Problem 1

(a)

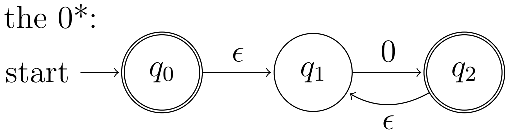
$$\mathrm{L}_1 = (+|-)?\,(\texttt{[0-9]*(.)?[0-9]*})$$

(b)

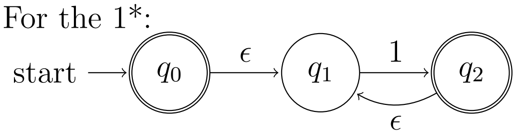
$$L_2 = ((a \mid b)\ a)^*\ \ (a \mid b)?$$

Problem 2

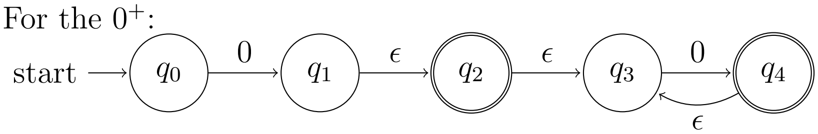
For the 0*:



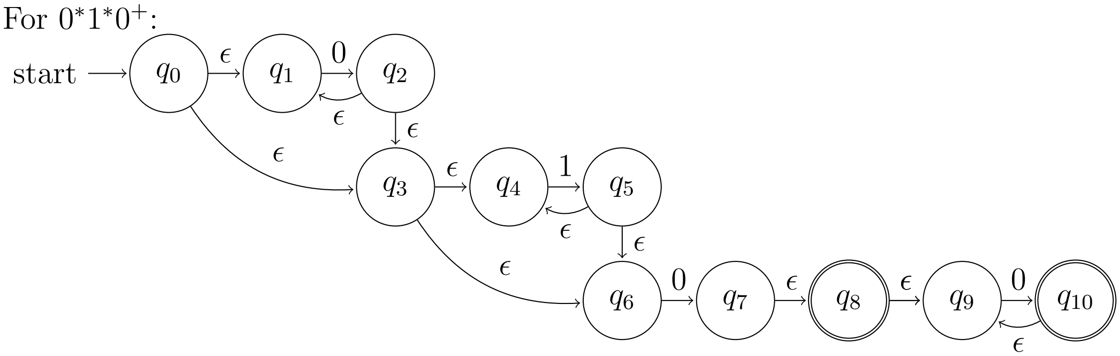
For the 1*:



For the 0^+:



For $0^*1^*0^+$:



Problem 3

(a)

Let's assume that L_1 is a regular language. Then, the Pumping Lemma must hold for L_1.

Let n be the pumping length given by the Pumping Lemma. We choose the string:

$$w = 0^n 1^{5n+1}$$

This string is in L_1 because:

$$i = n, \quad j = 5n + 1, \quad \text{and} \quad 5i = 5n < 5n + 1 = j$$

According to the Pumping Lemma, we can split w into xyz such that:

- $|xy| \leq n$
- $|y| \geq 1$

Since $|xy| \leq n$ and w starts with n zeros, both x and y consist only of zeros. Let:

$$x = 0^{n-t}, \quad y = 0^t, \quad \text{where} \quad 1 \leq t \leq n$$

and

$$z = 1^{5n+1}$$

So, $w = xyz = 0^{n-t}\, 0^t\, 1^{5n+1}$.

Now, we consider $k = 2$ to pump y:

$$w' = xy^2z = 0^{n-t}(0^t)^2 1^{5n+1} = 0^{n-t}0^{2t}1^{5n+1} = 0^{n+t}1^{5n+1}$$

For w' to be in L_1, it must satisfy $5i' < j'$, where:

$$i' = n + t, \quad j' = 5n + 1$$

$$5i' = 5(n + t) = 5n + 5t$$

Now, check if $5i' < j'$:

$$5i' - j' = (5n + 5t) - (5n + 1) = 5t - 1$$

Since $t \geq 1$, we have:

$$5t - 1 \geq 5(1) - 1 = 4$$

Therefore:

$$5i' - j' \geq 4 > 0 \implies 5i' > j'$$

This means $5i' < j'$ is not satisfied, so $w' \notin L_1$.

The Pumping Lemma requires that $xy^kz \in L$ for all $k \geq 0$. However, we found $k = 2$ such that $xy^2z \notin L_1$. This contradiction implies that L_1 does not satisfy the Pumping Lemma. Since assuming L_1 is regular leads to a contradiction, we conclude that L_1 is not a regular language.

(b)

We are given the language:

$$L = \left\{0^i 1^j : i, j \geq 0 \text{ and } (i \bmod 2) + 1 = j \bmod 3\right\}$$

with the alphabet $\Sigma = \{0, 1\}$. We need to write regular expression that represents L.

When $i \bmod 2 = 0$ **and** $j \bmod 3 = 1$

- i is even: $i = 2k$ for $k \geq 0$.
- $j = 1 \mod 3$: $j = 3m + 1$ for $m \geq 0$.

The regular expressions for these are:

- Even number of zeros: $(00)^*$
- Length of ones is equal to $1 \mod 3$: $(111)^*1$

So, the regular expression for this case is:

$$R_1 = (00)^*(111)^*1$$

When $i \bmod 2 = 1$ **and** $j \bmod 3 = 2$

- i is odd: $i = 2k + 1$ for $k \geq 0$.
- $j = 2 \mod 3$: $j = 3m + 2$ for $m \geq 0$.

The regular expressions for these are:

- Odd number of zeros: $(00)^*0$

- Length of ones congruent to 2 mod 3: $(111)^*11$

So, the regular expression for this case is:

$$R_2 = (00)^*0(111)^*11$$

The complete regular expression is the union of R_1 and R_2:

$$R = R_1 \cup R_2 = (00)^*(111)^*1 \cup (00)^*0(111)^*11$$

(c)
We are given the language:

$$L = \left\{0^i : \exists k \in \mathbb{N},\ i = k^2\right\}$$

over the alphabet $\Sigma = \{0\}$. We need to determine whether L is a regular language. Let's assume that L is a regular language. Then, it must satisfy the Pumping Lemma.

Let n be the pumping length given by the Pumping Lemma. We choose the string:

$$w = 0^{n^2}$$

This string is in L because:

$$i = n^2, \quad \text{and} \quad \exists k = n \text{ such that } i = k^2$$

According to the Pumping Lemma, we can split w into xyz such that:

- $|xy| \leq n$
- $|y| \geq 1$

Since $|xy| \leq n$ and w consists entirely of zeros, both x and y consist of zeros from the first n symbols of w. Let:

$$x = 0^p, \quad y = 0^q, \quad z = 0^{n^2-p-q}$$

where $p \geq 0$, $q \geq 1$, and $p + q \leq n$.

When $k = 2$:

$$w' = xy^2z = xyyz = 0^p 0^{2q} 0^{n^2-p-q} = 0^{n^2+q}$$

The length of w' is:

$$|w'| = n^2 + q$$

Since $q \geq 1$, $|w'| > n^2$.

We need to determine whether $|w'| = n^2 + q$ can still be a perfect square.

For w' to be in the language L, $|w'| = n^2 + q$ must also be a perfect square. Let us analyze the inequality:

$$n^2 + q \geq (n+1)^2$$

Expanding the right-hand side:

$$n^2 + q \geq n^2 + 2n + 1$$

Simplifying this gives:

$$q \geq 2n + 1$$

Since q is a portion of y with $|y| \leq n$, it follows that:

$$q \leq n$$

However, from the inequality $q \geq 2n + 1$, we see a clear contradiction:

$$n \geq 2n + 1$$

Simplifying, we get:

$$n - 2n \geq 1 \implies -n \geq 1,$$

which is impossible since $n \geq 1$.

Therefore, $|w'|$ is not a perfect square, and $w' \notin L$. This contradicts the Pumping Lemma, which requires that $xy^kz \in L$ for all $k \geq 0$. The contradiction implies that L does not satisfy the Pumping Lemma for regular languages. Therefore, L is not a regular language.

Collaborators: None