

б) Нечувствительность:

$$M[\tilde{\theta}_2] = \frac{1}{2} M[\max \tilde{x}_n] = \left\{ \begin{array}{l} \Psi = (F(x))^n = \left(\frac{1}{\theta} \int_0^x dx \right)^n = \left(\frac{x-\theta}{\theta} \right)^n \\ \Psi = n \cdot \frac{(x-\theta)^{n-1}}{\theta^n} \cdot \{0, 2\theta\} \end{array} \right.$$

$$= \frac{1}{2} \int_0^{2\theta} \frac{x \cdot n \cdot (x-\theta)^{n-1}}{\theta^n} dx = \frac{n}{2\theta^n} \int_0^{2\theta} x \cdot (x-\theta)^{n-1} dx =$$

$$= \frac{1}{2\theta^n} \left(2\theta^{n+1} - \theta^{n+1} \frac{1}{n+1} \right) = \theta - \frac{\theta}{2(n+1)} = \frac{\theta(2n+2) - \theta}{2(n+1)}$$

$$= \frac{2n \cdot \theta + \theta}{2(n+1)} = \theta \cdot \frac{2n+1}{2n+2} = \frac{1}{2} \cdot \frac{2n+1}{n+1} \cdot \theta - \text{нечувств.}$$

$$\tilde{\theta}_2 = 2 \cdot \frac{n+1}{2n+1} \cdot \tilde{\theta}_2 = \frac{n+1}{2n+1} \cdot \text{Error max}(x_i) - \text{нечувств.}$$

Состоятельность:

$$D[\tilde{\theta}_2] = \frac{(n+1)^2}{(2n+1)^2} \cdot D[\max(x_i)] = \left\{ M[\max(x_i)^2] = \int_0^{2\theta} \frac{n x^2 \cdot (x-\theta)^{n-1}}{\theta^n} dx = \right.$$

$$= \frac{4n^2 + 8n + 2}{(n+1)(n+2)} \theta^2 \Rightarrow \left(\frac{4n^2 + 8n + 2}{(n+1)(n+2)} - \left(\frac{2n+1}{n+1} \right)^2 \cdot \theta \right) \cdot \left(\frac{n+1}{2n+1} \right)^2 =$$

$$= \left(\frac{n+1}{2n+1} \right)^2 \cdot \frac{n}{(n+1)^2 (n+2)} \theta^2 = \frac{n}{(2n+1)^2 (n+2)} \cdot \theta^2 \xrightarrow{n \rightarrow \infty} 0 \Rightarrow$$

\Rightarrow состоят. \oplus

с) Эфферентность

$$D[\tilde{\theta}_1] = \frac{1}{27n} \theta^2$$

$$D[\tilde{\theta}_2] = \frac{n}{(2n+1)^2 (n+2)} \theta^2$$

$$\frac{\theta^2}{27n} < \frac{n \cdot \theta^2}{(2n+1)^2 (n+2)}$$

$$4n^3 + 12n^2 + 9n + 2 < 27n^2$$

$$4n^3 - 15n^2 + 9n + 2 < 0$$

\Rightarrow Если $n \geq 30$ оценка $\tilde{\theta}_2$ эфферентнее $\tilde{\theta}_1$

д) Точный довер. интервал:
 X_n^2 выборка. $g \sim R[0, 2\theta]$

$$f(x_n, \theta) = \frac{x_{\max} - 1}{\theta}$$

$$P(f < t) = P(x_{\max} < \theta t + \theta) = (F(\theta t + \theta))^n =$$

$$= \left\{ F(x) = \begin{cases} 0, & x \leq 0 \\ \frac{x}{\theta} - 1, & 0 \leq x \leq 2\theta \\ 1, & x > 2\theta \end{cases} \right\} = \begin{cases} 0, & t \leq 0 \\ \frac{t}{1}, & 0 \leq t \leq 1 \\ 1, & t > 1 \end{cases}$$

$$\alpha + \beta = 1:$$

$$t_1 = g_{\frac{\alpha}{2}} = \sqrt{\frac{\alpha}{2}} = \sqrt{\frac{1-\beta}{2}}$$

$$t_2 = g_{1-\frac{\alpha}{2}} = \sqrt{1-\frac{\alpha}{2}} = \sqrt{\frac{1+\beta}{2}}$$

$$P\left(t_1 < \frac{x_{\max}}{\theta} - 1 < t_2\right) = \beta$$

$$\left\{ \frac{x_{\max}}{1 + \sqrt{\frac{1+\beta}{2}}} < \theta < \frac{x_{\max}}{1 + \sqrt{\frac{1-\beta}{2}}} \right\}$$

е) Асимпт. доверительный интервал:

$$\text{ОММ: } \tilde{\theta}_1 = \frac{2}{3} \bar{X} = \frac{2}{3} \bar{x}_1 = g(\bar{x}_1)$$

$$g(x_1) = \frac{2}{3} x_1 = \theta \quad \text{и } g' = \frac{2}{3}$$

$$K_{11} = x_2 - x_1^2$$

$$\tilde{\mu}_2 = x_2 - x_1^2 = \frac{s^2(n-1)}{n}$$

$$\frac{\sqrt{n} (\tilde{\theta}_1 - \theta)}{\frac{\frac{2}{3} \bar{x}_1 \cdot \sqrt{\frac{n-1}{n}}}{2s \sqrt{n-1}}} = \frac{\sqrt{n} (\tilde{\theta}_1 - \theta)}{2s \sqrt{n-1}} \sim N(0, 1)$$

$$P\left(t_1 < \frac{3n(\tilde{\theta}_1 - \theta)}{2s\sqrt{n-1}} < t_2\right) \approx \beta$$

$$\frac{2st_1\sqrt{n-1}}{3n} < \tilde{\theta}_1 - \theta < \frac{2st_2\sqrt{n-1}}{3n}$$

$$P\left(\tilde{\theta}_1 - \frac{2st_2\sqrt{n-1}}{3n} < \theta < \tilde{\theta}_1 - \frac{2st_1\sqrt{n-1}}{3n}\right) = \beta$$

$$f \sim p(x) = \frac{1}{\theta} \cdot \mathbb{I}(0, 2\theta) \quad (N5)$$

$$M[\xi] = \int_0^{2\theta} x \cdot \frac{1}{\theta} dx = \frac{1}{\theta} \left. \frac{x^2}{2} \right|_0^{2\theta} = \frac{1}{\theta} \left(\frac{4\theta^2}{2} - \frac{0^2}{2} \right) =$$

$$= \frac{2}{\theta} \cdot \theta = 2; \quad M[\xi^2] = \frac{1}{\theta} \left. \frac{x^3}{3} \right|_0^{2\theta} = \frac{1}{\theta} \cdot \frac{8\theta^3 - 0^3}{3} = \frac{8}{3} \theta^2$$

$$D[\xi] = \frac{8}{3} \theta^2 - \frac{4}{\theta} \theta^2 = \left(\frac{8 - 12}{3} \right) \theta^2 = -\frac{4}{3} \theta^2$$

ОММ:

$$\bar{x}_1 = \tilde{x}_1 = \frac{1}{n} \cdot \sum_{i=1}^n x_i = \bar{x} \Rightarrow \frac{1}{\theta} \cdot \frac{3}{2} \theta = \bar{x} \Rightarrow \tilde{\theta}_1 = \frac{2}{3} \bar{x}$$

$$M[\tilde{\theta}_1] = \frac{2}{3} M[\bar{x}] = \frac{2}{3} \cdot \frac{1}{n} \left[\sum_{i=1}^n M[x_i] \right] = \frac{2}{3n} \cdot n \cdot (M[\xi]) = \theta - \text{несм.} \oplus$$

$$D[\tilde{\theta}_1] = D\left[\frac{2}{3} \bar{x}\right] = \frac{4}{9} \cdot \frac{1}{n^2} \cdot n D[\xi] = \frac{1}{n} \cdot \theta^2 \cdot \frac{4}{3} \xrightarrow{n \rightarrow \infty} 0$$

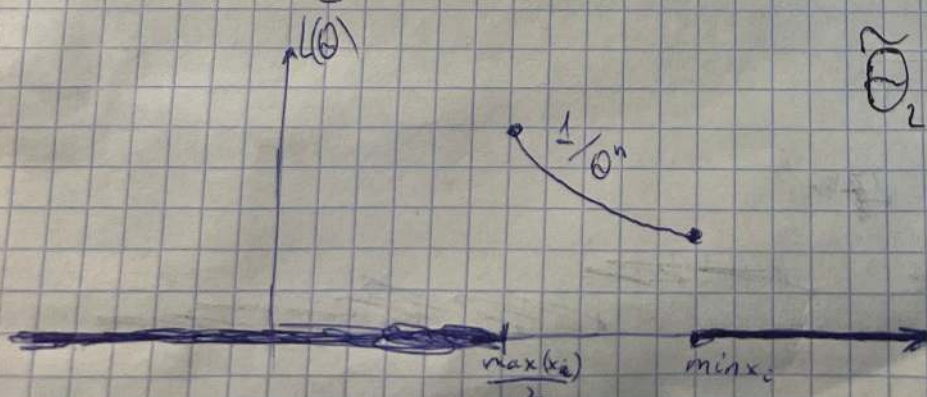
\$\Rightarrow\$ Consistent. \$\oplus\$

ММП

$$p(x, \theta) = \frac{1}{\theta} \cdot \mathbb{I}(0, 2\theta)$$

$$L(\theta) = \prod_{i=1}^n p(x_i, \theta) = \frac{1}{\theta^n} \cdot \mathbb{I}(\forall i, 0 < x_i < 2\theta) =$$

$$= \frac{1}{\theta^n} \begin{cases} \frac{\max(x_i)}{2} < \theta, \min x_i > 0 \\ \frac{\max(x_i)}{2} < \theta < \min x_i \end{cases}$$



$$\tilde{\theta}_2 = \frac{x_{\max}}{2}$$