Main topics

- basics of combinatorics
- basics of probability
- basics of statistics
    - not included (mean, median, modus, normal distribution etc.)

This document summarise most important things from this topic from mathematicator course. Exercises are not included in this repository (writen in paper)

Combinatorics

Rule of product

Rule of product / multiplication principle = if there are n ways of doing something, and m ways of doing another thing after that, then there are n \times m ways to perform both of thse actions

examples:

- 2 persons, 3 ice cream flavors (banana, apple, orange) - how many pairs I can create?

$$2 \times 3 = 6$$

Rule of sum

Addition principle / rule of sum = if there are n choices for one action, and m choices for another action and the _***two actions cannot be done at the same time***, then there are n+m ways to choose one of these actions

examples:

- 3 north shops, 2 south shops -> need to decide which shop to visit, because I can't to visit north and south at the same time, so I can choose which schop to visit today

$$3 + 2 = 5$$

Factorials

Are useful when we try to calculate combinations of something. For instance, in how many ways we can order 5 people in line?

$$5 \times 4 \times 3 \times 2 \times 1 = 120$$

or we can write it rather like:

$$5! = 120$$

Factorial is one-to-one function for \mathbb{N_0} (natural number), so number must be 0,1,2,3,4 etc...

and because factorial is one-to-one function we can do this:

$$(x+2)! = 10!$$

$$x + 2 = 10$$

$$n! = m!$$

$$n = m$$

Partial permutation

- order matter
- without repetition
- For instance we have 5 people and we have to choose 2 people. 1 person will get candy, second milk. How many permutations we have if order matter and no repetition is allowed?

$$n = 5 \\ k = 2$$

$$\text{We have 5 options for first pick and 4 for second pick}$$

$$5 \times 4 = 20$$

$$\text{so it looks like factorial without tail}$$

$$P(n,k) = \frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1}$$

$$\text{we get 3 from 5 - 2 so...}$$

$$P(n,k) = \frac{n!}{(n-k)!}$$

Permutations with repetition

- almost same as previous, but now order matter and repetition allowed
- We have 5 people and we pick 2 persons, both of them get candy and also the same person can get candy twice, so it doesn't matter if the person is pick first or second

$$n = 5$$
$$k = 2$$

$$\text{5 options for the firct pick and 5 options for second pick}$$

$$5 \times 5 = 25$$

$$5^2 = 25$$

$$n^k$$

Permutation

- order matter, we always use whole set of n
- so the k = n
- example: We have 5 dogs and we want to know in how many ways we can sort them in line

$$n = 5$$
$$k = 5$$

$$5 \times 4 \times 3 \times 2 \times 1 = 120$$

$$5! = 120$$

$$P(n) = n!$$

Combination

- order doesn't matter
- I will perform partial permutation and then divide it by permutation from k

$$C(n,k) = \binom{n}{k} = \frac{n!}{k! \cdot (n-k)!}$$

Binomial coefficient

- some interestik properties of binomial coefficient

$$\binom{n}{k}$$

- we can write it like equation in ombination, but we can leverage this:

$$\binom{n}{0} = 1$$

$$\binom{n}{1} = n$$

$$\binom{n}{n} = 1$$

$$\binom{n}{k} = \binom{n}{n-k}$$

$$\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$$

Probability

Union

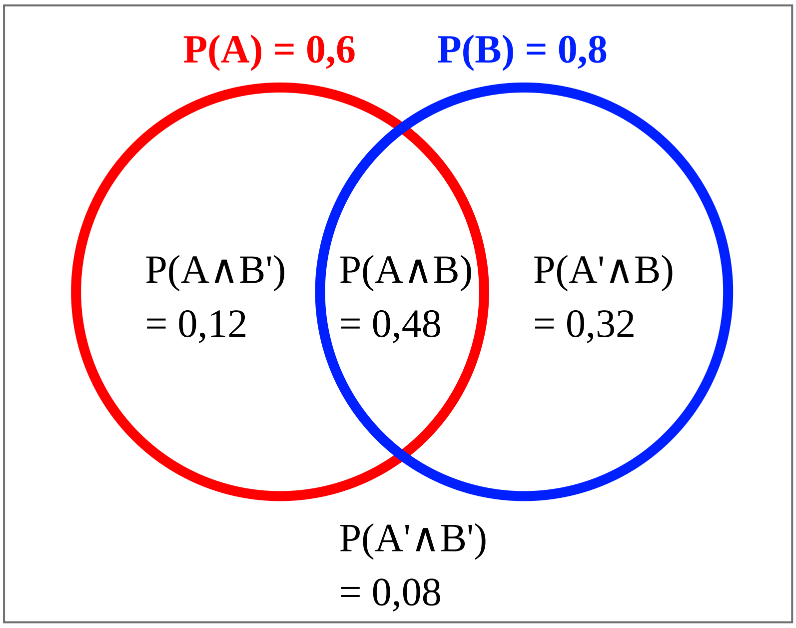
- all elements that are in the set (A) and (B), or both

$$A \cup B$$

Intersection

- all elements that are common to the sets (A) and (B)

$$A \cap B$$



Probability of event A and probability of event B

$$P(A) = 0,6$$
$$P(B) = 0,8$$

Probability of event A happen and B not happen

- The probability of event B not happen is 1 - P(B), so P(B′) = 1 - P(B)

$$P(A \wedge B') = P(A) \cdot P(B') = 0,6 \cdot 0,2 = 0,12$$

Probability of A and B happen at the same time (independent event)

$$P(A \wedge B) = P(A) \cdot P(B) = 0,6 \cdot 0,8 = 0,48$$

Probability that event A not happen and B happen

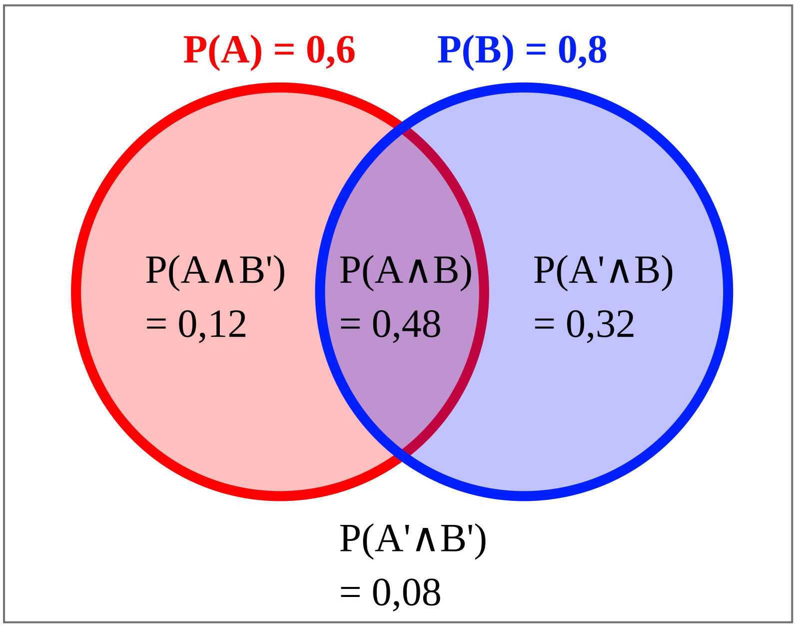
$$P(A' \wedge B) = P(A) \cdot P(B) = 0,4 \cdot 0,8 = 0,32$$

Probability that A and B not happen

$$P(A' \wedge B') = P(A) \cdot P(B) = 0,4 \cdot 0,2 = 0,08$$

Sum of all probability should be 1

$$P(A' \wedge B') + P(A' \wedge B) + P(A \wedge B) + P(A \wedge B') = 1$$



Probability that event A or B happen

- So we can write it like:

$$P(A \cup B) = P(A \wedge B\prime) + P(A \wedge B) + P(A\prime \wedge B)$$

$$P(A \cup B) = 0,12 + 0,48 + 0,32$$

$$P(A \cup B) = 0,92$$

- or we can calculate that both event not happen and subtract this from 1

$$P(A \cup B) = 1 - P(A\prime \wedge B\prime)$$

$$P(A \cup B) = 1 - 0,08$$

$$P(A \cup B) = 0,92$$

- but usually this method is used:
    - We have to subtract P(A∧B) otherwise the probability can be 1+, which is not possible

$$P(A \cup B) = P(A) + P(B) - P(A \wedge B)$$

$$P(A \cup B) = 0,6 + 0,8 - 0,48$$

$$P(A \cup B) = 0,92$$