> $p_1 := p_{11}\cdot x + p_{10}$

$$p_1 := p_{11}x + p_{10} \tag{1}$$

> $p_2 := p_{21}\cdot x + p_{20}$

$$p_2 := p_{21}x + p_{20} \tag{2}$$

> $q_1 := q_{11}\cdot x + q_{10}$

$$q_1 := q_{11}x + q_{10} \tag{3}$$

> $q_2 := q_{21}\cdot x + q_{20}$

$$q_2 := q_{21}x + q_{20} \tag{4}$$

> $r_1 := r_{11}\cdot x + r_{10}$

$$r_1 := r_{11}x + r_{10} \tag{5}$$

> $r_2 := r_{21}\cdot x + r_{20}$

$$r_2 := r_{21}x + r_{20} \tag{6}$$

> $p := collect(p_1\cdot p_2, x)$

$$p := p_{11}p_{21}x^2 + (p_{10}p_{21} + p_{11}p_{20})x + p_{10}p_{20} \tag{7}$$

$q := collect(q_1\cdot q_2, x)$

$$q := q_{11}q_{21}x^2 + (q_{10}q_{21} + q_{11}q_{20})x + q_{10}q_{20} \tag{8}$$

$r := collect(r_1\cdot r_2, x)$

$$r := r_{11}r_{21}x^2 + (r_{10}r_{21} + r_{11}r_{20})x + r_{10}r_{20} \tag{9}$$

$P := collect(q'\cdot r - q\cdot r', x)$

$$P := (-(q_{10}q_{21} + q_{11}q_{20})r_{11}r_{21} + q_{11}q_{21}(r_{10}r_{21} + r_{11}r_{20}))x^2 + (-2q_{10}q_{20}r_{11}r_{21} + 2q_{11}q_{21}r_{10}r_{20})x + (q_{10}q_{21} + q_{11}q_{20})r_{10}r_{20} - q_{10}q_{20}(r_{10}r_{21} + r_{11}r_{20}) \tag{10}$$

$Q := collect(r'\cdot p - r\cdot p', x)$

$$Q := (-(r_{10}r_{21} + r_{11}r_{20})p_{11}p_{21} + r_{11}r_{21}(p_{10}p_{21} + p_{11}p_{20}))x^2 + (2r_{11}r_{21}p_{10}p_{20} - 2r_{10}r_{20}p_{11}p_{21})x + (r_{10}r_{21} + r_{11}r_{20})p_{10}p_{20} - r_{10}r_{20}(p_{10}p_{21} + p_{11}p_{20}) \tag{11}$$

$R := collect(p'\cdot q - p\cdot q', x)$

$$R := (-(p_{10}p_{21} + p_{11}p_{20})q_{11}q_{21} + p_{11}p_{21}(q_{10}q_{21} + q_{11}q_{20}))x^2 + (-2p_{10}p_{20}q_{11}q_{21} + 2p_{11}p_{21}q_{10}q_{20})x + (p_{10}p_{21} + p_{11}p_{20})q_{10}q_{20} - p_{10}p_{20}(q_{10}q_{21} + q_{11}q_{20}) \tag{12}$$

$$\mathrm{M} := \begin{bmatrix} coeff(p,x,0) & coeff(q,x,0) & coeff(r,x,0) \\ coeff(p,x,1) & coeff(q,x,1) & coeff(r,x,1) \\ coeff(p,x,2) & coeff(q,x,2) & coeff(r,x,2) \end{bmatrix}$$

$$M := \begin{bmatrix} p_{10}p_{20} & q_{10}q_{20} & r_{10}r_{20} \\ p_{10}p_{21} + p_{11}p_{20} & q_{10}q_{21} + q_{11}q_{20} & r_{10}r_{21} + r_{11}r_{20} \\ p_{11}p_{21} & q_{11}q_{21} & r_{11}r_{21} \end{bmatrix} \tag{13}$$

$\Delta := |M|$

$$\Delta := p_{10}p_{20}q_{10}q_{21}r_{11}r_{21} + p_{10}p_{20}q_{11}q_{20}r_{11}r_{21} - p_{10}p_{20}q_{11}q_{21}r_{10}r_{21} - p_{10}p_{20}q_{11}q_{21}r_{11}r_{20} \tag{14}$$

$$- p_{10}p_{21}q_{10}q_{20}r_{11}r_{21} + p_{10}p_{21}q_{11}q_{21}r_{10}r_{20} - p_{11}p_{20}q_{10}q_{20}r_{11}r_{21} + p_{11}p_{20}q_{11}q_{21}r_{10}r_{20}$$
$$+ p_{11}p_{21}q_{10}q_{20}r_{10}r_{21} + p_{11}p_{21}q_{10}q_{20}r_{11}r_{20} - p_{11}p_{21}q_{10}q_{21}r_{10}r_{20} - p_{11}p_{21}q_{11}q_{20}r_{10}r_{20}$$

$g := collect(p_2 \cdot q_1 \cdot r_1, x)$

$$g := p_{21}q_{11}r_{11}x^3 + \left(\left(p_{20}q_{11} + p_{21}q_{10}\right)r_{11} + p_{21}q_{11}r_{10}\right)x^2 + \left(p_{20}q_{10}r_{11} + \left(p_{20}q_{11} + p_{21}q_{10}\right)r_{10}\right)x + p_{20}q_{10}r_{10} \quad (15)$$

$h := collect(p_1 \cdot q_2 \cdot r_2, x)$

$$h := p_{11}q_{21}r_{21}x^3 + \left(\left(p_{10}q_{21} + p_{11}q_{20}\right)r_{21} + p_{11}q_{21}r_{20}\right)x^2 + \left(p_{10}q_{20}r_{21} + \left(p_{10}q_{21} + p_{11}q_{20}\right)r_{20}\right)x + p_{10}q_{20}r_{20} \quad (16)$$

$T := [coeff(g,x,2)\cdot coeff(h,x,0) + coeff(g,x,0)\cdot coeff(h,x,2), coeff(g,x,3)\cdot coeff(h,x,0) + coeff(g,x,0)\cdot coeff(h,x,3), -coeff(g,x,3)\cdot coeff(h,x,1) - coeff(g,x,1)\cdot coeff(h,x,3), 4]$

$$T := \left[\left(\left(p_{20}q_{11} + p_{21}q_{10}\right)r_{11} + p_{21}q_{11}r_{10}\right)p_{10}q_{20}r_{20} + p_{20}q_{10}r_{10}\left(\left(p_{10}q_{21} + p_{11}q_{20}\right)r_{21} + p_{11}q_{2}\cdots\right)\right] \quad (17)$$

$$DD := \left[\left[coeff(P,x,2), coeff(Q,x,2), coeff(R,x,2), 0\right], \left[-coeff(P,x,1), -coeff(Q,x,1), -coeff(R,x,1), 0\right], \left[-coeff(P,x,0), -coeff(Q,x,0), -coeff(R,x,0), 0\right], \left[\frac{\left(\Delta - coeff(P,x,1)\cdot coeff(p,x,1)\right)\cdot coeff(q,x,1)\cdot coeff(r,x,1)}{8}, \frac{\left(\Delta - coeff(Q,x,1)\cdot coeff(q,x,1)\right)\cdot coeff(r,x,1)\cdot coeff(p,x,1)}{8}, \frac{\left(\Delta - coeff(R,x,1)\cdot coeff(r,x,1)\right)\cdot coeff(p,x,1)\cdot coeff(q,x,1)}{8}, -\frac{\Delta}{8}\right]\right]$$

$DD :=$ (18)

$$\begin{bmatrix} \cdots & 0 & \cdots \\ \cdots & 0 & \cdots \\ \cdots & 0 & \cdots \\ \cdots p_{10}p_{21}q_{11}q_{21}r_{10}r_{20} + \frac{1}{8}p_{11}p_{20}q_{10}q_{20}r_{11}r_{21} - \frac{1}{8}p_{11}p_{20}q_{11}q_{21}r_{10}r_{20} - \frac{1}{8}p_{11}p_{21}q_{10}q_{2(} & \cdots \end{bmatrix}$$

$$V := \begin{bmatrix} p_{21}\cdot p_{10} - p_{11}\cdot p_{20} \\ q_{21}\cdot q_{10} - q_{11}\cdot q_{20} \\ r_{21}\cdot r_{10} - r_{11}\cdot r_{20} \\ \left(p_{21}\cdot p_{10} - p_{11}\cdot p_{20}\right)\cdot\left(q_{21}\cdot q_{10} - q_{11}\cdot q_{20}\right)\cdot\left(r_{21}\cdot r_{10} - r_{11}\cdot r_{20}\right) \end{bmatrix}$$

$$V := \begin{bmatrix} p_{10}p_{21} - p_{11}p_{20} \\ q_{10}q_{21} - q_{11}q_{20} \\ r_{10}r_{21} - r_{11}r_{20} \\ (p_{10}p_{21} - p_{11}p_{20})\,(q_{10}q_{21} - q_{11}q_{20})\,(r_{10}r_{21} - r_{11}r_{20}) \end{bmatrix} \tag{19}$$

$verify(T \bullet DD \bullet V, 0, equal)$

$$true \tag{20}$$