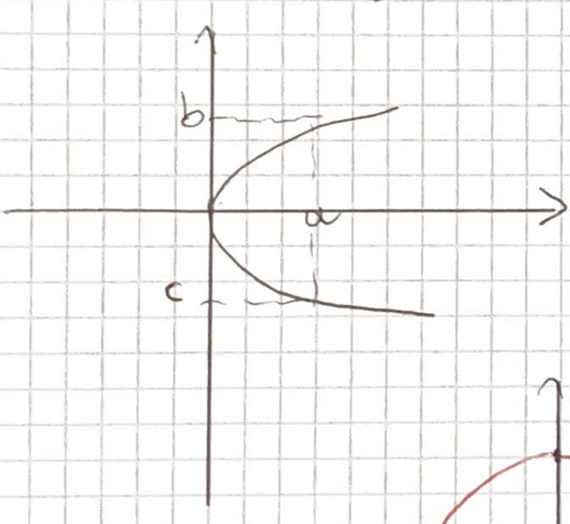
ESERCITAZIONE 11/10/2022

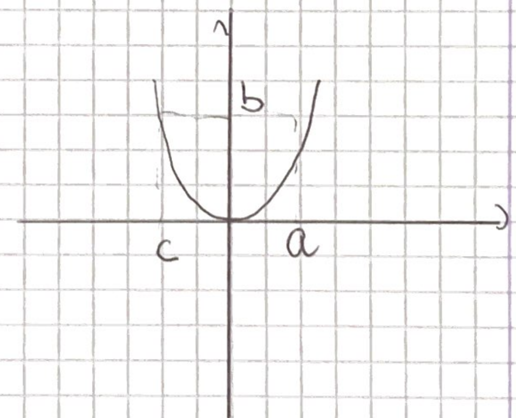
DEFINIZIONE DI FUNZIONE

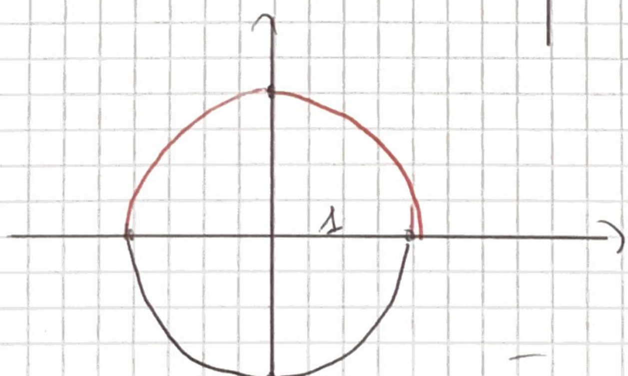
Una funzione f è un insieme di coppie ordinate (x, y) in cui non ce ne siano due con lo stesso primo membro

(a, b) (a, c) non è una funzione

(a, b) (c, b) è una funzione







la circonferenza $x^2+y^2=1$ non è una funzione $y = -\sqrt{1-x^2}$ $y = \sqrt{1-x^2}$ (semicirconferenze) sono funzioni.

Esercizio

Sia $\varphi(x) = |x-3| + |x-1| \quad \forall x \in \mathbb{R}$

Calcolare $\varphi(0) \qquad \varphi(3)$

Determinare tutti i $t \,/\, \varphi(t+2) = \varphi(t)$

$\varphi(0) = |0-3| + |0-1| = 3+1 = 4$

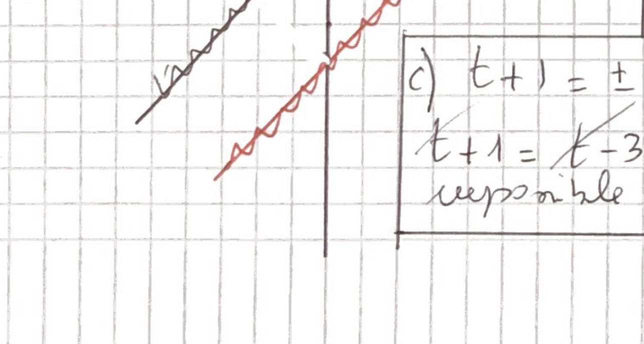
$\varphi(3) = |3-3| + |3-1| = 2$

$|t+2-3| + |t+2-1| = |t-3| + |t-1|$

$|t-1| + |t+1| = |t-3| + |t-1|$

$\Rightarrow \quad |t+1| = |t-3|$

a) $- \; y = |t+1| \qquad - \; y = |t-3|$



b) $|t+1| = |t-3|$

$(t+1)^2 = (t-3)^2$

$t^2 + 1 + 2t = t^2 - 6t + 9$

$8t = 8 \Rightarrow t = 1$

c) $t+1 = \pm(t-3)$

$\cancel{t}+1 = \cancel{t}-3$
impossibile

$t+1 = -t+3$

$2t = 2$

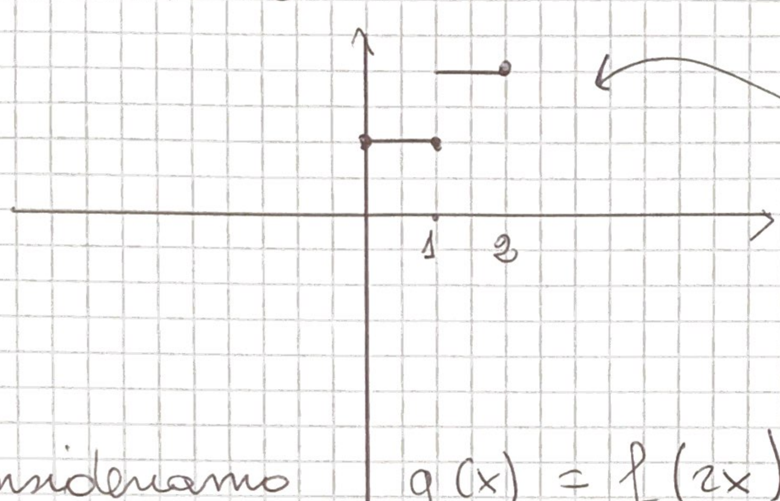
$t = 1$

Esercizio

Sia f definita nel modo seguente

$f(x) = 1$ se $0 \leq x \leq 1$; $f(x) = 2$

se $1 < x \leq 2$



Consideriamo $g(x) = f(2x)$

Se $0 \leq x \leq 1 \Rightarrow g(x)$ diventa

$f(0 \leq x \leq 2)$ tutto il grafico

Se $1 < x \leq 2 \Rightarrow f(2 < x \leq 4)$ $\nexists$

Esercizio

Sia $g(x) = \sqrt{4-x^2}$ $\forall\, |x| \le 2$

infatti $4-x^2 \ge 0 \Rightarrow -2 \le x \le 2 \Rightarrow$

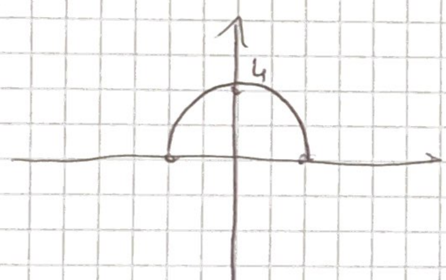
$|x| \le 2$

Verificare le seguenti formule

a) $g(-x) = g(x)$

$\sqrt{4-(-x)^2} \overset{?}{=} \sqrt{4-x^2} \Rightarrow \sqrt{4-x^2} = \sqrt{4-x^2}$

$\forall\, |x| \le 2$ funzione pari



$g(x) = \sqrt{4-x^2} \Rightarrow$

$y = \sqrt{4-x^2} \Rightarrow$

$$\begin{cases} y^2 = 4-x^2 \\ y \ge 0 \end{cases} \Rightarrow$$

$$\begin{cases} x^2+y^2 = 4 \\ y \ge 0 \end{cases}$$

$g(a-2) = \sqrt{4a-a^2}$ ⑤

$\to$ $0 \leq a \leq 4$

$\downarrow$

$\sqrt{4-(a-2)^2} = \sqrt{\cancel{4}-a^2+4a-\cancel{4}}$

Determinare il Dominio delle seguenti funzioni

6

a) $y = \sqrt{x-2}$

b) $y = \sqrt{|x-2|}$

c) $y = \sqrt{|x|-2}$

d) $y = \dfrac{1}{\sqrt{|x|-2}}$

e) $y = \sqrt{\log x + 1}$

f) $y = \log\left(\sqrt{x^2-6x+5}\right)$

g) $y = \sin\left(x - \sqrt{1-2x}\right)$

h) $y = \dfrac{1}{e^x - 6}$

i) $y = \dfrac{1}{1-\sqrt{x-2}}$

l) $y = \log\left(2x - \sqrt{x^2-1}\right)$

m) $y = \sqrt{x+1} + \sqrt{1-x}$

n) $y = \sqrt{\log(2-x) - \log(x+1)}$

o) $y = \sqrt{1-2\log_4 x} - \dfrac{1}{\sqrt{|x-1|}}$

7

b) $y = \sqrt{|x-2|}$ D: $\mathbb{R}$

d) $y = \dfrac{1}{\sqrt{|x|-2}}$

$\begin{cases} |x|-2 \geq 0 \\ \sqrt{|x|-2} \neq 0 \end{cases} \Rightarrow |x|-2 > 0 \Rightarrow x < -2 \cup x > 2$

$(-\infty, -2) \cup (2, +\infty)$

f) $y = \log\left(\sqrt{x^2-6x+5}\right)$

$\begin{cases} x^2-6x+5 \geq 0 \\ \sqrt{x^2-6x+5} > 0 \end{cases} \Rightarrow x^2-6x+5 > 0$

$x = 3 \pm 2 \begin{cases} 5 \\ 1 \end{cases} \Rightarrow x < 1 \cup x > 5$

$(-\infty, 1) \cup (5, +\infty)$

g) $y = \sin\left(x - \sqrt{1-2x}\right) \Rightarrow 1-2x \geq 0$

$\Rightarrow x \leq \frac{1}{2}$ $\left(-\infty, \frac{1}{2}\right]$

l) $y = \log\left(2x - \sqrt{x^2-1}\right)$

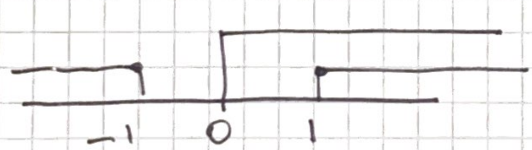
$$\begin{cases} 2x - \sqrt{x^2-1} > 0 \\ x^2 - 1 \geq 0 \end{cases} \Rightarrow \begin{cases} \sqrt{x^2-1} < 2x \\ x \leq -1 \cup x \geq 1 \end{cases}$$

n pari

$$\sqrt[n]{g(x)} < f(x) \rightarrow \begin{cases} g(x) \geq 0 \\ f(x) > 0 \\ g(x) < [f(x)]^n \end{cases}$$

$$\sqrt[n]{g(x)} > f(x) \Rightarrow \begin{cases} g(x) \geq 0 \\ f(x) < 0 \end{cases} \cup \begin{cases} f(x) \geq 0 \\ g(x) > [f(x)]^2 \end{cases}$$

$$\Rightarrow \begin{cases} x^2 - 1 \geq 0 \\ 2x > 0 \\ x^2 - 1 < 4x^2 \end{cases} \rightarrow \begin{cases} x \leq -1 \cup x \geq 1 \\ x > 0 \\ 3x^2 > -1 \quad \forall x \end{cases}$$



$x \geq 1$

$[1, +\infty)$

secondo livello

a) $y = \dfrac{\sqrt{\log(1-tgx)}}{sen^2x - \log x + \cos^2 x} = \dfrac{\sqrt{\log(1-tgx)}}{1 - \log x}$

$$\begin{cases} x > 0 \\ 1 - tgx > 0 \\ \log(1-tgx) \geq 0 \\ \log x \neq 1 \end{cases} \qquad \begin{cases} x > 0 \\ 1 - tgx > 0 \\ 1 - tgx \geq 1 \\ x \neq 10 \end{cases} \Rightarrow 1 - tgx \geq 1$$

$$\begin{cases} x > 0 \\ tgx \leq 0 \\ x \neq 10 \end{cases} \qquad \frac{\pi}{2} + k\pi < x \leq \pi + k\pi \quad k \in \mathbb{Z}^+$$

$x \neq 10$

b) $y = \sqrt[3]{\log^2 x + 1 + \sqrt{1+x^2}}$

D: $x > 0$

c) $sen[\log(1+\cos^2 x)]$ $\quad \mathbb{R}$

d) $y = x^{\frac{x \ln x}{x-1}}$

$$f(x)^{g(x)} = e^{g(x) \ln f(x)}$$

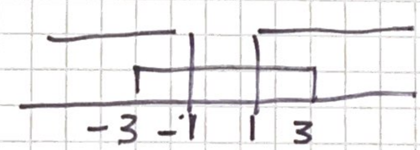
$$e^{\frac{x \ln x}{x-1} \cdot \ln x} \Rightarrow \begin{cases} x > 0 \\ x \neq 1 \end{cases}$$

e) $y = \sqrt{1 + \log_{\frac{1}{2}} (|x|-1)}$

$$\begin{cases} 1 + \log_{1/2} (|x|-1) \geq 0 \\ (|x|-1) > 0 \end{cases}$$

$$\begin{cases} \log_{1/2} (|x|-1) \geq -1 \\ |x| > 1 \end{cases} \Rightarrow \begin{cases} |x|-1 \leq 2 \Rightarrow |x| < 3 \\ x < -1 \cup x > 1 \end{cases}$$

$$\begin{cases} -3 < x < 3 \\ x < -1 \cup x > 1 \end{cases}$$



$(-3, -1) \cup (1, 3)$

Determinare il campo di esistenza delle seguenti funzioni:

a) $y = \sqrt{x+1}$

b) $y = \sqrt[3]{x+1}$

c) $y = \frac{1}{4-x^2}$

d) $y = \sqrt{x^2-2}$

e) $y = x\sqrt{x^2-2}$

f) $y = \sqrt{2+x-x^2}$

g) $y = \sqrt{-x} + \frac{1}{\sqrt{2+x}}$

h) $y = \sqrt{x-x^2}$

i) $y = \ln \frac{2+x}{2-x}$

l) $y = \ln \frac{x^2-3x+2}{x+1}$

m) $y = \arccos \frac{2x}{x+1}$

n) $y = \operatorname{arcsen}\left(\ln \frac{x}{10}\right)$

o) $y = \sqrt{\operatorname{sen} 2x}$

p) $y = \sqrt{\frac{1-x}{1-\log_2 |x|}}$

q) $y = \log_{10}(x^2-1)$