How does the temperature of a liquid change its dynamic viscosity?

Investigation of the relationship between the dynamic viscosity and temperature of liquids

1. Personal interest in this investigation

I love approaching real-life situations from the physical and mathematical perspective. Phenomena such as rainbows, surface tension, lower boiling point of water in the mountains have grabbed my attention since an early age. Once I was making a pudding, I observed that the stirring process of the pudding in a pot became easier when the temperature of the pudding increased. I thought the “easier stirring” of the pudding was caused by the lower density when it was heated. I wondered whether other liquids could also macroscopically seem less viscous at higher temperature and by what it could be caused. My mother told me that oil acts similarly – it becomes less viscous when it is heated up. I observed this when I was frying a slice of meat on a pan and it seemed to me that the density and macroscopic structure of the oil could not change as much as the pudding changed when it was heated up. I browsed the internet where I found that the change of the density of oil with temperature is not so significant. This spurred my suspicion that the change in the stirring must have been, at least partially, caused by some other factors. As liquids are also broadly used in industry, I found this phenomena both interesting and important. Petrol stations must surely know in which temperature interval they can keep oil in the pipelines, so it is not too much or too little viscous. Viscosity is also important in food industry and medicaments – the cough syrups are usually very viscous to soothe the throat but must have the ability to be drunk[1]. There is an obvious need to know more about the viscosity of liquids and how it depends on parameters of the surroundings. For this reason, I decided to research the dependence of liquids’ viscosity on its temperature and chose it as a topic for this investigation.

[1] St. Rosemary Educational Institution, 2017

2. Theoretical background

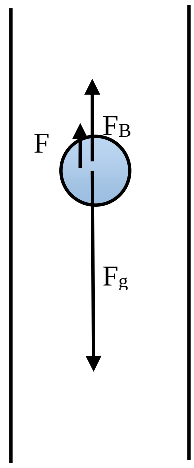

Figure 1. Forces acting on the metal sphere in the liquid

Viscosity is a measure of a fluid's resistance to flow. It describes the internal friction of a moving fluid.[2] This viscosity is also referred to as dynamic viscosity. Kinematic viscosity is the dynamic viscosity divided by the density of the liquid. In this investigation term 'viscosity' will always refer to the dynamic viscosity.

In order to determine the temperature effect of viscosity, the viscosity for a given measurement and set of parameters has to be firstly calculated.

There is a gravitational force acting on the sphere pointing to the centre of the Earth. In this case, the gravitational field is homogeneous and this force points simply downwards. This can be expressed as:

$$F_g = mg \tag{1}$$

where m is mass of the sphere and g is gravitational acceleration.

A moving object in a fluid experiences buoyant force that acts in the opposite direction than is direction of the motion of the object. The force acts upwards in this case. According to the Archimedes' principle, this force might be expressed as:

$$F_B = V\rho g \tag{2}$$

where ρ is density of the liquid and V is volume of the sphere[3]. As the sphere in the liquid should reach velocities up to tens of centimetres per second, flow in the liquid will be considered laminar. From the Stoke's law, the drag force acting on the sphere can be expressed as:

$$\mathrm{F} = 6\pi\mu r v \tag{3}$$

where μ is viscosity of the liquid, r is diameter of the sphere and v is the terminal velocity that should be reached[4]. The net force acting on the sphere would be:

$$F_{net} = F_g - (F_B + F) \tag{4}$$

[2] Definition of viscosity, 2017

[3] Buoyancy, 2017

[4] Tsokos, 2014

Assuming the sphere reaches the terminal velocity, the net force acting on it would and the acceleration of the sphere would be both zero. The gravitational force would be compensated by the buoyant force and the drag force of the laminar flow. Hence:

$$F_g = F_B + F \tag{5}$$

Therefore, the viscosity can be easily calculated as:

$$\mu = \frac{gV(\beta - \alpha)}{6\pi rs} t \tag{6}$$

where V is volume of the sphere, β is density of the sphere, r is its diameter, α is density of the liquid and s is the given distance that the sphere crosses over time t. This determined formula provides calculation of the viscosity for given parameters, nevertheless, formula *6* does not include temperature of the liquid and thus seems to be independent on it.

3. Hypothesis

My hypothesis is that the viscosity of a liquid should definitely decrease with an increase in temperature, although I cannot anticipate the relation. I already know that atoms in liquids do not form a crystalline lattice. Molecules are in less dense structures than in the solid objects and can exchange their positions. When a metal sphere is thrown into a liquid, molecules of the sphere have to push the liquids' molecules aside which is macroscopically perceived as the ball's fall. The higher the temperature of the liquid, the higher the kinetic energy of the atoms and molecules. Hence, molecules with the higher kinetic energy would move faster and should be more easily pushed to sides and allow the sphere to move through the liquid. The change in density of the liquid caused by change in its temperature should affect the buoyant force acting on the sphere, but should not affect viscosity of the liquid.

4. Method

The purpose of this experiment is to **investigate the relation between the dynamic viscosity and temperature of a liquid**. A metal sphere will be released at the surface level in a measuring cylinder which will contain a liquid that will be investigated. The sphere will be carefully released, so its initial velocity with which it enters the liquid is zero. The gravitational force that the ball experiences will make it gradually fall through the liquid as its density is higher than the density of the liquid. Time interval required for the ball to travel a certain distance between two fixed points of the cylinder will be measured, which is substantial for calculation of the viscosity. This will be done at different temperatures and thus a dependency of the liquid's viscosity on the temperature can be determined.

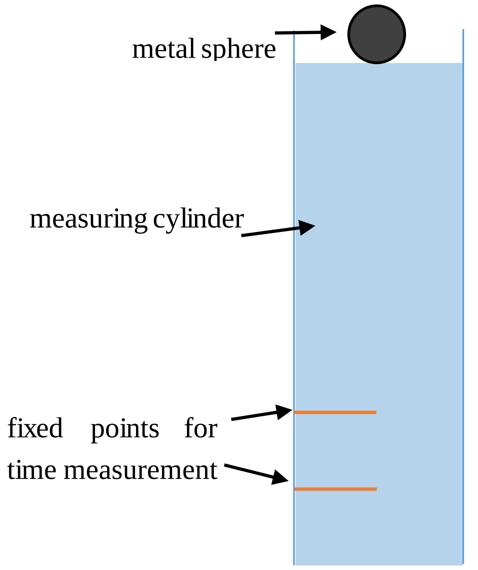

Figure 2 Scheme of the experiment

A. Experimental variables

The **independent variable** in this study is the temperature. The liquid was initially heated up to a certain level and the measurements were being made while it was cooling down until it approximately reached the room temperature.

The **dependent variable** is the viscosity of the liquid that is also point of this investigation.

The **controlled variables** include the same volume of the liquid in the measuring cylinder, the zero initial speed with which the sphere is released, using the same sphere and the same measuring cylinder. The ball's fall was always considered vertical. It is also assumed that the room temperature and the atmospheric pressure remain constant at the place where the experiment took place.

B. Preliminary work

Liquids. Two different liquids (oil and honey) will be used to compare their viscosity dependence on the temperature. These two liquids were chosen because the second one is much more viscous than the first one.

Measuring cylinder. One measuring cylinder with the volume of 500 ml will be used for all measurements.

Drop height. The ball will always be released at the surface level so the initial velocity with which the ball enters the liquid is zero.

Time measurement. Measuring time with a stopwatch could result in relatively big errors as the human reaction is not immediate and the measured time interval could be far below one second. Therefore, a camera capable of making 1000 frames per second will be used to record the fall of the ball to determine the time with a higher precision.

Data presentation. The data will be analysed in program 'Movie Maker' in which the video record can be slowed down up to 8 times of its length if necessary. Analysis of the ball's motion will be made in program 'Tracker'.

Temperature. The temperature of the liquid was measured by an electric thermometer.

C. Experimental assumptions

1) The temperature of the liquid is constant during the fall of the sphere and the liquid is evenly heated (the temperature of the liquid is the same at every point in the cylinder).
2) When a measurement is made the ball has already reached the terminal velocity.
3) The ball in the cylinder does not rotate and does not touch the walls. Its trajectory is a line and is not deflected by any angle.
4) Density of the liquid does not change with temperature and remains constant.
5) The camera is set perpendicular to the measuring cylinder, meaning that every measurement is taken from the right angle.

D. Safety and environmental issues

This experiment was not be dangerous and did not do any harm. Nevertheless, it was still needed to be careful, especially when dealing with the hot liquids whose temperature could rise up to 80°C. For this reason a pair of safety gloves was used when manipulating with the hot liquids as to diminish a potential danger. In order to minimise contact with the hot liquids a neodymium magnet was used to take the metal sphere out of the cylinder. From the environmental perspective this experiment was clean because no toxic or harmful substances or materials were used. It was only necessary to properly clean the apparatus, so it can be used by other students without some complications.

5. Gathering raw data

The table below specifies uncertainties of the measured quantities.

Phyiscal quantity	Value of the physical quantity	Absolute uncertainty (Δx)	Percentage uncertainty (δx)
Mass of the sphere [kg]	0.0102	10^{-4}	0.985
Radius of the sphere [m]	0.00675	10^{-4}	0.747
Volume of the sphere [m^3]	$1.28*10^{-6}$	$3*10^{-7}$	2.22
Density of the sphere [kg/m^3]	7880	253	3.21
Density of the honey [kg/m^3]	1250	1.87	0.150
Density of the oil [kg/m^3]	865	1.30	0.150

Table 1. Absolute and percentage uncertainties of the measured quantities.

The absolute uncertainty in time measurement is $\Delta t = 0.001$s and is determined by the precision of the camera. The percentage uncertainty of the time measurement can be calculated as:

$$\delta t = 100\frac{\Delta t}{t}\% \tag{7}$$

where t a specific measured time and varies for every temperature. The percentage uncertainty in time measurement is inversely proportional to the value of time and therefore is much lower in honey than in oil. The uncertainty in temperature changes with the value of temperature because the rate of heat transfer depends on the current difference between the temperature of the surroundigs and temperature of the liquid. This percentage uncertainty was estimated as:

$$\delta T = 5\%\Omega T \tag{8}$$

where ΩT is the difference between the temperature of the sample and temperature of the surroundings and is not to be misinterpreted with the absolute uncertainty. Table below shows the value of dynamic viscosity for appropriate sample at a given temperature. One line in the table represents one video record of the ball's fall.

	Oil		Honey	
Number of measurement	Temperature T(K)	Viscosity μ(Pa.s)	Temperature T(K)	Viscosity μ(Pa.s)
1	294.75	0.49	296.15	82.55
2	302.25	0.47	306.65	38.78
3	308.25	0.46	309.65	29.01
4	313.55	0.45	313.85	23.84
5	323.15	0.44	320.15	15.04
6	329.15	0.44	326.25	9.95
7	338.65	0.43	330.65	7.15
8	348.95	0.42	339.25	4.52
9	355.15	0.42	344.85	3.09
10	361.95	0.42	350.35	0.86

Table 2. Values of dynamic viscosity of oil and honey at different temperatures. The values of viscosity are adjusted to two decimal places because that is the precision of the temperature measurements.

The absolute and percentage uncertainties obtained for the viscosity of oil and honey for the lowest and highest temperature are in the table below:

Oil			Honey		
Temperature (K)	δμ(%)	Δμ(Pa. s)	Temperature (K)	δμ(%)	Δμ(Pa. s)
294.75	8.54	0.04	296.15	6.81	5.00
361.95	8.90	0.04	350.35	7.35	0.12

Table 3. Absolute and percentage uncertainty of the lowest and highest temperature

6. Data presentation and analysis

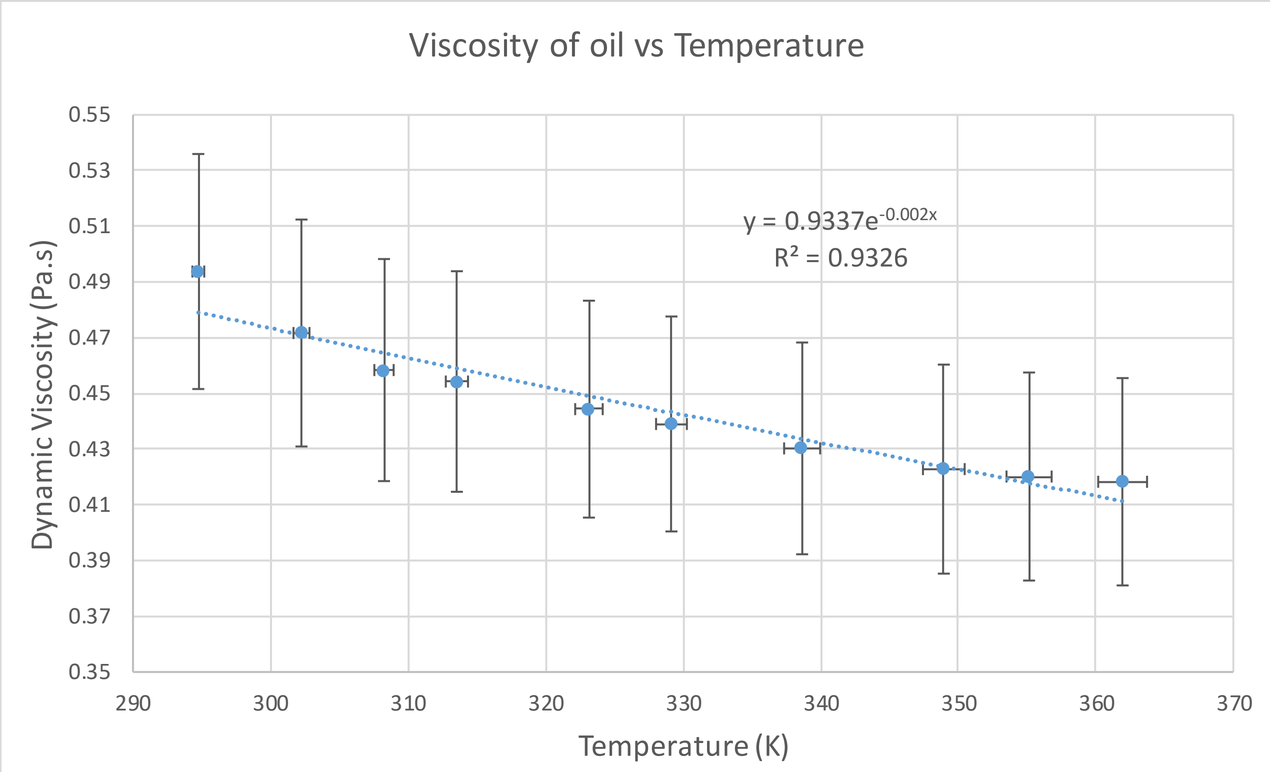

Figure 3. Dynamic viscosity of oil versus temperature

It is difficult to estimate the relation because the R^2 visibility gains similar values for different sort of functions and does not more concretely specify which relation it should be. Furthermore, the uncertainties are rather high. This is the result of the particular uncertainties that were already mentioned. Nevertheless, uncertainty of the dynamic viscosity can be significantly minimised if the liquid in which the ball is thrown is more viscous and thus the ball's fall would take more time.

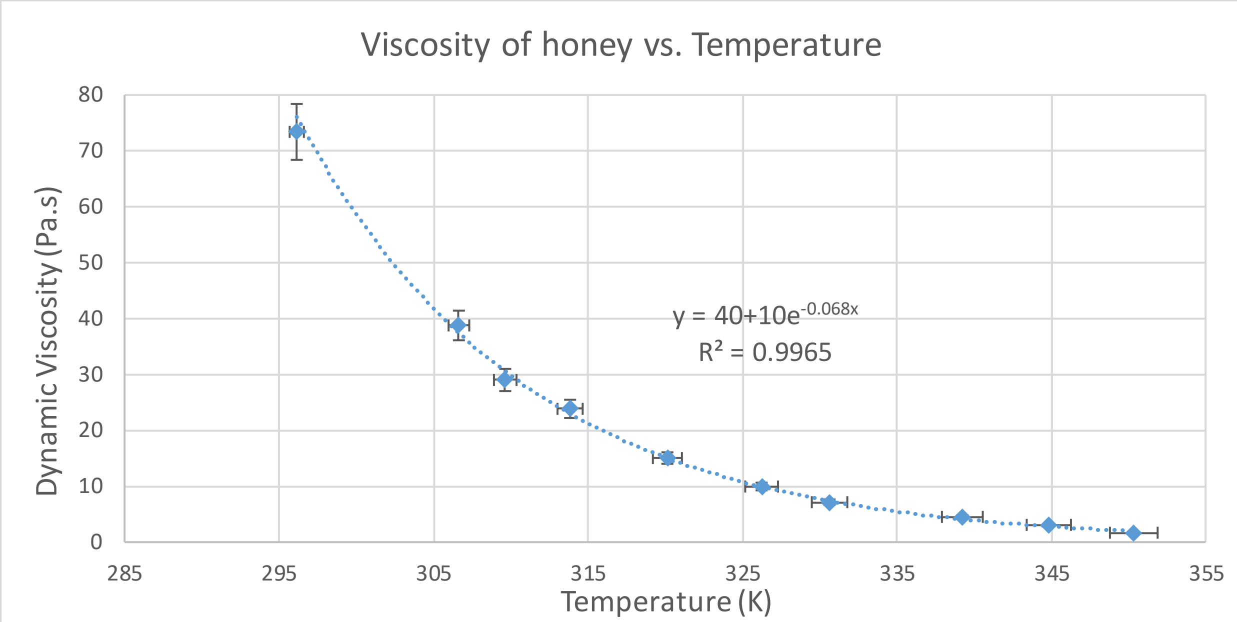

Figure 4 Dynamic viscosity of honey versus temperature

The relation between the viscosity and the temperature seems exponential as the R^2 visibility is very close to 1 and significantly decreases for other types of functions. The closer this value approaches 1, the closer the experimental data portray the exponential relation. The graph will be manipulated so the y-axis shows the logarithmic value of the viscosity. If this relation is linear, then the exponential nature will be confirmed.

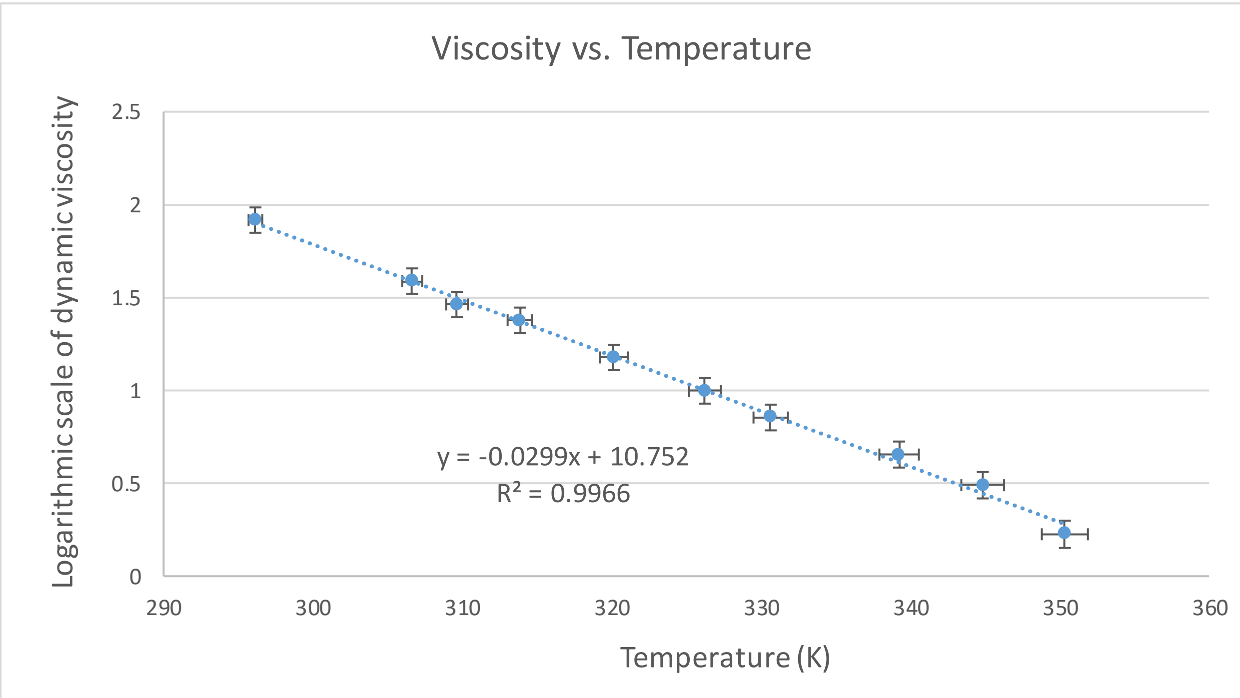

Figure 5. Logarithmic scale of dynamic viscosity of honey versus temperature

The linear trend of the best-fit line in the logarithmic scale of viscosity justifies the that the relation between the viscosity and temperature is exponential.

7. Does the sphere reach terminal velocity?

The metal sphere would reach the terminal in an infinite time but it should already approach this value close enough after a short time if the liquids are viscous enough. Hence, tracking of the sphere was performed to find out if the ball indeed reaches the terminal velocity. During the measurements, there will be always some small net force acting on the sphere pointing downwards that should give the ball an acceleration:

$$a = \frac{F_{net}}{m} \tag{9}$$

This net force is the result of the same forces that were already mentioned, however, their equilibrium is not reached yet. Therefore, the net force will be calculated as the difference between the gravitational force minus the sum of the buoyant force and the drag force from the laminar flow. This results in equation:

$$F_{net} = F_g - F_b - F_d \tag{10}$$

Considering that the acceleration is the rate of change of velocity in an infinitely small time and using the adequate equations for the other forces mentioned in the above part (equations 1, 2, 3), it is obtained:

$$m\frac{dv}{dt} = mg - Vg\rho - 6\pi\mu dv \tag{11}$$

After some time spent on this problem, equation *11* led to the result that velocity of the sphere should be given by the formula:

$$v = \frac{mg - Vg\rho}{6\pi\mu d}(1 - e^{-t\frac{6\pi\mu d}{m}}) \tag{12}$$

Further, considering the difference in the displacement of the ball, which in this case is the distance travelled by the sphere is the integral of velocity with reference to time, it should be satisfied by the following equation:

$$s = \int v\,dt = \frac{mg - Vg\rho}{6\pi\mu d}(t + \frac{m}{6\pi\mu d}e^{-t\frac{6\pi\mu d}{m}}) \tag{13}$$

This theoretical model includes a linearly dependent tern on time and an exponentially dependent term on time. Exponent in equation 13 contains a minus sign which means that with increasing time, this term should quickly approach zero value. Hence, after a relatively short period of time, the distance travelled by the sphere should be proportional to time. If this is indeed observed, then the velocity of the sphere will have already approached the terminal velocity and can be calculated as the gradient.

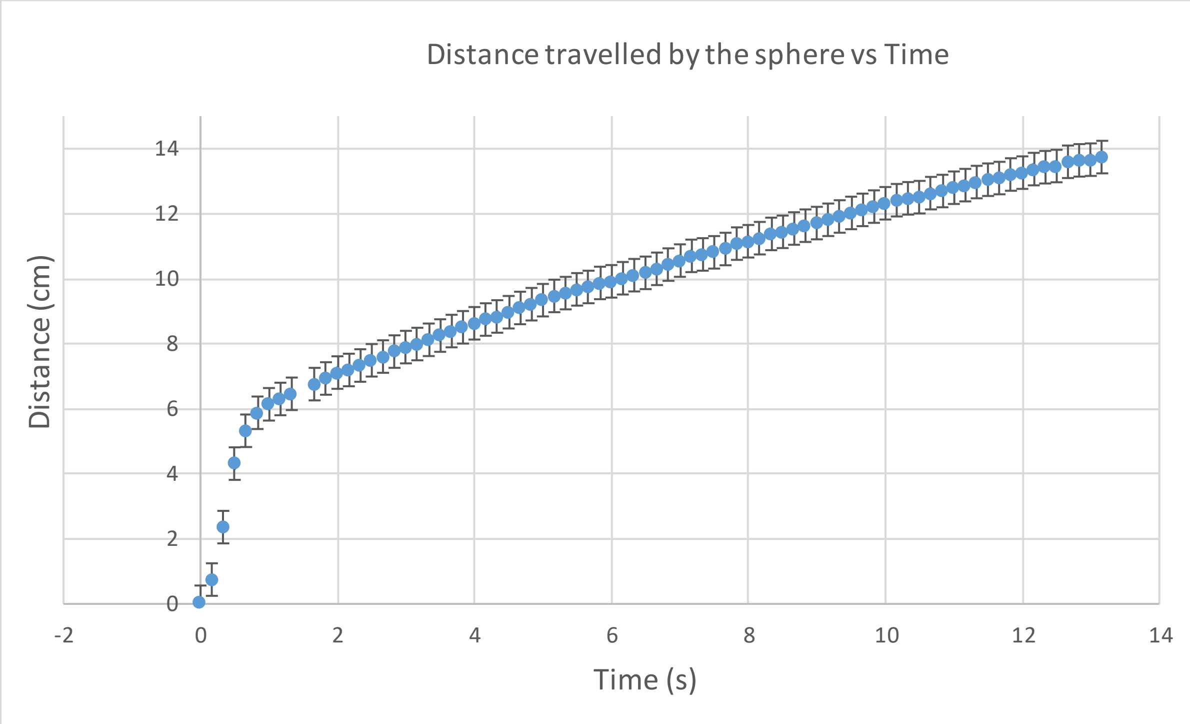

Figure 6. Distance travelled by the sphere over time in honey at temperature 344.85 K

The graph above justifies that the sphere in honey already approaches the terminal velocity after approximately 2 seconds. The terminal velocity can be calculated as the gradient of two data points in which the curve already seems to portray a linear function. The absolute uncertainty of the terminal velocity will be the difference between the maximum and minimum gradient of these points divided by two.

The following two data points were chosen:

Data point number	Time (s) $\Delta t = 0.001s$	Distance (cm) $\Delta s = 0.5cm$
1	2.00	7.11
2	12.00	13.26

Table 4. Two data points of the sphere's fall that will be used to determine the terminal velocity

The maximum and minimum gradients of the two data points are illustrated in the next figure:

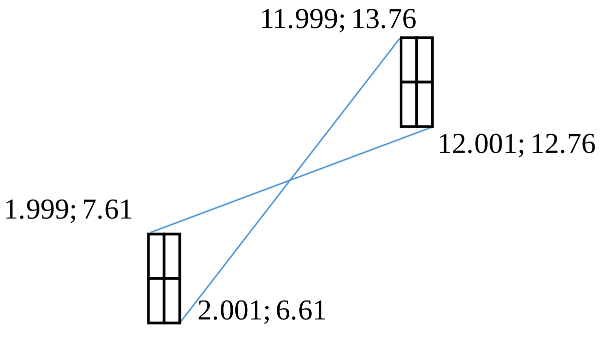

Figure 7. Maximum and minimum gradients of the selected data points

Hence, the absolute uncertainty of the terminal velocity can be calculated as:

$$\Delta v = \frac{grad_{max} - grad_{min}}{2} \tag{14}$$

Which results in: $\Delta v = \pm 0.100\ \mathrm{cm/s}$ and the real terminal velocity of the sphere in honey at temperature 344.85 K is located in the interval:

$$v = \frac{\Delta s}{\Delta t} \pm \Delta v = \frac{13.26cm - 7.11cm}{12s - 2s} \pm 0.100\frac{cm}{s} = (0.615 \pm 0.100)\frac{cm}{s} \tag{15}$$

8. Evaluation

Measurements that were taken in the oil and honey show that the dynamic viscosity of these two liquids is exponentially dependent on the temperature of the liquid. This corresponds with an exponential model provided by Wikipedia[5] that works for Newtonian liquids such as the investigated honey and water. For further study, it would be interesting to investigate this dependency in non-newtonian liquids and gases.

There are more uncertainties in this investigation that could be minimised. Diameter of the sphere was measured as precisely as it could be with the school laboratory equipment. Although uncertainty in this measurement was relatively small, this error became much more significant when the volume of the sphere and its density were calculated. Furthermore, only one measurement was made for a given temperature which could cause random errors caused by the human factor. The ball could have been deflected by an angle which would increase the trajectory it had to cross, or could touch

[5]Temperature dependence of liquid viscosity, 2017

the wall of the cylinder, lose some energy and start rotating. Taking more measurements would decrease this random error. The camera was not perpendicular to the two fixed points which determined the measured distance that the sphere had to travel. Hence, they were not observed from exactly right angle and this caused a systematic error in the time measurement. Nonetheless, this error is very small and this method could be hardly improved. Although the highest available cylinder was chosen for the purpose of this investigation, it can still limit the study. It was empirically proven that the sphere reached the terminal velocity in the honey. Due to the lower viscosity of oil, it is likely that the terminal velocity was not reached in this liquid. Using a longer cylinder would solve this problem and should then provide more precise measurements in oil.
Moreover, the liquid might not be equally heated up. The temperature could vary throughout the liquid and it is likely to be lower in the upper parts of the cylinder due to its cooling down by the surroundings, and higher in the lower parts of the cylinder. In addition, the temperature of the liquid decreases during the experiment and so the measurements had to be performed as fast as possible in order to minimise this effect.

9. Conclusion

The hypothesis that the higher temperature of a liquid should decrease its viscosity was confirmed and experimental relation was determined. This relation corresponds with the theoretical model which satisfied my aim of this investigation. Overall, I am glad to have chosen this topic as I could comprehend this phenomena from the theoretical part into a higher extent and with my own experimental data could therefore successfully answer the question of this investigation.

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