Probability Distribution and Parameter Values	Probability Mass Function	Moment-Generating Function	Mean $E(X)$	Variance Var(X)	Examples
Bernoulli $0 < p < 1$ $q = 1 - p$	$p^x q^{1-x},\ x = 0, 1$	$q + pe^t,$ $-\infty < t < \infty$	p	pq	Experiment with two possible outcomes, say success and failure, $p = P(\text{success})$
Binomial $n = 1, 2, 3, \ldots$ $0 < p < 1$	$\binom{n}{x} p^x q^{n-x},$ $x = 0, 1, \ldots, n$	$(q + pe^t)^n,$ $-\infty < t < \infty$	np	npq	Number of successes in a sequence of n Bernoulli trials, $p = P(\text{success})$
Geometric $0 < p < 1$ $q = 1 - p$	$q^{x-1}p,$ $x = 1, 2, \ldots$	$\frac{pe^t}{1 - qe^t}$ $t < -\ln(1-p)$	$\frac{1}{p}$	$\frac{q}{p^2}$	The number of trials to obtain the first success in a sequence of Bernoulli trials
Hypergeometric $x \le n, x \le N_1$ $n - x \le N_2$ $N = N_1 + N_2$ $N_1 > 0,\ N_2 > 0$	$\frac{\binom{N_1}{x}\binom{N_2}{n-x}}{\binom{N}{n}}$		$n\left(\frac{N_1}{N}\right)$	$n\left(\frac{N_1}{N}\right)\left(\frac{N_2}{N}\right)\left(\frac{N-n}{N-1}\right)$	Selecting n objects at random without replacement from a set composed of two types of objects
Negative Binomial $r = 1, 2, 3, \ldots$ $0 < p < 1$	$\binom{x-1}{r-1} p^r q^{x-r},$ $x = r, r+1, \ldots$	$\frac{(pe^t)^r}{(1 - qe^t)^r},$ $t < -\ln(1-p)$	$\frac{r}{p}$	$\frac{rq}{p^2}$	The number of trials to obtain the rth success in a sequence of Bernoulli trials
Poisson $\lambda > 0$	$\frac{\lambda^x e^{-\lambda}}{x!},$ $x = 0, 1, \ldots$	$e^{\lambda(e^t - 1)}$ $-\infty < t < \infty$	λ	λ	Number of events occurring in a unit interval, events are occurring randomly at a mean rate of λ per unit interval
Uniform $m > 0$	$\frac{1}{m},\ x = 1, 2, \ldots, m$		$\frac{m+1}{2}$	$\frac{m^2 - 1}{12}$	Select an integer randomly from $1, 2, \ldots, m$

Probability Distribution and Parameter Values	Probability Density Function	Moment-Generating Function	Mean $E(X)$	Variance Var(X)	Examples
Beta $\alpha > 0$ $\beta > 0$	$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1},$ $0 < x < 1$		$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}$	$X = X_1/(X_1 + X_2)$, where X_1 and X_2 have independent gamma distributions with same θ
Chi-square $r = 1, 2, \ldots$	$\frac{x^{r/2-1}e^{-x/2}}{\Gamma(r/2)2^{r/2}},$ $0 < x < \infty$	$\frac{1}{(1-2t)^{r/2}},\ t < \frac{1}{2}$	r	$2r$	Gamma distribution, $\theta = 2$, $\alpha = r/2$; sum of squares of r independent $N(0,1)$ random variables
Exponential $\theta > 0$	$\frac{1}{\theta} e^{-x/\theta},\ 0 \le x < \infty$	$\frac{1}{1-\theta t},\ t < \frac{1}{\theta}$	θ	θ^2	Waiting time to first arrival when observing a Poisson process with a mean rate of arrivals equal to $\lambda = 1/\theta$
Gamma $\alpha > 0$ $\theta > 0$	$\frac{x^{\alpha-1}e^{-x/\theta}}{\Gamma(\alpha)\theta^\alpha},$ $0 < x < \infty$	$\frac{1}{(1-\theta t)^\alpha},\ t < \frac{1}{\theta}$	$\alpha\theta$	$\alpha\theta^2$	Waiting time to αth arrival when observing a Poisson process with a mean rate of arrivals equal to $\lambda = 1/\theta$
Normal $-\infty < \mu < \infty$ $\sigma > 0$	$\frac{e^{-(x-\mu)^2/2\sigma^2}}{\sigma\sqrt{2\pi}},$ $-\infty < x < \infty$	$e^{\mu t + \sigma^2 t^2/2}$ $-\infty < t < \infty$	μ	σ^2	Errors in measurements; heights of children; breaking strengths
Uniform $-\infty < a < b < \infty$	$\frac{1}{b-a},\ a \le x \le b$	$\frac{e^{tb} - e^{ta}}{t(b-a)},\ t \ne 0$ $1,\ t = 0$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	Select a point at random from the interval $[a, b]$

集合理论

$$(A \cup B) \cup C = A \cup (B \cup C)$$
$$(A \cap B) \cap C = A \cap (B \cap C)$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \qquad (A \cup B)' = A' \cap B'$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \qquad (A \cap B)' = A' \cup B'$$

$$_nP_r = \frac{n(n-1)\cdots(n-r+1)(n-r)\cdots(3)(2)(1)}{(n-r)\cdots(3)(2)(1)} = \frac{n!}{(n-r)!}.$$

Definition 1.2-2
Each of the $_nP_r$ arrangements is called a **permutation of n objects taken r at a time**.

Definition 1.2-6
Each of the $_nC_r$ unordered subsets is called a **combination of n objects taken r at a time**, where

$$_nC_r = \binom{n}{r} = \frac{n!}{r!\,(n-r)!}.$$

$$\binom{n}{n_1, n_2, \ldots, n_s} = \frac{n!}{n_1!\,n_2!\cdots n_s!}. \qquad {}_{n-1+r}C_r = \frac{(n-1+r)!}{r!\,(n-1)!}.$$

条件概率

$$P(A' \mid B) = 1 - P(A \mid B) \qquad P(A \cap B \cap C) = P(A)P(B \mid A)P(C \mid A \cap B).$$

$$P(B \mid A_1 \cup A_2 \cup A_3) = \frac{P(A_1 \cap B) + P(A_2 \cap B) + P(A_3 \cap B)}{P(A_1) + P(A_2) + P(A_3)}$$

Events A and B are **independent** if and only if $P(A \cap B) = P(A)P(B)$. Otherwise, A and B are called **dependent** events.

$$P(B_k \mid A) = \frac{P(B_k)P(A \mid B_k)}{\sum_{i=1}^{m} P(B_i)P(A \mid B_i)},$$

$$P(A) = \sum_{i=1}^{m} P(B_i \cap A) = \sum_{i=1}^{m} P(B_i)P(A \mid B_i)$$

离散型随机变量分布

(a) $f(x) > 0, \qquad x \in S;$
(b) $\sum_{x \in S} f(x) = 1;$
(c) $P(X \in A) = \sum_{x \in A} f(x),$

$$E[u(X)] = \sum_{x \in S} u(x)f(x).$$

$$\sigma^2 = E[X(X-1)] + E(X) - [E(X)]^2,$$

连续型随机变量分布

均匀型分布

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(t)\,dt, \qquad -\infty < x < \infty. \qquad \mu = E(X) = \int_{-\infty}^{\infty} x f(x)\,dx,$$

$$\sigma^2 = \mathrm{Var}(X) = E[(X-\mu)^2] = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx,$$

The mean, variance, and moment-generating function of X, which is $U(0,1)$, are not difficult to calculate. (See Exercise 3.1-1.) They are, respectively,

X~U(0,1)

$$\mu = \frac{a+b}{2}, \qquad \sigma^2 = \frac{(b-a)^2}{12},$$

$$M(t) = \begin{cases} \dfrac{e^{tb} - e^{ta}}{t(b-a)}, & t \ne 0, \\ 1, & t = 0. \end{cases}$$

The **$(100p)$th percentile** is a number π_p such that the area under $f(x)$ to the left of π_p is p. That is,

$$p = \int_{-\infty}^{\pi_p} f(x)\,dx = F(\pi_p).$$

Exponential Distribution

$$M'(t) = \frac{\theta}{(1-\theta t)^2} \qquad M''(t) = \frac{2\theta^2}{(1-\theta t)^3}.$$

Chi Distribution

We say that X has a **chi-square distribution with r degrees of freedom**, which we abbreviate by saying that X is $\chi^2(r)$. The mean and the variance of this chi-square distribution are, respectively,

$$\mu = \alpha\theta = \left(\frac{r}{2}\right)2 = r \quad \text{and} \quad \sigma^2 = \alpha\theta^2 = \left(\frac{r}{2}\right)2^2 = 2r.$$

Normal Distribution

If X is $N(\mu, \sigma^2)$, then $Z = (X-\mu)/\sigma$ is $N(0,1)$.

If the random variable X is $N(\mu, \sigma^2)$, $\sigma^2 > 0$, then the random variable $V = (X-\mu)^2/\sigma^2 = Z^2$ is $\chi^2(1)$.

Additional

Let's summarize the formulas for the four categories of sampling. Assuming that we have a set with n elements, and we want to draw k samples from the set, then the total number of ways we can do this is given by the following table.

ordered sampling with replacement	n^k
ordered sampling without replacement	$P^n_k = \frac{n!}{(n-k)!}$
unordered sampling without replacement	$\binom{n}{k} = \frac{n!}{k!(n-k)!}$
unordered sampling with replacement	$\binom{n+k-1}{k}$