$$\lim_{x\to 0} \frac{\operatorname{sen}(3x)}{x}$$

$$\lim_{x\to 0} \frac{\operatorname{sen}(\operatorname{sen}(x))}{x}$$

$$\lim_{x\to 0} \frac{\operatorname{sen}(x^2)}{x^2}$$

$$\lim_{x\to 0} (\cos(x))^{\frac{1}{x^2}}$$

$$\lim_{x\to 0} (1+\sin(x))^{\frac{1}{x}}$$

$$\lim_{x\to 0^{\pm}} e^{\frac{1}{x}}$$

$$\lim_{x\to 0} \frac{\ln(1+\sin(x))}{x\cos(x)}$$

$$\lim_{x\to +\infty} x\sin\left(\frac{1}{x}\right)$$

$$\lim_{x\to 0} x\log(x)$$

$$\lim_{x\to 0} x^x$$

(3) $\lim_{n\to+\infty} \frac{3x^2+1}{x}$

(4) $\lim_{n\to+\infty} \frac{x}{3x^2+1}$

(5) $\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x^2}$

(6) $\lim_{x\to-\infty} \frac{x^6-15x^3}{3x^2-2x+1}$

(7) $\lim_{x\to+3} \frac{x^2-x-6}{x^2-9}$

(8) $\lim_{x\to+\infty} \frac{x^5+3x^2+3x+1}{2x^2-x}$

(9) $\lim_{x\to+\infty} \sqrt{x-1}-\sqrt{x-2}$

(10) $\lim_{x\to+\infty} \log(2x)-\log(x-1)$

(11) $\lim_{x\to+\infty} e^{(2+x)}-e^{x}$

(12) $\lim_{x\to+\infty} e^{\left(\frac{x+7}{x-1}\right)}$

(1) $\lim_{x\to 1} \frac{1}{(x-1)^2} \overset{\text{sostituisco}}{=} \frac{1}{0^+} = +\infty$

(2) $\lim_{x\to+\infty} \sqrt[3]{\underbrace{\arctan x}_{\to\frac{\pi}{2}}} = \sqrt[3]{\lim_{x\to+\infty} \arctan(x)} = \sqrt[3]{\frac{\pi}{2}}$

poiché $\sqrt[3]{\ }$ è continua

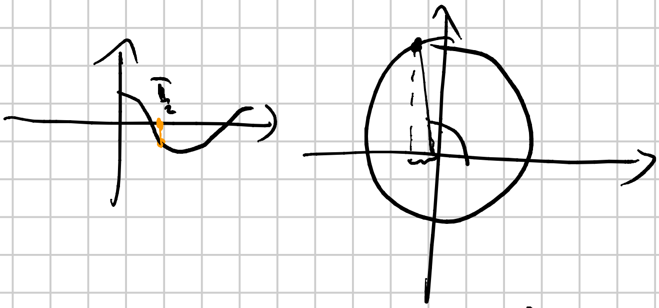
(3) $\lim_{x\to+\infty} \underbrace{x}_{+\infty}\,\underbrace{2^x}_{+\infty} = +\infty$

(4) $\lim_{x\to 0^{\pm}} \frac{3x+1}{x} = \frac{1}{0} = \text{ind.} = \pm\infty$

(5) $\lim_{x\to 0} \frac{1}{1+x^2} = \frac{1}{1+0^2} = 1$

(6) $\lim_{x\to+\infty} \frac{1}{1+x^2} = \frac{1}{1+(+\infty)^2} = \frac{1}{1+\infty} = \frac{1}{+\infty} = 0^+$

(7) $\lim_{x\to\frac{\pi}{2}^+} \frac{1}{\cos x} = \frac{1}{\cos(\frac{\pi^+}{2})} = \frac{1}{0^-} = -\infty$



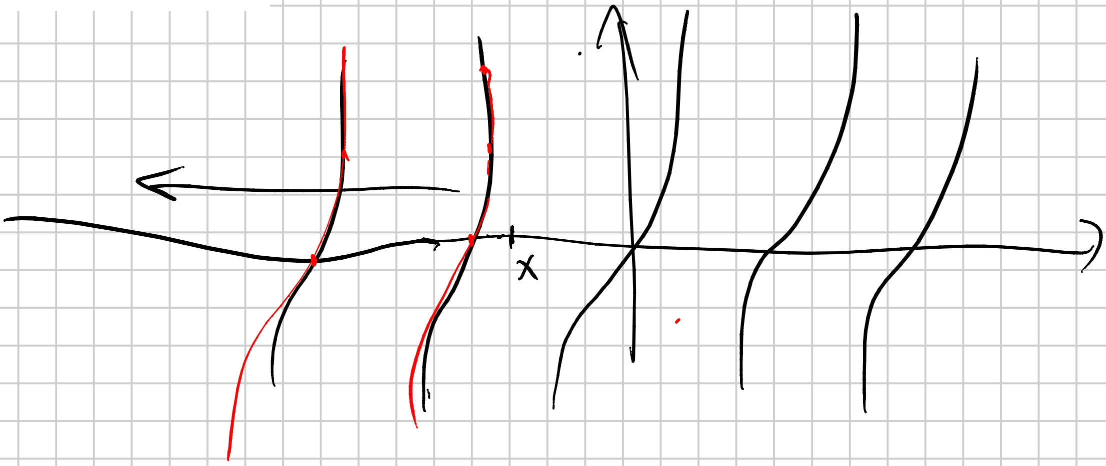
(8) $\lim_{x\to\frac{\pi}{2}^-} \frac{1}{\cos x} = \frac{1}{\cos(\frac{\pi^-}{2})} = \frac{1}{0^+} = +\infty$

(9) $\lim_{x\to-\infty} \arcsin \frac{1}{1-x^2} = \arcsin\left(\frac{1}{1-(-\infty)^2}\right) = \arcsin\left(\frac{1}{1-\infty}\right) = \arcsin\left(\frac{1}{-\infty}\right) = \arcsin(0^-) = 0$

(10) $\lim_{x\to-\infty} \ln(1+e^{-x}) = \ln\left(1+e^{-(-\infty)}\right) = \ln\left(1+e^{+\infty}\right) = \ln(+\infty) = +\infty$

(11) $\lim_{x\to\frac{\pi}{2}^-} (\tan(x))^x = \left(\tan\left(\left(\frac{\pi}{2}\right)^-\right)\right)^{\frac{\pi}{2}^-} = (+\infty)^{\frac{\pi}{2}} = +\infty$

(12) $\lim_{x\to-\infty} \tan(x) =$ NON ESISTE



(3) $\lim_{x\to+\infty} \frac{3x^2+1}{x}$ sol. $= \frac{\infty}{\infty}$ $\quad \lim_{x\to+\infty} \frac{x^{\cancel{2}}\left(3+\frac{1}{x^2}\right)}{\cancel{x}} = \lim_{x\to+\infty} \overset{+\infty}{x} \cdot \underbrace{\left(3+\overset{0}{\cancel{\frac{1}{x^2}}}\right)}_{3} = +\infty \cdot 3 = +\infty$

(4) $\lim_{x\to+\infty} \frac{x}{3x^2+1}$ sol. $= \frac{\infty}{\infty}$ $\quad \lim_{x\to+\infty} \frac{\cancel{x}}{x^{\cancel{2}}\left(3+\underset{0}{\frac{1}{x^2}}\right)} = \frac{1}{+\infty \cdot 3} = \frac{1}{+\infty} = 0^+$

(5) $\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x^2}$ sol. $= \frac{\sqrt{1}-1}{0} = \frac{0}{0}$ $\quad$ idea: RAZIONALIZZARE: $(a-b)\cdot(a+b) = a^2-b^2$

$\left(\sqrt{1+x}-1\right)\left(\sqrt{1+x}+1\right) = \left(\sqrt{1+x}\right)^{\cancel{2}} - 1^2$

$= \cancel{1}+x-\cancel{1} = x$

(6) $\lim_{x\to-\infty} \frac{x^6-15x^3}{3x^2-2x+1} = \frac{+\infty}{+\infty}$ $\quad \lim_{x\to-\infty} \frac{\overset{x^4}{\cancel{x^6}}\overset{1}{\left(1-\cancel{\frac{15}{x^3}}\right)}}{\cancel{x^2}\underset{3}{\left(3-\cancel{\frac{2}{x}+\frac{1}{x^2}}\right)}} = +\infty$

(7) $\lim_{x\to+3} \frac{x^2-x-6}{x^2-9} = \frac{0}{0}$ $\rightsquigarrow$ scomporre!!

$\lim_{x\to 0^\pm} \frac{\sqrt{1+x}-1}{x^2} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1} = \lim_{x\to 0^\pm} \frac{\cancel{x}}{x^{\cancel{2}}\left(\sqrt{x+1}+1\right)}$

$= \frac{1}{0^\pm\cdot(1+1)} = \pm\infty$

$$x^2 - 9 = (x+3)(x-3)$$
$$x^2 - x - 6 = (x-3)(x+2)$$

$$\lim_{x \to 3} \frac{x^2 - x - 6}{x^2 - 9} = \lim_{x \to 3} \frac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}(x+3)} = \frac{3+2}{3+3} = \boxed{\frac{5}{6}}$$

(8) $$\lim_{x\to+\infty} \frac{x^5 + 3x^2 + 3x + 1}{2x^2 - x} = \lim_{x\to+\infty} \frac{x^{\cancel{5}\,3}\left(1 + \frac{3}{x^3} + \frac{3}{x^4} + \frac{1}{x^5}\right)}{\cancel{x^2}\left(2 - \frac{1}{x}\right)} = +\infty \cdot \frac{1}{2} = +\infty$$

(9) $$\lim_{x\to+\infty} \sqrt{x-1} - \sqrt{x-2} = \lim_{x\to+\infty} \left(\sqrt{x-1} - \sqrt{x-2}\right) \cdot \frac{\left(\sqrt{x-1} + \sqrt{x-2}\right)}{\left(\sqrt{x-1} + \sqrt{x-2}\right)} = \lim_{x\to+\infty} \frac{\overbrace{\cancel{x}-1 - \cancel{x} + 2}^{=1}}{\underbrace{\sqrt{x-1}}_{+\infty} + \underbrace{\sqrt{x-2}}_{+\infty}} = 0^+$$

$\log\left(\frac{a}{b}\right) = \log a - \log b$

(10) $$\lim_{x\to+\infty} \log(2x) - \log(x-1) = \lim_{x\to+\infty} \log\left(\frac{2x}{x-1}\right) = \lim_{x\to+\infty} \log\left(\frac{\cancel{x}\cdot 2}{\cancel{x}\left(1 - \frac{1}{x}\right)}\right) = \log(2)$$

$\log(x)$ è una funz. continua

(11) $$\lim_{x\to+\infty} e^{(2+x)} - e^x = \lim_{x\to+\infty} e^2 \cdot e^x - e^x = \lim_{x\to+\infty} \underbrace{e^x}_{+\infty} \cdot \underbrace{\left(e^2 - 1\right)}_{>0} = +\infty$$

$A \cdot B - B$
$= B \cdot (A - 1)$

(12) $$\lim_{x\to+\infty} e^{\left(\frac{x+7}{x-1}\right)} = e^1$$

e^x è una funzione continua

$$\lim_{x\to+\infty} \frac{x+7}{x-1} = \lim_{x\to+\infty} \frac{\cancel{x}\left(1 + \frac{7}{x}\right)}{\cancel{x}\left(1 - \frac{1}{x}\right)} = 1$$

$$\lim_{x\to+\infty} \operatorname{sen}\left(\frac{x^2+2}{x^2+1}\right) = \operatorname{sen}(1)$$

sen è una funz. continua