重要公式及结论:

① Set Theroy: $(A\cup B)\cup C = A\cup(B\cup C)$; $A\cap(B\cup C) = (A\cap B)\cup(A\cap C)$; $(A\cup B)' = A'\cap B'$

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*: $P(A\cup B) = P(A) + P(B) - P(A\cap B)$; $(A\cap \cancel{B}) = A\cup(B\cap B') = (A\cap B)\cup($

$P(A\cup B\cup C) = P(A)+P(B)+P(C) - P(A\cap B) - P(A\cap C) - P(B\cap C) + P(A\cap B\cap C)$; $P(A\cup B\cup C) = P(A'\cap B'\cap C')'$

$_nP_r = \frac{n!}{(n-r)!}$ [排列] $_nC_r = \binom{n}{r} = C_n^r = \frac{n!}{r!(n-r)!}$ [组合]

$\binom{n}{n_1, n_2 \dots n_s} = \frac{n!}{n_1!\,n_2!\,n_3!\cdots n_s!}$ [ ] $_{n-1+r}C_r = \frac{(n-1+r)!}{r!(n-1)!}$ [

② Conditional Proability:

*: $P(A|B) = \frac{P(A\cap B)}{P(B)}$ [条件概率] $P(A'|B) = 1 - P(A|B)$ [B情况下A不发生的概率]

$P(A\cap B\cap C) = P(A)P(B|A)P(C|A\cap B)$, $P(B|A_1\cup A_2\cup A_3) = \frac{P(A_1\cap B)+P(A_2\cap B)+P(A_3\cap B)}{P(A_1)+P(A_2)+P(A_3)}$

Independent: $P(A\cap B) = P(A)P(B)$, Totally Proability: $P(A) = \sum_{i=1}^{m} P(B_i\cap A) = \sum_{i=1}^{m} P(B_i)\cdot P(A|B_i)$

Bayes' Therom: $P(B_k|A) = \frac{P(B_k)\cdot P(A|B_k)}{\sum_{i=1}^{m} P(B_i)\cdot P(A|B_i)}$ *: 分母可用全概率公式计算，在A发生下，B_k发生的可能

Mutually

Pairwise Independent: ① Each 2 of them are pairwise mutually Independent ②: $P(A\cap B\cap C) = P(A)P(B)P(C)$

Some Conclusion: ① A, B are Independent → [A, B'], [A', B], [A', B'], each of them Independent.

② A, B, C are mutually Independent → [A, (B∩C)], [A', (B∩C')], [A, (B∪C)], [A', B', C'] Independent.

③ Some Important Concept: ① Space S, a function $X: S \to \bar{S} \subseteq R$ that assign one real number $X(s) = x$ to $s \in S$

② 期望: $E(x) = \int_{\bar{S}} x\cdot(pmf/pdf)\,(dx)$; 对 c 是常数下，有 $E(c) = c$; $E[cg(X)] = cE[g(x)]$, $E[c_1g_1(x)+c_2g_2(x)] = c_1E(g_1(x)) + c_2E(g_2(x))$

③ 方差: $E[(X-E[X])^2] = E(X^2) - [E(x)]^2$; $Var(c) = 0$; $Var(cx) = c^2 Var X$

④ Mgf: $M^{(r)}(t) = \sum_{x\in\bar{S}} x^r \cdot e^{tx} \cdot f(x)$. ⑤ Cdf: $F(x): R \to [0,1] \Rightarrow F(x) = P(x \le x) = \int_{-\infty}^{x} f(t)(dt)$

⑥ Relation between pdf and cdf: pdf: $f(x)$ = cdf: $F'(x)$

⑦ Relation between normal and χ^2: $\frac{(x-\mu)^2}{\sigma^2} \sim \chi(1)$

离散型随机变量：　　补：Cdf of Binomial: $P(X\le x)=\sum_{y=0}^{[x]}\binom{n}{y}p^y(1-p)^{n-y}$

变量类型：		pmf	cdf	mgf	Mean/E(X)	Var(X)	
Bernoulli	$0<p<1;\ q=1-p$	$p^x q^{1-x}$; $x=0,1$	/	$q+pe^t$ $t\in(-\infty,+\infty)$	p	$p\cdot q\ /\ p(1-p)$	
Binomial:	$n=1,2,3\ldots$ $p\in(0,1)$	$\binom{n}{x}p^x\cdot q^{n-x}$ $C_n^x\cdot p^x\cdot q^{n-x}$	/	$(q+pe^t)^n$ $t\in(-\infty,+\infty)$	np	$n\cdot p\cdot q\ /np(1-p)$	
Geometric:	$0<p<1, q=1-p$	$q^{x-1}\cdot p$ $x=1,2,3\ldots$	/	$\frac{pe^t}{(1-qe^t)}$ $t<-\ln(1-p)$	$\frac{1}{p}$	$\frac{1-p}{p^2}=\frac{q}{p^2}$	事件A发生首次进行的试验次数X
Nective Binomial: Success Numbers	$r=1,2,3\ldots;\ 0<p<1$	$\binom{x-1}{r-1}\cdot p^r\cdot q^{x-r}$ $x=r,r+1,r+2\ldots$	/	$\frac{(pe^t)^r}{(1-qe^t)^r}$ $t<-\ln(1-p)$	$\frac{r}{p}$	$r\cdot\frac{1-p}{p^2}=\frac{rq}{p^2}$	第r次成功时所进行的试验次数X

☆：判别：Geometric：一直失败到第1次成功的试验X；Negative Binomial：出现第r次成功的试验次数X

Poisson:	$\lambda>0$	$\frac{\lambda^x\cdot e^{-\lambda}}{x!}$, $x=1,2,3\ldots$	/	$e^{\lambda(e^t-1)}$, $t\in R$,	λ	λ

☆：$\sum_{x=0}^{\infty}x\cdot\frac{\lambda^x}{x!}\cdot e^{-\lambda}=E(x)=\lambda$；$E(x^2)=\lambda^2+\lambda$；$E(x^3)=\lambda^3+3\lambda^2+\lambda$；$E(x^4)=\lambda^4+6\lambda^3+7\lambda^2+\lambda$.

对于 $Y=a+bX$，证明 $Y\sim N$ ⇒ 利用Y的Mgf ⇒ $E(e^{tY})=e^{ta}E(e^{tbx})=\exp[(a+\mu b)t+\frac{1}{2}\sigma^2b^2t^2)]$ ⇒ $Y\sim N(a+b\mu,b^2\sigma^2)$

		pdf	cdf	mgf	Mean	Var
Exponential:	$\theta>0;\ x\ge0;$	$\frac{1}{\theta}\cdot e^{-\frac{x}{\theta}}$	$1-e^{-\frac{x}{\theta}}$	$\frac{1}{1-\theta t}$, $t<\frac{1}{\theta}$	θ	θ^2

☆：①$\lambda=\frac{1}{\theta}$；λ是在单位时间某事发生的概率（数量） ② $P(X>a+b\mid X>a)=e^{-\frac{1}{\theta}b}=P(X>b)$

③ $E(X^k)=\frac{k!}{\lambda^k}$ ④分布意义是在某段时间内，某事件第一次发生的概率

Gamma:	$\alpha>0,\theta>0;$	$\frac{x^{\alpha-1}\cdot e^{-\frac{x}{\theta}}}{\Gamma(\alpha)\theta^\alpha}$	/	$\frac{1}{(1-\theta t)^\alpha}$; $t<\frac{1}{\theta}$	$\alpha\theta$	$\alpha\theta^2$

☆：① $E(x^k)=\alpha(\alpha+1)\cdots(\alpha+k)\theta^k$；② $\theta=\frac{1}{\lambda}$ ③意义是在某段时间内，某事第α次发生的概率

Normal:	$\mu\in R,\ \sigma>0;$	$\frac{1}{\sigma\sqrt{2\pi}}\cdot \exp(\frac{-(x-\mu)^2}{2\sigma^2})$	查表	$\exp(\mu t+\frac{\sigma^2t^2}{2})$	μ	σ^2

☆：① $E(x)=0$, $E(x^2)=\sigma^2$, $E(x^3)=0$, $E(x^4)=3\sigma^4$ ②奇数次中心矩都为0，因为是奇函数.

③和~~Gamma~~ Standard 的相互转化：$Y=\frac{x-\mu}{\sigma}\sim N(0,1)$，pdf: $f(x)=\frac{1}{\sqrt{2\pi}}\cdot e^{-\frac{x^2}{2}}$ ⇒ $\int_{-\infty}^{+\infty}x^2\cdot e^{-\frac{x^2}{2}}d(x)=\sqrt{2\pi}$.

④意义是大量事件中某特定事件发生的概率 ⑤ 100th percentile: $P(X\le\pi_p)=p$ ⑥ upper point: $P(X\ge z_\alpha)=\alpha$.

Uniform:	$-\infty<a<b<+\infty;$	$\frac{1}{b-a}$, $a\le x\le b$	$\begin{cases}0, & x<a\\ \frac{x-a}{b-a}, & a\le x\le b\\ 1, & x>b\end{cases}$	$\frac{e^{tb}-e^{ta}}{t(b-a)}$, $t\ne0$ 1, $t=0$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
CHi-Squore: $(\theta=2,\ \alpha=\frac{r}{2})$	$r=1,2,3\ldots$	$\frac{x^{\frac{r}{2}-1}\cdot e^{-\frac{x}{2}}}{\Gamma(\frac{r}{2})\cdot2^{\frac{r}{2}}}$	/	$\frac{1}{(1-2t)^{\frac{r}{2}}}$, $t<\frac{1}{2}$	r,	$2r$　$X\sim\chi^2(r)$

$E(x)=r$, $E(x^2)=r^2+2r$, $E(x^3)=\sqrt{\frac{8}{r}}$, $E(x^4)=3+\frac{12}{r}$