GEOMETRIA 80103 (2022)

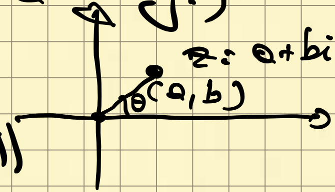
ING. INFORMATICA

SETTIMANA 2
GIOVEDI

Numeri complessi:

Forme: $z = a+bi$ (forma alg.)

$= $ cartesiana



forma trig. $= |z|(\cos(\theta)+i\sin(\theta))$

$|z| = \boxed{\sqrt{a^2+b^2}}$

$= |z|\cdot e^{i\theta}$ – forma esponenziale.

$\left(e^{i\theta} = \cos(\theta)+i\sin(\theta)\right)$

$\boxed{e^{i\pi} = -1}$ Eulero

Esempio 1:

$$|(1+3i)\cdot(\cos(7)+i\sin(7))|$$

$$|z_1\cdot z_2| = |z_1|\cdot|z_2|$$

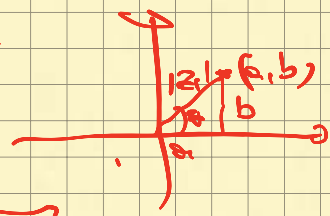
$$= |1+3i|\cdot|\cos(7)+i\sin(7)| = \sqrt{1^2+3^2}\cdot$$

$$\cdot\sqrt{(\cos(7))^2+(\sin(7))^2} = \sqrt{10}\cdot\sqrt{1} = \sqrt{10}.$$

$$\boxed{|\cos(\theta)+i\sin(\theta)| = 1}$$

Esempio 2: Scrivere nella forma $a+bi$ il numero: $\dfrac{(2+2i)^7}{(-1+\sqrt{3}\,i)^9}$

Soluzione: $e^{z_1}\cdot e^{z_2} = e^{z_1+z_2}$



$$z_1 = \overset{\boxed{a}}{2}+\overset{\boxed{b}}{2}\,i = |z_1|\cdot e^{i\theta} =$$

$$\boxed{\cos(\theta) = \frac{a}{|z_1|}} \qquad \boxed{\sin(\theta) = \frac{b}{|z_1|}}$$

$$|z_1| = \sqrt{2^2+2^2} = 2\sqrt{2}.$$

$$\cos(\theta) = \frac{2}{2\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \sin(\theta) = \frac{2}{2\sqrt{2}} = \frac{\sqrt{2}}{2}.$$

$$\theta = 45^\circ = \pi/4.$$

$$z_1 = 2\sqrt{2}\left(\cos(\pi/4)+i\sin(\pi/4)\right) = 2\sqrt{2}\,e^{i\pi/4}$$

$$z_2 = -\overset{a}{1}+\overset{b}{\sqrt{3}}\,i = |z_2|\cdot e^{i\theta}$$

$$|z_2| = \sqrt{(-1)^2+(\sqrt{3})^2} = 2$$

$$\cos(\theta) = \frac{a}{|z_2|} = -\frac{1}{2} \qquad \sin(\theta) = \frac{b}{|z_2|} = \frac{\sqrt{3}}{2}$$

$$\theta = 120^\circ = \frac{2\pi}{3}$$

$$z_2 = 2\cdot\left(\cos\left(\frac{2\pi}{3}\right)+i\sin\left(\frac{2\pi}{3}\right)\right) = 2\cdot e^{i\frac{2\pi}{3}}$$

$$\frac{(2+2i)^7}{(-1+\sqrt{3}i)^9} = \frac{\left(2\sqrt{2}\cdot e^{i\frac{\pi}{4}}\right)^7}{\left(2\cdot e^{i\cdot\frac{2\pi}{3}}\right)^9} = \frac{\left(2^{3/2}\cdot e^{i\pi/4}\right)^7}{\left(2\cdot e^{i\frac{2\pi}{3}}\right)^9}$$

$$\underline{\underline{(z^k)^m = z^{k\cdot m}}}$$

$$= \frac{2^{3/2\cdot 7}\cdot e^{i\pi/4\cdot 7}}{2^9\cdot e^{i\frac{2\pi}{3}\cdot 9}} =$$

$$= \boxed{\frac{2^{21/2}}{2^9}}\cdot\frac{e^{i\frac{7\pi}{4}}}{e^{i6\pi}} =$$

$$e^{i6\pi} = 1$$

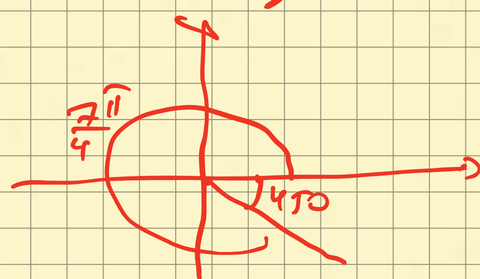
$$\cos(6\pi) + i\sin(6\pi)$$

$$\boxed{e^{i\cdot(2k\pi)} = 1}$$

$$= 2^{21/2-9}\cdot e^{i\frac{7\pi}{4}} = 2^{3/2}\, e^{i\frac{7\pi}{4}}$$

$$= 2\sqrt{2}\left(\cos\left(\overset{315^\circ}{\frac{7\pi}{4}}\right) + i\sin\left(\overset{315^\circ}{\frac{7\pi}{4}}\right)\right)$$

$$= 2\sqrt{2}\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)$$



$$= \boxed{2 - 2i}$$

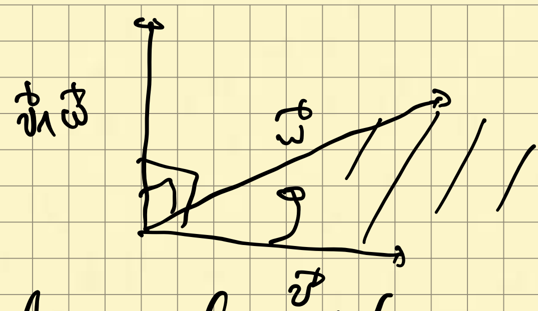
Vettori: Prodotto vettoriale:

$$\vec{v}\wedge\vec{w} = (y_1 z_2 - y_2 z_1,\ -x_1 z_2 + x_2 z_1,\ x_1 y_2 - x_2 y_1)$$

$\vec{v} = (x_1, y_1, z_1)$

$\vec{w} = (x_2, y_2, z_2)$

(1)

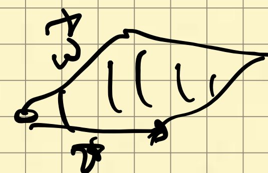


la regola del mano destra.

(2) Si $\vec v \wedge \vec w = (0,0,0) \iff \vec v$ e $\vec w$ sono parallele

$\iff$ hanno la stessa retta di supporto.

(3) $|\vec v \wedge \vec w|$ = l'area del parallelogramo



Esempio: Sia $\vec v = (1,1,1)$ e $\vec w = (2,1,1)$

(1) Trovate un vettore $\perp$ a $\vec v$ e $\vec w$ con modulo $= 2$.

$$\vec v \wedge \vec w = \left(1\cdot 1 - 1\cdot 1, -(1\cdot 1 - 1\cdot 2), 1\cdot 1 - 1\cdot 2\right)$$

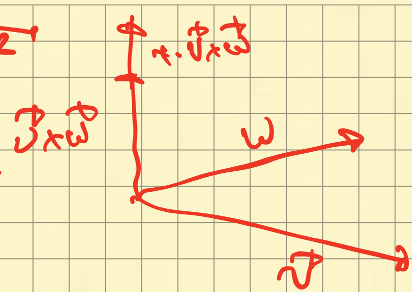
$$= (0, 1, -1)$$

Il nostro vettore deve essere

$$t \cdot \vec v \wedge \vec w = (0, t, -t)$$

$$2 = |t\cdot(\vec{v}\wedge\vec{w})| = \sqrt{0^2+t^2+(-t)^2}$$

$$= \sqrt{2t^2} = \sqrt{2}\,|t|$$



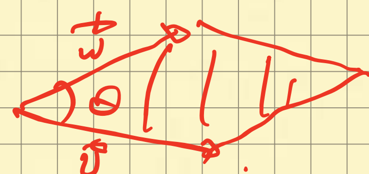
$$|t| = \sqrt{2} \iff t = \pm\sqrt{2}$$

$$\vec{v}_1 = (0, \sqrt{2}, -\sqrt{2})$$

$$\vec{v}_2 = (0, -\sqrt{2}, \sqrt{2}).$$

<u>Esempio:</u> Calcolare il angolo tra due vettori $\vec{v} = (2,-2,1)$ e $\vec{w} = (1,0,-1)$

<u>Soluzione:</u>



Area del parall

$$= |\vec{v}\wedge\vec{w}|$$

$$= |\vec{v}|\cdot|\vec{w}|\cos(\theta)$$

$$\cos(\theta) = \frac{|\vec{v}\wedge\vec{w}|}{|\vec{v}|\cdot|\vec{w}|}$$

$$\vec{v}\wedge\vec{w} = (2,-2,1)\wedge(1,0,-1)$$

$$= \big((-2)(-1) - 1\cdot 0,\ -(2\cdot(-1) - 1\cdot 1),\ 2\cdot 0 - (-2)\cdot 1\big)$$

$$= (2, 3, 2).$$

$$|\vec{v}\wedge\vec{w}| = \sqrt{2^2+3^2+2^2} = \sqrt{17}$$

$$|\vec{v}| = |(2,-2,1)| = \sqrt{2^2+(-2)^2+1^2} = \sqrt{9}$$

$$|\vec{w}| = |(1,0,-1)| = \sqrt{1^2+0^2+(-1)^2} = \sqrt{2}$$

$$\cos(\theta) = \frac{\sqrt{17}}{\sqrt{9}\cdot\sqrt{2}} = \sqrt{\frac{17}{18}} < 1$$

$$\boxed{\theta = \arccos\left(\sqrt{\frac{17}{18}}\right)} \approx \text{vicino di } 0.$$