

**Logic concepts:** proposition,  $P$ , logical operators, **not**  $P$ ,  $P$  **and**  $Q$ ,  $P$  **or**  $Q$ , truth table, logically equivalent statements,  $(P \equiv Q)$ , conditional proposition,  $(P \Rightarrow Q)$ , contrapositive, predicates, quantifiers, ‘for all’,  $\forall x$ , ‘there exists’,  $\exists x$ , negation of statements with quantifiers, direct argument, contrapositive argument, proof by contradiction, the principle of mathematical induction,

**Logic skills:** Decide if statement is proposition, compute truth tables for compound propositions, determine if propositions are logically equivalent, determine the truth of predicate propositions with quantifiers, negate propositions involving quantifiers, use direct argument, contrapositive argument and proof by contradiction, use proof by mathematical induction.

**Set concepts:** set,  $x \in A$ ,  $x \notin A$ ,  $X = Y$ ,  $\mathbb{N}$ ,  $\mathbb{Z}$ ,  $\mathbb{Q}$ ,  $\mathbb{R}$ ,  $\mathbb{C}$ ,  $\emptyset$ ,  $A \subseteq B$ , properties of set inclusion,  $P(A)$ ,  $A \cup B$ ,  $A \cap B$ , Venn diagram,  $X - A$ , universal set,  $U - A$ ,  $\sim A$ , set identities from logical equivalences, symmetric difference,  $|A|$  for  $A$  finite,  $|A \cup B|$ ,  $|A \cup B \cup C|$ ,  $A \times B$ ,

**Set skills:** determine if a given object is or is not an element of a given set, draw Venn diagrams of set relationships, apply  $|A \cup B|$  and  $|A \cup B \cup C|$  identities, apply  $|A \times B| = |A||B|$  identity, convert logical rules to set theory.

**Relation concepts:** binary relation  $R \subset A \times B$ , digraph and matrix of a relation, predicate description of relation, reflexive relation, symmetric relation, antisymmetric relation, transitive relation, closure of relation with respect to a property, equivalence relation, partition of a set, partitions determine equivalence relations and vice versa, partial order, poset, Hasse diagram, total order, lexicographic order, inverse of a relation, composition of relations, digraph of composition,

**Relation skills:** Convert binary relations from subsets to digraphs to matrices, determine if relation on a set is reflexive, symmetric, antisymmetric, transitive, compute transitive closure of relation, convert equivalence relations to partitions and vice versa, compute Hasse diagrams of partial orders, determine if partial order is a total order, order product sets lexicographically, compute inverse of relation, compute composition of relations.

**Function concepts:** function, digraph of a function, matrix of a function,  $b = f(a)$ , domain, codomain, subsets of  $A$  as functions from  $A$  to  $\{0, 1\}$ , range, graph of a function, equality of functions, surjective, injective, bijective, invertible function, invertible if and only if bijective, composition of functions,  $g \circ f$ ,  $g \circ f \neq f \circ g$ ,  $h \circ (g \circ f) = (h \circ g) \circ f$ ,  $\text{id}_A$ , properties of

identity,  $|x|$ ,  $\lfloor x \rfloor$ ,  $(\text{mod})n$ , pigeonhole principle.

**Function skills:** determine if a relation is a function or not, draw digraph of function, test functions for equality, determine if a function is surjective, injective, bijective, determine if a function is invertible, compute function inverses, compute function compositions, compute with absolute value function  $|x|$ , floor function  $\lfloor x \rfloor$  and  $(\text{mod})n$  functions, apply pigeonhole principle and its extension.

**Combinatorics concepts:** addition principle of counting, difference rule, product principle of counting, permutation,  $n!$ ,  ${}^nP_r$ ,  ${}^nC_r$  or  $\binom{n}{r}$ , number of subsets of size  $r$ , binomial theorem, Pascal's triangle, selections order important or not and replacement occurring or not,  $r$ -selection from  $n$ , the number of  $r$ -selections from  $n$ , multinomial coefficients, arrangements of objects with some identical, number of ways of partitioning a set into ordered subsets,

**Combinatorics skills:** apply addition principle, difference rule and product principle of counting, enumerate permutations and combinations, compute with  $n!$ ,  ${}^nP_r$  and  ${}^nC_r$  or  $\binom{n}{r}$ , enumerate subsets of a given size, compute coefficients using binomial theorem, solve enumeration problems involving sampling with and without replacement where order is or is not important, compute multinomial coefficients, apply multinomial coefficients to count arrangements and ordered partitions.

**Probability concepts:** outcome, sample space, event, probability measure, probability of an outcome, probability of an event, equally likely outcomes, addition rule for probabilities, the complement rule, conditional probability, the law of total probability, Bayes' formula, independent events, Bernoulli trials, the binomial distribution.

**Probability skills:** assign probability measure to sample space, compute probability of events using combinatorics, apply the addition rule for probabilities and the complement rule, compute conditional probability, apply the law of total probability, apply Bayes' formula, determine if events are independent, determine probability of  $k$  successes in  $n$  Bernoulli trials.