APM 4663/5663—Fall 2024
Small Assignment #1
Due date: Sunday, September 15, 2024

Instructions: First watch the corresponding short videos on proof techniques, quantifiers, and induction in Moodle, then answer the questions below (a couple of sentences suffice for each problem; you don't need to have scratch work for these assignments). Upload your solutions to Moodle.

1. (2 pts.) Consider the following statement: "For every integer n, $n^2 + n$ is even."
   (a) How do you start to prove that this statement is true? *(Indicate in a sentence the variable that you fix and what you need to show. Do not actually prove the statement.)*
   (b) What do you need to prove to show that this statement is false? *(Negate the above statement and simplify so that your answer does not contain any negations.)*
2. (3 pts.) Consider the following statement: "If n^2 is even, then n is even."
   (a) State what you assume and what you need to show if you want to use the direct proof method.
   (b) State what you assume and what you need to show if you want to use the contrapositive proof method.
   (c) State what you assume and what you need to show if you want to use proof by contradiction (indirect proof).
3. (2 pts.) Consider the following statement: "For every positive integer n, $n^2 + n$ is even."
   We want to prove this statement using induction on n.
   (a) What is the base case we would need to check? *(Just state what needs to be checked. No need to actually check it.)*
   (b) What is the induction hypothesis, and what do we need to prove for the induction step? *(Just state the induction hypothesis and what we need to show. Do not prove it.)*

You don't need to actually prove/disprove the statements in the above problems.

Small Assignment 1

Sunday, September 15, 2024 5:07 PM

Matthew P. Horvath Jr.

1) "For every Integer n, $n^2 + n$ is even"

a) I would fix n to be a variable belonging to all Integers.
I would then show that for all of n, n^2+n will always be even.
This would be a universal qualifier, All on n needs to work.

b) To show its false, we would want to fix n to be an Integer, and show that $n^2 + n$ is odd.
This would be an Existential quantifier, as we need only 1 n to evaluate out s.t. n^2+n is odd to prove the w.t.s. to be False.

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2) "If n^2 is even, then n is even"

a) For Direct Proof: $P \rightarrow Q$
Assume n^2 is even
W.T.S. n is even

b) For Contrapositive: $P \rightarrow Q \iff \neg P \rightarrow \neg Q$
Assume n^2 is odd (not even)
W.T.S. n is odd (not even)

c) For contradiction (Indirect):
Assume n^2 is even
W.T.S. n is odd (not even)

3) "For every positive Integer n, n^2+n is even"

* using Induction on n

a) Positive Integers → 1, 2, 3 ..... (N^+)

$\forall n \in N^+ : P(n)$ → universal

we can get to/start with step $\underline{1}$

$\boxed{\text{Base Case}}$ → $P(1)$ Assume $n=1$

w.T.S. n^2+n is even

b) Our steps are +1, so we can use $n+1$

$P(N) \rightarrow P(N+1)$

$\boxed{\text{IH:}}$ Assume $P(N)$ is true

w.T.S. $P(N+1)$ is True