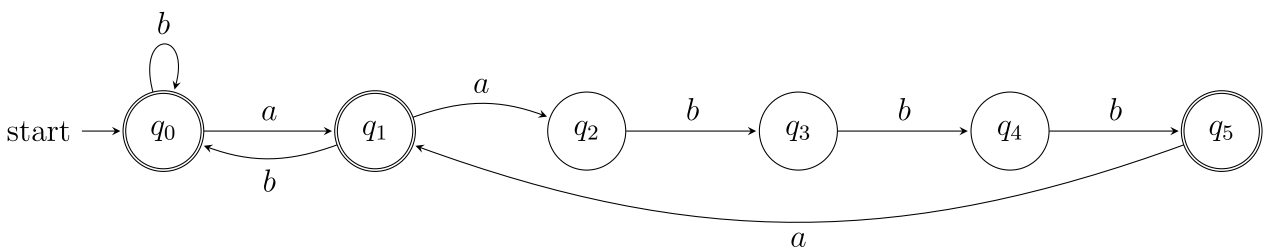
Problem 1

The DFA recognizes the language of all binary strings that end with at least two consecutive "0"s, which can be written as:

$$L(M_1) = \{w \in \{0,1\}^* \mid w \text{ ends with at least two "0"s}\}$$

Problem 2



Problem 3

We define the NFA as the 5-tuple $M = (Q, \Sigma, \delta, q_0, F)$, where:

$$Q = \{q_0, q_1, q_2\}, \quad \Sigma = \{a, b\}, \quad \delta \text{ is as follows:}$$

$$\delta(q_0, a) = \{q_1, q_2\}$$
$$\delta(q_1, \epsilon) = \{q_0\}$$
$$\delta(q_2, b) = \{q_1\}$$

$$q_0 \text{ is the start state}, \quad F = \{q_1\} \text{ is the accepting state.}$$

To show that the string "aab" is accepted, we trace the computation:

$$q_0 \xrightarrow{a} q_1 \xrightarrow{\epsilon} q_0 \xrightarrow{a} q_2 \xrightarrow{b} q_1$$

Since q_1 is the accepting state, the string "aab" is accepted.

Problem 4

Given an NFA with states $Q = \{q_0, q_1, q_2\}$, input alphabet $\Sigma = \{a, b\}$, and transition function:

$$\begin{aligned}\delta(q_0, a) &= \{q_1, q_2\} \\ \delta(q_1, \epsilon) &= \{q_0\} \\ \delta(q_2, b) &= \{q_1\}\end{aligned}$$

The start state is q_0, and the accept state is q_1.

The DFA's start state is the ϵ-closure of the NFA's start state q_0, which is $\{q_0\}$.

From state $\{q_0\}$ with input a:

$$\begin{aligned}q_0 &\xrightarrow{a} \{q_1, q_2\} \\ \epsilon\text{-closure}(\{q_1, q_2\}) &= \{q_0, q_1, q_2\}\end{aligned}$$

So, $\{q_0\} \xrightarrow{a} \{q_0, q_1, q_2\}$.

From state $\{q_0\}$ with input b:

$$q_0 \text{ has no transition on } b.$$

So, $\{q_0\} \xrightarrow{b} \emptyset$.

From state $\{q_0, q_1, q_2\}$ with input a:

$$\begin{aligned}&q_0 \xrightarrow{a} \{q_1, q_2\} \\ &q_1 \text{ has no transition on } a \\ &q_2 \text{ has no transition on } a \\ \epsilon\text{-closure}(\{q_1, q_2\}) &= \{q_0, q_1, q_2\}\end{aligned}$$

So, $\{q_0, q_1, q_2\} \xrightarrow{a} \{q_0, q_1, q_2\}$.

From state $\{q_0, q_1, q_2\}$ with input b:

$$\begin{aligned}&q_0 \text{ has no transition on } b \\ &q_1 \text{ has no transition on } b \\ &q_2 \xrightarrow{b} \{q_1\} \\ \epsilon\text{-closure}(\{q_1\}) &= \{q_0, q_1\}\end{aligned}$$

So, $\{q_0, q_1, q_2\} \xrightarrow{b} \{q_0, q_1\}$.

From state $\{q_0, q_1\}$ with input a:

$$q_0 \xrightarrow{a} \{q_1, q_2\}$$
$$q_1 \text{ has no transition on } a$$
$$\epsilon\text{-closure}(\{q_1, q_2\}) = \{q_0, q_1, q_2\}$$

So, $\{q_0, q_1\} \xrightarrow{a} \{q_0, q_1, q_2\}$.

From state $\{q_0, q_1\}$ with input b:

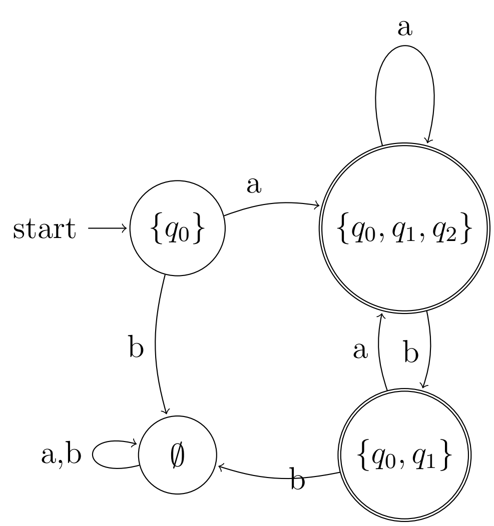
$$q_0 \text{ and } q_1 \text{ have no transitions on } b.$$

So, $\{q_0, q_1\} \xrightarrow{b} \emptyset$.

From state $\emptyset$ with any input: remain in $\emptyset$.

DFA State	a	b
$\{q_0\}$	$\{q_0, q_1, q_2\}$	$\emptyset$
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_1\}$
$\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$	$\emptyset$
$\emptyset$	$\emptyset$	$\emptyset$

The accept states are those that contain the NFA's accept state q_1:
$\{q_0, q_1, q_2\}$ $\{q_0, q_1\}$



Problem 5

Since A and B are regular languages, there exist DFAs $M_A = (Q_A, \Sigma, \delta_A, q_{A0}, F_A)$ and $M_B = (Q_B, \Sigma, \delta_B, q_{B0}, F_B)$ that recognize A and B, respectively.

We construct a new DFA $M = (Q, \Sigma, \delta, q_0, F)$ to recognize $A \cap B$ as follows:

$$Q = Q_A \times Q_B = \{(q_a, q_b) \mid q_a \in Q_A,\ q_b \in Q_B\}$$

$$\Sigma \quad \text{(same as the alphabets of } M_A \text{ and } M_B\text{)}$$

$$\delta\big((q_a, q_b), a\big) = \big(\delta_A(q_a, a),\ \delta_B(q_b, a)\big), \quad \forall a \in \Sigma$$

$$q_0 = (q_{A0},\ q_{B0})$$

$$F = F_A \times F_B = \{(q_a, q_b) \mid q_a \in F_A,\ q_b \in F_B\}$$

- The state set Q consists of all possible pairs of states from M_A and M_B.
- The transition function δ simulates both M_A and M_B simultaneously by updating the state pair according to their individual transitions.
- A string is accepted by M if and only if it is accepted by both M_A and M_B, since (q_a, q_b) is an accepting state only when both $q_a \in F_A$ and $q_b \in F_B$.

The constructed DFA M recognizes the intersection $A \cap B$. Since we can construct such a DFA for any regular languages A and B, the class of regular languages is closed under intersection.

Collaborators: None