

APM 4663/5663—Fall 2024
Small Assignment #1
Due date: Sunday, September 15, 2024

Instructions: First watch the corresponding short videos on proof techniques, quantifiers, and induction in Moodle, then answer the questions below (a couple of sentences suffice for each problem; you don't need to have scratch work for these assignments). Upload your solutions to Moodle.

1. (2 pts.) Consider the following statement: “For every integer n , $n^2 + n$ is even.”
 - (a) How do you start to prove that this statement is true? (*Indicate in a sentence the variable that you fix and what you need to show. Do not actually prove the statement.*)
 - (b) What do you need to prove to show that this statement is false? (*Negate the above statement and simplify so that your answer does not contain any negations.*)
2. (3 pts.) Consider the following statement: “If n^2 is even, then n is even.”
 - (a) State what you assume and what you need to show if you want to use the direct proof method.
 - (b) State what you assume and what you need to show if you want to use the contrapositive proof method.
 - (c) State what you assume and what you need to show if you want to use proof by contradiction (indirect proof).
3. (2 pts.) Consider the following statement: “For every positive integer n , $n^2 + n$ is even.”

We want to prove this statement using induction on n .

 - (a) What is the base case we would need to check? (*Just state what needs to be checked. No need to actually check it.*)
 - (b) What is the induction hypothesis, and what do we need to prove for the induction step? (*Just state the induction hypothesis and what we need to show. Do not prove it.*)

You don't need to actually prove/disprove the statements in the above problems.