## Problem 1

This is the sequence of configurations that the Turing Machine  $M_1$  enters when started on the input 110#11.

$$(q_1, \overset{\downarrow}{1} 10\#11)$$

$$(q_3, x \stackrel{\downarrow}{1} 0 \# 11)$$

$$(q_3, x1 \overset{\downarrow}{0} \# 11)$$

$$(q_3, x10 \overset{\downarrow}{\#} 11)$$

$$(q_5, x10 \# \stackrel{\downarrow}{1} 1)$$

$$(q_6, x10 \overset{\downarrow}{\#} x1)$$

$$(q_7, x1 \overset{\downarrow}{0} \# x1)$$

$$(q_7, x \stackrel{\downarrow}{1} 0 \# x 1)$$

$$(q_7, \overset{\downarrow}{x} 10 \# x1)$$

$$(q_1, x \stackrel{\downarrow}{1} 0 \# x 1)$$

$$(q_3, xx \overset{\downarrow}{0} \# x1)$$

$$(q_3, xx0 \overset{\downarrow}{\#} x1)$$

$$(q_5, xx0 \# \overset{\downarrow}{x} 1)$$

$$(q_5, xx0\#x \stackrel{\downarrow}{1})$$

$$(q_6, xx0 \# \overset{\downarrow}{x} x)$$

$$(q_6, xx0 \overset{\downarrow}{\#} xx)$$

$$(q_7, xx \stackrel{\downarrow}{0} \# xx)$$

$$(q_7, x \stackrel{\downarrow}{x} 0 \# xx)$$

$$(q_1, xx \stackrel{\downarrow}{0} \# xx)$$

$$(q_2, xxx \stackrel{\downarrow}{\#} xx)$$

$$(q_4, xxx # \stackrel{\downarrow}{x} x)$$

$$(q_4, xxx # x \stackrel{\downarrow}{x} x)$$

Here we can see that it stcuks in q4 state and can't go to the state of  $q_accept$ . Hence 110#11 goes to reject state.

## Problem 2

- (a) Scan for '#': Move right until the '#' symbol is found. If '#' is not found, reject.

  Compare symbols symmetrically: Repeat until all symbols before '#' are processed:
  - 1. Move left from '#' to find the first '0' or '1'), replace with 'X', and remember its value.
  - 2. Move right to '#', then move right to find the first '0' or '1' after '#'.
  - 3. Compare this symbol with the remembered value:
    - If they match, mark it.
    - If they do not match, then reject.

Check for extra symbols: After all symbols before '#' are marked, move right from '#' to check for any unmarked symbols. If unmarked symbols are found, reject; else, accept.

(b) The Turing machine M is defined as follows:

 $Q = \{q_0, q_1, q_2, q_3, q_{\text{accept}}, q_{\text{reject}}\}$  is the set of states.

 $\Sigma = \{0, 1, \#\}$  is the input alphabet.

 $\Gamma = \{0, 1, \#, \_\}$  is the tape alphabet, where  $\_$  represents the blank symbol.

 $q_0$  is the initial state.

 $q_{\text{accept}}$  is the accept state.

 $q_{\text{reject}}$  is the reject state.

 $\delta$  is the transition function, defined as follows:

| Current State                                 | Current Symbol | New Symbol | Direction | New State   |
|---|----------------|------------|-----------|---|
| q0  | 0              | X          | R         | q1  |
| q0  | 1              | X          | R         | q2  |
| q0  | #              | #          | R         | q2 $q6$   |
| q1  | 0              | 0          | R         | q1  |
| q1  | 1              | 1          | R         | q1  |
| q1  | X              | X          | R         | q1  |
| q1  | #              | #          | R         | q3  |
| q2  | 0              | 0          | R         | $   \begin{array}{c}     q2 \\     q2 \\     q2   \end{array} $ |
| q2  | 1              | 1          | R         | q2  |
| $ \begin{array}{c} q2\\ q2\\ q2 \end{array} $ | X              | X          | R         | q2  |
| q2  | #<br>X         | #<br>X     | R         | q5  |
| q3  | X              | X          | R         | q3  |
| q3  | 0              | X          | L         | q4  |
| q4  | 0              | 0          | L         | $\begin{array}{c} q4 \\ q4 \end{array}$                         |
| q4  | 1              | 1          | L         | q4  |
| q4  | X              | X          | L         | q4  |
| $\begin{array}{c} q4\\q4\\q4\end{array}$      | #              | #          | L         | q4 $q4$   |
| q4  | _              | _          | R         | q0  |
| q5  | X              | X          | R         | q5  |
| q5  | 1              | X          | L         | q4  |
| q6  | X              | X          | R         | q6  |
| q6  | _              | _          | S         | $q\_accept$   |

Collaborators: None