

Task 1

$$h(M) = 88$$

Choose $p = 17$ and $q = 11$

$$187 = 17 \cdot 11$$

$$N = 187$$

$$160 = 2,5$$

$$F(N) = (p-1)(q-1) = 160$$

$$e \begin{cases} 1 < e < f(N) \\ \text{coprime with } N, F(N) \end{cases}$$

$e(7, 187)$ - public key

Inverse modulo Extended Euclidian

$$d \cdot e \pmod{f(N)} = 1$$

$$7 \cdot d \pmod{160} = 1$$

$$160 = 7 \cdot 22 + 6 \quad \rightarrow \quad 6 = 160 - 7 \cdot 22$$

$$7 = 6 \cdot 1 + 1 \quad \rightarrow \quad 1 = 7 - 6 \cdot 1$$

$$1 = 7 - (160 - 7 \cdot 22) \cdot 1$$

$$1 = 7 - \cancel{160} + 7 \cdot 22$$

$$1 = 7 + 7 \cdot 22$$

$$1 = 7 \cdot 23$$

$$7^{-1} \pmod{160} = 23$$

$$\overbrace{2 + 2 \cdot 3} = 8$$

$$2 \cdot 4 = 8$$

$e(7, 187)$ - public key

$d(23, 187)$ - private key

Signin's private key:

$$s = h(M)^d \bmod n$$

$$s = 88^{23} \bmod 187$$

$$s = 11$$

Verify the signature of the message M

$$s^e \bmod n$$

$$11^7 \bmod 187 = 88$$

In this case, we will assume that the signature is genuine

Task 2.1

$$n = p \cdot q = 851$$

$$851 \bmod 2 = 1$$

$$851 \bmod 3 = 2$$

$$851 \bmod 5 = 1$$

$$851 \bmod 7 = 4$$

$$851 \bmod 11 = 4$$

$$851 \bmod 13 = 6$$

$$851 \bmod 17 = 1$$

$$851 \bmod 19 = 15$$

$$851 \bmod 23 = 0$$

$$p = 23$$

$$q = \frac{851}{23} = 37$$

$$p = 23, q = 37$$

Task 2.2.

$$p = 3, q = 11, n = 33, e = 7, d = 3$$

$$c = m^e \bmod n$$

$$m = 2$$

$(7, 33)$ - public key

encrypt:

$$2^7 \bmod 33 = 29$$

re-encrypt:

$$29^7 \bmod 33 = 17$$

$$17^3 \bmod 33 = 8$$

$$8^3 \bmod 33 = 2$$

Task 3.2

The code for this task is in ex2_3.py

The appropriate key is 19.

The cipher text includes the name of the AES algorithm.