

$$p := p_2 \cdot x^2 + p_1 \cdot x + p_0$$

$$p := p_2 x^2 + p_1 x + p_0 \quad (1)$$

$$q := q_2 \cdot x^2 + q_1 \cdot x + q_0$$

$$q := q_2 x^2 + q_1 x + q_0 \quad (2)$$

$$r := r_2 \cdot x^2 + r_1 \cdot x + r_0$$

$$r := r_2 x^2 + r_1 x + r_0 \quad (3)$$

$$P := \text{collect}(q' \cdot r - q \cdot r', x)$$

$$P := (-q_1 r_2 + q_2 r_1) x^2 + (-2 q_0 r_2 + 2 q_2 r_0) x - q_0 r_1 + q_1 r_0 \quad (4)$$

$$Q := \text{collect}(r' \cdot p - r \cdot p', x)$$

$$Q := (r_2 p_1 - r_1 p_2) x^2 + (2 r_2 p_0 - 2 r_0 p_2) x + r_1 p_0 - r_0 p_1 \quad (5)$$

$$R := \text{collect}(p' \cdot q - p \cdot q', x)$$

$$R := (-p_1 q_2 + p_2 q_1) x^2 + (-2 p_0 q_2 + 2 p_2 q_0) x - p_0 q_1 + p_1 q_0 \quad (6)$$

$$M := \begin{bmatrix} \text{coeff}(p, x, 0) & \text{coeff}(q, x, 0) & \text{coeff}(r, x, 0) \\ \text{coeff}(p, x, 1) & \text{coeff}(q, x, 1) & \text{coeff}(r, x, 1) \\ \text{coeff}(p, x, 2) & \text{coeff}(q, x, 2) & \text{coeff}(r, x, 2) \end{bmatrix}$$

$$M := \begin{bmatrix} p_0 & q_0 & r_0 \\ p_1 & q_1 & r_1 \\ p_2 & q_2 & r_2 \end{bmatrix} \quad (7)$$

$$\Delta := |M|$$

$$\Delta := p_0 q_1 r_2 - p_0 q_2 r_1 - p_1 q_0 r_2 + p_1 q_2 r_0 + p_2 q_0 r_1 - p_2 q_1 r_0 \quad (8)$$

$$\begin{aligned} \mu := (j, k, l, m) & \quad \text{coeff}(P, x, j) \cdot \text{coeff}(p, x, k) \cdot \text{coeff}(p, x, l) \cdot \text{coeff}(q, x, m) \cdot \text{coeff}(r, x, m) + \text{coeff}(Q, x, j) \\ & \quad \cdot \text{coeff}(q, x, k) \cdot \text{coeff}(q, x, l) \cdot \text{coeff}(p, x, m) \cdot \text{coeff}(r, x, m) + \text{coeff}(R, x, j) \cdot \text{coeff}(r, x, k) \cdot \text{coeff}(r, x, l) \\ & \quad \cdot \text{coeff}(p, x, m) \cdot \text{coeff}(q, x, m) \end{aligned}$$

$$\begin{aligned} \mu := (j, k, l, m) & \quad \text{coeff}(P, x, j) \cdot \text{coeff}(p, x, k) \cdot \text{coeff}(p, x, l) \cdot \text{coeff}(q, x, m) \cdot \text{coeff}(r, x, m) + \text{coeff}(Q, x, j) \\ & \quad \cdot \text{coeff}(q, x, k) \cdot \text{coeff}(q, x, l) \cdot \text{coeff}(p, x, m) \cdot \text{coeff}(r, x, m) + \text{coeff}(R, x, j) \cdot \text{coeff}(r, x, k) \cdot \text{coeff}(r, x, l) \\ & \quad \cdot \text{coeff}(p, x, m) \cdot \text{coeff}(q, x, m) \end{aligned} \quad (9)$$

$$\Psi_0 := 4 \cdot \mu(0, 0, 0, 2) + \mu(2, 1, 1, 0) + \mu(2, 0, 0, 1)$$

$$\begin{aligned} \Psi_0 := & 4 \left(-q_0 r_1 + q_1 r_0 \right) p_0^2 q_2 r_2 + 4 \left(r_1 p_0 - r_0 p_1 \right) q_0^2 p_2 r_2 + 4 \left(-p_0 q_1 + p_1 q_0 \right) r_0^2 p_2 q_2 + \left(-q_1 r_2 \right. \\ & + q_2 r_1 \left. \right) p_1^2 q_0 r_0 + \left(p_1 r_2 - p_2 r_1 \right) q_1^2 p_0 r_0 + \left(-p_1 q_2 + p_2 q_1 \right) r_1^2 p_0 q_0 + \left(-q_1 r_2 + q_2 r_1 \right) p_0^2 q_1 r_1 \\ & + \left(p_1 r_2 - p_2 r_1 \right) q_0^2 p_1 r_1 + \left(-p_1 q_2 + p_2 q_1 \right) r_0^2 p_1 q_1 \end{aligned} \quad (10)$$

$$\Psi_1 := -\frac{\Delta}{2} \cdot p_1 \cdot q_1 \cdot r_1 - \mu(1, 0, 2, 1)$$

$$\Psi_1 := -\frac{(p_0 q_1 r_2 - p_0 q_2 r_1 - p_1 q_0 r_2 + p_1 q_2 r_0 + p_2 q_0 r_1 - p_2 q_1 r_0) p_1 q_1 r_1}{2} - (-2 q_0 r_2 \quad (11)$$

$$\begin{aligned}
& + 2q_2r_0)p_0p_2q_1r_1 - (2p_0r_2 - 2p_2r_0)q_0q_2p_1r_1 - (-2p_0q_2 + 2p_2q_0)r_0r_2p_1q_1 \\
\psi_2 := & -4 \cdot \mu(2, 2, 2, 0) - \mu(0, 1, 1, 2) - \mu(0, 2, 2, 1) \\
\psi_2 := & -4 \left((-q_1r_2 + q_2r_1)p_2^2q_0r_0 - 4(p_1r_2 - p_2r_1)q_2^2p_0r_0 - 4(-p_1q_2 + p_2q_1)r_2^2p_0q_0 - (-q_0r_1 \right. \quad (12) \\
& + q_1r_0)p_1^2q_2r_2 - (r_1p_0 - r_0p_1)q_1^2p_2r_2 - (-p_0q_1 + p_1q_0)r_1^2p_2q_2 - (-q_0r_1 + q_1r_0)p_2^2q_1r_1 \\
& \left. - (r_1p_0 - r_0p_1)q_2^2p_1r_1 - (-p_0q_1 + p_1q_0)r_2^2p_1q_1 \right)
\end{aligned}$$

$$C_{inv} := \begin{bmatrix} -\frac{p_0}{\Delta} & -\frac{p_1}{2 \cdot \Delta} & \frac{p_2}{\Delta} & 0 \\ -\frac{q_0}{\Delta} & -\frac{q_1}{2 \cdot \Delta} & \frac{q_2}{\Delta} & 0 \\ -\frac{r_0}{\Delta} & -\frac{r_1}{2 \cdot \Delta} & \frac{r_2}{\Delta} & 0 \\ \psi_0 & \psi_1 & \psi_2 & -16 \cdot \Delta \end{bmatrix}$$

$C_{inv} :=$ (13)

$$\begin{bmatrix} \dots \\ \dots \\ \dots \\ 4(-q_0r_1 + q_1r_0)p_0^2q_2r_2 + 4(r_1p_0 - r_0p_1)q_0^2p_2r_2 + 4(-p_0q_1 + p_1q_0)r_0^2p_2q_2 + (-q_1r_0 + q_0r_1)p_1^2q_2r_2 + 4(p_1r_2 - p_2r_1)q_2^2p_0r_0 + 4(-p_1q_2 + p_2q_1)r_2^2p_0q_0 - (-q_0r_1 + q_1r_0)p_2^2q_1r_1 - (r_1p_0 - r_0p_1)q_1^2p_2r_2 - (-p_0q_1 + p_1q_0)r_1^2p_2q_2 - (-q_0r_1 + q_1r_0)p_2^2q_1r_1 - (r_1p_0 - r_0p_1)q_2^2p_1r_1 - (-p_0q_1 + p_1q_0)r_2^2p_1q_1 \end{bmatrix}$$

$$\begin{aligned}
DD := & \left[\left[\text{coeff}(P, x, 2), \text{coeff}(Q, x, 2), \text{coeff}(R, x, 2), 0 \right], \right. \\
& \left[-\text{coeff}(P, x, 1), -\text{coeff}(Q, x, 1), -\text{coeff}(R, x, 1), 0 \right], \\
& \left[-\text{coeff}(P, x, 0), -\text{coeff}(Q, x, 0), -\text{coeff}(R, x, 0), 0 \right], \\
& \left[\frac{(\Delta - \text{coeff}(P, x, 1) \cdot \text{coeff}(p, x, 1)) \cdot \text{coeff}(q, x, 1) \cdot \text{coeff}(r, x, 1)}{8}, \right. \\
& \frac{(\Delta - \text{coeff}(Q, x, 1) \cdot \text{coeff}(q, x, 1)) \cdot \text{coeff}(r, x, 1) \cdot \text{coeff}(p, x, 1)}{8}, \\
& \left. \left. \frac{(\Delta - \text{coeff}(R, x, 1) \cdot \text{coeff}(r, x, 1)) \cdot \text{coeff}(p, x, 1) \cdot \text{coeff}(q, x, 1)}{8}, -\frac{\Delta}{8} \right] \right] \\
DD := & \quad \quad \quad (14)
\end{aligned}$$

$$\begin{aligned}
a &:= \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \\
a &:= \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}
\end{aligned}
\tag{15}$$

$$b := \text{Cinv} \cdot a :$$

$$\begin{aligned}
D_p &:= p_1^2 - 4 \cdot p_0 \cdot p_2 \\
D_p &:= -4 p_0 p_2 + p_1^2
\end{aligned}
\tag{16}$$

$$\begin{aligned}
D_q &:= q_1^2 - 4 \cdot q_0 \cdot q_2 \\
D_q &:= -4 q_0 q_2 + q_1^2
\end{aligned}
\tag{17}$$

$$\begin{aligned}
D_r &:= r_1^2 - 4 \cdot r_0 \cdot r_2 \\
D_r &:= -4 r_0 r_2 + r_1^2
\end{aligned}
\tag{18}$$

$$c := \begin{bmatrix} 2 \cdot \Delta^4 \cdot D_p \cdot b[2] \cdot b[3] + 2 \cdot \Delta^2 \cdot b[1] \cdot b[4] \\ 2 \cdot \Delta^4 \cdot D_q \cdot b[1] \cdot b[3] + 2 \cdot \Delta^2 \cdot b[2] \cdot b[4] \\ 2 \cdot \Delta^4 \cdot D_r \cdot b[1] \cdot b[2] + 2 \cdot \Delta^2 \cdot b[3] \cdot b[4] \\ \Delta^4 \cdot (D_q \cdot D_r \cdot b[1]^2 + D_p \cdot D_r \cdot b[2]^2 + D_p \cdot D_q \cdot b[3]^2) + b[4]^2 \end{bmatrix} :$$

$$A := \text{expand}(DD \cdot c) :$$

$$W := \text{collect}(A[4], a_4) :$$

$$\text{verify}(\text{coeff}(W, a_4, 2), -32 \cdot \Delta^3, \text{equal})$$

$$\text{true} \tag{19}$$

$$\text{verify}(\text{coeff}(W, a_4, 1), 4 \cdot ((\Delta^3 \cdot (p_0 \cdot q_1 \cdot r_1 + p_1 \cdot q_0 \cdot r_1 + p_1 \cdot q_1 \cdot r_0) + \Delta^2 \cdot \Psi_0) \cdot a_1 - \Delta^2 \cdot \mu(1, 0, 2, 1) \cdot a_2 + (-\Delta^3 \cdot (p_2 \cdot q_1 \cdot r_1 + p_1 \cdot q_2 \cdot r_1 + p_1 \cdot q_1 \cdot r_2) + \Delta^2 \cdot \Psi_2) \cdot a_3), \text{equal})$$

$$\text{true} \tag{20}$$