SVR-Wavelet Adaptive Model for Forecasting Financial Time Series

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***Abstract*—There is a necessity to anticipate and identify changes in events points to a new direction in the stock exchange markets in line with the analysis of the oscillations of prices of financial assets. This need leads to argue about new alternatives in the prediction of financial time series using machine learning methods. This paper aims to describe the development of the SVR-wavelet model, an adaptive and hybrid prediction model, which integrates wavelet models and Support Vector Regression (SVR), for prediction of financial time series, particularly applied to Foreign Exchange Market (FOREX), obtained from a public knowledge base. The method consists of using the Discrete Wavelets Transform (DWT) to decompose data from FOREX time series, that are used as SVR input variables to predict new data. The adjusted series are compared with traditional models such as ARIMA and ARFIMA Model. In Addition, statistical tests like normality and unit root are performed to prove that the series in question have non-linear distribution and also to verify the level of correlation between the periods of the series.**

Keywords-SVR; wavelets; FOREX; financial time series; ARIMA; ARFIMA

I. INTRODUCTION

One of the objectives of the analysis of financial time series is the evaluation of the risks inherent in the investments of financial portfolios. This risk is usually measured in terms of asset price changes [1].

With the economic globalization, the companies that operate in this market are inevitably exposed to the risks of oscillation of the exchange rates both, positively and negatively. Consequently, the forecast of these financial securities has an important meaning for multinational companies [2].

Several methods and models have been developed to forecast financial securities, which leads to arguments about new alternatives in the prediction of financial time series [2].

Recently a growing number of studies have appeared on the use of machine learning methods in the forecasting of financial time series. Some features explain its use in this area. First, they are self-adaptive models, so they learn from examples. Add to this its ability to recognize nonlinear patterns in time series data. This characteristic is found in the movements of the Financial Market [3]. Moreover, machine learning methods can infer missing data in the training process even if the patterns present noises [4].

The objective of this paper is to describe the development of the SVR-*wavelet* model, an adaptive and hybrid prediction model, which integrates *wavelet* models and Support Vector Regression (SVR), for prediction of financial time series, using Discrete Wavelets Transform (DWT), to decompose Foreign Exchange Market (FOREX) time series, obtained from a public knowledge base, that are used as SVR input variables to predict new data. The adjusted series are compared with the traditional models, such as Autoregressive Integrated Moving Average (ARIMA) and Autoregressive Fractionally Integrated Moving Average (ARFIMA) models. In Addition, statistical tests like as normality and unit root are performed to prove that the series in question have non-linear distribution and also to verify the level of correlation between the periods of the series.

II. MATHEMATICAL MODELS

According to Morettin [2], financial time series can be modeled by Integrated Autoregressive Moving Average (ARIMA) processes. As well, financial time series can be modeled through Fractionally Integrated Autoregressive Move (ARFIMA) or ARIMA Fractional processes [5].

ARIMA models are designated as ARIMA(p, d, q) where the parameters p, d, and q are non-negative integers, p is the order of the autoregressive model, d is the degree of differencing and q is the order of the moving-average model [2].

ARIMA models are widely used in finance for the purpose of elaborating forecast models. This model is defined as [2]:

$$\phi(B)\,\nabla^d X_t = \theta(B)a_t,\ d \in N^* \qquad (1)$$

where ϕ is the autoregressive coefficients vector, θ is the moving average coefficients vector, a_t is the white noise at the instant t, and $\nabla^d = (1 - B)^d$ is the difference operator of order d. A commonly used model is ARIMA(1, 1, 1) = $(1 - \phi B)\,\nabla X_t = (1 - \theta B)a_t$ [2].

The same way, ARFIMA models are also traditionally used to forecast financial time series. This model is likewise defined as [5]:

$$\phi(B)\,\nabla^d X_t = \theta(B)a_t,\ d \in (-½, ½) \qquad (2)$$

although the order of differencing, d, takes fractional values, $d \in (-½, ½)$.

III. Wavelet Transform

Wavelets are mathematical functions that extend data intervals, separating them into different frequency and scale components, allowing the analysis of each component in its corresponding scale. The idea of approximation, through the superposition of functions, has its origin in the works of Joseph Fourier, who in the nineteenth century discovered that he could use sines and cosines to represent other functions [6].

The innovation in relation to Fourier is that the basis of the Fourier functions is dependent of the frequency, but not of the time. *Wavelets* are dependent on both domains, frequency, via dilation, and time, via translation [7].

The principle of *wavelet* analysis is to express or approximate a signal or function by a family of functions generated by dilations and translations of a mother *wavelets*.

Wavelet is a $\psi(.)$ function, a small wave that raises and decays over a finite period of time. A *wavelet* $\psi(.)$ function, defined on the real axis $(-\infty, \infty)$, must satisfy three properties [8]:

1) $\int_{(-\infty, \infty)} \psi(u)\, du = 0$
2) $\int_{(-\infty, \infty)} \psi^2(u)\, du = 1$
3) $C_\psi \equiv \int_{(0, \infty)} \{[\psi(f)^2 / f]\}\, df,\ 0 < C_\psi < \infty$

Usually the $\psi(.)$ function is referenced as the mother *wavelet*. Indexed families of *wavelets* can be created by the use of dilations and translations of the mother *wavelet* [9]:

$$\psi_{\lambda, t}(u) = (1 / \sqrt{\lambda})\psi\, [(u - t) / \lambda] \tag{3}$$

where $\lambda > 0$ and t is finite. The λ coefficient adjusts the signal from one dilatation to the next one, and t shifts the signal over time.

The Discrete Wavelet Transform (DWT) works on a set of time series $x(.)$, where t is normally finite and $t = 0, 1, \ldots,$ N-1 [9]. The analysis is performed on the time series for dilations and discrete translations of the mother *wavelet* $\psi(.)$.

The Multiresolution Analysis (MRA), uses orthonormal *wavelet* bases as an analysis tool. It leads to a rapid hierarchical method for calculating the *wavelet* coefficients of a given function and can be written as [10]:

$$x(t) = \sum_k a_k\, \phi(t - k) \tag{4}$$

where the coefficients a_k are in fact the coefficients $\langle x, \phi_{0,k}\rangle$. With these indexes the coefficients $\langle x, \psi_{-1,k}\rangle$ and $\langle x, \phi_{-1,k}\rangle$ are calculated. This process is repeated until the coefficients of the desired k^{th} level are found [9], [10].

The analysis by *wavelets* eliminates the noises contained in the series, employing filters and denoising techniques or *wavelet* shrinkage to obtain a cleaner version of the original time series.

IV. Support Vector Regression

Nonlinear models such as Support Vector Regression (SVR) proved to be effective in predicting time series [3].

The mathematical definition for the Support Vector Regression (SVR), using the Support Vector Machines (SVM) for predicting time series can be expressed by [11] [12]:

$$\Upsilon_t = \omega_t\, \Phi(x_t) + b \tag{5}$$

where $\Upsilon_t \in \Re$ is the predicted value of the time series, $x_t \in \Re^D$ is the input vector of the regressor and consists of the historical data, $x_t = \{y_{t-D}, y_{t-D+1}, \ldots, y_{t-1}\}^T$, and $b \in \Re$ is the bias term, $\omega \in \Re^M$ is the weight vector of x_t and $\Phi(x_t)$ $\Re^D \rightarrow \Re^M$ (M > D) is the feature space that transforms the input vector $x_t \in \Re^D$ through of the higher-dimensional space $\Phi(x_t) \in \Re^M$. The SVR regressor is calculated as:

$$\Upsilon_t(x_t) = \sum_{(i=1,\, N)} \alpha_i\, K(x_t - x_i) + b \tag{6}$$

where α_i ($i = 1, \ldots,$ N) are non-negative Lagrange multipliers and $K(x_t, x_i) = \Phi(x_t)\, \Phi(x_i)$ are the Kernel functions, such as the Kernel RBF [13] [14]:

$$K(x_t, x_i) = exp(-\sigma\, \|x_t\, .\, x_i\|^2) \tag{7}$$

V. Specifying of the SVR-Wavelet Model

The SVR-*wavelet* is an adaptive and hybrid prediction model, which integrates *wavelet* models and Support Vector Regression (SVR).

This model uses the Discrete Wavelet Transform (DWT) to perform the time series data decomposition, allowing high and low frequency components, contained in the original data, to be separated.

After this decomposition, the approximation and detail components are used as SVR input variables to predict new data.

The analysis of *wavelet* makes it possible to eliminate the noise contained in the series, adding to this the fact that nonlinear models such as Support Vector Machines (SVM) are efficient and effective in predicting time series.

In this way, the SVR-*wavelet* hybrid model can be considered an adaptive optimizer where the coefficients are properly changed to minimize the mean square error for the lowest possible value.

Fig. 1 illustrates the proposed model for predicting time series using SVR-*wavelet* model.

Given a time series $\{X_t\}$, $t = 0, \ldots, N - 1$, the goal is to predict T^{th} steps ahead $x_{(T,\ T+1)}$, $T \in N^*$. The proposed prediction system involves three steps: First, through of the MODWT expressed by the MRA, a reduction is carried out on the input data set to the level J_0. The original series is then represented as the sum of the approximation components $\tilde{N}_{J0}$ and of the detail *wavelets* $\tilde{a}_J$, $J = 1, \ldots, J_0$.

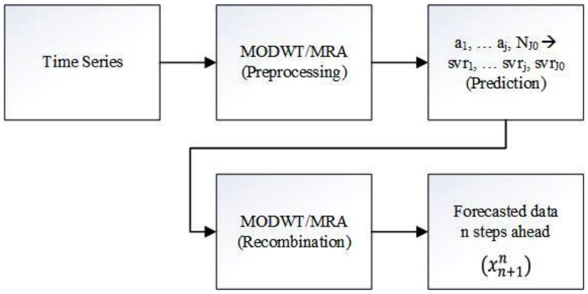

Figure 1. Illustration of the SVR-*wavelet* model proposed.

Next, instead of predicting the original series, the components of the MRA are forecasted independently on each resolution scale over an SVR.

Finally, the approximation components $\tilde{N}_{J0}$ and the detail *wavelets* $\tilde{a}_J$, J = 1, ..., J_0 are recombined to generate an aggregate forecast. This is done using the addition structure of the MRA decomposition.

VI. Applying the SVM-Wavelet Model

Were selected financial time series of the Foreign Exchange Market (FOREX), global market for decentralized currency trading [15]. In terms of trading volume, it is by far the largest market in the world [16]. The main participants in this market are the largest international banks and they are responsible for defining the relative values of the different currencies.

The currency pairs AUD-JPY, CHF-JPY, EUR-JPY, GBP-JPY and EUR-CHF were chosen from a public knowledge base [17], between 01/01/2003 to 12/30/2014, around 3000 values per series with intervals of 1 day, 1 hour and 15 minutes.

Using data mining algorithms, the time series data were treated by eliminating irrelevant data from the set of results to be processed [18], which included verifying price distortions in a given period of the series and adjusting the data formatting.

Some statistical tests such as normality and unit root tests are performed to prove that the series in question have non-linear distribution and also to verify the level of correlation between the periods of the series, of which this paper stands out: Mean Error (ME), Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Mean Percentage Error (MPE), Mean Absolute Percentage Error (MAPE), Mean Absolute Scaled Error (MASE), Autocorrelation of errors at lag 1 (ACF1), Akaike Information Criterion (AIC), Second-Order AIC (AICc) and Bayesian Information Criterion (BIC) [5] [2] [19].

Among the criteria for model selection, the criteria based on the Maximum-Likelihood Function (MLE) are the most used, with emphasis on the Akaike Information Criterion (AIC), the Second Order AIC Criterion (AICc) and the Bayesian Information Criterion (BIC) [20] [21] [22].

A. Forecast Quality

According to the presented methodology, the results found in the application of the SVR-*wavelet* model are detailed.

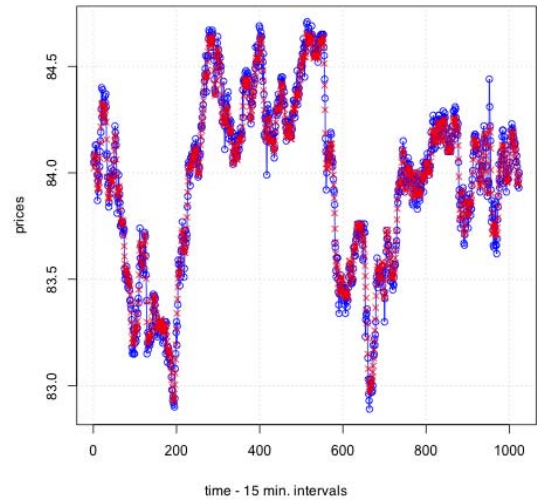

Figure 2. Original AUD-JPY series (°) overlapped by the predicted series (×) using SVR-*wavelet* model.

Fig. 2 shows the graphic of the original series with its points highlighted with small blue circles (°), and the graphic of the series predicted by the proposed SVR-*wavelet* model, enhanced with small red Xs (×) on its points, superimposed on the original series for the relationship between the Canadian Dollar (AUD) and the Japanese Yen (JPY) with a 15-minute intervals.

Analyzing the superimposed graphics is observed that they are congener for a very good forecast, which is proven by the metrics adopted.

Adopting certain metrics, such as ME or RMSE or even MAE, exposed in the Table 1, between the points of the original series and the points of the predicted series, using the SVR-*wavelet* model, obtains very small results, ME = 3.1926e−03, RMSE = 4.6395e−02 and MAE = 3.4644e−02.

Table 1 presents the results of the metrics for the forecast of financial FOREX securities, for SVR-*wavelet*, ARIMA and ARFIMA models, for the relationship between the currency pair AUD-JPY with a 15-minute intervals, according to the method shown in Fig. 1.

The same behavior, demonstrated by Table 1, could be observed with the other currency pairs AUD-JPY, CHF-JPY, EUR-JPY, GBP-JPY and EUR-CHF with intervals of 1 day, 1 hour and 15 minutes.

TABLE I. Results of the Metrics Adopted to Compare The SVR-Wavelet, ARIMA and Arfima Models

Model	ME	RMSE	MAE	MPE	MAPE
	MASE	**ACF1**	**AIC**	**AIC$_C$**	**BIC**
SVR-*wavelet*	3.1926e-03	4.6395e-02	3.4644e-02	3.6930e-03	4.1279e-02
	7.1037e-01	1.5910e-01	-3.3745e+03	-3.3745e+03	-3.3548e+03
ARIMA	6.2895e-05	0.0676809	0.0486665	4.00302e-05	0.0579770
	0.9979129	0.0046305	-2601.179	-2601.155	-2581.453
ARFIMA	-3.841e-05	0.0675356	0.0486958	-0.0001124	0.05801219
	0.9985219	0.0014146	-2605.58	-2605.556	-2585.854

TABLE II. AIC, AICC AND BIC VALUES OBTAINED FOR THE SVR-WAVELET, ARIMA MODELS AND ARFIMA

Series	AIC	AIC_C	BIC
	SVR-*wavelet* model		
AUD-JPY 15 min.	-3374.500	-3374.500	-3354.800
	ARIMA MODEL		
AUD-JPY 15 min.	-2601.179	-2601.155	-2581.453
	ARFIMA MODEL		
AUD-JPY 15 min.	-2605.580	-2605.556	-2585.854

B. Selection of the Best Model

The AIC, like the AICc and BIC, considers the best model to be the one with the lowest values, which can be seen in the Table 2, that compares the values of the AIC, AICc and BIC methods between the SVR-*wavelet*, ARIMA and ARFIMA models, for the relationship between the currency pair AUD-JPY with a 15-minute intervals.

When compared to the ARIMA and ARFIMA models, the lines with the lowest values of AIC, AICc and BIC are those belonging to the SVR-*wavelet* model, highlighted by Table 2, which corroborates that the model based on a SVR-*wavelet* system, for prediction of FOREX time series is more appropriate than those modeled by the ARIMA and ARFIMA models.

An identical behavior, established by Table 2, was observed with the other currency pairs AUD-JPY, CHF-JPY, EUR-JPY, GBP-JPY and EUR-CHF with intervals of 1 day, 1 hour and 15 minutes.

VII. CONCLUSION

This paper highlighted some of the most relevant methods used for regression, such as the mathematical models traditionally used as financial time series regressors, and machine learning, in particular the Support Vector Regression and the Wavelet Transform, as suitable alternatives for forecasting non-linear and non-stationary.

Also, this paper showed a new prediction model, portrayed by hybrid models, in particular the SVR-*wavelet* prediction model, which integrates *wavelet* and SVR models in the predicting of financial time series of FOREX securities.

The SVR-*wavelet* model has shown to be capable and efficient in predicting of the financial time series of FOREX securities. Their performance, when compared to other models, such as traditional mathematical models, has been shown to be significantly better in predicting non-linear and non-stationary time series. Some performance measures, such as ME, RMSE, MAE, MPE, MAPE and MASE, also demonstrated that the SVR-*wavelet* model surpassed the traditional models ARIMA and ARFIMA.

Similarly, this study integrated *wavelets* and SVR models culminating in a new hybrid model to predict non-linear and non-stationary time series, such as the financial time series of FOREX securities. After combining these two innovative approaches and applying them to a new prediction model, a robust prediction was achieved, as verified by the AIC, AICc and BIC metrics, when comparing it with the traditional methods ARIMA and ARFIMA.

However, time series predictions still require considerable academic research to ensure that it reaches maturity and can realize its full potential. Some research may increase the contributions of this work, such as comparative studies between hybrid models using Deep Learning or application with others time series, such as commodities, or even studies of Candlestick graphical patterns, through hybrid models, appear as relevant themes of future research.

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