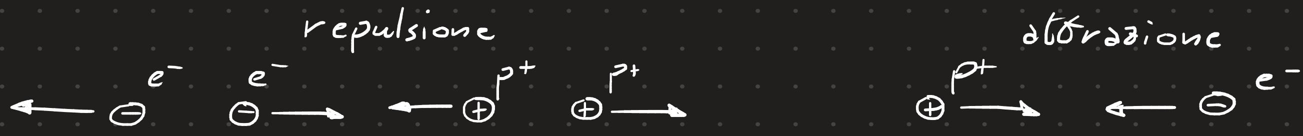
Elettrostatica: fenomeno in cui non c'è movimento



I fenomeni elettrostatici sono descrivibili come iterazioni tra protoni ed elettroni.

Legge di Coulomb

$$\vec{F}_{e_{q_2}} = \boxed{\frac{1}{4\pi\varepsilon_0}} \frac{q_1 \cdot q_2}{r^2} \vec{u}$$

$$\frac{1}{4\pi\varepsilon_0} = 9 \cdot 10^9 \frac{N \cdot m^2}{C^2}$$

Se q_1 e q_2 hanno lo stesso segno $\vec{F}$ e $\vec{u}$ sono concordi, se invece hanno segno opposto $\vec{F}$ e $\vec{u}$ sono di verso opposto.

La legge di Coulomb, in formula, è simile alla formula di attrazione gravitazionale, con la differenza che la seconda è solo attrattiva.

Valore carica di un elettrone/protone:

$$|e^-| = |p^+| \cong 1,6 \cdot 10^{-19} \; C$$

Esercizio

Si consideri un atomo di idrogeno semplice, calcolare la forza elettrostatica e la forza gravitazionale.

$r = 0,53 \cdot 10^{-10} \; m$

$m_e = 9,109 \cdot 10^{-31} \; Kg$

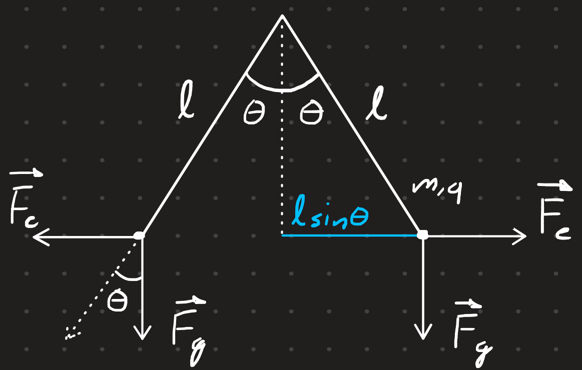
$m_p = 1,67 \cdot 10^{-27} \; Kg$

$\gamma = 6,67 \cdot 10^{-11} \; \frac{N \cdot m^2}{Kg}$

$$\frac{F_e}{F_g} = \frac{\frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{\cancel{r^2}}}{\gamma \cdot \frac{m_e \cdot m_p}{\cancel{r^2}}} \simeq 2,3 \cdot 10^{39}$$

Forza elettrostatica più forte della forza gravitazionale.

Esercizio



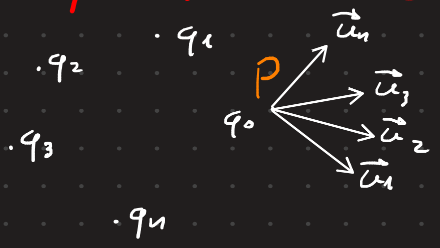
$q(\theta) = ?$

$$\tan\theta = \frac{F_e}{F_g} = \frac{\frac{1}{4\pi\varepsilon_0} \frac{q^2}{(2l\sin\theta)^2}}{m \cdot g}$$

$$q = 2l\sin\theta \sqrt{4\pi\varepsilon_0 \cdot mg \cdot tg\theta} \simeq \theta^{3/2}$$

per angoli piccoli

Campo elettrostatico



$$\vec{F}_{q_0} = \sum_{K=1}^{N} \vec{F}_K = \sum_{K=1}^{N} \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_K \cdot q_0}{r_K^2} \vec{u}_K$$

$$= q_0 \cdot \underbrace{\frac{1}{4\pi\varepsilon_0} \sum_{K=1}^{N} \frac{q_K}{r_K^2} \vec{u}_K}_{\vec{E}_P}$$

$$\vec{E}_{q_0} = \lim_{q \to 0} \frac{\vec{F}_{q_0}}{q_0}$$

Se ho solo una carica

$$\vec{E}_P = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \vec{u}$$

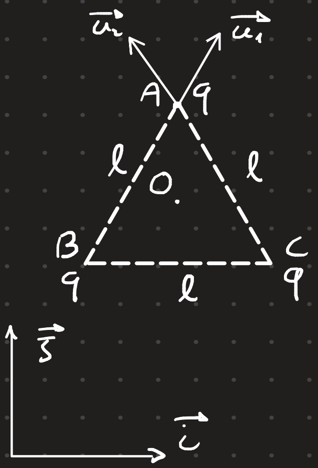
Il segno della carica mi dà il verso del campo

$$[E] = N/C = V/m$$

→ volt su metro

Il campo elettrostatico è definito in tutti i punti dello spazio,
tranne nelle cariche che lo generano dove vale infinito.

Esercizio



Calcolare il campo elettrico e forza in A, e campo elettrico nel punto O.

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{l^2} \vec{u}_1$$

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{l^2} \vec{u}_2$$

$$\vec{u}_1 = \frac{1}{2}\vec{i} + \frac{\sqrt{3}}{2}\vec{j}$$

$$\vec{u}_2 = -\frac{1}{2}\vec{i} + \frac{\sqrt{3}}{2}\vec{j}$$

$$\vec{E}_A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{l^2}(\vec{u}_1 + \vec{u}_2) = \frac{1}{4\pi\varepsilon_0} \frac{q}{l^2} \sqrt{3} \ \vec{j}$$

↓ componente verticale

$$\vec{F}_A = q \vec{E}_A$$

Nel punto O il campo è nullo per la simmetria del problema.

$$\vec{E}_{O,1} = \frac{1}{4\pi\varepsilon_0} \frac{q}{l^2/\sqrt{3}} \left(\frac{\sqrt{3}}{2} \vec{i} + \frac{1}{2}\vec{j}\right)$$

$$\vec{E}_{O,3} = \frac{1}{4\pi\varepsilon_0} \frac{q}{l^2/\sqrt{3}} \left(-\frac{\sqrt{3}}{2} \vec{i} + \frac{1}{2}\vec{j}\right)$$

$$\vec{E}_{O,2} = \frac{1}{4\pi\varepsilon_0} \frac{q}{l^2/\sqrt{3}} \left(-\vec{j}\right)$$

$$\vec{E}_O = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = 0$$

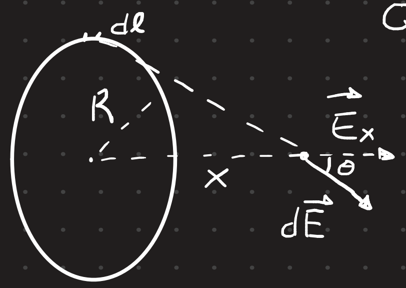
Oggetto caricato elettrostaticamente.
Quanto vale il campo elettrostatico generato dall'oggetto?

$$\rho \equiv \frac{dq}{d\tau} \quad \text{densità di carica}$$

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\vec{u} = \frac{1}{4\pi\varepsilon_0}\frac{\rho\, d\tau}{r^2}\vec{u}$$

$$\boxed{\vec{E}_P = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho\, d\tau}{r^2}\vec{u}}$$

Esercizio anello



Q con densità uniforme

$$\boxed{\lambda = \frac{dq}{dl}} = \frac{Q}{2\pi R} \qquad Q = \lambda \cdot 2\pi R$$

densità di carica lineare

$$dE_x = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\lambda\, dl}{x^2+R^2}\cdot\underbrace{\frac{x}{\sqrt{x^2+R^2}}}_{\cos\theta}$$

$$E_P = \int dE_x = \frac{1}{4\pi\varepsilon_0}\frac{\lambda x}{(x^2+R^2)^{3/2}}\int dl \;\; = 2\pi R$$

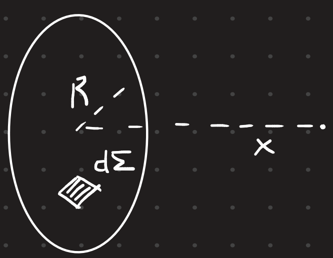
$$\vec{E}_P = \frac{1}{4\pi\varepsilon_0}\frac{Q\,x}{(x^2+R^2)^{3/2}}\vec{i} \underset{x>>R}{\simeq} \boxed{\frac{1}{4\pi\varepsilon_0}\frac{Q}{x^2}} \quad \frac{x}{|x|}$$

questo termine mi dà solo il verso

legge di Coulomb

$$\vec{E}_{anello} = \frac{1}{4\pi\varepsilon_0}\frac{Q\,x}{(x^2+R^2)^{3/2}}\vec{i}$$

Esercizio disco

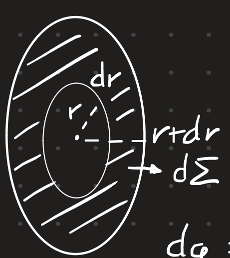


$$\boxed{\sigma = \frac{dq}{d\Sigma}} = \frac{Q}{\pi R^2}$$

densità superficiale

$$\vec{E}_P = \frac{1}{4\pi\varepsilon_0}\cdot\int_\Sigma \frac{\sigma}{r^2}\vec{u}\, d\Sigma$$

Divido la mia superficie in anelli infinitesimi.



$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq\cdot x}{(x^2+r^2)^{3/2}} = \frac{1}{4\pi\varepsilon_0}\frac{\sigma\, 2\pi r\, x\, dr}{(x^2+r^2)^{3/2}}$$

$$dq = \sigma\cdot d\Sigma$$

$$d\Sigma = \pi(r+dr)^2 - \pi r^2 = 2r\pi\, dr$$

$$E = \frac{\sigma \cancel{\pi} x}{4\cancel{\pi}\varepsilon_0} \int_0^R \frac{2r\,dr}{(x^2+r^2)^{3/2}} = \frac{\sigma x}{4\varepsilon_0}\left[\frac{-2}{\sqrt{x^2+r^2}}\right]_0^R$$

$$= \frac{\sigma x}{2\varepsilon_0}\left(\frac{1}{|x|} - \frac{1}{\sqrt{x^2+R^2}}\right)$$

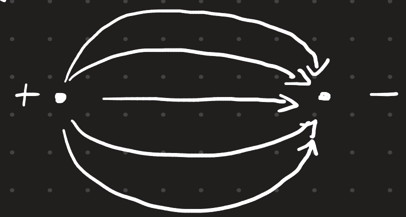
Cosa succede se il piano è infinito?

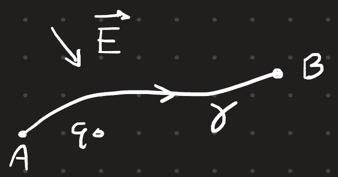
$$E = \frac{\sigma x}{2\varepsilon_0}\left(\frac{1}{|x|} - \frac{1}{\sqrt{x^2+R^2}}\right) \underset{R>>0}{\simeq} \frac{\sigma}{2\varepsilon_0}\cdot\frac{x}{|x|} \longrightarrow \text{campo elettrostatico costante}$$

Linee di campo

→ le linee non si incontrano mai, eccetto nelle cariche

- Nella zona in cui le linee sono più dense il campo è più forte





$$L_{AB} = \int_\gamma \vec{F}_{q_0}\cdot d\vec{s} = q_0 \int_A^B \underbrace{\vec{E}\,d\vec{s}}_{\text{tensione campo elettrico}}$$

con $\vec{F}_{q_0} = q_0\cdot\vec{E}$

Un campo è conservativo quando l'integrale non dipende dal percorso, ma dal punto iniziale a quello finale.

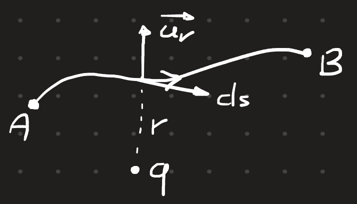
$$L_{AB} = -\Delta U = U_A - U_B = q_0\,\underbrace{(V_A - V_B)}_{\text{differenza di potenziale}}$$

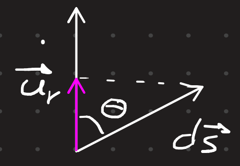
$$V_A - V_B = \int_A^B \vec{E}\cdot d\vec{s} \iff \boxed{V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{s}}$$

$$C_\gamma(\vec{E}) = \oint_\gamma \vec{E}\cdot d\vec{s} = 0$$

(percorso chiuso)

Dimostriamo che il campo elettrostatico è conservativo:





$d\vec{s}$ è lo spostamento infinitesimo

$$\vec{u}_r \cdot d\vec{s} = ds \cdot \cos\theta = dr$$

$$\int_A^B \vec{E} \cdot d\vec{s} = \int_A^B \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \vec{u}_r \cdot d\vec{s} = \int_A^B \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} dr =$$

$$= \left[\frac{1}{4\pi\varepsilon_0} \frac{-q}{r} \right]_A^B = \frac{1}{4\pi\varepsilon_0} \frac{q}{r_A} - \frac{1}{4\pi\varepsilon_0} \frac{q}{r_B} = V_A - V_B$$

Il campo Coulombiano è un campo conservativo: non dipende dal percorso, ma solo del punto finale ed iniziale.

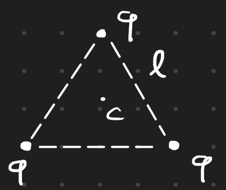
$$\boxed{V(r) = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}} + K$$

formula del potenziale Coulombiano

$$[V] = \frac{1J}{1C} = 1 \text{ Volt}$$

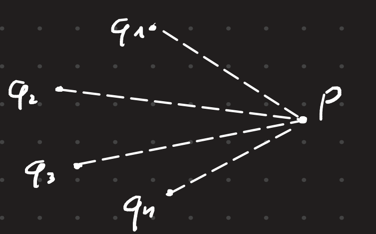
$V(\infty) = 0$ (è una scelta per eliminare K)

Esercizio



$$\overline{qc} = r = l \frac{\sqrt{3}}{3}$$

$$V = \left(\frac{1}{4\pi\varepsilon_0} \frac{q}{l\frac{\sqrt{3}}{3}} \right) \cdot 3$$

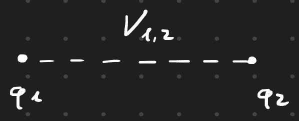


$$V_P = \sum \frac{1}{4\pi\varepsilon_0} \frac{q_k}{r_k}$$

$$V_P = \int d\tau \frac{1}{4\pi\varepsilon_0} \frac{\rho}{r}$$

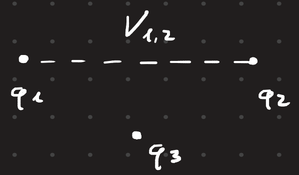
densità di carica

Energia potenziale di un sistema di cariche



$$L_{ext} = -\int_{\infty}^{V_{1,2}} \vec{F} \cdot d\vec{s} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 \cdot q_2}{r_{1,2}}$$

il lavoro è opposto al campo elettrico



$$U_{TOT} = \frac{1}{4\pi\varepsilon_0} \left(\frac{q_1 \cdot q_2}{r_{1,2}} + \frac{q_1 \cdot q_3}{r_{1,3}} + \frac{q_2 \cdot q_3}{r_{2,3}} \right)$$

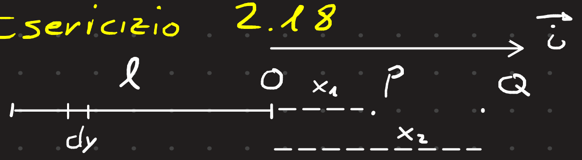
$$= \frac{1}{4\pi\varepsilon_0} \sum_{i,j} \frac{1}{2} \frac{q_i \cdot q_j}{r_{ij}}$$

energia spesa per arrivare ad una determinata configurazione di cariche

$$L_{AB} = \int_A^B \vec{F} \, d\vec{s} = \frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2 = -q_0 (V_B - V_A)$$

$$\frac{1}{2} m v_B^2 + q V_B = \frac{1}{2} m v_A^2 + q V_A$$

Esercizio 2.18



$\ell = 20 cm$
$q = 6 \cdot 10^{-8} C = 60 nC$
$x_1 = 15 cm$
$x_2 = 20 cm$

1° modo

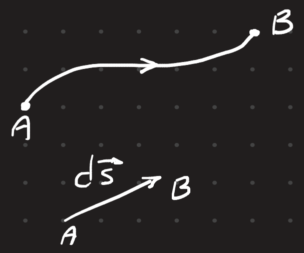
formula campo lineare

$$L_{ext} = \int_P^Q \left(-\vec{F}_e\right) d\vec{s} \qquad \vec{F}_e = q_0 \frac{1}{4\pi\varepsilon_0} \int_{-\ell}^{0} \frac{\lambda \, dy}{(x-y)^2} \vec{i}$$

2° Metodo

$$L_{ext} = +\Delta U = U_Q - U_P = q_0 (V_Q - V_P)$$

$$V(x) = \frac{1}{4\pi\varepsilon_0} \int_{-\ell}^{0} \frac{\lambda \, dy}{(x-y)} = \frac{\lambda}{4\pi\varepsilon_0} \Big[-\ln(x-y) \Big]_{-\ell}^{0} = \frac{\lambda}{4\pi\varepsilon_0} \ln\left(\frac{x+\ell}{x}\right)$$



$$\overbrace{V_B - V_A}^{dV} = -\int_A^B \vec{E}\cdot d\vec{s}$$

$$\vec{E} = E_x\vec{i} + E_y\vec{j} + E_z\vec{k}$$

$$d\vec{s} = dx\,\vec{i} + dy\,\vec{j} + dz\,\vec{k}$$

$$dV = -\vec{E}\cdot d\vec{s} =$$

$$= -\vec{E}_x d\vec{x} - \vec{E}_y d\vec{y} - \vec{E}_z d\vec{z}$$

$V_B - V_A$

$\forall\, d\vec{s}$

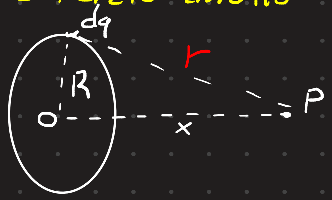
$$dV = V(\vec{x}+d\vec{s}) - V(\vec{x}) = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$$

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_x = -\frac{\partial V}{\partial z}$$

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) = \text{NABLA}$$

$$\vec{\nabla} V = \left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}\right) = \text{gradiente di } V \implies \vec{E} = \vec{\nabla} V$$

Esercizio anello



$$V_p = ?$$

$$\lambda = \frac{dq}{dl} = \frac{Q}{2\pi R} = \text{cost}$$

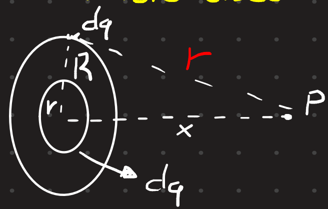
$$V_P = \frac{1}{4\pi\varepsilon_0}\int\frac{dQ}{r} = \frac{1}{4\pi\varepsilon_0}\int\frac{\lambda\,dl}{\sqrt{x^2+R^2}} =$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{\sqrt{x^2+R^2}}\,2\pi R = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{x^2+R^2}}$$

$$E_z - E_y = 0$$

$$E_x = -\frac{\partial V}{\partial x} = \cancel{-}\frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{(x^2+R^2)^3}}\left(\cancel{-}\frac{1}{\cancel{2}}\right)\cancel{2}x = \frac{1}{4\pi\varepsilon_0}\frac{Q\cdot x}{(x^2+R^2)^{3/2}}$$

Esercizio disco



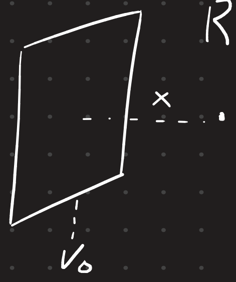
$$dq = \sigma\,d\Sigma = \sigma\,2\pi r\,dr$$

$$dV(dq) = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\sqrt{x^2+r^2}}$$

$$V_P = \int\frac{1}{4\pi\varepsilon_0}\frac{dq}{\sqrt{x^2+r^2}} = \int_0^R\frac{1}{4\pi\varepsilon_0}\frac{\sigma\,2\pi r\,dr}{\sqrt{x^2+r^2}} = \frac{\sigma}{4\varepsilon_0}\left[2\sqrt{x^2+r^2}\right]_0^R =$$

$$= \frac{\sigma}{2\varepsilon_0}\left[\sqrt{x^2+R^2} - |x|\right]$$

$$E_x = -\frac{\partial V}{\partial x} = \frac{\sigma}{2\varepsilon_0}\left[-\frac{x}{\sqrt{x^2+R^2}} + \frac{|x|}{x}\right]$$



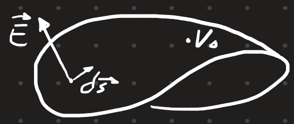
$$R \gg x \qquad \vec{E} = \frac{\sigma}{2\varepsilon_0}\,\vec{u} = \frac{\sigma}{2\varepsilon_0}\,\frac{|x|}{x}\,\vec{u} = \sigma\, 2\pi r\, dr$$

$$V = V_0 \cdot - \frac{\sigma}{2\varepsilon_0}|x|$$

$$\vec{E} = -\frac{\partial V}{\partial x} = \frac{\sigma}{2\varepsilon_0}\,\frac{|x|}{x} = E_x$$

Superfici equipotenziali

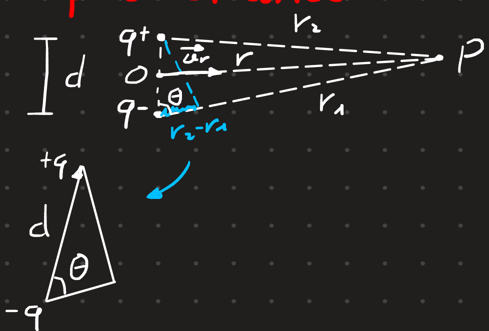
$$V(x,y,z) = V_0$$



$$dV = 0 = -\vec{E}\cdot d\vec{s} \qquad \vec{E} \perp d\vec{s}$$

- Come le linee di campo anche queste non si intersecano mai, per ogni punto passa una sola superficie equipotenziale.

Dipolo elettrico



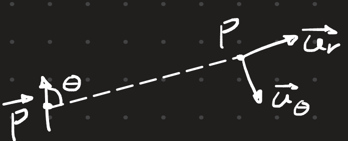
$$V_P = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{r_1} + \frac{1}{4\pi\varepsilon_0}\,\frac{-q}{r_2} = \frac{q}{4\pi\varepsilon_0}\left(\frac{r_2 - r_1}{r_1 \cdot r_2}\right)$$

Se $r \gg d \qquad r_1 \cdot r_2 \simeq r^2 \qquad r_2 - r_1 \simeq d\cos\theta$

$$V_P \simeq \frac{q}{4\pi\varepsilon_0}\,\frac{d\cos\theta}{r^2} = \boxed{\frac{\vec{p}\cdot\vec{u}_r}{4\pi\varepsilon_0 \cdot r^2}}$$

$\vec{p} = q\vec{d} \equiv$ momento di dipolo

$$\vec{p} = \sum_k \vec{r}_k\, q_k \qquad Q = \sum_k q_k$$

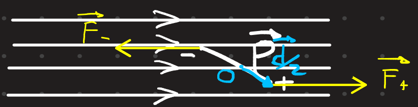


$$V_P = \frac{1}{4\pi\varepsilon_0}\,\frac{p\cos\theta}{r^2}$$

$$E_r = -\frac{\partial V}{\partial r} = \frac{1}{4\pi\varepsilon_0}\,\frac{2p\cos\theta}{r^3}$$

$$E_\theta = -\frac{1}{r}\,\frac{\partial V}{\partial \theta} = \frac{1}{4\pi\varepsilon_0}\,\frac{p\sin\theta}{r^3}$$

$$\boxed{\vec{E} = \frac{p}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\,\vec{u}_r + \sin\theta\,\vec{u}_\theta\right)}$$



$\vec{E}$ uniforme

$\vec{F} = q\vec{E}$ (il segno della forza dipende dalla carica + o -)

$$\vec{M} = \frac{\vec{d}}{2} \times \vec{F}_+ + \left(-\frac{\vec{d}}{2}\right) \times \vec{F}_- = \frac{\vec{d}}{2} \times q\vec{E} + \left(-\frac{\vec{d}}{2}\right) \times \left(-q\vec{E}\right) = q\vec{d} \times \vec{E}$$

momento di dipolo

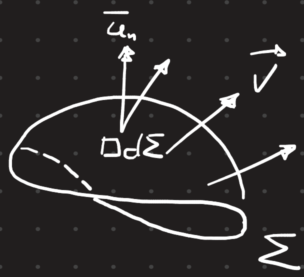
$$\boxed{\vec{M} = \vec{p} \times \vec{E}}$$

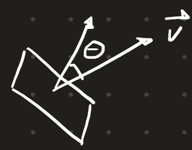
$$U = U_+ + U_- = qV_A - qV_B = q(V_A - V_B) \simeq -q\vec{E} \cdot \vec{d}$$

$$\boxed{U = -\vec{p} \cdot \vec{E}} \qquad U_{min} = -|p||E| \qquad U_{max} = +|p||E|$$

- **Def. di flusso**

$$\Phi_\Sigma(\vec{v}) = \int_\Sigma \vec{v} \cdot \vec{u}_n \, d\Sigma$$





$$\Phi_\Sigma(\vec{v}) = v \cdot \Sigma \cdot \cos\theta$$

Teorema di Gauss

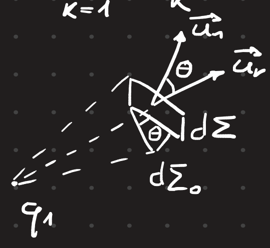
$$\Phi_\Sigma(\vec{E}) = \oint_\Sigma \vec{E} \cdot \vec{u}_n \cdot d\Sigma = \underbrace{\sum}_{\text{sommatoria}} \frac{q_{interne}}{\varepsilon_0}$$

(campo * normale * superficie)

Σ è chiusa

Dimostrazione

$$\vec{E} = \sum_{k=1}^{N} \vec{E}_k \qquad \vec{E}_k \text{ è generato da } q_k$$



$$\vec{E}_1 \cdot u_n \, d\Sigma = \frac{1}{4\pi\varepsilon_0} \frac{q_1}{r^2} \overbrace{\vec{u}_r \cdot \vec{u}_n}^{\cos\theta} \, d\Sigma$$

$$= \frac{1}{4\pi\varepsilon_0} q_1 \frac{d\Sigma_0}{r^2} = \frac{1}{4\pi\varepsilon_0} q_1 \, d\Omega$$

angolo solido

$$d\theta = \frac{dl}{r}$$

$$\oint_\Sigma \vec{E} \cdot \vec{u}_n \, d\Sigma = \frac{q_1}{4\pi\varepsilon_0} \oint_\Sigma d\Omega = \frac{q_1}{\cancel{4\pi}\varepsilon_0} \cancel{4\pi} = \frac{q_1}{\varepsilon_0}$$

$$\Phi(\vec{E}) = \Phi_{\Sigma_1}(\vec{E}) + \Phi_{\Sigma_2}(\vec{E}) = \frac{q_1}{4\pi\varepsilon_0}\Omega_1 - \frac{q_1}{4\pi\varepsilon_0}\Omega_1 = 0$$

Esercizio

$\cdot\ \rho = \frac{dq}{d\tau} = \text{cost}$ densità di carica costante nella sfera

$\vec{E} = E(r)\,\vec{u}_r$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho\, d\tau}{r^2}\,\vec{u}_r$$

$$\Phi_{S_r}(\vec{E}) = E(r)\,4\pi r^2 = \frac{\rho \frac{4}{3}\pi R^3}{\varepsilon_0} = \frac{q_{int}}{\varepsilon_0} \qquad r \geq R$$

$$Q = \rho\frac{4}{3}\pi$$

$$E(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \qquad \text{campo esterno}$$

$$\Phi_{S_r}(\vec{E}) = E(r)\,4\pi r^2 = \frac{\rho\frac{4}{3}\pi r_1^3}{\varepsilon_0} = \frac{Q\, r_1^3}{\varepsilon_0 R^3}$$

$$E(r) = \begin{cases} \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}\,r & r < R \\ \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} & r \geq R \end{cases}$$

$\rho = \text{cost}$

$$E(r) = \begin{cases} \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}\,r & r < R & \text{interno} \\ \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} & r \geq R & \text{esterno} \end{cases}$$

$$\boxed{V(r) = -\int E(r)\,dr}$$

$$V(r) = \begin{cases} -\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}\frac{r^2}{2} + C_1 & r < R & \text{interno} \\ \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} + C_2 & r \geq R & \text{esterno} \end{cases}$$

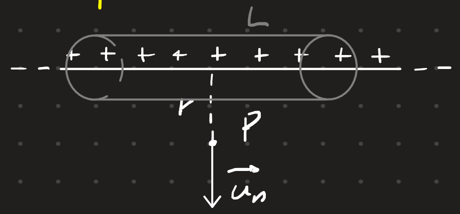
Come trovo C_1 e C_2?

$$V(\infty) = 0 = C_2$$

$$\frac{Q}{R^{\cancel{3}}}\frac{\cancel{R^2}}{2}$$

$$V(R_+) = V(R_-) \iff \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} = \frac{-1}{8\pi\varepsilon_0}\frac{Q}{R} + C_1 \implies C_1 = \frac{3}{8\pi\varepsilon_0}\frac{Q}{R}$$

Esempi di utilizzo del Th. di Gauss



$\lambda = cost = \frac{dq}{dl}$

$dq = \lambda\, dl$

$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda\, dl}{r^2}\,\vec{u}_r$

$\phi = \oint \vec{E}\cdot\vec{u}_n\, d\Sigma$

$\vec{E} = E_{(r)}\,\vec{u}_r$

$\phi(\vec{E}) = E_{(r)}\, 2\pi r \cancel{L} \overset{Gauss}{=} \frac{\lambda \cancel{L}}{\varepsilon_0}$ → cariche interne

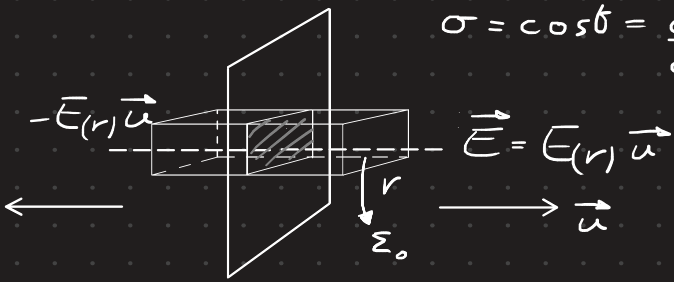
Nelle superfici il campo elettrico è perpendicolare alla normale: il flusso fa 0.

$E_{(r)} = \frac{1}{2\pi\varepsilon_0}\,\frac{\lambda}{r}$

$\ln c_1 = \ln r_0$

$V_{(r)} = -\int E_{(r)}\, dr = -\frac{1}{2\pi\varepsilon_0}\left(\ln r + c_1\right) = -\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r}{r_0}$

$\sigma = cost = \frac{dq}{d\Sigma}$



$\phi(\vec{E}) = 0 + 2\cancel{\Sigma_0}\vec{E}_{(r)} \overset{Gauss}{=} \frac{\sigma\cancel{\Sigma_0}}{\varepsilon_0} \implies E_{(r)} = \frac{\sigma}{2\varepsilon_0}$

Due proprietà fondamentali dell'elettrostatica viste fino ad ora

$C_\gamma(\vec{E}) = \oint_\gamma \vec{E}\, d\vec{s} = 0$ • Campo conservativo → posso derivare il potenziale

$\phi_\Sigma(\vec{E}) = \oint_\Sigma \vec{E}\,\vec{u}_n\, d\Sigma = \frac{q_{int}}{\varepsilon_0}$ • Posso ricavare il campo

Voglio riscrivere queste due equazioni usando delle derivate

Th. della Divergenza

superficie / volume

$$\oint_\Sigma \vec{v}\cdot\vec{u}_n\, d\Sigma = \int_\tau \underbrace{\vec{\nabla}\cdot\vec{v}}_{divergenza}\, d\tau$$

superficie che è bordo di un volume

$\Sigma = \partial\tau$

$\vec{\nabla}\cdot\vec{v} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$

Assomiglia al Th. fondamentale del calcolo integrale

$f_{(b)} - f_{(a)} = \int_a^b f'(x)\, dx$

$$\oint_\Sigma \vec{E}\,\vec{u}_n\,d\Sigma = \frac{q_{int}}{\varepsilon_0}$$

$$\int_\tau \vec{\nabla}\cdot\vec{E}\,d\tau = \int_\tau \frac{\rho}{\varepsilon_0}\,d\tau$$ (esprimo la carica come l'integrale della densità di carica)

Quindi posso riscrivere il teorema di Gauss come

$$\Phi_\Sigma(\vec{E}) = \oint_\Sigma \vec{E}\,\vec{u}_n\,d\Sigma = \frac{q_{int}}{\varepsilon_0} \iff \vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

Teorema del rotore

$$\underbrace{\oint_\gamma \vec{v}\cdot d\vec{s}}_{\text{linea}} = \underbrace{\int_\Sigma (\underbrace{\vec{\nabla}\times\vec{v}}_{\text{rotore di } v})\,\vec{u}_n\,d\Sigma}_{\text{superficie}}$$

Σ

$\gamma = \partial\Sigma$

abbiamo una linea che è bordo di una superficie

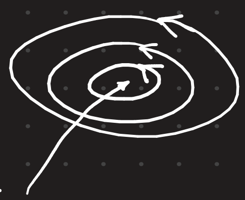
$$\oint_\gamma \vec{E}\,d\vec{s} = 0 = \int_\Sigma (\vec{\nabla}\times\vec{v})\,\vec{u}_n\,d\Sigma \qquad \forall \Sigma / \gamma = \partial\Sigma$$

$$\Longrightarrow \vec{\nabla}\times\vec{E} = 0$$

Posso riscrivere la circuitazione come

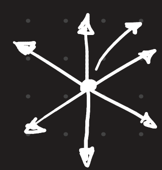
$$C_\gamma(\vec{E}) = \oint_\gamma \vec{E}\,d\vec{s} = 0 \iff \vec{\nabla}\times\vec{E} = 0$$

Spiegazione geometrica



$\vec{\nabla}\times\vec{v} \neq 0$

rotore non nullo:
punti in cui le linee di campo girano intorno.
Nel campo elettrico le linee di campo non si uniscon
perciò è sempre 0



$\vec{\nabla}\cdot\vec{v} \neq 0$

divergenza non nulla:
punti in cui entrano o escono le linee di campo

$$C_\gamma(\vec{E}) = \oint_\gamma \vec{E}\, d\vec{s} = 0 \iff \vec{\nabla} \times \vec{E} = 0 \iff \vec{E} = -\vec{\nabla} V$$

$$\Phi_\Sigma(\vec{E}) = \oint_\Sigma \vec{E} \cdot \vec{u}\, d\Sigma = \frac{q_{int}}{\varepsilon_0} \iff \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \iff \vec{\nabla} \cdot (\vec{\nabla} V) = -\frac{\rho}{\varepsilon_0}$$

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho}{\varepsilon_0}$$

eq. di Poisson $\longrightarrow \Delta V = -\frac{\rho}{\varepsilon_0}$

$$\Delta V = -\frac{\rho}{\varepsilon_0} \implies V = \frac{1}{4\pi\varepsilon_0} \int_\tau \frac{\rho\, d\tau}{r}$$

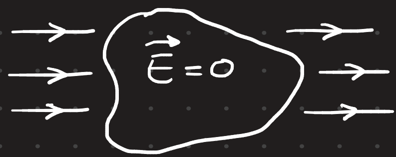
Conduttori

fenomeno di induzione

Proprietà in condizioni elettrostatiche

1 - $\vec{E}_{interno} = 0$

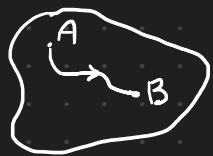
Le cariche sul conduttore si modificano per creare un campo uguale e opposto al campo esterno



2 - La carica in eccesso si deposita sulla superficie (Dim. con Th. di Gauss)

Siccome il campo interno è 0, qualsiasi per qualsiasi superficie interna che possiamo prendere il flusso è 0.

3 - Tutto il conduttore è allo stesso potenziale

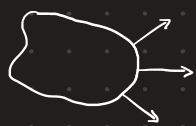


$$V_A - V_B = -\int_B^A \vec{E} \cdot d\vec{s} = 0$$

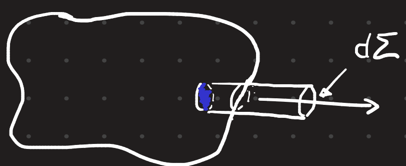
il campo è nullo

4 - Teorema di Coulomb

$$\vec{E}_{sup} = \frac{\sigma}{\varepsilon_0} \vec{u}_n$$



Dimostrazione



$$\sigma = \frac{dq}{d\Sigma} \implies dq = \sigma\, d\Sigma$$

$$\Phi(\vec{E}) = \underbrace{0}_{\text{laterale}} + \underbrace{0}_{\text{base interna}} + E\,\cancel{d\Sigma} = \frac{\sigma\,\cancel{d\Sigma}}{\varepsilon_0}$$

Esempio 4.1

Come si dispongono le cariche sulle due sfere?

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{R_1} \qquad V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{R_2}$$

$$\begin{cases} \dfrac{q_1}{R_1} = \dfrac{q_2}{R_2} \\ q_1 + q_2 = q \end{cases}$$

Esercizio 3.20

$R = 10\,cm$
$q = 8 \cdot 10^{-3}\,C$
$\rho(r) = br$

- Calcolare b
- Campo elettrostatico $E(r)$
- Differenza di potenziale tra il centro O e la superficie della sfera

$$q = \int_\tau \rho\, d\tau = \int_0^R br \cdot 4\pi r^2 dr = $$
$$= \int_0^R 4b\pi r^3 dr = \pi b R^4$$
$$\Longrightarrow b = \frac{q}{\pi R^4}$$

$$V = \frac{4}{3}\pi r^3$$
$$dV = 4\pi r^2\, dr$$

la carica è l'integrale della densità sul volume!!!

$$\vec{E} = E(r)\,\vec{u}_n$$

$$\Phi_S(E) = \vec{E}(r)\, 4\pi r^2 = \frac{q_{int}(r)}{\varepsilon_0}$$

$r \geq R \qquad q_{int}(r) = q$

$r < R \qquad q_{int}(r) = \int_0^r 4\pi b\, y^2 dy = \pi b r^4 = q\frac{r^4}{R^4}$

$$\vec{E}(r) = \begin{cases} \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2} & r \geq R \\ \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{R^4} r^2 & r < R \end{cases}$$

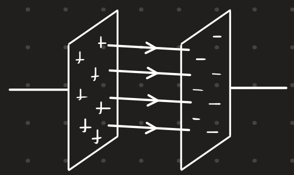
$$V_0 - V_R = -\int_R^0 \vec{E}(r)\, dr = \int_0^R \frac{1}{4\pi\varepsilon_0}\frac{q}{R^4} r^2\, dr = \frac{1}{12\pi\varepsilon_0}\frac{q}{R}$$

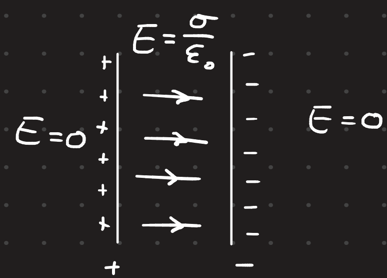
• Conduttori cavi

Per il Th. di Gauss la carica totale interna è 0.

$$C_\gamma(\vec E) = \int_A^B \vec E\, d\vec s + \cancel{\int_B^A \vec E\, d\vec s}$$

• CONDENSATORI





$$C = \text{capacità} = \left|\frac{Q}{\Delta V}\right| \qquad [C] = \frac{1C}{1V} = 1F \ \text{(Farad)}$$

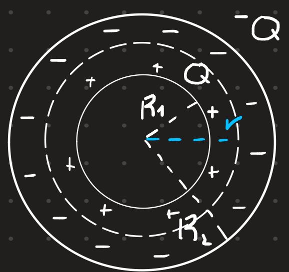
Esercizio

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 \Sigma}$$

$$\Delta V = \int_A^B \vec E\, d\vec s = \frac{Q}{\varepsilon_0 \Sigma}\int ds = \frac{Q}{\varepsilon_0 \Sigma}\, d$$

$$C = \frac{Q}{\Delta V} = \frac{\cancel{Q}}{\frac{\cancel{Q}}{\varepsilon_0 \Sigma}\, d} = \varepsilon_0 \frac{\Sigma}{d}$$

Esercizio



$$E(r)\, 4\pi r^2 = \frac{Q}{\varepsilon_0} \qquad E(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

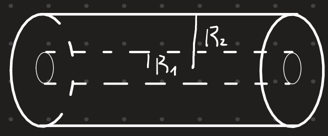
$$\Delta V = \left|\int_{R_1}^{R_2} E(r)\, dr\right| = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$R_2 - R_1 = d \ll R_{1,2}$

$R_1 \sim R_2 \sim R$

$$C = \frac{Q}{\Delta V} = \frac{\cancel{Q}}{\frac{\cancel{Q}}{4\pi\varepsilon_0}\frac{R_2 - R_1}{R_1 R_2}} = \varepsilon_0 \frac{4\pi R_2 R_1}{R_2 - R_1} \underset{d \ll R_2 - R_1}{\simeq} \varepsilon_0 \frac{4\pi R^2}{d}$$

Esercizio



$$E(r)\, 2\pi r L = \frac{Q}{\varepsilon_0}$$

$$E(r) = \frac{1}{2\pi\varepsilon_0} \frac{Q/L}{r}$$

$d \ll R_1, R_2$

$R_2 = R_1 + d$

$$\ln \frac{R_1 + d}{R_1} = \ln\left(1 + \frac{d}{R_1}\right) \simeq \frac{d}{R_1}$$

$$\Delta V = \int E\, dr = \frac{1}{2\pi\varepsilon_0} \frac{Q}{L} \ln \frac{R_2}{R_1}$$

$$C = \frac{Q}{\Delta V} = \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{R_2}{R_1}\right)} \simeq \varepsilon_0 \frac{2\pi L}{d/R_1} = \varepsilon_0 \frac{2\pi R_1 L}{d}$$

Serie

$$\frac{1}{C_T} = \frac{1}{C_1} + \dots + \frac{1}{C_N}$$

Parallelo

$$C_N = C_1 + \dots + C_N$$

Energia condensatori

$$dL_{ext} = +dq\,\Delta V = +dq\,\frac{q}{C}$$

$$L_{TOT} = \int_0^Q \frac{q}{C}\, dq = \frac{1}{2}\frac{Q^2}{C} = U_e$$

energia elettrostatica accumulata sul condensatore

$$U_e = \frac{1}{2} Q\,\Delta V = \frac{1}{2} C\,\Delta V^2$$

Per il condensatore piano la capacità vale

$$C = \varepsilon_0 \frac{\Sigma}{d}$$

$$U_e = \frac{1}{2} C \Delta V^2 = \frac{1}{2}\varepsilon_0 \frac{\Sigma}{d}\Delta V^2 = \frac{1}{2}\varepsilon_0 \frac{\Sigma}{\cancel{d}} E^2 d^{\cancel{2}} = \frac{1}{2}\varepsilon_0 E^2 \overbrace{\Sigma d}^{\tau \text{ volume}}$$

$\Delta V = E \cdot d$ solo per condensatore piano o quando E è costante

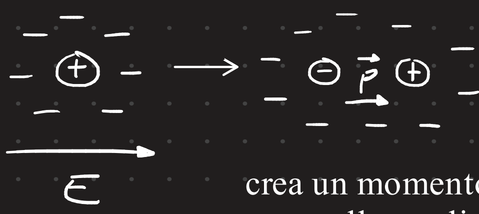
$$u_e = \frac{dU_e}{d\tau} = \frac{1}{2}\varepsilon_0 E^2$$ (densità di energia elettrostatica)

$$U_e = \int_\tau \frac{1}{2}\varepsilon_0 E^2\, d\tau$$

energia accumulata nella regione di spazio grazie al campo elettrico

Campo in materiali isolanti: dielettrici

- Polarizzazione per spostamento

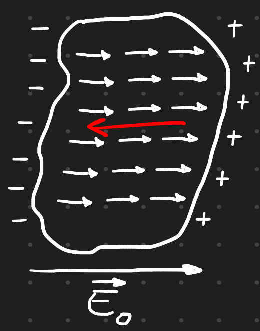


$\langle \vec{p} \rangle \neq 0$

crea un momento di dipolo
non nullo negli atomi

- Polarizzazione per orientamento

$\langle \vec{p} \rangle \neq 0$



carica di
polarizzazione
q_P

genera un campo elettrico che si sommerà
al campo esterno (di verso opposto)

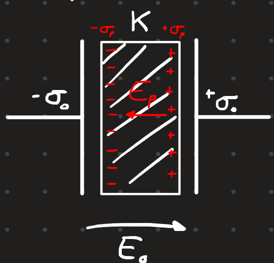
$\vec{P}$ = vettore di polarizzazione $= \dfrac{d\vec{p}}{d\tau}$

$$\sigma_p = \vec{P} \cdot \vec{u}_n$$

densità superficiale
di carica di polarizzazione

$$\vec{P} = \varepsilon_0 \frac{K-1}{K} \vec{E}_0$$

K = costante dielettrica > 1



$$E_0 = \frac{\sigma_0}{\varepsilon_0}$$

$$P = \boxed{\frac{K-1}{K}\sigma_0 = \sigma_p}$$

$$E_p = \frac{\sigma_p}{\varepsilon_0}$$

$$E_0 = \frac{\sigma_0}{\varepsilon_0}$$

$$E_p = \frac{\sigma_p}{\varepsilon_0}$$

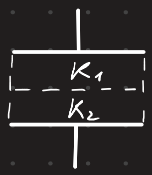
$K\varepsilon_0 = \varepsilon$

$$E = E_0 - E_p = \frac{\sigma_0}{\varepsilon_0} - \frac{K-1}{K}\frac{\sigma_0}{\varepsilon_0} = \frac{\sigma_0}{K\varepsilon_0} = \frac{E_0}{K}$$

campo elettrico nel dielettrico:
campo elettrico nel vuoto diviso
la costante dielettrica

$$\Delta V = \frac{\Delta V_0}{K} \qquad C = K \varepsilon_0 \frac{\Sigma}{d}$$

Esercizio



$\Sigma = 600 \, cm^2$ $\quad \Delta V = 2 \, kV$

$d = 5 \, mm$

$K_1 = 5 \quad K_2 = 2$

1) $C = ?$ 2) $q = ?$ 3) $U_e = ?$

$$C_T = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{K_1 \varepsilon_0 \frac{\Sigma}{d/2}} + \frac{1}{K_2 \varepsilon_0 \frac{\Sigma}{d/2}}$$

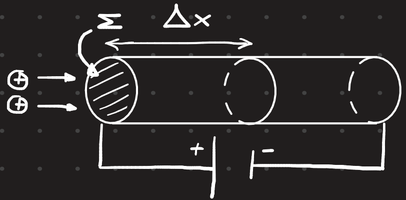
$$\Delta V = q_T \cdot C_T \longrightarrow q_T = \frac{\Delta V}{C_T}$$

$$U_e = \frac{1}{2} q \Delta V$$

Elettrodinamica

$V_1 > V_2$

$\mathcal{E} = \Delta V$ (il generatore mantiene costante la differenza di potenziale)



$i = \frac{dq}{dt} = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t}$ = CORRENTE ELETTRICA

$[i] = \frac{1C}{1s} = 1A$

$\langle \vec{v} \rangle \neq 0$ $\quad \langle \vec{v} \rangle = \vec{v}_d$ = velocità di deriva

in Δt ogni carica percorre $\Delta x = \vec{v}_d \cdot \Delta t$

$\Delta q = n\, q^+ \Delta \tau = n\, q^+ d\Sigma \cos\theta \,\Delta x = n\, q^+ d\Sigma \cos\theta\, \vec{v}_d \Delta t$

$\frac{\Delta q}{\Delta t} = di = n\, q^+ v_d\, d\Sigma \cos\theta = \vec{J} \cdot \vec{u}_n \cdot d\Sigma$

$\vec{J} = n\, q^+ \vec{v}_d$ = densità di corrente $\quad [J] = \frac{1A}{1m^2}$

$i = \int_\Sigma \vec{J}\, \vec{u}_n\, d\Sigma = \Phi_\Sigma(\vec{J})$

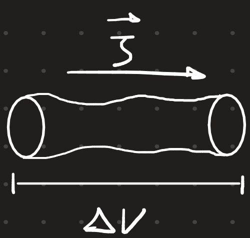
$\vec{J} = n_+ q^+ \vec{v}_+ - n_- |q^-| \vec{v}_-$

Legge di Ohm

$\vec{J} = \sigma \vec{E} = \frac{1}{\rho} \vec{E}$

σ = conducibilità propria di ogni materiale $= \frac{1}{\rho}$

ρ = resistività



$\Delta V = \int_A^B \vec{E}\, d\vec{s} = \int_A^B \rho \vec{J}\, d\vec{s} = \int_A^B \rho J\, ds = \int_A^B \rho \frac{i}{\Sigma}\, ds = i \int_A^B \frac{\rho}{\Sigma}\, ds$

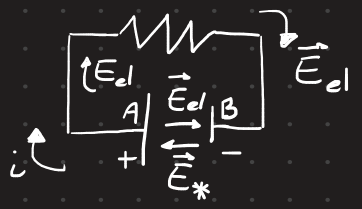
$R = \int_A^B \frac{\rho}{\Sigma}\, ds = \frac{\rho\, l}{\Sigma}$ (se ρ, Σ costanti)

$[\rho] = \Omega \cdot m$

$\Delta V = R \cdot i$

$[R] = \frac{1V}{1A} = 1\Omega$

Legge di Ohm generalizzata

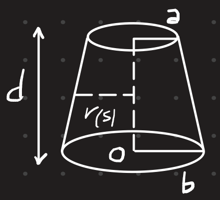


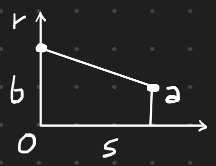
E_* = campo elettromotore

$\vec{E}_* = -\vec{E}_{el}$ (all'interno del generatore)

$$C_{(\vec{E})} = \oint_\gamma \vec{E}\, d\vec{s} = \int_A^B E_{el}\, d\vec{s} + \int_B^A 0 \cdot d\vec{s} = Ri$$

Esercizio 5.6





ρ data

$R = ?$

$$R = \int_0^d \frac{\rho\, ds}{\Sigma} = \rho \int_0^d \frac{ds}{\Sigma(s)}$$

$$r - b = \frac{a-b}{d-0}(s-0) \qquad r = b + \frac{a-b}{d}\, s$$

$$\Sigma = \pi r^2 = \pi\left(b + \frac{a-b}{d}\, s\right)^2$$

$$R = \rho \int_0^d \frac{ds}{\pi\left(b+\frac{a-b}{d}s\right)^2} = \frac{\rho}{\pi}\left[-\frac{1}{b+\frac{a-b}{d}} \times \frac{d}{a-b}\right]_0^d$$

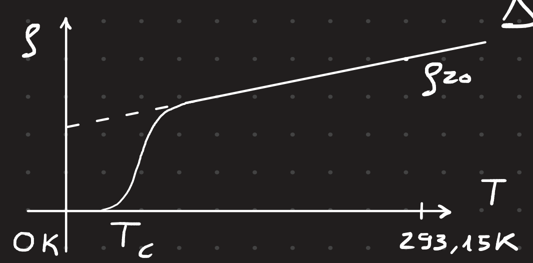
$$= \frac{\rho d}{\pi a b}$$

Effetti termici sulla resistenza

$$\rho(T) = \rho_{20}(1 + \alpha \Delta T)$$

ρ_{20} = resistività a 20 °C

$\Delta T = T - 20\,°C$



T_c = temperatura critica

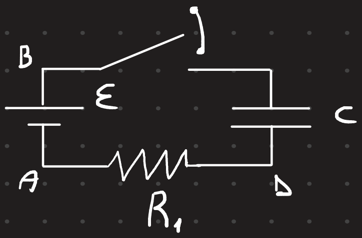
$$dL = dq \cdot \Delta V$$

$$P = \frac{dL}{dq} = \boxed{\frac{dq}{dt}}\,\Delta V = i\,\Delta V = R \cdot i^2 \implies \text{effetto Joule}$$

$$E_J = \int P\, dt = \int_0^{T_1} R i^2\, dt$$

Carica e scarica di un condensatore (circuito RC)

Carica



$$V_B - V_A = (V_B - V_D) + (V_D - V_A)$$

$$\varepsilon = \frac{q(t)}{C} + R\, i_{(t)}$$

$$\varepsilon = \frac{q(t)}{C} + R \frac{dq(t)}{dt}$$

$$\varepsilon - \frac{q}{C} = R\frac{dq}{dt} \Rightarrow dt = \frac{R}{\varepsilon - \frac{q}{C}}\, dq = \frac{R}{\frac{\varepsilon C - q}{C}}\, dq = \frac{CR}{\varepsilon C - q}\, dq$$

$$\Rightarrow \frac{dt}{RC} = \frac{dq}{\varepsilon C - q}$$

$$\int_0^t \frac{dt}{RC} = \int_0^q \frac{dq}{\varepsilon C - q}$$

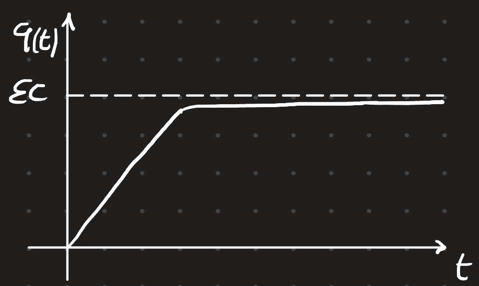
al tempo t = 0 la carica è 0 siccome il circuito è aperto

$$\frac{t}{RC} = \Big[-\ln(\varepsilon C - q)\Big]_0^q = -\ln(\varepsilon C - q) + \ln(\varepsilon C) = -\ln\frac{\varepsilon C - q(t)}{\varepsilon C}$$

$$-\frac{t}{RC} = \ln\left(1 - \frac{q(t)}{\varepsilon C}\right) \qquad e^{-t/RC} = 1 - \frac{q}{\varepsilon C}$$

$$q_{(t)} = \varepsilon C\left[1 - e^{-t/\tau}\right] \qquad \tau = RC \quad [\Omega \cdot F = s]$$

costante di tempo



$$t = 4{,}5\tau$$

$$e^{-t/\tau} \ll 1$$

$$\varepsilon = \frac{q}{C} + Ri$$

condensatore resistenza

$$P_{gen} = \varepsilon \cdot i(t) = \frac{1}{C}\, q(t)\, i(t) + R\, i_{(t)}^2$$

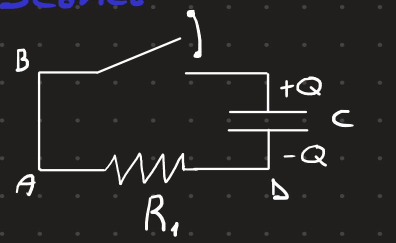
$$i(t) = \frac{dq}{dt} = \frac{\varepsilon}{R} e^{-t/\tau}$$

$$L_{gen} = \int_0^{+\infty} P_{gen}(t)\, dt = \int_0^{+\infty} \frac{1}{C} q \frac{dq}{\cancel{dt}} \cancel{dt} + \int_0^{+\infty} R i^2\, dt =$$

$$\frac{1}{2}\frac{q^2}{C} = \frac{1}{2} C \varepsilon^2 \equiv U_e$$

$$\int_0^{+\infty} R i^2 dt = \frac{\varepsilon^2}{R}\int_0^{+\infty} e^{-\frac{2t}{\tau}}\, dt = \frac{\varepsilon^2}{R}\left[-\frac{\tau}{2} e^{-\frac{2t}{\tau}}\right] = \frac{\varepsilon^2}{R}\frac{\tau}{2} = \frac{1}{2} C \varepsilon^2$$

Scarica



$t=0 \rightarrow Q =$ carica in C

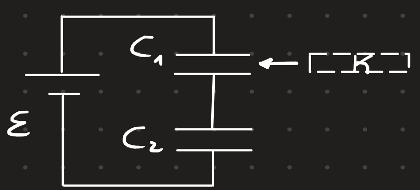
$$\overset{\varepsilon}{\overset{\shortparallel}{0}} = \frac{q(t)}{C} + R\,\frac{dq(t)}{dt}$$

$$-\frac{q}{C} = R\,\frac{dq}{dt} \Rightarrow -\frac{dt}{RC} = \frac{dq}{q}$$

$$-\int_0^t \frac{dt}{RC} = \int_0^q \frac{dq}{q}$$

$$-\frac{t}{RC} = \ln\frac{q}{Q} \qquad q(t) = Q e^{-t/\tau}$$

Esercizio 4.10



$C_1 = 500\ pF$
$C_2 = 100\ pF$
$\varepsilon = 400V$
$K = 4$
a) Δq energia dal generatore
b) variazione ΔV_1 (ai capi di C_1)
c) energia fornita dal generatore

ⓐ $\varepsilon = \dfrac{q}{C_T} \longrightarrow q = \varepsilon C_T$ q prima dielettrico

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 C_2}{C_1 + C_2}$$

$$C_T' = \frac{K C_1 C_2}{K C_1 + C_2}$$

$$\Delta q = C_T' \varepsilon - C_T \varepsilon = (C_T' - C_T)\varepsilon$$

ⓑ

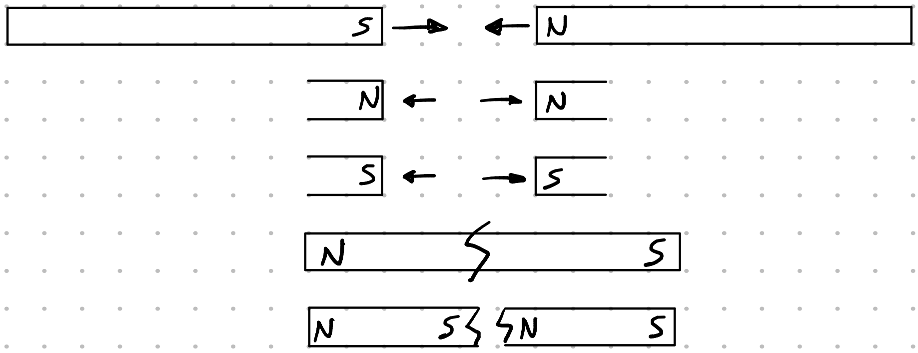
$$V_1 = \frac{Q}{C_1} = \frac{C_T}{C_1}\varepsilon \qquad \Delta V_1 = V_1' - V_1$$

$$V_1' = \frac{Q'}{C_1'} = \frac{C_T'\varepsilon}{K C_1}$$

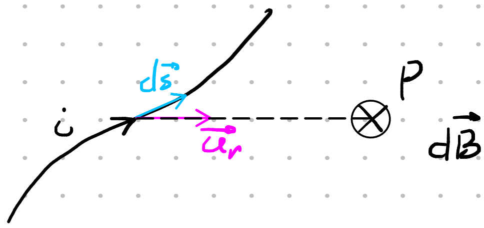
ⓒ

$$E_{gen} = \Delta U_e = \frac{1}{2} C_T' \varepsilon^2 - \frac{1}{2} C_T \varepsilon^2 = \frac{1}{2}(C_T' - C_T)\varepsilon^2$$

Magnetismo



I^a legge di Laplace



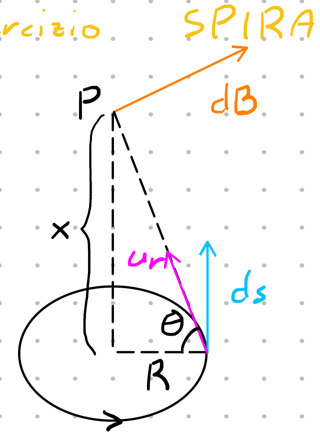
B = campo magnetico

$[B]$ = Tesla (T)

$$\frac{\mu_0}{4\pi} = 10^{-7}\ \frac{T \cdot m}{A}$$

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi}\,\frac{i\,|d\vec{s}\times\vec{u}_r|}{r^2}}$$

Esercizio SPIRA CIRCOLARE



ds è entrante nel foglio

$$dB_z = \frac{\mu_0}{4\pi}\,\frac{i\,|d\vec{s}\times\vec{u}_r|}{r^2}\cos\theta$$

$$= \frac{\mu_0}{4\pi}\, i\, \frac{d\vec{s}}{R^2+x^2}\,\frac{R}{\sqrt{x^2+R^2}}$$

$$\vec{B}_z = \int dB_z = \frac{\mu_0}{4\pi}\, i\, \frac{R}{(x^2+R^2)^{3/2}} \int d\vec{s} =$$

$2\pi R$

$$= \frac{\mu_0}{4\pi}\,\frac{i\,2\pi R}{(x^2+R^2)^{3/2}}$$

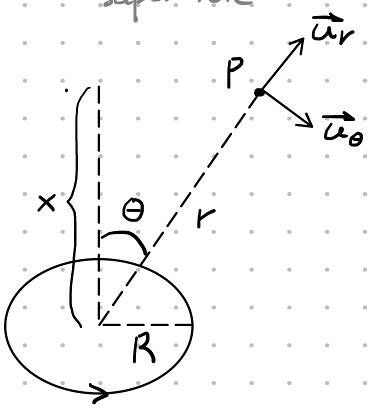
$m = i\,\Sigma = i\,R^2\pi \equiv$ momento di dipolo $\qquad \vec{m} = i\,\Sigma\,\vec{u}_n$

normale alla superficie

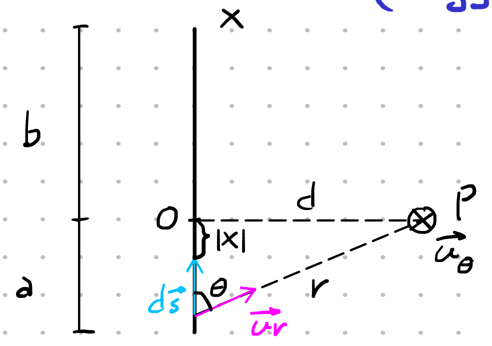
$$B_z = \frac{\mu_0}{2\pi}\,\frac{m}{(x^2+R^2)^{3/2}} \simeq r^3$$

$$\vec{B} = \frac{\mu_0}{4\pi}\,\frac{m}{r^3}\left(2\cos\theta\,\vec{u}_r + \sin\theta\,\vec{u}_r\right) \qquad r \gg R$$

per qualsiasi punto P



Filo infinito (Legge di Biot-Savart) $\vec{B} = \frac{\mu_0 i}{2\pi d}\vec{u}_\theta$



a, b, i, d dati

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i|d\vec{s}\times\vec{u}_r|}{r^2}$$

$$= \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{d^2/\sin^2\theta}\vec{u}_\theta \qquad r = \frac{d}{\sin\theta}$$

$ds = dx \qquad x_\theta = -\frac{d}{tg\theta} \qquad dx = \frac{d}{\sin^2\theta}d\theta$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,\cancel{d}/\cancel{\sin^2\theta}\,\sin\theta}{\cancel{d^2}/\cancel{\sin^2\theta}}d\theta\,\vec{u}_\theta$$

$\cos\theta_0 = \frac{a}{\sqrt{a^2+d^2}}$

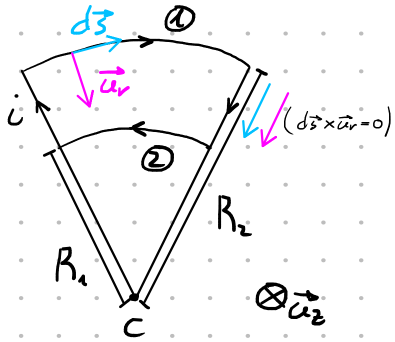
$\cos\theta_1 = \frac{b}{\sqrt{b^2+d^2}}$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{i\vec{u}_\theta}{d}\int_{\theta_0}^{\theta_1}\sin\theta\,d\theta = \frac{\mu_0}{4\pi}\frac{i}{d}\vec{u}_\theta(\cos\theta_0 - \cos\theta_1)$$

filo infinito $\qquad \theta_0 \to 0 \qquad \theta_1 \to \pi$

$$\boxed{\vec{B} = \frac{\mu_0 i}{2\pi d}\vec{u}_\theta}$$

Esercizio 7.9



$\vec{B}_c = ?$

$\vec{m} = ?$

$m = i\theta\left(\frac{R_2^2 - R_1^2}{2}\right)$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i|d\vec{s}\times\vec{u}_r|}{r^2} \qquad \int ds = \theta R_2$$

$$d\vec{B}_1 = \frac{\mu_0}{4\pi}\frac{i\,ds}{R_2^2}\vec{u}_z \implies \vec{B}_1 = \frac{\mu_0}{4\pi}\frac{i\theta}{R_2}\vec{u}_z$$

$$\vec{B}_2 = -\frac{\mu_0}{4\pi}\frac{i\theta}{R_1}\vec{u}_z$$

$$B_c = \frac{\mu_0 i\theta}{4\pi}\left(\frac{1}{R_2} - \frac{1}{R_1}\right)\vec{u}_z$$

Riassunto legge di Laplace

$$d\vec{B} = \frac{\mu_0}{4\pi}\,\frac{i\,|d\vec{s}\times\vec{u}_r|}{r^2} \Longrightarrow \frac{\mu_0}{4\pi}\,\frac{dq\,\vec{v}\times\vec{u}_r}{r^2}$$

$$i = \frac{dq}{dt} \qquad i\,d\vec{s} = \frac{dq}{dt}\cdot d\vec{s} = dq\cdot\vec{v}$$

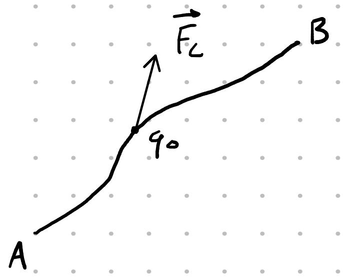
velocità di deriva cariche

$$B = \frac{\mu_0}{4\pi}\,\frac{q\,\vec{v}\times\vec{u}_r}{r^2} \qquad |\vec{v}| << c \text{ (velocità luce)}$$

$$\vec{F}_e = q\vec{E} \longrightarrow$$ carica in presenza di un campo esterno

carica q in
un campo esterno

Forza di Lorentz



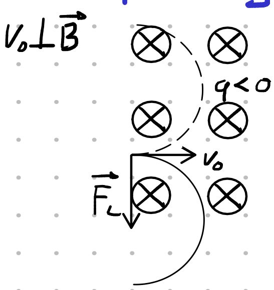
$$\boxed{\vec{F}_L = q_0(\vec{v}\times\vec{B})} \qquad \vec{F}_L = q_0(\cancel{\vec{E}}+\vec{v}\times\vec{B}) \quad \vec{E}=0$$

$$L_{AB} = \int_A^B \vec{F}_L\, d\vec{s} = \int_A^B q_0\overbrace{(\vec{v}\times\vec{B})\,\vec{v}}^{0}\,dt = 0$$

$$\Delta E_K = \frac{1}{2}(\vec{v}_B - \vec{v}_A)\,m$$

$$\Longrightarrow |\vec{v}_A| = |\vec{v}_B|$$

Campo magnetico uniforme



$\vec{B}$

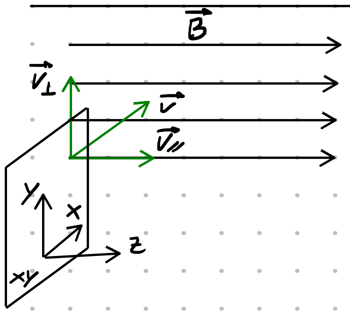
$$|\vec{F}_L| = q\,v_0\times\vec{B}$$

verso il basso

forza centrifuga

$$F_c = m\frac{v_0^2}{R} = |F_L| = qv_0B$$

$$R = \frac{mv_0}{qB} \qquad \omega = \frac{V_0}{V} = \frac{qB}{m} \qquad T = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

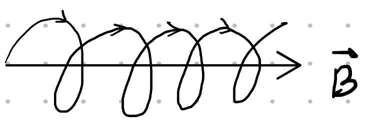


$$\vec{v} \not\perp \vec{B} \qquad \vec{v} = \vec{v}_{//} + \vec{v}_\perp$$

$$\vec{F}_L = q\vec{v}\times\vec{B} = q\vec{v}_\perp\times\vec{B} + \cancel{q\vec{v}_{//}\times\vec{B}} = 0$$ paralleli

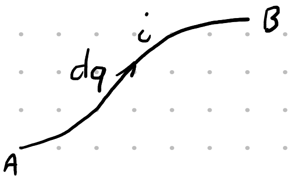
$$= q\vec{v}\times\vec{B}$$

$$\vec{E}_z = 0 \qquad z(t) = z_0 + v_{//}t$$ moto rettilineo uniforme lungo z



moto circolare lungo xy

II^a legge di Laplace



$$d\vec{F} = dq\,\vec{v}_d \times \vec{B} \qquad dq\,\vec{v}_d = i\,d\vec{s}$$

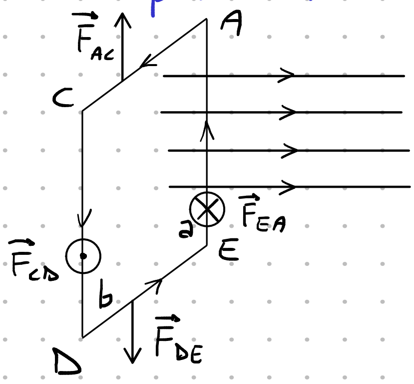
$$\boxed{d\vec{F} = i\,d\vec{s} \times \vec{B}}$$

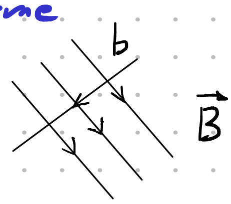
Forza subita dal filo percorso dalla corrente in presenza di un campo magnetico

$$\vec{F} = \int_A^B i\,d\vec{s} \times \vec{B}$$ con $\vec{B}$ costante e uniforme

$$\Longrightarrow\ i\left(\int_A^B d\vec{s}\right) \times \vec{B} = i\,\vec{AB} \times \vec{B}$$

Circuito piano in un campo uniforme

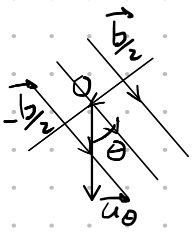




$$\left.\begin{aligned}\vec{F}_{AC} &= i\,\vec{AC} \times \vec{B}\\ \vec{F}_{DE} &= i\,\vec{DE} \times \vec{B}\end{aligned}\right\}\ \text{si annullano}$$

$$\vec{F}_{EA} = i\,\vec{EA} \times \vec{B} \qquad \text{entrante}$$

$$\vec{F}_{CD} = i\,\vec{CD} \times \vec{B} = -\vec{F}_{EA} \qquad \text{uscente}$$



Il centro di massa è 0
le due forze generano un momento

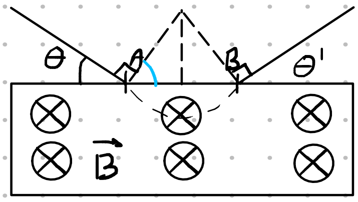
$$\vec{M} = \frac{\vec{b}}{2} \times \vec{F}_{AE} - \frac{\vec{b}}{2} \times \vec{F}_{CD} = \vec{b} \times \vec{F}_{AE}$$

$$|\vec{M}| = b \cdot F_{AE} \sin\theta = i\,a\,b\,B \sin\theta \qquad \vec{m} = i\,a\,b\,\vec{u}_n = i\,\Sigma\,\vec{u}_n$$

$$\vec{M} = \vec{m} \times \vec{B}$$

momento di dipolo magnetico

Esercizio 6.1



$$d = 2R\cos\left(\boxed{\frac{\pi}{2} - \theta}\right)$$

$$R = \frac{mv_0}{qB}$$

Riassunto

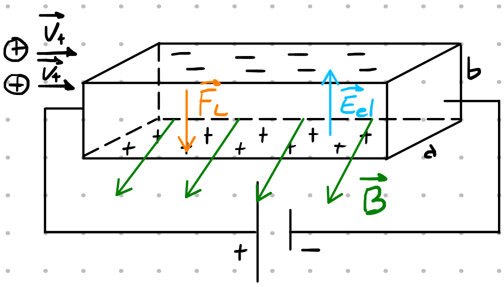
$$\vec{F}_L = q\left(\vec{E} + v \times \vec{B}\right)$$

$$d\vec{F} = i\,d\vec{s} \times \vec{B}$$

$$\vec{m} = i\,\Sigma\,\vec{u}_n \qquad \vec{M} = \vec{m} \times \vec{B} \qquad U_m = \vec{m}\,\vec{E} \qquad \text{magnetico}$$

$$\vec{M}_e = \vec{p} \times \vec{E} \qquad U_e = -\vec{p}\,\vec{E} \qquad \text{elettrico}$$

Effetto Hall



B perpendicolare al momento delle cariche

$$\vec{F}_L = q_+ \vec{v} \times \vec{B}$$

La faccia superiore si carica negativamente, quella inferiore positivamente. Così facendo si forma un campo elettrostatico verso l'alto.

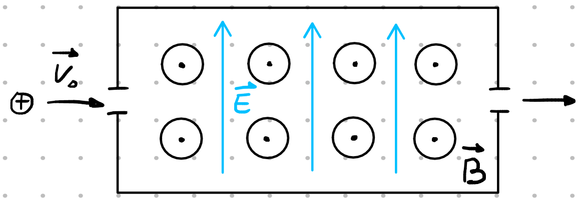
$$\vec{E}_H = \frac{\vec{F}_L}{q_+} = \vec{v} \times \vec{B} \qquad \text{campo di Hall}$$

$\Longrightarrow$ situazione a regime: $\vec{E}_H + \vec{E}_{el} = 0$

$$|\vec{E}_{el}| = |\vec{E}_H| = v \cdot \vec{B} \cdot \cancel{\sin 90°}^{\,=1} = \frac{J}{nq_+} B = \frac{i}{nabq_+} \vec{B}$$

$$\Delta V_H = |E_H|\, b = \frac{iB}{naq_+}$$

Selettore di velocità



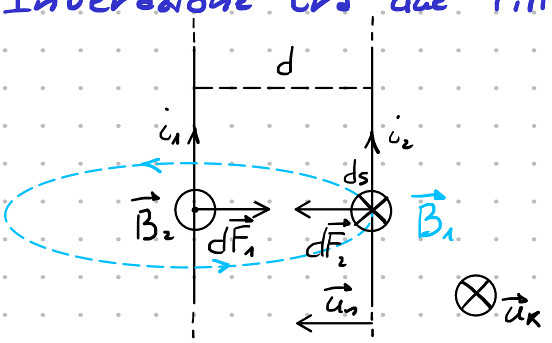
$$\vec{F}_L = q\left(E + v_0 \times \vec{B}\right) = 0$$ (la forza è 0 per avere un moto rettilineo uniforme)

$$E = -v_0 \times \vec{B}$$

$$E = v_0 B \cancel{\sin\theta}^{\,=1} = v_0 B \Longrightarrow v_0 = \frac{E}{B}$$

$$1\,T = \frac{1\,V}{m\, m/s} = \frac{Vs}{m^2}$$

Interazione tra due fili percorsi da corrente



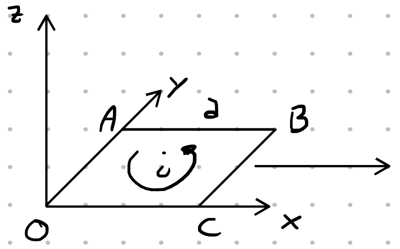
$$\vec{B}_1 = \frac{\mu_0}{2\pi d} i_1 \vec{u}_K$$

$$d\vec{F}_2 = i_2\, d\vec{s} \times \vec{B}_1 = \frac{\mu_0 i_1 i_2}{2\pi d}\, ds\, (\vec{u}_n)$$

$$\frac{dF}{ds} = \frac{\mu_0 i_1 i_2}{2\pi d}$$

Esercizio 6.23

$a = 20\,cm$
$i = 5A$
$\vec{B} = \lambda x \vec{u}_z$
λ costante
$\vec{F} = ?$



$\vec{F}_{BC}$

$d\vec{F} = i\, d\vec{s} \times d\vec{B}$

II^a legge di Laplace

$\vec{F}_{AO} = 0$ per $x=0$ il campo è nullo

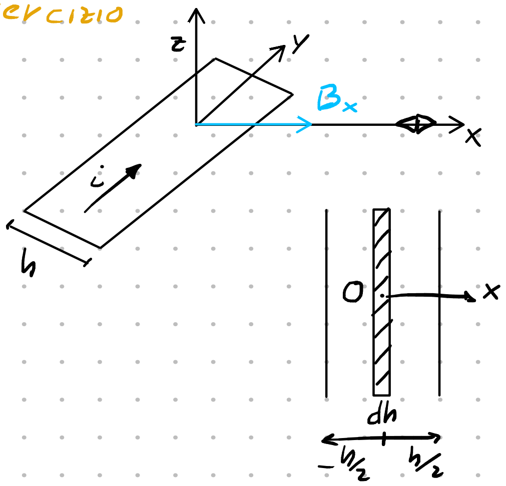
$\vec{F}_{BC} = i \cdot a \vec{u}_y \times \lambda a \vec{u}_z = \lambda i a^2 \vec{u}_x$

$$\vec{F}_{OC} = \int_0^a i\, d\vec{s} \times (\lambda x \vec{u}_z) = \lambda i \int_0^a x\, dx (-\vec{u}_y)$$

$$= \lambda i \frac{a^2}{2} (-\vec{u}_y)$$

$\vec{F}_{AB} = -\vec{F}_{OC} \implies \vec{F}_{AB} + \vec{F}_{OC} = 0$

Esercizio



$\frac{di}{i} = \frac{dh}{h} \implies di = \frac{dh}{h} \cdot i$

$$B = \int_{-h/2}^{+h/2} \frac{\mu_0}{2\pi} \frac{di}{(x-y)} = \int_{-h/2}^{+h/2} \frac{\mu_0}{2\pi} \frac{i}{(x-y)h}\, dh = \dots$$

distanza filo/punto

$\vec{M} = \vec{m} \times \vec{B} = 0 \quad \vec{m} \perp \vec{B}$