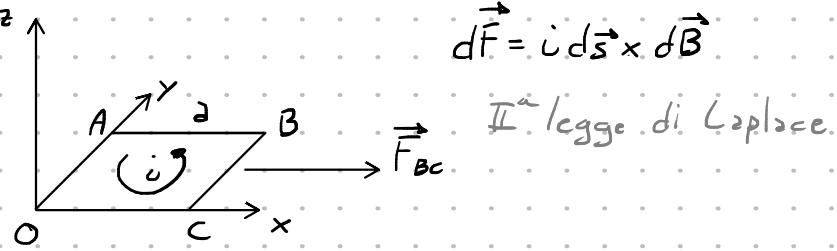
$$a = 20 \text{ cm}$$

$$i = 5 \text{ A}$$

$$\vec{B} = \mu_0 \times \vec{u}_z$$

μ_0 costante

$$\vec{F} = ?$$



$$d\vec{F} = i d\vec{s} \times d\vec{B}$$

II^a legge di Laplace.

$$\vec{F}_{AO} = 0 \quad \text{per } x=0 \text{ il campo è nullo}$$

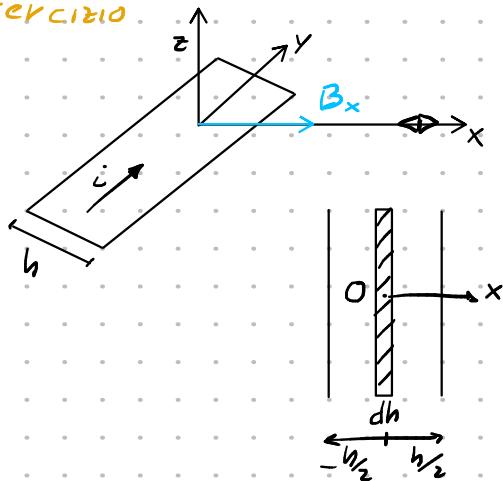
$$\vec{F}_{BC} = i \cdot a \vec{u}_y \times \mu_0 a \vec{u}_z = \mu_0 i a^2 \vec{u}_x$$

$$\vec{F}_{OC} = \int_0^a i d\vec{s} \times (\mu_0 a \vec{u}_z) = \mu_0 i \int_0^a x dx (-\vec{u}_y)$$

$$= \mu_0 i \frac{a^2}{2} (-\vec{u}_y)$$

$$\vec{F}_{AB} = -\vec{F}_{OC} \Rightarrow \vec{F}_{AB} + \vec{F}_{OC} = 0$$

Esercizio



$$\frac{di}{i} = \frac{db}{b} \Rightarrow di = \frac{db}{b} \cdot i$$

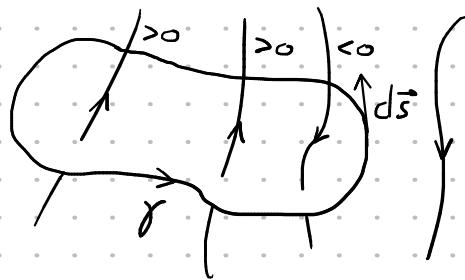
$$B = \int_{-h/2}^{+h/2} \frac{\mu_0}{2\pi} \frac{di}{(x-y)} = \int_{-h/2}^{+h/2} \frac{\mu_0}{2\pi} \frac{i}{(x-y)/h} db = \dots$$

distanza filo/punto

$$\vec{M} = \vec{m} \times \vec{B} = 0 \quad \vec{m} \perp \vec{B}$$

Legge di Ampere

$$C_r(\vec{B}) = \oint_r \vec{B} \cdot d\vec{s} = \mu_0 \sum_K i_K^{(conc)}$$



Dimostrazione

$$\vec{B} = \sum_K \vec{B}_K$$

$$C_r(\vec{B}) = \sum_K \oint_r \vec{B}_K \cdot d\vec{s}$$

Considero solo un filo

$$K=1 \quad \vec{B}_1 = \frac{\mu_0 i}{2\pi r} \vec{u}_\theta \quad \Leftrightarrow \quad \vec{B}_1 \cdot d\vec{s} = \frac{\mu_0 i}{2\pi r} d\vec{s} \cdot \vec{u}_\theta = \frac{\mu_0 i}{2\pi r} \underbrace{ds \cdot \cos\alpha}_{\pm dl} = \pm \frac{\mu_0 i}{2\pi} \frac{dl}{r} = \pm \frac{\mu_0 i}{2\pi} d\theta$$

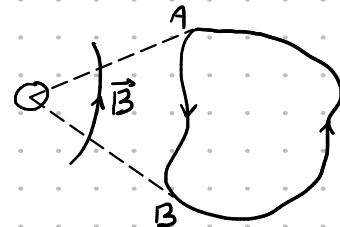
definizione di angolo in radianti (arco/raggio)

$$\vec{B} \cdot d\vec{s} = \pm \frac{\mu_0 i}{2\pi} d\theta = \frac{\mu_0 (\pm) i}{2\pi} d\theta$$

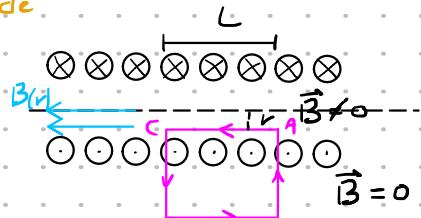
$$\oint \vec{B} \cdot d\vec{s} = \frac{\mu_0 (\pm) i}{2\pi} \oint d\theta$$

$$i \text{ concatenato} \quad \oint d\theta = 2\pi$$

$$(\pm) i \oint d\theta = -i \int_A^B d\theta + i \int_B^A d\theta = -i\theta + i\theta = 0$$



Esempio solenoide



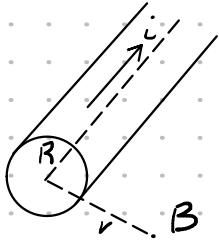
$$C_r(\vec{B}) = \oint_A^C \vec{B} \cdot d\vec{s} + \oint_C^D \vec{B} \cdot d\vec{s} + \oint_D^E \vec{B} \cdot d\vec{s} + \oint_E^A \vec{B} \cdot d\vec{s} = B_{(i)} \oint_A^C d\vec{s} = B_{(i)} L$$

$$C_r(\vec{B}) = \overset{\text{Ampere}}{\mu_0} \sum_K i_K^{(conc)} = \mu_0 N_i$$

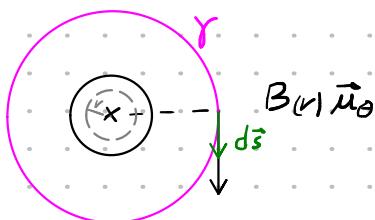
numero di "avvolgimenti" nel tratto L

$$B_{(i)} L = \mu_0 N_i \rightarrow B = \mu_0 n i \quad n = \frac{N}{L}$$

Esercizio 7.2



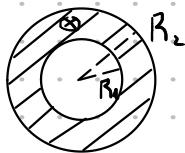
$B(r) = ?$
 \mathcal{I} uniforme



$$C_r(\vec{B}) = \oint_r \vec{B}(r) d\vec{s} = B(r) \oint_r d\vec{s} = B(r) \cdot 2\pi r$$

$$C_r(\vec{B}) \stackrel{\text{Ampere}}{=} \mu_0 i_c = \begin{cases} \mu_0 i & r \geq R \\ \mu_0 \mathcal{I} \pi r^2 & r < R \end{cases} \quad \mathcal{I} = \frac{i}{\pi R^2}$$

$$B(i) = \begin{cases} \frac{\mu_0 i}{2\pi r} & r \geq R \\ \frac{\mu_0 i}{2\pi R^2} r & r < R \end{cases}$$



$$\mathcal{I} = \frac{i}{\pi(R_2^2 - R_1^2)}$$

$$C(B) = B \cdot 2\pi r = \begin{cases} \mu_0 i & r \geq R_2 \\ \mathcal{I} \pi (r^2 - R_1^2) & R_1 < r < R_2 \\ 0 & r \leq R_1 \end{cases}$$

Ripassa leggi di Maxwell

- Campo elettrostatico

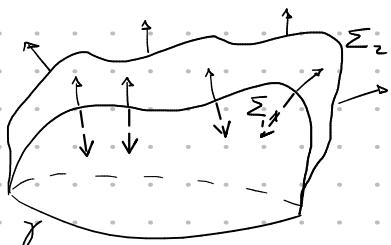
$$C_r(\vec{E}) = 0$$

$$C_r(\vec{B}) = \mu_0 i_{\text{conc}}$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\Phi_{\Sigma}(\vec{E}) = \frac{q_{int}}{\epsilon_0}$$

$$\Phi_{\Sigma}(\vec{B}) = 0$$



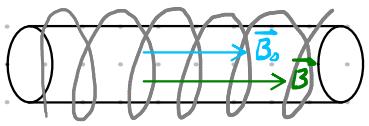
$$\partial \Sigma_1 = \partial \Sigma_2 = \gamma$$

$$\Sigma = \Sigma_1 \cup \Sigma_2$$

$$\begin{aligned} \Phi_{\Sigma}(\vec{B}) &= 0 = \Phi_{\Sigma_1}(\vec{B}) + \Phi_{\Sigma_2}(\vec{B}) \\ &= \Phi_{\Sigma_2}(\vec{B}) - \Phi_{\Sigma_1}(\vec{B}) \end{aligned}$$

$$\Phi_{\Sigma_1}(\vec{B}) = \Phi_{\Sigma_2}(\vec{B}) = \Phi_{\gamma}(\vec{B})$$

$\partial X = \text{borde di } X$



- $K_m < 1$ diamagnetic
- $K_m > 1$ paramagnetic
- $K_m \gg 1$ ferromagnetic

$$\vec{B}_0 = \mu_0 n i \vec{u}_n$$

$$\vec{B} = K_m \vec{B}_0 =$$

$$= \vec{B}_0 + (K_m - 1) \vec{B}_0 =$$

$$= \mu_0 \vec{H} + \mu_0 (K_m - 1) \vec{H} = \mu_0 (\vec{H} + \vec{M})$$

\vec{M} = magnetizzazione

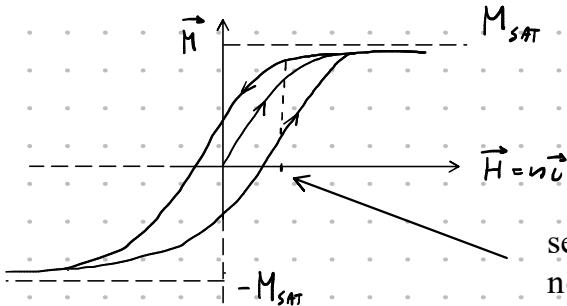
$$\vec{M} = (K_m - 1) \vec{H} = \chi_m \vec{H} \quad \vec{H} = \frac{\vec{B}_0}{\mu_0}$$

$$\vec{B} = \mu_0 (\vec{H} + \vec{M})$$

solenoido i_m = corrente di magnetizzazione

$$\vec{B} = \mu_0 n i + \mu_0 n' \vec{\chi}_{m,i} = K_m \mu_0 n i \quad \mu_0 \rightarrow K_m \mu_0 = \mu$$

Ciclo di isteresi:



$$\vec{M} = \chi_m \vec{H}$$

(In parole povere, processo di magnetizzazione dei materiali)

se conosco solo il campo magnetizzante, non conosco anche la magnetizzazione

Ancora Maxwell (Forme differenziali)

$$C_r(\vec{E}) = 0 \iff \vec{\nabla} \times \vec{E} = 0$$

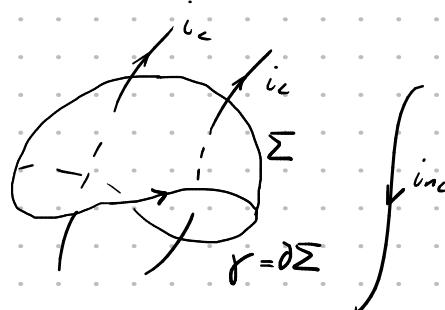
$$\Phi(\vec{E}) = \frac{q_{int}}{\epsilon_0} \iff \vec{\nabla} \cdot \vec{E} = \frac{q}{\epsilon_0}$$

$$C_r(\vec{B}) = \mu_0 i_{conc} \iff \vec{\nabla} \times \vec{B} = \mu_0 \vec{j}$$

$$\Phi_z(\vec{B}) = 0 \iff \vec{\nabla} \cdot \vec{B} = 0$$

$$C_r(\vec{B}) = \iint_{\Sigma} (\vec{\nabla} \times \vec{B}) \cdot \vec{n} d\Sigma$$

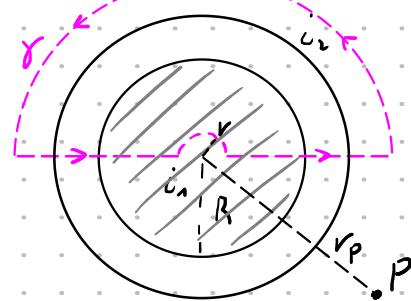
$$\mu_0 i_{conc} = \mu_0 \iint_{\Sigma} \vec{j} \cdot \vec{n} d\Sigma$$



$$\Phi_{\Sigma}(\vec{B}) = \iint_{\Sigma} \vec{\nabla} \cdot \vec{B} d\Sigma$$

TH DI GAUSS

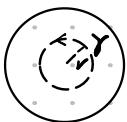
Esercizio



$$\begin{aligned} R &= 2 \text{ cm} \\ r &= R \sqrt{2} \\ B_r &= 20 \mu\text{T} \\ C_\gamma &= 0 \\ r_p &= 10 \text{ cm} \end{aligned}$$

$$\begin{aligned} i_1 &=? \\ B_p &=? \end{aligned}$$

①



$$C_f(\vec{B}) = B(r) 2\pi r \stackrel{\text{Ampere}}{=} \mu_0 \frac{i_1}{\pi R^2} \pi r^2$$

$$i_1 = 2\pi r B(r) \frac{R^2}{\mu_0 r^2}$$

②

$$B_p = \frac{\mu_0(i_1 + i_2)}{2\pi r_p} = \frac{\mu_0 \cancel{\pi} i_1}{2\pi r_p}$$

$$i_{\text{conc}} = \frac{i_1}{\pi R^2} \left(\frac{\pi R^2}{2} - \frac{\pi r^2}{2} \right) = i_1 \left(\frac{1}{2} - \frac{r^2}{2R^2} \right) =$$

$$C_f = \mu_0 \left(\frac{i_2}{2} + i_1 \left(\frac{1}{2} - \frac{r^2}{2R^2} \right) \right) = 0$$

$$i_2 = -i_1 \left(1 - \frac{r^2}{R^2} \right) = i_1 \frac{15}{16}$$

Maxwell

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla} \times \vec{E} = 0$$

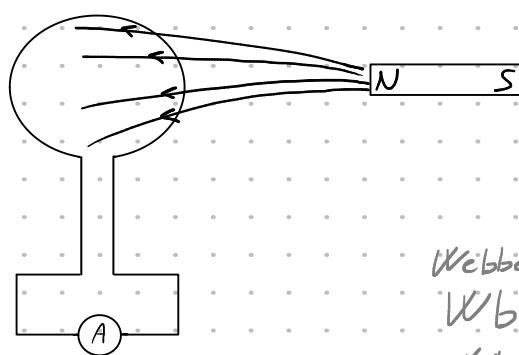
$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

Legge di Faraday-Lenz

Circuito collegato ad un amperometro

Muovendo la calamità, o un altro circuito viene creata una forza elettromotrice indotta nel circuito



Webber

$$Wb = T \cdot m^2 = [\Phi(\vec{B})]$$

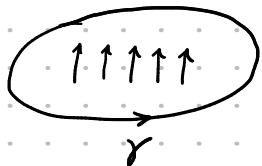
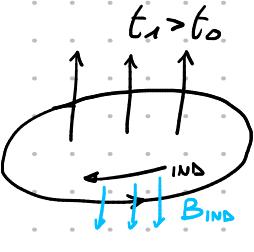
$$\frac{Wb}{s} = V$$



$$C_f(\vec{E}) = - \frac{d\Phi_r(\vec{B})}{dt}$$

$$\Phi_r(\vec{B}) = \int_{\Sigma} \vec{B} \cdot \vec{n} \cdot d\Sigma$$

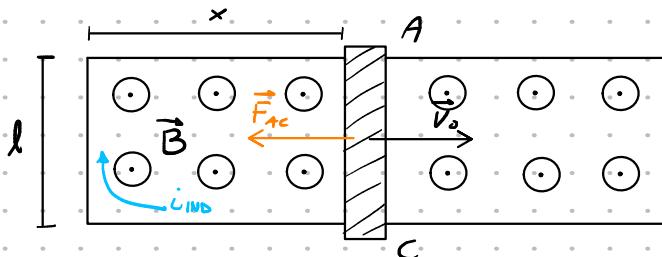
$$i_{IND} = - \frac{1}{R} \frac{d\Phi_r(\vec{B})}{dt}$$

t_0  \Rightarrow 

B aumenta

- 1) $\text{Q} > 0 \Rightarrow \phi \text{ aumenta} \Rightarrow -\frac{d\phi}{dt} < 0 \Rightarrow i_{IND} < 0$
- 2) $\text{Q} < 0 \Rightarrow \phi \text{ diminuisce} \Rightarrow -\frac{d\phi}{dt} > 0 \Rightarrow i_{IND} > 0$

Generatore fem costante



Campo costante, varia la superficie!

$$\vec{v}_o = \text{cost}$$

$$\phi(\vec{B}) = B \sum = B l x(t)$$

$$f_{em} = C_x(\vec{E}) = -\frac{d\phi}{dt} = -Bl \frac{dx}{dt}(t) = -Blv_o$$

$$i_{IND} = -\frac{Blv_o}{R}$$

$$\vec{F}_{AC} = i_{IND} \vec{AC} \times \vec{B}$$

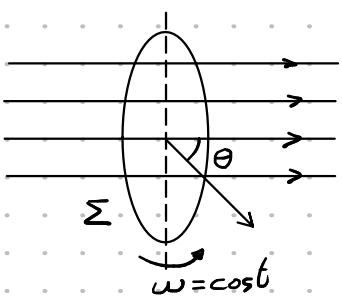
Lavoro per mantenere la bacchetta a velocità costante

$$\vec{F}_{ext} = -\vec{F}_{AC} \quad \vec{F}_{ext} = i l B \quad l \perp B \quad l \times B \Rightarrow l \cdot B \sin^{\theta}$$

$$P_{ext} = F_{ext} \cdot v_o = i_{IND} l B v_o = \left(\frac{Blv_o}{R} \right)^2$$

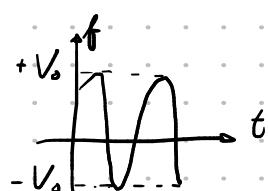
$$P_S = R i_{IND}^2 = \cancel{R} \left(\frac{Blv_o}{R} \right)^2 = P_{ext}$$

Fem alternata



$$\phi(B) = \vec{B} \cdot \vec{\Sigma} = N B \sum \cos \theta(t)$$

$$f_{em} = -\frac{d\phi}{dt} = +N B \sum \sin \theta(t) \cdot \frac{d\theta}{dt} = \underbrace{N B \sum}_{V_o} \omega \sin(\omega t)$$



$$i(t) = \frac{V_o}{R} \sin(\omega t) = i_o \sin(\omega t)$$

$$\langle P_S \rangle = \frac{1}{T} \int dt R i(t)^2 = \frac{1}{2} R i_o^2 = R i_{eff}^2 \quad i_{eff} = \frac{i_o}{\sqrt{2}} \quad V_{eff} = \frac{V_o}{\sqrt{2}}$$

$$2\pi\nu = \omega \\ \nu = 50 \text{ Hz}$$

$$C_r = - \frac{d\Phi_r(\vec{B})}{dt}$$

Coefficiente di auto-induzione (L)



\rightarrow I^a legge di Laplace

$$\text{autoflusso} \quad \Phi_{\Sigma}(\vec{B}) = \int_{\Sigma} \vec{B} \cdot \vec{n}_n \cdot d\Sigma = \int_{\Sigma} \left(\oint_r \frac{\mu_0}{4\pi r} \frac{id\vec{s} \times \vec{n}_n}{r^2} \right) \cdot \vec{n}_n \cdot d\Sigma \quad \xrightarrow{\text{dipende dalla geometria del circuito}}$$

$$= i_o \cdot L$$

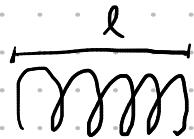
$$L = \frac{\Phi_r(\vec{B})}{i_o}$$

$$[L] = \frac{Tm^2}{A} \equiv H \text{ (Henry)}$$

$$i_{\text{ind}}(t) = - \frac{1}{R} \frac{d\Phi_r(\vec{B})}{dt} = - \frac{1}{R} \frac{d}{dt} (L i_o(t)) = - \frac{L}{R} \frac{di_o(t)}{dt}$$

$$i_{\text{ind}}(t) = - \frac{L}{R} \frac{di(t)}{dt}$$

$$f_{\text{em}}(t) = -L \frac{di}{dt}$$

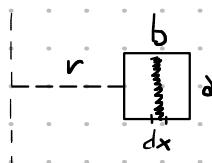
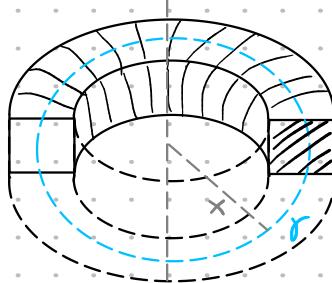


N spire

$$\Phi(\vec{B}) = N \cdot B \cdot \Sigma = N \mu_0 \frac{N}{l} i \Sigma$$

$$L = \frac{\Phi(\vec{B})}{i} = \mu_0 \frac{N^2}{l} \Sigma \quad C = \epsilon_0 \frac{\Sigma}{d}$$

Esempio 8.6



N spire

$$C_r(\vec{B}) = \oint_r \vec{B} \cdot d\vec{s} = B(x) 2\pi x \stackrel{\text{Ampere}}{=} \mu_0 N i_o$$

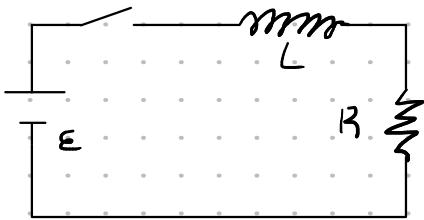
$$B(x) = \frac{\mu_0 N i_o}{2\pi x}$$

campo magnetico non costante

$$\Phi(\vec{B}) = N \int_r^{r+b} B(x) a dx = \frac{\mu_0 N^2 i_o a}{2\pi} \int_r^{r+b} \frac{dx}{x} = \frac{\mu_0 N^2 i_o a}{2\pi} \ln\left(\frac{r+b}{r}\right)$$

$$L = \frac{\mu_0 N^2 a}{2\pi} \ln\left(\frac{r+b}{r}\right)$$

Circuito RL



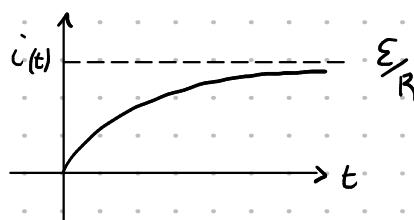
$$V = R \cdot i$$

$$E - L \frac{di(t)}{dt} = R \cdot i(t)$$

$$E - R \cdot i = L \frac{di(t)}{dt} \Rightarrow \int_0^t dt = \int_0^t L \frac{di}{E - R \cdot i} \Rightarrow t = -\frac{L}{R} \ln \frac{E - R \cdot i}{E}$$

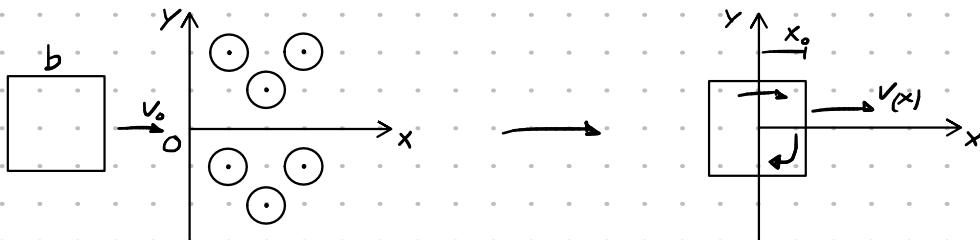
$$-\frac{R}{L} t = \ln \frac{E - R \cdot i}{E} \Rightarrow \frac{E - R \cdot i}{E} = e^{-\frac{R}{L} t} \Rightarrow E \left[1 - e^{-\frac{R}{L} t} \right] = R \cdot i$$

$$i(t) = \frac{E}{R} \left(1 - e^{-\frac{R}{L} t} \right) \quad Z_{LR} = \frac{L}{R} \quad Z_{RC} = R \cdot C$$



Esercizio

$$\begin{aligned} b &= 20 \text{ cm} \\ m &= 4g \\ R &= 25 \Omega \\ v_0 &= 0,05 \text{ m/s} \\ B &= 0,5 \text{ T} \end{aligned}$$



- a) $v(x) = ?$
 b) $v_t = ?$, spira completamente entrata
 c) $W_{diss} = ?$

Per calcolare la velocità devo prima calcolare la forza. Per la forza mi serve la corrente indotta. Quindi calcolo il flusso

$$\Phi(B) = B \cdot \overbrace{b \cdot x}^{\text{superficie}}$$

$$i_{IND} = -\frac{1}{R} \frac{d\Phi(B)}{dt} = -\frac{Bb}{R} \frac{dx}{dt} = -\frac{Bb}{R} v(x)$$

$$F = m \ddot{x}$$

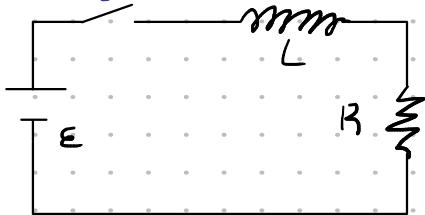
$$\vec{F}_{alt} = i_{IND} \vec{B} \times \vec{B} \quad \vec{F}_{alt} = |i_{IND}| b B = \frac{B^2 b^2}{R} v(x) = -m \frac{dv}{dx}$$

$$\frac{B^2 b^2}{R} \frac{dx}{dt} = -m \frac{dv}{dt} \quad \frac{dv}{dx} = -\frac{B^2 b^2}{R m} = \alpha$$

$$\int_{v_0}^v dv = -\frac{B^2 b^2}{R m} \int_0^x dx \quad \Rightarrow \quad v_x = v_0 - \frac{B^2 b^2}{R m} x = v(b) > 0$$

$$W_{diss} = \frac{1}{2} m v_0^2 - \frac{1}{2} m v_x^2$$

Energia magnetica



$$\mathcal{E} = L \frac{di}{dt} + Ri$$

$$W_{\text{gen}} = \int_0^T P_{\text{gen}} dt = \int_0^{i(t)} \left(L \frac{di}{dt} + Ri \right) dt + \int_0^T R i^2 dt$$

$$W_{\text{gen}} = \frac{1}{2} L i_{(t)}^2 + W_3$$

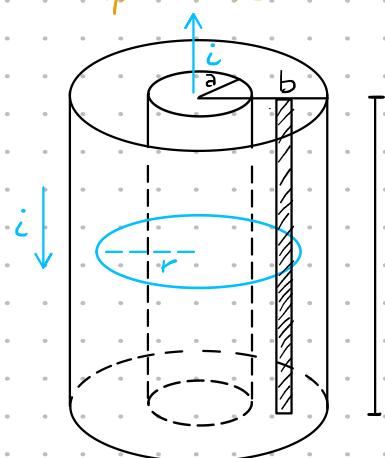
$$U_m = \frac{1}{2} L i^2$$

$$U_e = \frac{1}{2} \frac{Q^2}{C} \rightarrow u_e = \frac{dU_e}{dt} = \frac{1}{2} \varepsilon_0 \dot{E}^2$$

$$L = \mu_0 \frac{N^2}{l} \sum \quad B = \mu_0 \frac{N}{l} i \Rightarrow i = \frac{Bl}{\mu_0 N}$$

$$U_m = \frac{1}{2} \frac{\mu_0 N^2}{l} \sum \frac{B^2 l^2}{\mu_0 N^2} = \frac{B^2}{2\mu_0} \sum l \Rightarrow U_m = \frac{dU_m}{dl} = \frac{B^2}{2\mu_0}$$

Esempio 8.6

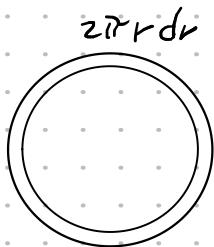


$$U_m = \int u_m dz = \int \frac{B^2}{2\mu_0} dz$$

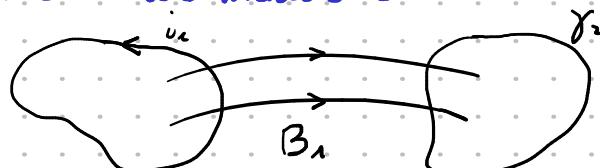
$$C_r(\vec{B}) = B 2\pi r \xrightarrow{\text{Ampero}} \mu_0 i \quad a < r < b$$

$$B = \frac{\mu_0 i}{2\pi r}$$

$$U_m = \int_a^b \frac{\left(\frac{\mu_0 i}{2\pi r}\right)^2}{2\mu_0} 2\pi l r dr = \frac{\mu_0 i^2 l}{4\pi} \int_a^b \frac{1}{r} dr = \frac{\mu_0 i^2 l}{4\pi} \ln \frac{b}{a}$$



Coeff. di mutua induzione



$$\overline{\Phi}_{J_2}(\vec{B}_1) = M_{12} i_1$$

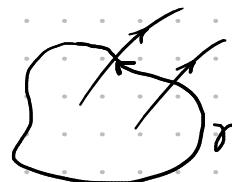
$$\overline{\Phi}_{J_1}(\vec{B}_2) = M_{21} i_2$$

$$M_{12} = M_{21}$$

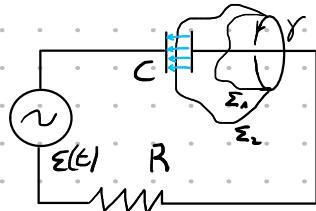
Legge di Faraday

$$Q = \int i_{\text{IND}}(t) dt = - \int \frac{1}{R} \frac{d\phi(\vec{B})}{dt} dt$$

$$= - \frac{\Delta \phi}{R} = \frac{\phi_{\text{IND}} - \phi_{\text{FIN}}}{R} = Q$$



Legge di Ampere-Maxwell



$$C_r(\vec{B}) = \mu_0 \Phi_{\Sigma}(\vec{B})$$

$$\sum_1 \Rightarrow C(\vec{B}) = \mu_0 i_c(t)$$

$$\sum_2 \Rightarrow C(\vec{B}) = \mu_0 i_s$$

$$i_s = \frac{dq_c}{dt} = \frac{d}{dt} (C \Delta V^2) = \frac{d}{dt} \left[\epsilon_0 \sum_i \frac{\Delta V_i}{d} \right]$$

$$i_s = \epsilon_0 \frac{d}{dt} (\sum E) = \epsilon_0 \frac{d}{dt} \Phi_{\Sigma}(\vec{E})$$

$$C_r(\vec{B}) = \mu_0 (i_c + i_s) = \mu_0 \left(i_c + \epsilon_0 \frac{d \phi_e(\vec{E})}{dt} \right)$$

$$C(\vec{E}) = - \frac{d \phi_B}{dt}$$

$$\phi(\vec{B}) = 0 \quad \Phi(\vec{E}) = \frac{q_{int}}{\epsilon_0}$$

Esercizio

$$t_1 = 133 \text{ } \mu\text{s}$$

$$i_1 = 0,75 i_{\infty}$$

$$L = 0,4 \text{ mH}$$

$$x = ? \quad R = ? \quad \frac{U_m(t_1)}{U_m(\infty)} = \frac{\cancel{x} L i_1^2}{\cancel{x} L i_{\infty}^2} = \frac{3}{16}$$

$$i(t) = i_{\infty} \left(1 - e^{-\frac{t}{x}} \right) \quad i_1 = \frac{3}{16} i_{\infty} = i_{\infty} \left(1 - e^{-\frac{t_1}{x}} \right)$$

$$R = \frac{L}{x}$$

