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# Proposition 1

Prove that $\forall P, Q \in \mathbb{B}.(P \implies Q) \iff (\neg P \vee Q)$.
**Proof:**

We need to prove this equivalence by proving both directions.
Let's start with:
$P \implies Q \Rightarrow \neg P \vee Q$
Assume $P \implies Q$ holds. By the definition of implication, this means that:

- If $P$ is true, then $Q$ is true.
- If $P$ is false, the implication holds regardless of $Q$'s truth value.

Now, we show that $\neg P \vee Q$ holds by considering two cases:

- **Case 1:** $P$ is false. In this case, $\neg P$ is true, and thus $\neg P \vee Q$ holds regardless of the value of $Q$.
- **Case 2:** $P$ is true. Since $P \implies Q$, we know $Q$ must also be true. Therefore, $\neg P \vee Q$ holds because $Q$ is true.

Thus, we have shown that $P \implies Q \Rightarrow \neg P \vee Q$.
Now let's continue to prove $\neg P \vee Q \Rightarrow P \implies Q$
Assume $\neg P \vee Q$ holds. We will show that $P \implies Q$ follows by considering two cases:

- **Case 1:** $P$ is false. In this case, $\neg P$ is true, and therefore $\neg P \vee Q$ holds regardless of the value of $Q$. Since $P$ is false, $P \implies Q$ is true.
- **Case 2:** $P$ is true. Since $\neg P \vee Q$ holds, and $P$ is true, $Q$ must also be true. Therefore, $P \implies Q$ holds.

Thus, we have shown that $\neg P \vee Q \Rightarrow P \implies Q$.

Since we have proven both directions, we conclude that:

$$P \implies Q \iff \neg P \vee Q$$

# Proposition 2

Prove that $\forall n \in \mathbb{N}. \sum_{i=0}^{n} i^3 = \frac{1}{4}n^2(n+1)^2$.
**Proof:**

We will use mathematical induction on $n$ to prove this statement.
**Base Case:** For $n = 0$, the left-hand side is $\sum_{i=0}^{0} i^3 = 0^3 = 0$. The right-hand side is $\frac{1}{4} \times 0^2 \times (0+1)^2 = 0$. Thus, the base case holds.

**Inductive Step:** Assume that the statement is true for some $k \in \mathbb{N}$,

$$\sum_{i=0}^{k} i^3 = \frac{1}{4}k^2(k+1)^2$$

We need to prove that the statement is true for $k+1$,

$$\sum_{i=0}^{k+1} i^3 = \frac{1}{4}(k+1)^2(k+2)^2$$

Starting from the inductive hypothesis:

$$\sum_{i=0}^{k+1} i^3 = \left(\sum_{i=0}^{k} i^3\right) + (k+1)^3$$

Substitute the inductive hypothesis:

$$\sum_{i=0}^{k+1} i^3 = \frac{1}{4}k^2(k+1)^2 + (k+1)^3$$

Factor out $(k+1)^2$ from the terms:

$$\sum_{i=0}^{k+1} i^3 = (k+1)^2\left(\frac{1}{4}k^2 + (k+1)\right)$$

Simplify the expression inside the parentheses:

$$\sum_{i=0}^{k+1} i^3 = (k+1)^2\left(\frac{k^2+4k+4}{4}\right)$$

$$= (k+1)^2 \left( \frac{(k+2)^2}{4} \right)$$
$$= \frac{1}{4}(k+1)^2(k+2)^2$$

Thus, the inductive step holds. By the principle of mathematical induction, the statement is true for all $n \in \mathbb{N}$.

**Collaborators: None**