$$p := p_2 \cdot x^2 + p_1 \cdot x + p_0$$

$$p := p_2 x^2 + p_1 x + p_0 \tag{1}$$

$$q := q_2 \cdot x^2 + q_1 \cdot x + q_0$$

$$q := q_2 x^2 + q_1 x + q_0 (2)$$

$$r \coloneqq r_2 \cdot x^2 + r_1 \cdot x + r_0$$

$$r := r_2 x^2 + r_1 x + r_0 \tag{3}$$

 $P := collect(q' \cdot r - q \cdot r', x)$ 

$$P := (-q_1 r_2 + q_2 r_1) x^2 + (-2 q_0 r_2 + 2 q_2 r_0) x - q_0 r_1 + q_1 r_0$$
(4)

 $Q := collect(r' \cdot p - r \cdot p', x)$ 

$$Q := (r_2 p_1 - r_1 p_2) x^2 + (2 r_2 p_0 - 2 r_0 p_2) x + r_1 p_0 - r_0 p_1$$
 (5)

 $R := collect(p' \cdot q - p \cdot q', x)$ 

$$R := (-p_1q_2 + p_2q_1)x^2 + (-2p_0q_2 + 2p_2q_0)x - p_0q_1 + p_1q_0$$
(6)

$$\mathbf{M} := \begin{bmatrix} \mathit{coeff}(p, x, 0) & \mathit{coeff}(q, x, 0) & \mathit{coeff}(r, x, 0) \\ \mathit{coeff}(p, x, 1) & \mathit{coeff}(q, x, 1) & \mathit{coeff}(r, x, 1) \\ \mathit{coeff}(p, x, 2) & \mathit{coeff}(q, x, 2) & \mathit{coeff}(r, x, 2) \end{bmatrix}$$

$$M := \begin{bmatrix} p_0 & q_0 & r_0 \\ p_1 & q_1 & r_1 \\ p_2 & q_2 & r_2 \end{bmatrix} \tag{7}$$

 $\Delta := |M|$ 

$$\Delta := p_0 q_1 r_2 - p_0 q_2 r_1 - p_1 q_0 r_2 + p_1 q_2 r_0 + p_2 q_0 r_1 - p_2 q_1 r_0$$
(8)

$$\phi_{1} := -\frac{1}{16 \cdot \Delta} \left( coeff(P, x, 2)^{2} \cdot coeff(Q, x, 0) \cdot coeff(R, x, 0) + coeff(P, x, 2) \cdot coeff(P, x, 0) \cdot \left( coeff(Q, x, 2) \right) \right)$$

$$\cdot coeff(R, x, 0) + coeff(Q, x, 1) \cdot coeff(R, x, 1) + coeff(Q, x, 0) \cdot coeff(R, x, 2) \right) + coeff(P, x, 0)^{2} \cdot coeff(Q, x, 2)$$

$$\cdot x, 2) \cdot coeff(R, x, 2)$$

$$\begin{aligned} \mathbf{\psi}_{l} &\coloneqq -\left(\left(-q_{1}r_{2}+q_{2}r_{1}\right)^{2}\left(r_{1}p_{0}-r_{0}p_{1}\right)\left(-p_{0}q_{1}+p_{1}q_{0}\right)+\left(-q_{1}r_{2}+q_{2}r_{1}\right)\left(-q_{0}r_{1}+q_{1}r_{0}\right)\left(\left(p_{1}r_{2}\right)\right) \\ &-p_{2}r_{1}\right)\left(-p_{0}q_{1}+p_{1}q_{0}\right)+\left(2p_{0}r_{2}-2p_{2}r_{0}\right)\left(-2p_{0}q_{2}+2p_{2}q_{0}\right)+\left(r_{1}p_{0}-r_{0}p_{1}\right)\left(-p_{1}q_{2}\right) \\ &+p_{2}q_{1}\right)+\left(-q_{0}r_{1}+q_{1}r_{0}\right)^{2}\left(p_{1}r_{2}-p_{2}r_{1}\right)\left(-p_{1}q_{2}+p_{2}q_{1}\right)\right) / \left(16\left(p_{0}q_{1}r_{2}-p_{0}q_{2}r_{1}-p_{1}q_{0}r_{2}\right) \\ &+p_{1}q_{2}r_{0}+p_{2}q_{0}r_{1}-p_{2}q_{1}r_{0}\right) \end{aligned}$$

$$\phi_{2} := -\frac{1}{16 \cdot \Delta} \left( coeff(Q, x, 2)^{2} \cdot coeff(P, x, 0) \cdot coeff(R, x, 0) + coeff(Q, x, 2) \cdot coeff(Q, x, 0) \cdot \left( coeff(P, x, 2) \right) \right) + coeff(R, x, 0) + coeff(P, x, 1) \cdot coeff(R, x, 1) + coeff(P, x, 0) \cdot coeff(R, x, 2) + coeff(Q, x, 0)^{2} \cdot coeff(P, x, 2) \cdot coeff(R, x, 2) \right)$$

$$\Psi_{2} := -\left(\left(p_{1}r_{2} - p_{2}r_{1}\right)^{2}\left(-q_{0}r_{1} + q_{1}r_{0}\right)\left(-p_{0}q_{1} + p_{1}q_{0}\right) + \left(p_{1}r_{2} - p_{2}r_{1}\right)\left(r_{1}p_{0} - r_{0}p_{1}\right)\left(\left(-q_{1}r_{2} - q_{1}r_{2}\right)\right)\right)$$
(10)

```
+ \ q_2 r_l \big) \left( -p_0 q_1 + p_1 q_0 \right) + \left( -2 \, q_0 r_2 + 2 \, q_2 r_0 \right) \left( -2 \, p_0 q_2 + 2 \, p_2 q_0 \right) + \left( -q_0 r_l + q_1 r_0 \right) \left( -p_1 q_2 + q_1 r_0 \right) \left( -p_1 q
                                                + p_{2}q_{1}) + \left(r_{1}p_{0} - r_{0}p_{1}\right)^{2} \left(-q_{1}r_{2} + q_{2}r_{1}\right) \left(-p_{1}q_{2} + p_{2}q_{1}\right) / \left(16\left(p_{0}q_{1}r_{2} - p_{0}q_{2}r_{1} - p_{1}q_{0}r_{2}\right)\right) / \left(16\left(p_{0}q_{1}r_{2} - p_{0}q_{2}r_{1} - p_{1}q_{0}r_{2}\right)\right) + \left(16\left(p_{0}q_{1}r_{2} - p_{0}q_{2}r_{1} - p_{0}q_{2}r_{2}\right)\right) + \left(16\left(p_{0}q_{1}r_{2} - p_{0}q_{2}r_{2}\right)\right) + \left(16\left(p_{0}q_{1}r_{2} - p_{0}q_{2}r_{2}\right)\right) + \left(16\left(p_{0}q_{1}r_{2} - p_{0}q_{2}r_{2}\right)\right)
                                                +p_{1}q_{2}r_{0}+p_{2}q_{0}r_{1}-p_{2}q_{1}r_{0})
    \phi_3 := -\frac{1}{16 \cdot \Lambda} \left( coeff(R, x, 2)^2 \cdot coeff(Q, x, 0) \cdot coeff(P, x, 0) + coeff(R, x, 2) \cdot coeff(R, x, 0) \cdot \left( coeff(Q, x, 2) \right) \right)
                                           \cdot coeff(P,x,0) + coeff(Q,x,1) \cdot coeff(P,x,1) + coeff(Q,x,0) \cdot coeff(P,x,2)) + coeff(R,x,0)^{2} \cdot coeff(Q,x,0) \cdot coeff(P,x,0) + coeff(P,x,0) \cdot coeff(P,x,0) + coeff(P,x,0) \cdot coeff(P,x,0)
                                                (x,2) \cdot coeff(P,x,2)
\mathbf{\psi}_{3} := -\left(\left(-p_{1}q_{2} + p_{2}q_{1}\right)^{2}\left(r_{1}p_{0} - r_{0}p_{1}\right)\left(-q_{0}r_{1} + q_{1}r_{0}\right) + \left(-p_{1}q_{2} + p_{2}q_{1}\right)\left(-p_{0}q_{1} + q_{1}r_{0}\right)\right) + \left(-p_{1}q_{2} + p_{2}q_{1}\right)\left(-p_{0}q_{1} + q_{1}r_{0}\right) + \left(-p_{1}q_{2} + p_{2}q_{1}\right)\left(-p_{0}q_{1} + q_{2}r_{0}\right) + \left(-p_{1}q_{1} + q_{2}r_{0}\right) + \left(-p_{1}q_{1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        (11)
                                                + \, p_{1} \, q_{0} \big) \, \left( \left( p_{1} r_{2} - p_{2} r_{1} \right) \left( - q_{0} r_{1} + q_{1} r_{0} \right) + \left( 2 p_{0} r_{2} - 2 p_{2} r_{0} \right) \left( - 2 \, q_{0} r_{2} + 2 \, q_{2} r_{0} \right) + \left( r_{1} p_{0} \right) \right) \, dt + c \, dt
                                                -r_{0}p_{1}\left(-q_{1}r_{2}+q_{2}r_{1}\right)+\left(-p_{0}q_{1}+p_{1}q_{0}\right)^{2}\left(p_{1}r_{2}-p_{2}r_{1}\right)\left(-q_{1}r_{2}+q_{2}r_{1}\right)\right)\Big/\left(16\left(p_{0}q_{1}r_{2}+q_{2}r_{1}\right)\right)
                                                  -p_0q_2r_1-p_1q_0r_2+p_1q_2r_0+p_2q_0r_1-p_2q_1r_0)
                                                                                         coeff(P, x, 2) coeff(Q, x, 2)
                                                                                                                                                                                                                                                                                                                                                                                                                                               coeff(R, x, 2)
                                                                                   -coeff(P, x, 1) -coeff(Q, x, 1) -coeff(R, x, 1)
                                                                                   -coeff(P, x, 0) -coeff(Q, x, 0) -coeff(R, x, 0)
    C :=
 C :=
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        (12)
                                                                                                  \frac{\left(-q_{1}r_{2}+q_{2}r_{1}\right)^{2}\left(r_{1}p_{0}-r_{0}p_{1}\right)\left(-p_{0}q_{1}+p_{1}q_{0}\right)+\left(-q_{1}r_{2}+q_{2}r_{1}\right)\left(-q_{0}r_{1}+q_{1}r_{0}\right)\left(\left(-q_{0}r_{1}+q_{1}r_{0}\right)\right)}{\left(-q_{0}r_{1}+q_{1}r_{0}\right)}
    \mu := (j, k, l, m) \quad coeff(P, x, j) \cdot coeff(p, x, k) \cdot coeff(p, x, l) \cdot coeff(q, x, m) \cdot coeff(r, x, m) + coeff(Q, x, j)
                                           \cdot coeff(q,x,k) \cdot coeff(q,x,l) \cdot coeff(p,x,m) \cdot coeff(r,x,m) + coeff(R,x,j) \cdot coeff(r,x,k) \cdot coeff(r,x,l)
                                             \cdot coeff(p, x, m) \cdot coeff(q, x, m)
                                                                                                                                                                                       coeff(P, x, j) \cdot coeff(p, x, k) \cdot coeff(p, x, l) \cdot coeff(q, x, m) \cdot coeff(r, x, m) + coeff(Q, x, m)
                                    j) coeff(q, x, k) \cdot coeff(q, x, l) \cdot coeff(p, x, m) \cdot coeff(r, x, m) + coeff(r, x, l) \cdot coeff(r, x, l) \cdot
                                           l) \cdot coeff(p, x, m) \cdot coeff(q, x, m)
   \psi_0 := 4 {\cdot} \mu(0,0,0,2) + \mu(2,1,1,0) + \mu(2,0,0,1)
+ \left. q_{2} r_{1} \right) p_{1}^{2} q_{0} r_{0} + \left( p_{1} r_{2} - p_{2} r_{1} \right) q_{1}^{2} p_{0} r_{0} + \left( -p_{1} q_{2} + p_{2} q_{1} \right) r_{1}^{2} p_{0} q_{0} + \left( -q_{1} r_{2} + q_{2} r_{1} \right) p_{0}^{2} q_{1} r_{1}
```

$$\begin{split} &+\left(p_{1}r_{2}-p_{2}r_{1}\right)q_{0}^{2}p_{1}r_{1}+\left(-p_{1}q_{2}+p_{2}q_{1}\right)r_{0}^{2}p_{1}q_{1} \\ \psi_{1} :=&-\frac{\Delta}{2}\cdot p_{1}\cdot q_{1}\cdot r_{1}-\mu(1,0,2,1) \\ \psi_{I} :=&-\frac{\left(p_{0}q_{1}r_{2}-p_{0}q_{2}r_{1}-p_{1}q_{0}r_{2}+p_{1}q_{2}r_{0}+p_{2}q_{0}r_{1}-p_{2}q_{1}r_{0}\right)p_{1}q_{1}r_{1}}{2}-\left(-2q_{0}r_{2}\right) \\ &+2q_{2}r_{0}\right)p_{0}p_{2}q_{1}r_{1}-\left(2p_{0}r_{2}-2p_{2}r_{0}\right)q_{0}q_{2}p_{1}r_{1}-\left(-2p_{0}q_{2}+2p_{2}q_{0}\right)r_{0}r_{2}p_{1}q_{1} \\ \psi_{2} :=&-4\cdot\mu(2,2,2,0)-\mu(0,1,1,2)-\mu(0,2,2,1) \\ \psi_{2} :=&-4\left(-q_{1}r_{2}+q_{2}r_{1}\right)p_{2}^{2}q_{0}r_{0}-4\left(p_{1}r_{2}-p_{2}r_{1}\right)q_{2}^{2}p_{0}r_{0}-4\left(-p_{1}q_{2}+p_{2}q_{1}\right)r_{2}^{2}p_{0}q_{0}-\left(-q_{0}r_{1}\right)q_{1}^{2}p_{2}r_{2}-\left(-p_{0}q_{1}+p_{1}q_{0}\right)r_{1}^{2}p_{2}q_{2}-\left(-q_{0}r_{1}+q_{1}r_{0}\right)p_{2}^{2}q_{1}r_{1} \\ &-\left(r_{1}p_{0}-r_{0}p_{1}\right)q_{2}^{2}p_{1}r_{1}-\left(-p_{0}q_{1}+p_{1}q_{0}\right)r_{2}^{2}p_{1}q_{1} \\ Cinv :=&\begin{bmatrix} -\frac{P_{0}}{\Delta}-\frac{P_{1}}{2\cdot\Delta}\frac{P_{2}}{\Delta}&0\\ -\frac{r_{0}}{\Delta}-\frac{q_{1}}{2\cdot\Delta}\frac{q_{2}}{\Delta}&0\\ -\frac{r_{0}}{\Delta}-\frac{r_{1}}{2\cdot\Delta}\frac{r_{2}}{\Delta}&0\\ \psi_{0}&\psi_{1}&\psi_{2}&-16\cdot\Delta \end{bmatrix} \\ Cinv :=& (17) \end{split}$$

 $simplify(Cinv \cdot C)$