APM 4663/5663—Fall 2024
Hints for Assignment #3

1. Use the same proof method as on the similar problems from the first two assignments.

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3. Use the algorithm we discussed in class **starting form the given flow**. Use a queue as we did in class to find an augmenting path, then augment the flow. At the next iteration you should not find any augmenting paths. To find the s-t cut certifying optimality, the vertices reached in the last search should form the left side of the cut, every other vertex should be on the right side. Remember to add capacities when computing the capacity of the cut, and only from left to right.

4. Draw the graphs starting from small n until you cannot draw it without crossings. When that happens, try to move some of the vertices so you can avoid the crossings. When you don't seem to be able to do that, try to find either a subdivision of K_5 or $K_{3,3}$. I suggest that you pick vertices of high degree, and then try to find paths for the missing edges. Remember that no vertex can belong to different paths except the ends.

5. Use Hall's criteria to show that there is a matching saturating one part of the bipartition, then show that this will be a perfect matching.

6. Check Tutte's condition. First show that it is satisfied for $S = \emptyset$, then for arbitrary nonempty S. You will need to use our earlier theorem about the number of vertices of odd degree in any graph. When you check Tutte's condition for nonempty S, show that the number of edges between S and an odd component of $G-S$ must be at least 4, then show that it cannot be exactly 4 (so it is at least 5). Then count the edges between S and all odd components of $G - S$ in two ways.