The Sound Stops with the Passive Sonar Equation

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The bioSNR package is an open-source SONAR equation calculator. The calculator is capable of handling simple to intermediate level acoustic problems associated with bioacoustics and passive acoustic monitoring (PAM) systems.

This document gives quick examples of bioSNR's capabilities with the commonly utilized passive sonar equation.

```
#Stable - Install package from CRAN
install.packages("bioSNR")

#Unstable - Install package from Github repository
devtools::install_github("MattyD797/bioSNR")

#Attach package namespace to active libraries in Rstudio
library(bioSNR)
```

The Passive Sonar Equation

The passive sonar equation is utilized to describe the relationship, or sound-to-noise ratio (SNR), of an **underwater** sound from a source to receiver. It is only applicable when reflection and scattering aren't impacting factors, however, it is good for more general characteristic approaches, such as in determining detection ranges of a recording unit and acoustic space. It terms of bioacoustics, the equation would be written out as follows:

$$SL - TL - (NL - PG) \geq DT$$

- SL is the source level or dB of the animal call.
- TL is the transmission loss or propagation loss.
- NL is the noise level or background ambient noise.
- PG is the processing gain or directivity index which reduces the noise level.
- DT is the detection threshold or level of sound necessary to reach the reciever.

Source Level

Before we start utilizing the formula, let's understand its parts. The SL is typically measured 1 meter away from the source (~3 wavelengths). This also means that SL is the ratio of the transmitted intensity from the source to a reference intensity. Refer to the vignette decibels for a refresher on dBs or the introduction vignette for more information on intensity, power, and pressure.

If we take a simulated sound source, such as a clown fish called Nemo, we can better see the physics! Let's say for simplicity that Nemo emits a perfect omni-directional sound. We refer to this uniform prorogation of sound power as *spherical spreading*. By teh definition of SL, we can form this equation:

$$SL = 10log\frac{I_s}{I_{ref}}$$

- I_s is the intensity of the transmitted signal from 1 meter or at elast 3 wavelengths away.
- I_{ref} is the intensity of a sound with a root mean square pressure of 1 μPa.
- SL is the source level in $dB\ re\ 1\ \mu Pa$, but is actually the intensity of a 1 μPa signal.

If we remember the equation for itensity, we can easily see the relationship between pressure and intensity ($I = \frac{P_{rms}^2}{z}$). Just in case, a similar relationship between intensity and power can also be expressed as $I = \frac{P}{4\pi r^2}$. The SL can then be expressed in power or pressure as well:

$$SL = 10log_{10}\frac{I_s}{I_{ref}} = 10log_{10}\frac{P}{4\pi I_{ref}} = 10log_{10}P - 10log_{10}(4\pi I_{ref}) = 10log_{10}P + 170.8$$

$$... = 10log_{10}\frac{P_{rms}^2}{\rho c I_{ref}} = 10log_{10}P_{rms}^2 - 10log_{10}(\rho c * 6.7 * 10^{-19})$$

- Where I_{ref} is equal to $1\mu Pa$ (If you recall from the decibels document: $1\mu Pa = 6.7 * 10^{-19}\ \frac{W}{m^2}$)

Transmission Loss

Nemo's sound will eventually soften as the sound wave moves through the water. This propagation loss is caused by resistance of water particles to move and the properties of the water itself. The exact definition is the ratio of sound intensity at 1 m from a source to the sound intensity at distance r. Thus, transmission loss can be represented as:

$$TL = 10log\frac{I_s}{I_s R}$$

TL is the most complicated parameter in the passive sonar equation, but in a perfect world there are two parts to transmission loss 1) the geometric spreading and 2) absorption of the sound as it propagates. 1 is the main contributor.Geometric spreading of the sounds energy over a larger area lowers the intensity of the sound over an increasing distance. 2 is the absorption coefficient we discussed in the introduction and we are going to discuss what goes into calculating this measurement in more depth.

Geometric spreading is either spherical or cylindrical. These shapes are used to map spreading because water has a certain depth and, thus, the sphere starts becoming more and more resembling of a cylinder Think of a watermelon and cutting the top and bottom). The transition from spherical to cylindrical is not uniform, but to try to keep consistent, we will maintain a transition range of half the depth of the location of the sound source. Below is the geometric transmission loss for spherical and then for cyclindrical (before and after transition range):

$$TL_{spherical} = 20log_{10}r$$

$$TL_{cylindrical} = 10log_{10}r + 10log_{10}r_{trans}$$

- r is the propogation distance.
- r_{trans} is transition range or half the depth.

The absorption coefficient calculated using absorptionWater() is actually calculated by the viscous absorption, boric acid relaxation, and magnesium sulfate relaxation process. The equations for these are explained well in other literature and really beyond our need of understanding absorption in the context of bioacoustics, so I will not repeat them here. When we do add this parameter to the geometrical spreading our formula takes on the form:

$$TL = TL_{spherical|cylindrical}(r) + \alpha r$$

- α is the absorption coefficient calculated using the function absorptionWater().

Noise Level and Processing Gain

Noise Level (NL) is simply the ratio of the average background noise intensity and reference intensity that is the same as used in the SL (typically $I_{ref} = 6.7 * 10^{-19} \frac{W}{m^2}$). The equation is the same as SL, however, to get an estimate we can use a tool which is called a Wenz curve to get spectrum levels and it is adapted into the function specLvlGraph(). Below is the empirical Wenz curve for odontocetes (toothed whales), that describe Oceanic ambient noise as a function of frequency:
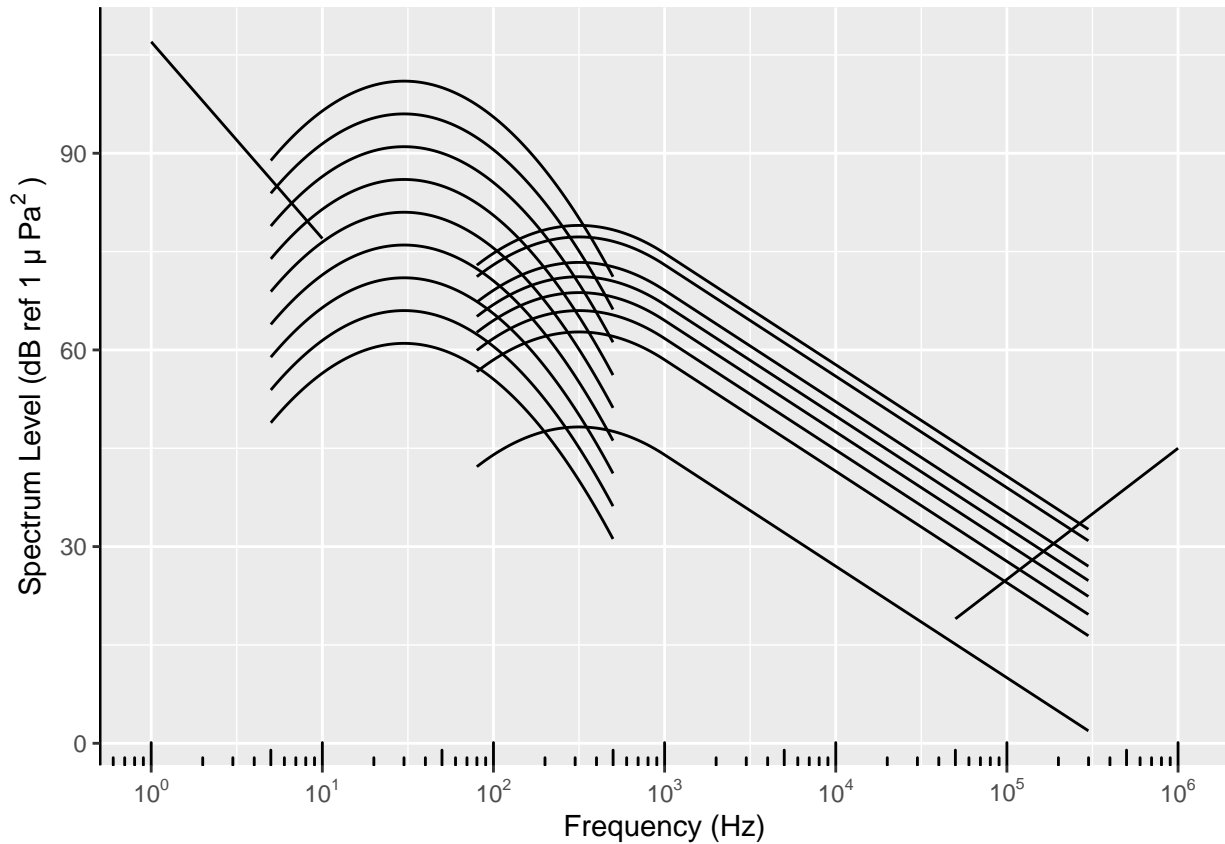
NL can then be found by the following formula:

$$NL = SpectrumLvl + 10log_{10}(Bandwidth)$$

- *Bandwidth* is equal to the difference from max to min frequency of the measured frequency band.

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An array of recorders can reduce the noise level by making independent measurements of the soundscape and making a combination of the soundscape to increase the signal-to-noise ratio, called in the passive sonar equation propagation gain (PG) or array gain (AG). This is typically specified on the recording array of use, but there are a variety of formulas to find the PG value called 'beamforming' and is beyond the scope of this package. PG can be assumed to be 1 $dB\,re\,1\mu Pa$ unless otherwise stated or known.

Examples   Each example will utilize the following information:

Nemo suddenly turned into a blue whale (*Balaenoptera musculus intermedia*) and made a call at a Rockhopper recorder. The call was made at 195 $dB\,re\,1\mu Pa$ at 1 meter and measured between [28,33] Hz frequency.

1) Using a Wenz curve, what would be the noise level in the frequency band of interest, considering moderate shipping, a sea state of 1, and a wind speed of 10 mph?

```
specLvlGraph(c(28,33), ship=4,seaState = 1, wSpeed = 10, boolR = T)
```
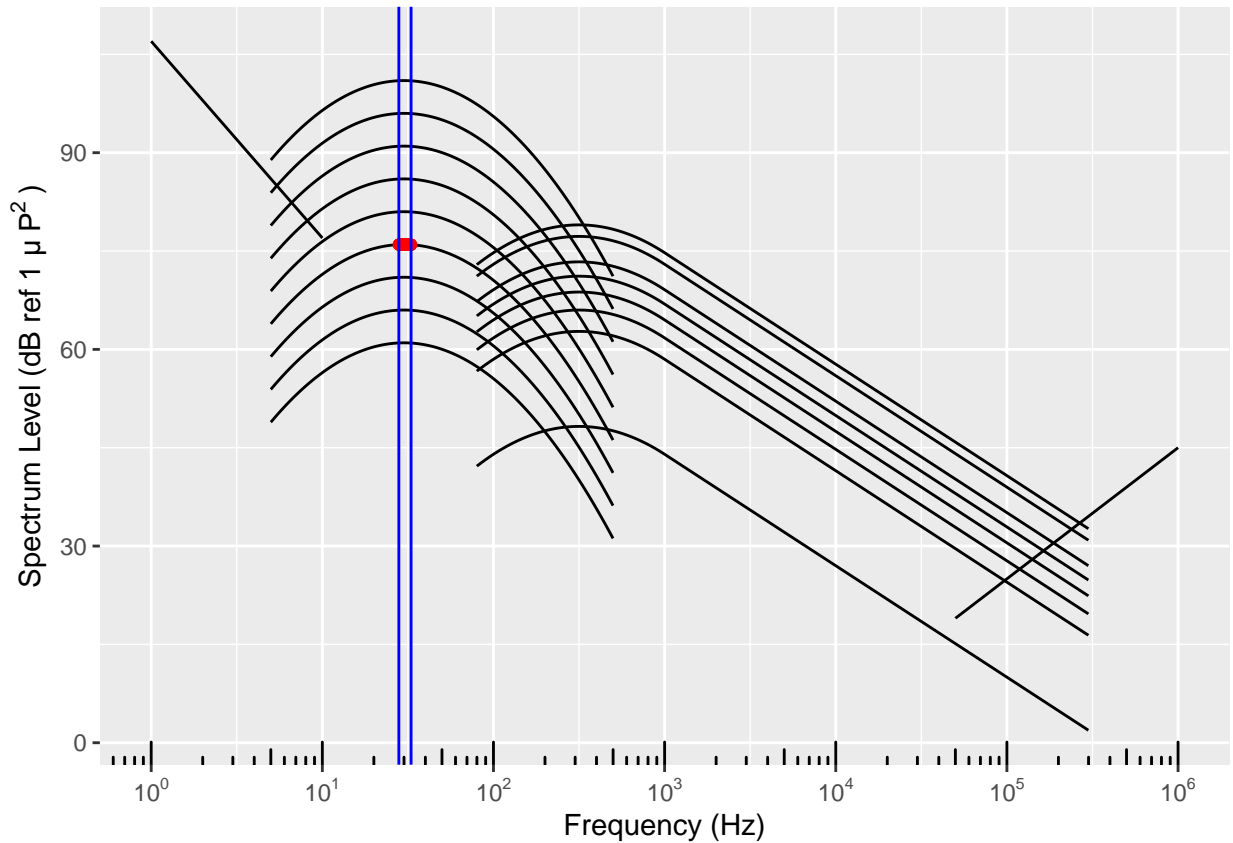


```
#> [1] "The noise level (NL) is: 82.9896990174959 dB ref 1 µP"
#> [1] 82.9897
```

- The NL is 82.9897 $dB\,re\,1\mu Pa$.

2) Given the above NL, we want to find the detection range of the call and we can neglect absorption at this distance (a=0) in the function rmax(). Given we want a DT of 10 $dB\,re\,1\mu Pa$. We can do this by finding the propagation distance in the formula for TL, rewriting the passive sonar equation: $TL = SL - NL - DT$. We can further break down the equation of $TL_{cylindrical}$:

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$$TL_{cylindrical} = 10log_{10}r + 10log_{10}r_{trans} + \alpha r$$

Into:

$$10log_{10}r + 10log_{10}r_{trans} + \alpha r = SL - NL - DT$$

```r
#Source Level
SL <- 195
#Noise Level - from above
NL <- 82.9897
#Detection Threshold
DT <- 10
#Transition Range <- depth/2
TR <- 2500

rmax(sl=189,nl=77,dt=10,d=10000, xaxis=1000000)
#> [1] 316978.6
```

The call has a detection range of 316978.6 meters. Nemo has some chords!