Number Systems

Decimal, Binary, Octal, and Hexadecimal

Decimal System

The ‘Decimal System’ is the one we are all familiar with. It is “Base 10” and uses 10 symbols you have become accustomed to: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

2	3	9

All number systems are built in a similar way based on ‘face value’ (the digit) and ‘place value’ (its position).

Write the following number in expanded form:

124_{10} =

5037_{10} =

Counting in the Decimal System (Base 10):

1, 2, …. , 9, 10, 11, 12, …. , 19, 20, 21, ….., 29, 30, 31, ….. , 99, 100, 101, …., 999, 1000, 1001

When we run out of ‘digits’, we ‘carry over’ to the next one and start again. The same is true for other systems, but some have more or less digits.

Binary System

- Represents numeric values using two symbols, typically 0 (off) and 1 (on)
- Used in computer systems as they are constructed of digital electronics – meaning their electronic circuits can exist in only one of two states: on or off.
- The Binary system is a "base 2" system (unlike the decimal system which is "base 10")
- Binary can be used to represent numbers and letters in programming based on predetermined designations (for example "ASCII" is commonly used)
- A single binary digit is a bit and a group of eight bits is a binary term or a byte.

Binary System				
1	1	0	1	1

Binary System				
1	0	1	1	0

Convert **from Binary to Decimal** by writing the following number in expanded form:

$1\,1\,0\,1\,0\,1\,0\,1_2$ =

$1\,1\,0\,0\,1\,1_2$ =

Decimal System – Binary System							
0			6			12	
1			7			13	
2			8			14	
3			9			15	
4			10			16	
5			11				

1st Method - How to convert **from Decimal to Binary** using Binary System Place Values:

Convert: 24_{10} =

Convert: 31_{10} =

2nd Method - How to convert **from Decimal to Binary**

Step-1: Divide the Number by 2 (as 2 is the base of the binary number system).

Step-2 : Collect the remainder.

Step-3: Divide the quotient again with 2.

Step-4: Repeat the step 2 & 3 until the quotient is 0.

Step-5: Start from bottom, read the sequence of remainders upwards to the top.

Practice

1) 110101_2 = ________________$_{10}$

2) 111100_2 = ________________$_{10}$

3) 98_{10} = ________________$_2$

4) 67_{10} = ________________$_2$

5) 10011_2 = ________________$_{10}$

6) 1111001_2 = ________________$_{10}$

7) 90_{10} = ________________$_2$

8) 41_{10} = ________________$_2$

Octal System

- Represents numeric values using EIGHT symbols: 0, 1, 2, 3, 4, 5, 6, 7
- The Octal system is a "base 8" system (unlike the decimal system which is "base 10")

Octal System			
2	1	6	5

Octal System			
1	4	3	0

Convert **from Octal to Decimal** by writing the following number in expanded form:

$3\,4\,1\,6_8$ =

$142_8 =$

$243_8 =$

Decimal System – Octal System							
0			6			12	
1			7			13	
2			8			14	
3			9			15	
4			10			16	
5			11				

1st Method – Convert **from Decimal System to Octal System** using Octal Place Values:

Convert: $96_{10} =$

Convert: $616_{10} =$

2^{nd} Method: How to convert **from Decimal to Octal**

Step-1: Divide the Number by 8 (as 8 is the base of the binary number system).

Step-2 : Collect the remainder.

Step-3: Divide the quotient again with 8.

Step-4: Repeat the step 2 & 3 until the quotient is 0.

Step-5: Start from bottom, read the sequence of remainders upwards to the top.

Practice

1) 77_8 = ________________$_{10}$

5) 55_8 = ________________$_{10}$

2) 53_8 = ________________$_{10}$

6) 100_8 = ________________$_{10}$

3) 124_{10} = ________________$_8$

7) 76_{10} = ________________$_8$

4) 312_{10} = ________________$_8$

8) 45_{10} = ________________$_8$

Hexadecimal System

- Hexadecimal system is a "base 16" system (unlike the decimal system which is "base 10")
- Represents numeric values using SIXTEEN symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F
- Each digit can only take up one space in the place value system so in order to use base sixteen, we need letters (A-F) to represent numbers 10-15
- **A = 10 , B = 11, C = 12, D = 13, E = 14, F = 15 (must memorize)**

Hexadecimal System			
2	E	8	A

Hexadecimal System			
1	5	F	B

Convert **from Hexadecimal to Decimal** by writing the following number in expanded form:

$3\ B\ D\ 6_{16}$ =

$9\ A\ 2_{16}$ =

$2\ 4\ F_{16}$ =

Decimal System – Hexadecimal System							
0			7			14	
1			8			15	
2			9			16	
3			10			17	
4			11			18	
5			12			19	
6			13			20	

<u>1^{st} Method – Convert **from Decimal System to Hexadecimal System** using Hexadecimal Place Values:</u>

Convert: 96_{10} =

Convert: 616_{10} =

2nd Method - How to convert **from Decimal to Hexadecimal**

Step-1: Divide the Number by 16 (as 16 is the base of the binary number system).

Step-2 : Collect the remainder.

Step-3: Divide the quotient again with 16.

Step-4: Repeat the step 2 & 3 until the quotient is 0.

Step-5: Start from bottom, read the sequence of remainders upwards to the top.

Practice

1) $3D_{16}$ = ________________$_{10}$

2) 164_{16} = ________________$_{10}$

3) 215_{10} = ________________$_{16}$

4) 645_{10} = ________________$_{16}$

5) AD_{16} = ________________$_{10}$

6) 412_{16} = ________________$_{10}$

7) 76_{10} = ________________$_{16}$

8) 389_{10} = ________________$_{16}$

Converting from Binary to Octal or Hexadecimal

Powers of 2	1	2	4	8	16	32	64	128	256
Powers of 8	1			8			64		
Powers of 16	1				16				256

Notice: Every “THIRD” power of 2 is a power of 8 and every “FOURTH” power of 2 is a power of 16

Because of this converting between binary to octal or hexadecimal is very easy.

BINARY TO OCTAL

If converting from binary to octal, divide binary number in to groups of 3 (starting from the right most digit) then convert each binary digit to its octal digit and combine the digits to create the new value.

If converting from octal to binary, turn each octal digit into its equivalent binary number (must be 3 bits long, include 0’s where necessary) and put them together.

BINARY TO HEXADECIMAL

If converting from binary to hexadecimal, divide binary number in to groups of 4 (starting from the right most digit) then convert each binary digit to its hexadecimal digit and combine the digits to create the new value.

If converting from hexadecimal to binary, turn each hexadecimal digit into its equivalent binary number (must be 4 bits long, include 0’s where necessary) and put them together.

Note: To Convert between Octal and Hexadecimal – change to binary first

Practice

1) Convert the following to binary or decimal, as indicated, **WITHOUT A CALCULATOR.**

a) $1101_2 = \underline{\qquad\qquad\qquad}_{10}$

b) $10101_2 = \underline{\qquad\qquad\qquad}_{10}$

c) $101101_2 = \underline{\qquad\qquad\qquad}_{10}$

d) $19_{10} = \underline{\qquad\qquad\qquad}_{2}$

e) $6_{10} = \underline{\qquad\qquad\qquad}_{2}$

f) $11_{10} = \underline{\qquad\qquad\qquad}_{2}$

2) Convert the following numbers to the system indicated.

a) $101101_2 = \underline{\qquad\qquad\qquad}_{10}$

b) $10011_2 = \underline{\qquad\qquad\qquad}_{10}$

c) $1100111_2 = \underline{\qquad\qquad\qquad}_{10}$

d) $96_{10} = \underline{\qquad\qquad\qquad}_{2}$

e) $173_{10} = \underline{\qquad\qquad\qquad}_{2}$

f) $412_8 = \underline{\qquad\qquad\qquad}_{10}$

g) $67_8 = \underline{\qquad\qquad\qquad}_{10}$

h) $204_8 = \underline{\qquad\qquad\qquad}_{10}$

i) $314_{10} = \underline{\qquad\qquad\qquad}_{8}$

j) $194_{10} = \underline{\qquad\qquad\qquad}_{8}$

k) $D39_{16} = \underline{\qquad\qquad\qquad}_{10}$

l) $ABC_{16} = \underline{\qquad\qquad\qquad}_{10}$

m) $179_{10} = \underline{\qquad\qquad\qquad}_{16}$

n) $983_{10} = \underline{\qquad\qquad\qquad}_{16}$

o) $100101_2 = \underline{\qquad\qquad\qquad}_{8}$

p) $1101111_2 = \underline{\qquad\qquad\qquad}_{8}$

q) $1101101110_2 = \underline{\qquad\qquad\qquad}_{8}$

r) $110011_2 = \underline{\qquad\qquad\qquad}_{16}$

s) $1100101011_2 = \underline{\qquad\qquad\qquad}_{16}$

t) $1001101101011_2 = \underline{\qquad\qquad\qquad}_{16}$

3) Which is the larger binary number: 10010_2 or 10100_2?

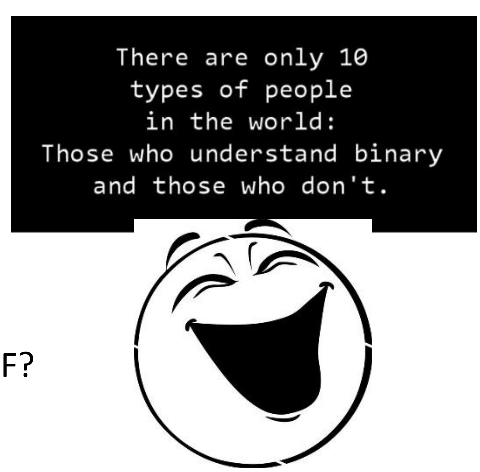
4) What is the largest five-digit binary number?

5) What octal number comes after 777_8?

6) What is the largest 3-digit hexadecimal number?

7) What is the smallest 5-digit hexadecimal number?

8) What hexadecimal number comes after: 98D? 125? BD? D2F?



Appendix B: ASCII Codes

Printable 8-bit ASCII codes

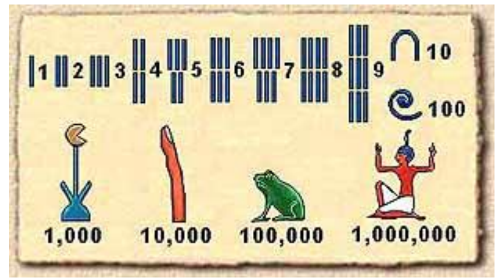
Decimal	Binary	Symbol	Decimal	Binary	Symbol	Decimal	Binary	Symbol
032	00100000	(space)	064	01000000	@	096	01100000	`
033	00100001	!	065	01000001	A	097	01100001	a
034	00100010	"	066	01000010	B	098	01100010	b
035	00100011	#	067	01000011	C	099	01100011	c
036	00100100	$	068	01000100	D	100	01100100	d
037	00100101	%	069	01000101	E	101	01100101	e
038	00100110	&	070	01000110	F	102	01100110	f
039	00100111	'	071	01000111	G	103	01100111	g
040	00101000	(	072	01001000	H	104	01101000	h
041	00101001	)	073	01001001	I	105	01101001	i
042	00101010	*	074	01001010	J	106	01101010	j
043	00101011	+	075	01001011	K	107	01101011	k
044	00101100	,	076	01001100	L	108	01101100	l
045	00101101	-	077	01001101	M	109	01101101	m
046	00101110	.	078	01001110	N	110	01101110	n
047	00101111	/	079	01001111	O	111	01101111	o
048	00110000	0	080	01010000	P	112	01110000	p
049	00110001	1	081	01010001	Q	113	01110001	q
050	00110010	2	082	01010010	R	114	01110010	r
051	00110011	3	083	01010011	S	115	01110011	s
052	00110100	4	084	01010100	T	116	01110100	t
053	00110101	5	085	01010101	U	117	01110101	u
054	00110110	6	086	01010110	V	118	01110110	v
055	00110111	7	087	01010111	W	119	01110111	w
056	00111000	8	088	01011000	X	120	01111000	x
057	00111001	9	089	01011001	Y	121	01111001	y
058	00111010	:	090	01011010	Z	122	01111010	z
059	00111011	;	091	01011011	[	123	01111011	{
060	00111100	<	092	01011100	\	124	01111100	|
061	00111101	=	093	01011101	]	125	01111101	}
062	00111110	>	094	01011110	^	126	01111110	~
063	00111111	?	095	01011111	_			

JUST FOR FUN: A HISTORY LESSON OF OTHER NUMBER SYSTEMS

Some of the earliest known number systems belong to the Egyptians, Babylonians, and Romans.

Imagine yourself in Egypt 5500 years ago, at a time when Egyptians used their fingers as standard counting units. To keep record of counted objects, the Egyptians used short straight strokes called 'tally marks'. Representing the numbers from 1 to 9 (the Egyptians had no number for 0).

The Egyptian system of numeration is additive. IE. the numeral "II" represents 1 + 1 = 2 and "III ∩" represents 1 + 1 + 1 + 10 = 13.



The Egyptians used Hieroglyphics for their numerals. The Egyptians used a **base 10 system** but they generally wrote their numbers from **right to left** or even top to bottom. This is acceptable due to its additive, rather than positional, nature. We could not do this since 12 and 21 are different to us since our system is based on position.

What number is being represented below?

Write the number 3254 in Egyptian numerals.

The Roman numeral system was developed between 500 B.C. and 100 A.D. is still used today. Although both the Roman and Egyptian systems used the addition principle, the Romans went a step further and used ***the subtraction principle*** as well. For example, instead of writing "IIII" for "4", the Romans would wrote "IV" with the understanding that the "I" was to be subtracted from the "V".

In the Roman system, the value of a numeral is found by starting at the left and adding the values of the succeeding symbols to the right – unless the value of a symbol is less than that of the symbol to its right! In that case, the smaller value is *subtracted* from the larger one.
Example: IX = 10 - 1 = 9 but XI = 10 + 1 = 11.

Roman Numeral	Number
I	1
V	5
X	10
L	50
C	100
D	500
M	1000

However – only the numbers 1, 10, and 100 can be subtracted and only from numbers no larger than two steps larger.
Example: 99 must be written as LXLIX not IC

What number is being represented below?

MCMXLIX

Write the number 3254 in Roman numerals.