$W = 0{,}17$

Пример 3) $\boxed{T11}$

$H_0: \xi \sim p_0(x) = 1\{(0,1)\}$

$H_1: \xi \sim p_1(x) = \frac{e}{e-1} \cdot e^{-x}\{(0,1)\}$

a) $n = 1 \quad \alpha$

$$l = \frac{L_1}{L_0} = \frac{\frac{e}{e-1} \cdot e^{-x}}{1} \geq C$$

$$e^{-x} \geq B \to x \leq A \text{ — вид } G_{кр.}$$

$$P(x \leq A | H_0) = \alpha$$

$$\int_0^A 1\,dx = A = \alpha \qquad \frac{G: x \leq \alpha}{\alpha_1 = \alpha}$$

$$W = P(x \leq A | H_1)$$

$$\int_0^{A=\alpha} \frac{e}{e-1} \cdot e^{-x}\,dx = \frac{e}{e-1}\left(1 - e^{-\alpha}\right)$$

$$\alpha_2 = 1 - W \quad \text{т.к. гипотеза простая}$$

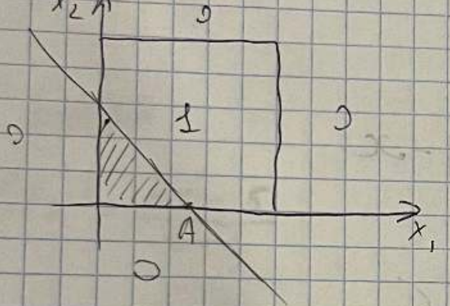
8) $n=2$

$$l = \frac{L_1}{L_0} = \frac{\left(\frac{e}{e-1}\right)^2 \cdot e^{-x_1} \cdot e^{-x_2}}{1 \cdot 1} \geq C$$

$$e^{-(x_1+x_2)} \geq B$$

$$x_1 + x_2 \leq A$$

$$P\left(x_1 + x_2 \leq A / H_0\right) = \alpha$$



$$\iint\limits_{x_1+x_2 \leq A} 1\, dx_1\, dx_2 = \frac{A^2}{2} = \alpha$$

$$A = \sqrt{2\alpha}$$

$G: x_1 + x_2 \leq \sqrt{2\alpha}$ $\qquad \alpha_1 = \alpha$

$$W = P\left(x_1 + x_2 \leq A / H_1\right) = \iint\limits_{x_1+x_2 \leq A} \left(\frac{e}{e-1}\right)^2 \cdot e^{-x_1-x_2}\, dx_1\, dx_2 =$$

$$= \left(\frac{e}{e-1}\right)^2 \cdot \int_0^A dx_1 \int_0^{A-x_1} e^{-x_1} \cdot e^{-x_2}\, dx_2 = \left(\frac{e}{e-1}\right)^2 \int_0^A e^{-x_1}\left(1 - e^{-(A-x_1)}\right) dx_1 =$$

$$= \left(\frac{e}{e-1}\right)^2 \cdot \int_0^A \left(e^{-x_1} - e^{-A}\right) \cdot dx_1 = \left(\frac{e}{e-1}\right)^2 \cdot \left(1 - e^{-A} - A \cdot e^{-A}\right)$$

$\alpha_2 = 1 - W$

c) $l = \frac{L_1}{L_0} = \prod_{i=1}^{m} \frac{p_1(x_i)}{p_0(x_i)} \geq c$

$$\ln l = \sum \underbrace{\ln \frac{p_1(x_i)}{p_0(x_i)}}_{=\eta_i} \geq \ln c$$

$$\frac{\sum \eta_i - n \cdot M \cdot \eta_i}{\sqrt{n D\eta_i}} \sim N(0,1)$$

$$P(\ln l \geq \ln c \,|\, H_0) = \alpha$$

$\cancel{H_0 \quad M_{\eta_1} =}$

$$\eta = \ln\left(\frac{e}{e-1} \cdot e^{-x}\right) = \ln\left(\frac{e}{e-1}\right) - x$$

$$\ln l = \sum \ln \frac{e}{e-1} - \sum x_i \geq \ln c$$

$$G: \sum x_i \leq A$$

$$P\left(\sum x_i \leq A \,|\, H_0\right) = \alpha$$

$$P\left(\frac{\sum x_i - nMx}{\sqrt{nDx}} \leq \frac{A - nMx}{\sqrt{nDx}} \,\Big|\, H_0\right) = \alpha$$

$Mx = \frac{1}{2}$

$Dx = \frac{1}{12}(b-a)^2 = \frac{1}{12}$

$$\frac{A - n\frac{1}{2}}{\sqrt{n \cdot \frac{1}{12}}} = u_\alpha =$$

$$A = n \cdot \frac{1}{2} + u_\alpha \cdot \sqrt{\frac{n}{12}}$$

$G: \sum x_i \le n \cdot \frac{1}{2} + u_\alpha \cdot \sqrt{\frac{n}{12}}$

$\alpha_1 = \alpha$

$\alpha_2 =$

$W = P(\sum x_i \le A | H_1) =$

$= P\left(\frac{\sum x_i - nMx}{\sqrt{nDx}} \le \frac{A - nMx}{\sqrt{nDx}} \Big| H_1\right)$

$Mx = \int_0^1 x \cdot \frac{e}{e-1} \cdot e^{-x} dx = \frac{e-2}{e-1}$

$Mx^2 = \frac{2e-5}{e-1}$

$Dx = \frac{e^2-3e+1}{(e-1)^2}$

$W = \int_{-\infty}^{B} \frac{1}{\sqrt{2\pi}} \cdot e^{-x^2/2} dx$

$B = \frac{\frac{n}{2} + u_\alpha \sqrt{\frac{n}{12}} - n\frac{e-2}{e-1}}{\sqrt{n \cdot \frac{e^2-3e+1}{(e-1)^2}}} = \frac{\sqrt{n}\overbrace{\left(\frac{1}{2} - \frac{e-2}{e-1}\right)}^{0,082} + u_\alpha \sqrt{\frac{1}{12}}}{\sqrt{n}\sqrt{\frac{e^2-3e+1}{(e-1)^2}}} \to \infty$

$\Downarrow$

$u_\alpha < 0$

$W \to 1$

$\Downarrow$

сост.

$\alpha_2 = 1 - W$

d) $G: x_{min} \le C$

$H_0: \xi \sim p(x) = 1 \{(0,1)\}$

$H_1: \xi \sim p(x) = \frac{e}{e-1} e^{-x} \{(0,1)\}$

$P(\vec{x}_n \in G | H_0) = \alpha$

$P(x_{min} \le C | H_0) = \alpha$

$H_0: \xi \sim R(0,1)$

$\xi \sim F_0(x)$ $\xi_1 \dots \xi_n$ — независ.

$\xi_{min} \sim 1 - (1 - F_0(x))^n$

$P(x_{min} \le C) = 1 - (1 - F(C))^n = \alpha$

$C = 1 - \sqrt[n]{1-\alpha}$

$G: x_{min} \le 1 - \sqrt[n]{1-\alpha}$

$\alpha_1 = \alpha$

$W = P(\vec{x}_n \in G | H_1) = P(x_{min} \le C | H_1) = 1 - (1 - F_1(C))^n =$

$= \left\{ F_1(c) = \int_0^c \frac{e}{e-1} e^{-x} dx = \frac{e}{e-1} \left(1 - e^{-1 + \sqrt[n]{1-\alpha}}\right) \right\} =$

$= 1 - \left(1 - \frac{e}{e-1} \left(1 - e^{\sqrt[n]{1-\alpha} - 1}\right)\right)^n$

$\alpha_2 = 1 - W$

Проверка на состоятельность:

$W \to 1$ — сост.

$n \to \infty$

$$1-\left(1-\frac{e}{e-1}\left(1-e^{\sqrt[n]{1-\alpha}-1}\right)\right)^n$$

$$e^{-1+\sqrt[n]{1-\alpha}}=e^{-1}\cdot e^{e^{\frac{1}{n}\cdot\ln(1-\alpha)}}=e^{-1}\cdot e^{1+\frac{1}{n}\ln(1-\alpha)+o\left(\frac{1}{n}\right)}=$$

$$=e^{\frac{1}{n}\ln(1-\alpha)+o\left(\frac{1}{n}\right)}=1+\frac{1}{n}\ln(1-\alpha)+o\left(\frac{1}{n}\right)$$

$$1-\left(1+\frac{e}{e-1}\frac{1}{n}\cdot\ln(1-\alpha)+o\left(\frac{1}{n}\right)\right)^n\to 1-e^{\frac{e\ln(1-\alpha)}{e-1}}=$$

$$=1-(1-\alpha)^{\frac{e}{e-1}}\nrightarrow 1$$

не состоят.