№5

$\xi \sim p(x) = \frac{1}{\theta}\{(\theta, 2\theta)\}$

$$M[\xi] = \int_{\theta}^{2\theta} x \cdot \frac{1}{\theta}\,dx = \frac{1}{\theta}\,\frac{x^2}{2}\Big|_{\theta}^{2\theta} = \frac{1}{\theta}\left(\frac{4\theta^2}{2} - \frac{\theta^2}{2}\right) =$$

$$= \underline{\underline{\frac{3}{2}\cdot\theta}} \quad ; \quad M[\xi^2] = \frac{1}{\theta}\,\frac{x^3}{3}\Big|_{\theta}^{2\theta} = \frac{1}{\theta}\cdot\frac{8\theta^3 - \theta^3}{3} = \frac{7}{3}\theta^2$$

$$D[\xi] = \frac{7}{3}\theta^2 - \frac{9}{4}\theta^2 = \left(\frac{28-27}{12}\right)\theta^2 = \frac{1}{12}\cdot\theta^2$$

ОММ:

$$\alpha_1 = \tilde{\alpha}_1 = \frac{1}{n}\cdot\sum_{i=1}^{n} x_i = \overline{x} \Rightarrow \frac{3}{2}\theta = \overline{x} \Rightarrow \tilde{\theta}_1 = \frac{2}{3}\overline{x}$$

$$M[\tilde{\theta}_1] = \frac{2}{3}M[\overline{x}] = \frac{2}{3n}M\left[\sum_{i=1}^{n} x_i\right] = \frac{2}{3n}\cdot n\cdot\left(M[\xi]\right) = \theta \text{ - несм. } \oplus$$

$$D[\tilde{\theta}_1] = D\left[\frac{2}{3}\overline{x}\right] = \frac{4}{9}\,\frac{1}{n^2}\cdot n\,D[\xi] = \frac{1}{n}\cdot\theta^2\cdot\frac{1}{27} \xrightarrow[n\to\infty]{} 0$$

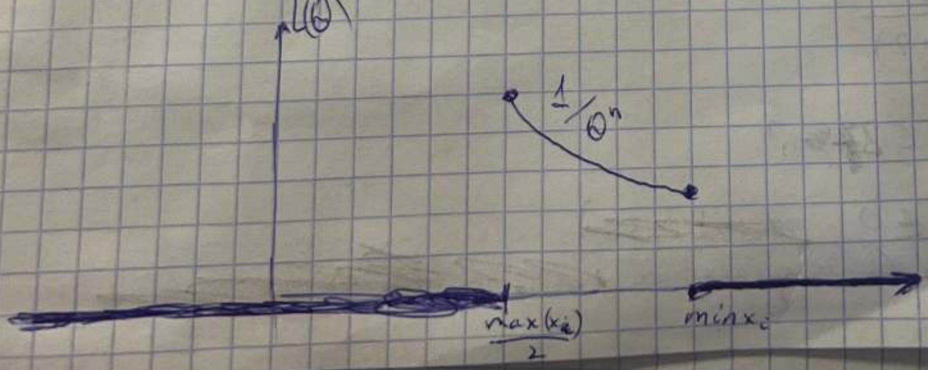
$\Rightarrow$ состоят. $\oplus$

ММП

$$p(x, \theta) = \frac{1}{\theta}\{(\theta, 2\theta)\}$$

$$L(\theta) = \prod_{i=1}^{n} p(x_i, \theta) = \frac{1}{\theta^n}\{\forall i,\ \theta < x_i < 2\theta\} =$$

$$= \frac{1}{\theta^n}\begin{cases} \frac{\max x_i}{2} < \theta,\ \min x_i > \theta \\ \frac{\max(x_i)}{2} < \theta < \min x_i \end{cases}$$



b) Несмещенность

$$M[\tilde{\theta}_2] = \frac{1}{2} M[\max \vec{x}_n] = \left\{ \begin{array}{l} \Psi_{max} = (F(x))^n = \left(\frac{1}{\theta}\int_\theta^x dx\right)^n = \left(\frac{x-\theta}{\theta}\right)^n \\ \psi = n \cdot \frac{(x-\theta)^{n-1}}{\theta^n} \quad \{\theta, 2\theta\} \end{array} \right\}$$

$$= \frac{1}{2}\int_\theta^{2\theta} \frac{x \cdot n \cdot (x-\theta)^{n-1}}{\theta^n} dx = \frac{1}{2\theta^n}\int_\theta^{2\theta} n \cdot x \cdot (x-\theta)^{n-1} dx =$$

$$= \frac{1}{2\theta^n}\left(2\theta^{n+1} - \theta^{n+1}/_{n+1}\right) = \theta - \frac{\theta}{2(n+1)} = \frac{\theta(2n+2) - \theta}{2(n+1)} =$$

$$= \frac{2n \cdot \theta + \theta}{2(n+1)} = \theta \cdot \frac{2n+1}{2n+2} = \frac{1}{2} \cdot \frac{2n+1}{n+1} \cdot \theta \text{ - смещ.}$$

$$\tilde{\tilde{\theta}}_2 = 2 \cdot \frac{n+1}{2n+1} \tilde{\theta}_2 = \frac{n+1}{2n+1} \cdot \max(x_i) \text{ - несмещ.}$$

Состоятельность:

$$D[\tilde{\tilde{\theta}}_2] = \frac{(n+1)^2}{(2n+1)^2} \cdot D[\max(\vec{x}_n)] = \left\{ M[\max(x_i)^2] = \int^{2\theta} \frac{n x^2 \cdot (x-\theta)^{n-1}}{\theta^n} dx = \right.$$

$$\left. = \frac{4n^2 + 8n + 2}{(n+1)(n+2)} \theta^2 \right\} \Rightarrow \left(\frac{4n^2+8n+2}{(n+1)(n+2)} - \left(\frac{2n+1}{n+1}\right)^2 \cdot \theta\right) \cdot \left(\frac{n+1}{2n+1}\right)^2 =$$

$$= \left(\frac{n+1}{2n+1}\right)^2 \cdot \frac{\cancel{4}n}{(n+1)^2(n+2)} \theta^2 = \frac{\cancel{4}n}{(2n+1)^2(n+2)} \cdot \theta^2 \xrightarrow[n\to\infty]{} 0 \Rightarrow$$

$\Rightarrow$ состоят. (+)

c) эффективность

$$D[\theta_1] = \frac{1}{27n}\theta^2 \qquad D[\tilde{\tilde{\theta}}_2] = \frac{\cancel{4}n}{(2n+1)^2(n+2)}\theta^2$$

$$\frac{\theta^2}{27n} < \frac{\cancel{4}n \cdot \theta^2}{(2n+1)^2(n+2)}$$

$$4n^3 + 12n^2 + 9n + 2 < 27n^2$$

$$4n^3 - 15n^2 + 9n + 2 \leq 0$$

$\Rightarrow$ Если $n \geq 3$ оценка θ_2 эффективнее

d) Точный довер. интервал:

$\vec{x}_n$ выборка. $\xi \sim R[\theta, 2\theta]$

$$f(\vec{x}_n, \theta) = \frac{x_{max}}{\theta} - 1$$

$$P(f < t) = P(x_{max} < \theta t + \theta) = (F(\theta t + \theta))^n =$$

$$= \left\{ F(x) = \begin{cases} 0, & x \le \theta \\ \frac{x}{\theta} - 1, & \theta \le x \le 2\theta \\ 1, & x > 2\theta \end{cases} \right\} = \begin{cases} 0, & t \le 0 \\ t^n, & 0 \le t \le 1 \\ 1, & t > 1 \end{cases}$$

$\alpha + \beta = 1$!

$$t_1 = q_{\alpha/2} = \sqrt[n]{\frac{\alpha}{2}} = \sqrt[n]{\frac{1-\beta}{2}}$$

$$t_2 = q_{1-\frac{\alpha}{2}} = \sqrt[n]{1 - \frac{\alpha}{2}} = \sqrt[n]{\frac{1+\beta}{2}}$$

$$P\left(t_1 < \frac{x_{max}}{\theta} - 1 < t_2\right) = \beta$$

$$\boxed{\frac{x_{max}}{1 + \sqrt[n]{\frac{1+\beta}{2}}} < \theta < \frac{x_{max}}{1 + \sqrt[n]{\frac{1-\beta}{2}}}}$$

e) Асимпт. доверительный интервал:

ОММ: $\tilde{\theta}_1 = \frac{2}{3}\overline{x} = \frac{2}{3}\overline{\alpha}_1 = g(\overline{\alpha}_1)$

$g(\alpha_1) = \frac{2}{3}\alpha_1 = \theta \quad \nabla g = \frac{2}{3}$

$K_{11} = \alpha_2 - \alpha_1^2$

$\tilde{\mu}_2 = \overline{\alpha}_2 - \overline{\alpha}_1^2 = \frac{s^2(n-1)}{n}$

$$\frac{\sqrt{n}\,(\tilde{\theta}_1 - \theta)}{\frac{2}{3}\frac{s}{1}\cdot\sqrt{\frac{n-1}{n}}} = \frac{3n(\tilde{\theta}_1 - \theta)}{2s\sqrt{n-1}} \to N(0,1)$$

$$P\left(t_1 < \frac{3n(\tilde{\theta}_1 - \theta)}{2S\cdot\sqrt{n-1}} < t_2\right) = \beta$$

$$\frac{2st_1\sqrt{n-1}}{3n} < \tilde{\theta}_1 - \theta < \frac{2st_2\sqrt{n-1}}{3n}$$

$$P\left(\tilde{\theta}_1 - \frac{2st_2\sqrt{n-1}}{3n} < \theta < \tilde{\theta}_1 - \frac{2st_1\sqrt{n-1}}{3n}\right) = \beta$$