APM 4663/5663—Fall 2024
Small Assignment #1
Due date: Sunday, September 15, 2024

Instructions: First watch the corresponding short videos on proof techniques, quantifiers, and induction in Moodle, then answer the questions below (a couple of sentences suffice for each problem; you don't need to have scratch work for these assignments). Upload your solutions to Moodle.

1. (2 pts.) Consider the following statement: "For every integer n, $n^2 + n$ is even."

   (a) How do you start to prove that this statement is true? *(Indicate in a sentence the variable that you fix and what you need to show. Do not actually prove the statement.)*

   (b) What do you need to prove to show that this statement is false? *(Negate the above statement and simplify so that your answer does not contain any negations.)*

2. (3 pts.) Consider the following statement: "If n^2 is even, then n is even."

   (a) State what you assume and what you need to show if you want to use the direct proof method.

   (b) State what you assume and what you need to show if you want to use the contrapositive proof method.

   (c) State what you assume and what you need to show if you want to use proof by contradiction (indirect proof).

3. (2 pts.) Consider the following statement: "For every positive integer n, $n^2 + n$ is even."
   We want to prove this statement using induction on n.

   (a) What is the base case we would need to check? *(Just state what needs to be checked. No need to actually check it.)*

   (b) What is the induction hypothesis, and what do we need to prove for the induction step? *(Just state the induction hypothesis and what we need to show. Do not prove it.)*

You don't need to actually prove/disprove the statements in the above problems.

You may resubmit the assignment by the end of Monday, Oct 7 with no penalty.

Small Assignment 1

Sunday, September 15, 2024 5:07 PM

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1) "For every Integer n, $n^2 + n$ is even"

1

a) I would fix n to be a variable belonging to all Integers. I would then show that ~~for all~~ specific n of n, n^2+n will always be even.

This would be a universal qualifier, All on n needs to work.

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b) To show its false, we would want to fix n to be an Integer, and show that n^2+n is odd.

What do you need to do?
Use the word in the table in the notes on quantifiers.

This would be an Existential quantifier, as we need only to evaluate out s.t. n^2+n is odd to prove the w.t.s. to be False.

2) "If n^2 is even, then n is even"

1

a) For Direct Proof: $P \to Q$

Assume n^2 is even

W.T.S. n is even

0

b) For Contrapositive: $P \to Q \iff \neg P \to \neg Q$

this is not the contrapositive

Assume n^2 is odd (not even)

W.T.S. n is odd (not even)

0

c) For contradiction (Indirect):

Assume n^2 is even

W.T.S. n is odd (not even)

Assume that the implication is false, i.e....

Show ?

3) " For every positive Integer n, n^2+n is even"

using Induction on n

a) ~~Positive Integers → 1, 2, 3 . . . . (N^+)~~

~~$\forall n \in N^+ : P(n)$ → universal~~

~~we can get to / ~~start~~ with step 1~~

~~$\boxed{\text{Base Case}}$ → $P(1)$ Assume $n = 1$~~

W.T.S. n^2+n is even

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Write out without a variable what you need to check

b) Our steps are +1, so we can use $n+1$

$P(N) \to P(N+1)$

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$\boxed{\text{IH:}}$ Assume $P(N)$ is true

W.T.S. $P(N+1)$ is True

Use the actual statements

Small Assignment 1 Corrections

Sunday, October 6, 2024 8:01 PM

1) "For every Integer n, n^2+n is even"

A) ✓

B) Correction:

To show this is false, we need to find a counter example s.t. for some n that is an integer, n^2+n is odd.

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2) "If n^2 is even, then n is even"

A) ✓

B) Correction:

For Contrapositive, Assume Q is false, show P is false

$P \to Q \iff \neg Q \to \neg P$

Assume: n is odd (Not even)

W.T.S.: n^2 is odd (Not even)

C) Correction:

Assume: n^2 is even, and n is odd

W.T.S: These assumptions will lead to a Contradiction, likely with seeing what an odd n squared is.

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3) "For every positive Integer n, n^2+n is even"

A) Correction: Using Induction on n, what is the base case?

Base Case: $n=1$

The base case would be checking if 1^2+1 is even.

B) Correction: what is the IH, and what do we need to prove for the Induction step?

IH: Assume n^2+n is even

I step: we want to prove this will hold for $n+1$, so we want to show $((n+1)^2+(n+1))$ is also even.