Fourier Transforms

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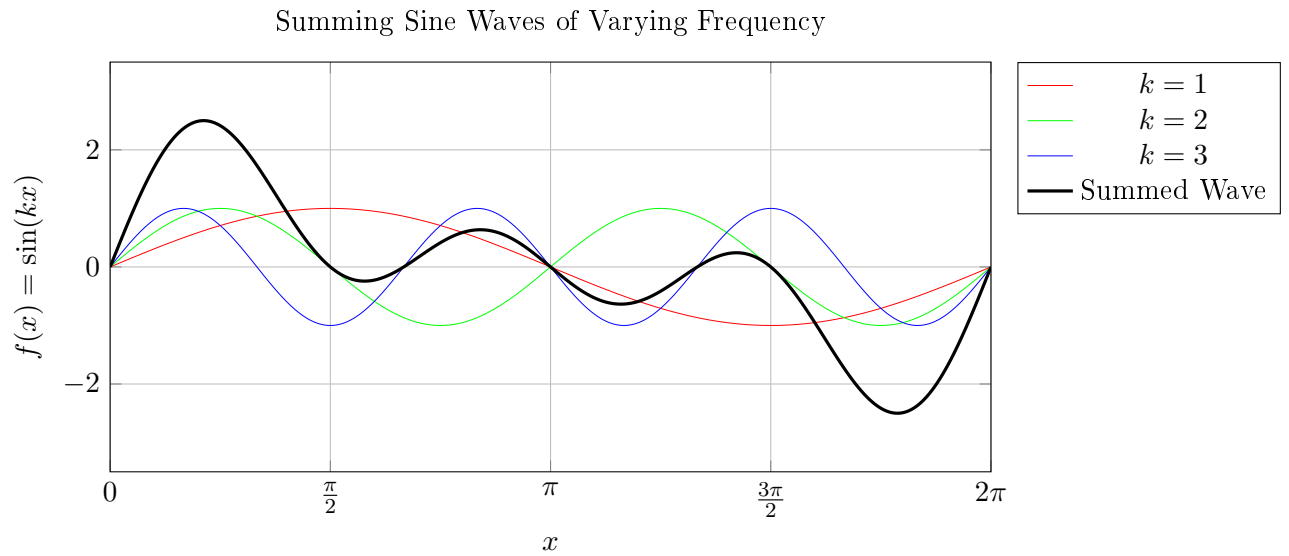# 1 Introduction

2 Fourier Series

2.1 Definition

A Fourier series is a way to represent a periodic function as a sum of sine and cosine functions. The general form is:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos(nx) + b_n \sin(nx)) \tag{1}$$

where a_n and b_n are the Fourier coefficients defined as:

$$a_n = \frac{2}{T} \int_0^T f(x) \cos\left(\frac{2\pi n x}{T}\right) dx \tag{2}$$

$$b_n = \frac{2}{T} \int_0^T f(x) \sin\left(\frac{2\pi n x}{T}\right) dx \tag{3}$$

Summing Sine Waves of Varying Frequency

3 Fourier Transforms

3.1 Definition

The Fourier transform of a function $f(x)$ is defined as

$$F(s) = \int_{-\infty}^{\infty} f(x)e^{-i\frac{2\pi}{T}x}dx \qquad (4)$$

where T is the period of the function $f(x)$.

3.2 Inverse Fourier Transform

3.3 Discrete Fourier Transform

3.4 FFT.js