Small Assignment 1

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1) "For every Integer n, $n^2 + n$ is even"

a) I would fix n to be a variable belonging to all Integers.
I would then show that for all of n, n^2+n will always be even.
This would be a universal qualifier, All on n needs to work.

b) To show its false, we would want to fix n to be an Integer, and show that $n^2 + n$ is odd.
This would be an Existential quantifier, as we need only 1 n to evaluate out s.t. n^2+n is odd to prove the w.t.s. to be False.

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2) "If n^2 is even, then n is even"

a) For Direct Proof: $P \rightarrow Q$
Assume n^2 is even
W.T.S. n is even

b) For Contrapositive: $P \rightarrow Q \iff \neg P \rightarrow \neg Q$
Assume n^2 is odd (not even)
W.T.S. n is odd (not even)

c) For contradiction (Indirect):
Assume n^2 is even
W.T.S. n is odd (not even)

3) "For every positive Integer n, n^2+n is even"

* using Induction on n

a) Positive Integers $\rightarrow$ 1, 2, 3 . . . . . (N^+)

$\forall n \in N^+ : P(n)$ $\rightarrow$ universal

we can get to/start with step $\underline{1}$

$\boxed{\text{Base Case}}$ $\rightarrow$ $P(1)$ Assume $n=1$

W.T.S. n^2+n is even

b) Our steps are $+1$, so we can use $n+1$

$P(N) \rightarrow P(N+1)$

$\boxed{\text{IH:}}$ Assume $P(N)$ is true

W.T.S. $P(N+1)$ is True