Assignment-2

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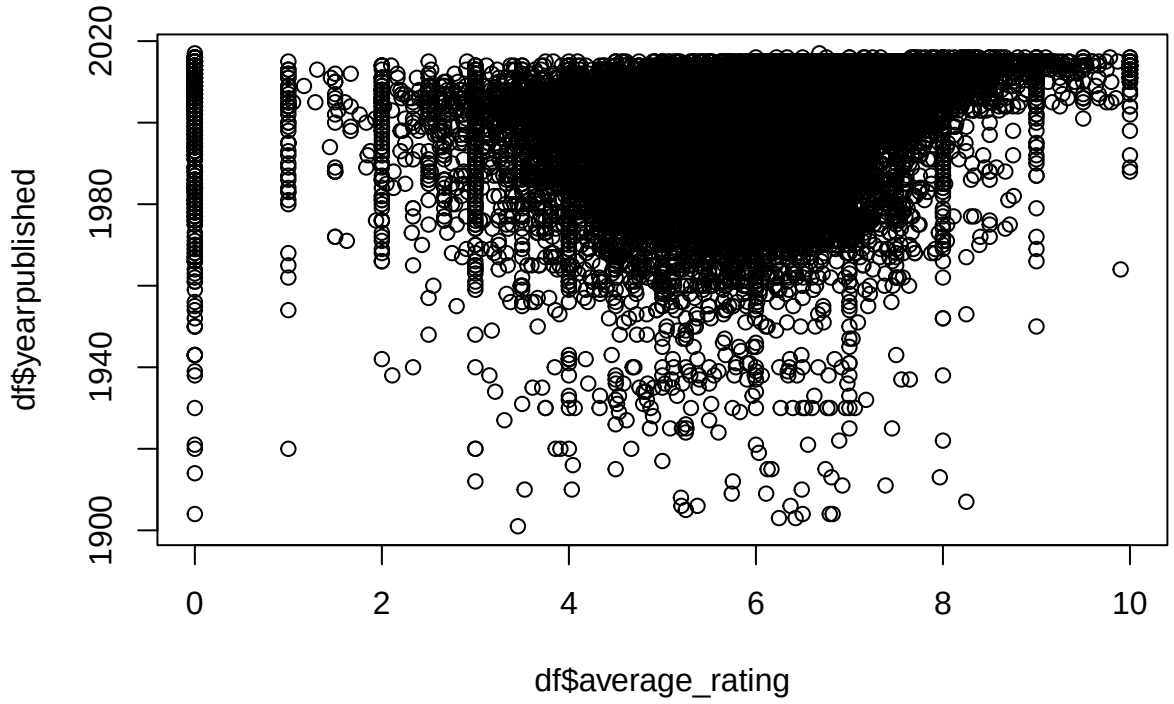
Contents

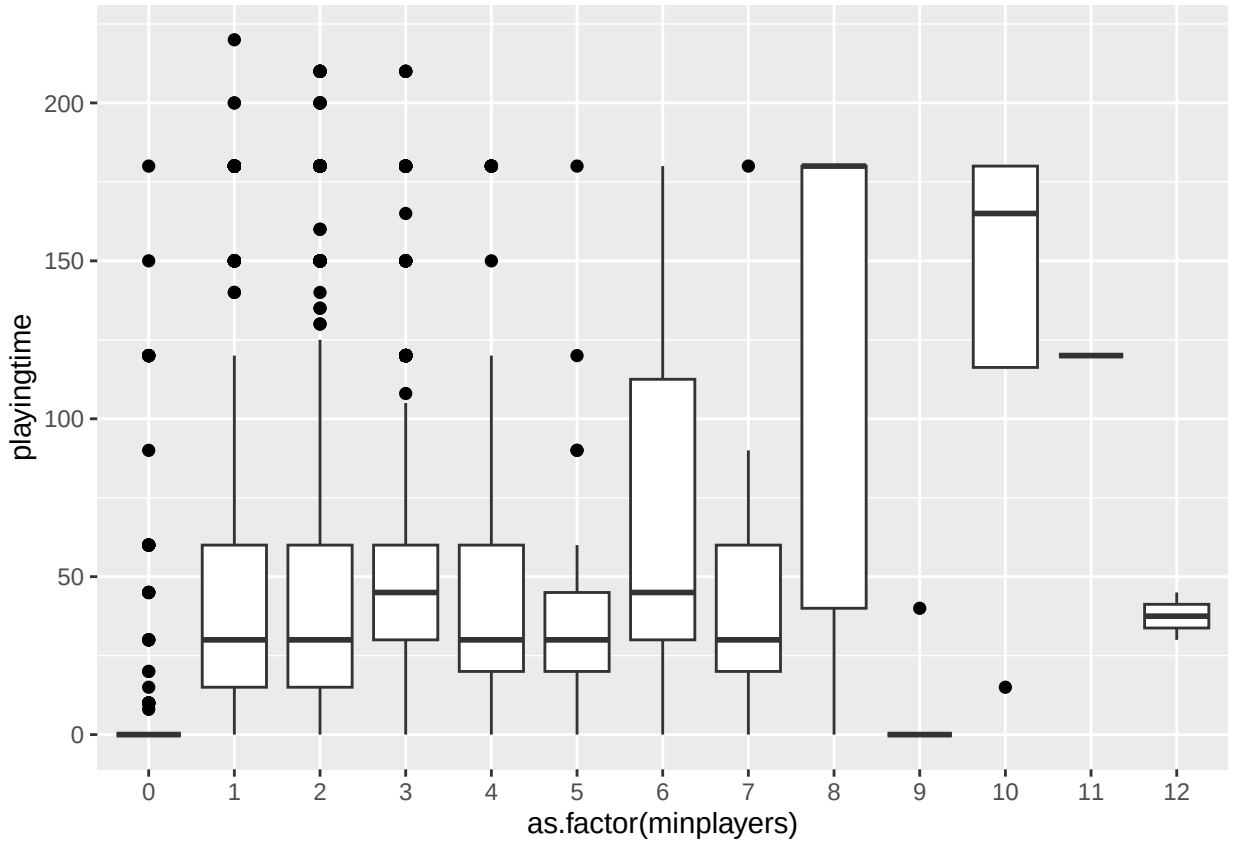
1 Excercise 1 - Risiko! is back, with friends

1.1 Point a)

```
df <- readRDS("~/Documents/GitHub/Computational-stat/Assignment 2/games_preprocessed.RDS")

plot(df$average_rating, df$yearpublished) # to see whether old board_games tend to be preferred more be
```




```
ggplot(data=df, aes(y=playingtime, x=as.factor(minplayers)))+ # As more player play are needed to start
  geom_boxplot(outlier.colour="black", outlier.shape=16,
               outlier.size=2, notch=FALSE)
```



Point b)

```
df= df[,-1:-2]
df=data.frame(scale(df))
```

```
sample_cov=cov(df)
cor(df)
```

```
##                yearpublished  minplayers  maxplayers playingtime     minage
## yearpublished     1.00000000  0.03585626  0.03112545 -0.06572597 0.09314918
## minplayers        0.03585626  1.00000000  0.19166864  0.04429612 0.09851689
## maxplayers        0.03112545  0.19166864  1.00000000 -0.01204945 0.05626623
## playingtime      -0.06572597  0.04429612 -0.01204945  1.00000000 0.28407403
## minage            0.09314918  0.09851689  0.05626623  0.28407403 1.00000000
## average_rating    0.21657330 -0.01705543 -0.02509443  0.18694736 0.16096705
## total_owners      0.04441719  0.02724071  0.01574204  0.13281225 0.15929911
## average_weight    0.04899808 -0.06353175 -0.09918355  0.43645196 0.21685937
##                average_rating total_owners average_weight
## yearpublished      0.21657330   0.04441719     0.04899808
## minplayers        -0.01705543   0.02724071    -0.06353175
## maxplayers        -0.02509443   0.01574204    -0.09918355
## playingtime        0.18694736   0.13281225     0.43645196
## minage             0.16096705   0.15929911     0.21685937
## average_rating     1.00000000   0.16819425     0.33090040
## total_owners       0.16819425   1.00000000     0.13106392
## average_weight     0.33090040   0.13106392     1.00000000
```

```
# We can get the same result using cov2cor
cov2cor(sample_cov)
```

```
##                yearpublished  minplayers  maxplayers playingtime     minage
## yearpublished     1.00000000  0.03585626  0.03112545 -0.06572597 0.09314918
## minplayers        0.03585626  1.00000000  0.19166864  0.04429612 0.09851689
## maxplayers        0.03112545  0.19166864  1.00000000 -0.01204945 0.05626623
## playingtime      -0.06572597  0.04429612 -0.01204945  1.00000000 0.28407403
## minage            0.09314918  0.09851689  0.05626623  0.28407403 1.00000000
## average_rating    0.21657330 -0.01705543 -0.02509443  0.18694736 0.16096705
## total_owners      0.04441719  0.02724071  0.01574204  0.13281225 0.15929911
## average_weight    0.04899808 -0.06353175 -0.09918355  0.43645196 0.21685937
##                average_rating total_owners average_weight
## yearpublished      0.21657330   0.04441719     0.04899808
## minplayers        -0.01705543   0.02724071    -0.06353175
## maxplayers        -0.02509443   0.01574204    -0.09918355
## playingtime        0.18694736   0.13281225     0.43645196
## minage             0.16096705   0.15929911     0.21685937
## average_rating     1.00000000   0.16819425     0.33090040
## total_owners       0.16819425   1.00000000     0.13106392
## average_weight     0.33090040   0.13106392     1.00000000
```

Since we have scaled all the columns with variance equal to 1, it implies that the covariance-variance matrix is equal to the correlation matrix. Fonte: https://math.stackexchange.com/questions/3780344/under-what-conditions-will-the-covariance-matrix-be-identical-to-the-correlation

1.2 Point c)

```
B=10

cov_var_sample=array(NA, dim=c(ncol(df),ncol(df),B))

for(i in 1:B){
  cov_var_sample[,,i]=cov(slice_sample(df,n=nrow(df), replace=TRUE))
}

theta_hat_bar=apply(cov_var_sample, c(1, 2), mean)
se_cov_var=apply(cov_var_sample, c(1, 2), sd)
```

Confidence intervals 1. A first method consist on using the fact that, as ths sample size n grows large $\frac{\hat{\theta}-\theta}{SE_B(\hat{\theta})} \approx N(0,1)$

```
lower_bound=sample_cov+qnorm(c(0.025), mean=0, sd=1)*se_cov_var
upper_bound=sample_cov+qnorm(c(0.975), mean=0, sd=1)*se_cov_var
```

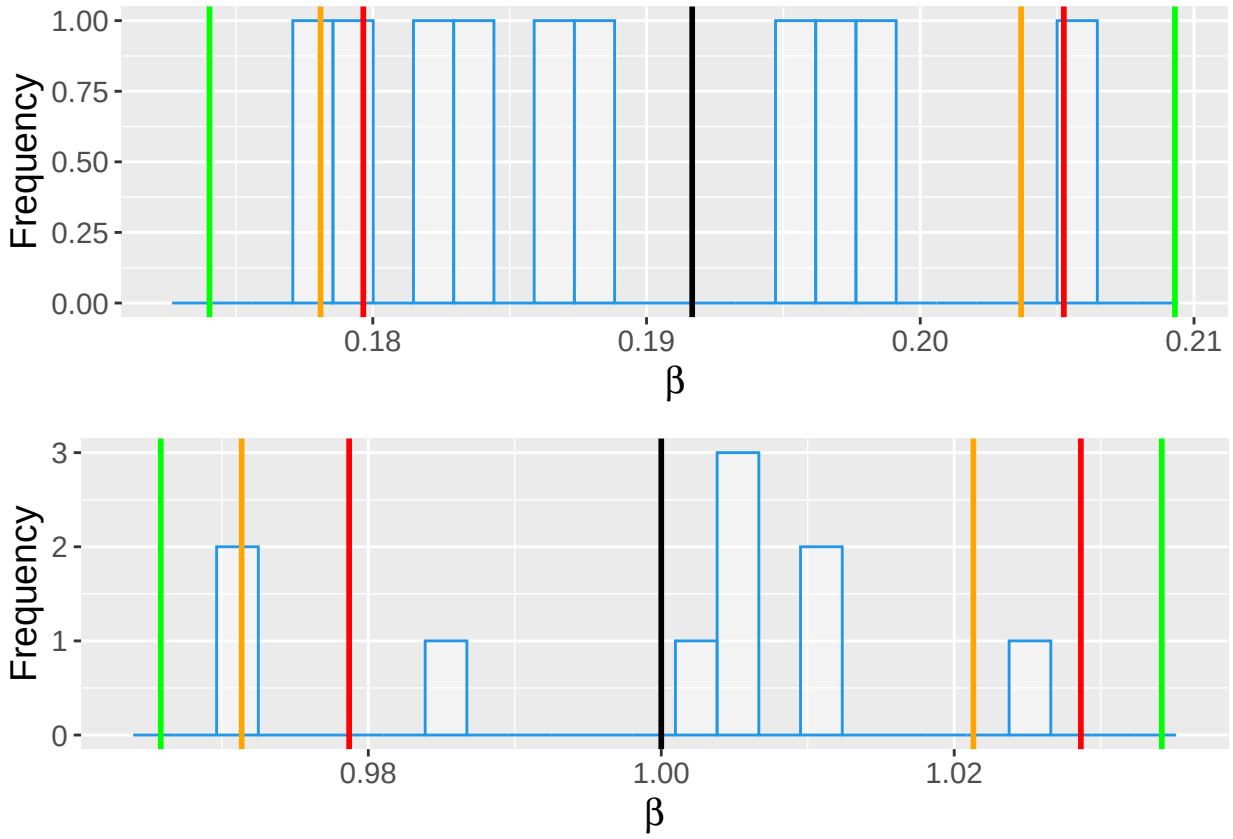
2. Use the quantile of the bootstrap sampling distribution, this method is considered more robust than the one before

```
boot_sample_variance <- data.frame(matrix(nrow = 6, ncol = 6))
colnames(boot_sample_variance) <- c("maxplayers", "playingtime", "minage", "average_rating", "total_owne

for(i in 1:6){
  for(j in 1:6){
    boot_sample_variance[i,j]=list(list(cov_var_sample[i,j,]))
  }
}
boot_sample_variance_ci <- data.frame(matrix(nrow = 6, ncol = 6))
colnames(boot_sample_variance_ci) <- c("maxplayers", "playingtime", "minage", "average_rating", "total_

for(i in 1:6){
  for(j in 1:6){
    boot_sample_variance_ci[i,j]=list(list(quantile(boot_sample_variance[i,j][[1]], probs=c(0.025,0.975)
  }
}
```

3. Using as assumption the fact that the behavior of the bias $\theta - \hat{\theta}$ is the same as $\hat{\theta} - \hat{\theta}^*$, we can compute the confidence interval of the bias-corrected estimate

```
boot_sample_variance_ci_bias <- data.frame(matrix(nrow = 6, ncol = 6))
colnames(boot_sample_variance_ci) <- c("maxplayers", "playingtime", "minage", "average_rating", "total_

for(i in 1:6){
  for(j in 1:6){
    boot_sample_variance_ci_bias[i,j]=list(list(2*sample_cov[i,j]-quantile(boot_sample_variance[i,j][[1
  }
}
```

```
plot_boot_sample_ci = function(row, column){
  ggplot()+
  geom_histogram(aes(x=boot_sample_variance[row,column][[1]]),alpha=.4,col=4,fill="white",bins=25)+
  geom_vline(aes(xintercept = sample_cov[row,column]) ,col=1, lwd = 1)+
  geom_vline(aes(xintercept = boot_sample_variance_ci[row, column][[1]][1]) ,col="orange", lwd = 1)+
  geom_vline(aes(xintercept = boot_sample_variance_ci[row, column][[1]][2]) ,col="orange", lwd = 1)+
  geom_vline(aes(xintercept = boot_sample_variance_ci_bias[row, column][[1]][1]) ,col="red", lwd = 1)+
  geom_vline(aes(xintercept = boot_sample_variance_ci_bias[row, column][[1]][2]) ,col="red", lwd = 1)+
  geom_vline(aes(xintercept = lower_bound[row, column]) ,col="green", lwd = 1)+
  geom_vline(aes(xintercept = upper_bound[row, column]) ,col="green", lwd = 1)+
  scale_linetype_manual( guide = guide_legend())+
  xlab(TeX("$\\beta$"))+
  ylab("Frequency") +
  theme(text = element_text(size=14), legend.position = "none")

}

 grid.arrange(plot_boot_sample_ci(2,3),plot_boot_sample_ci(1,1))
```

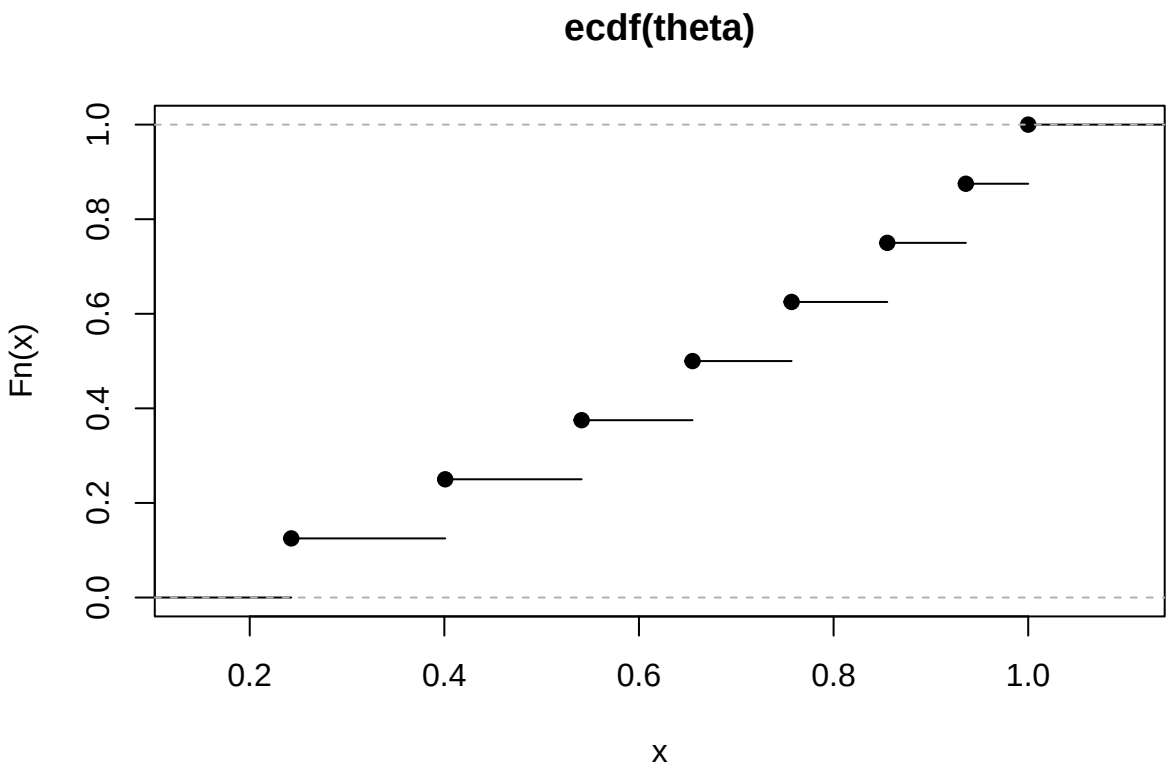


1.3 Point d)

```
#We know that these two values are equal
sum(eigen(sample_cov)$values) & sum(diag(sample_cov))
```

```
## [1] TRUE
```

```r
j_star=6
theta_j_star=sum(eigen(sample_cov)$values[1:j_star])/sum(eigen(sample_cov)$values)

theta=numeric(ncol(df))
for( i in 1:length(theta)){
  theta[i]=sum(eigen(sample_cov)$values[1:i]/sum(eigen(sample_cov)$values))
}

plot(ecdf(theta))
```



1.4 Point e)

```r
sample(theta, replace=TRUE)
```

```
## [1] 1.0000000 0.2426729 0.5410761 1.0000000 0.5410761 0.7568552 0.5410761
## [8] 0.8551548
```

```r
boot_eigen = function(){

}
```

1.5 Point f)

Make things more feasible

```
set.seed(abs(636-555-3226))
ind <- sample(1:nrow(df),5000,FALSE)
sub_data <- df[ind,]
```

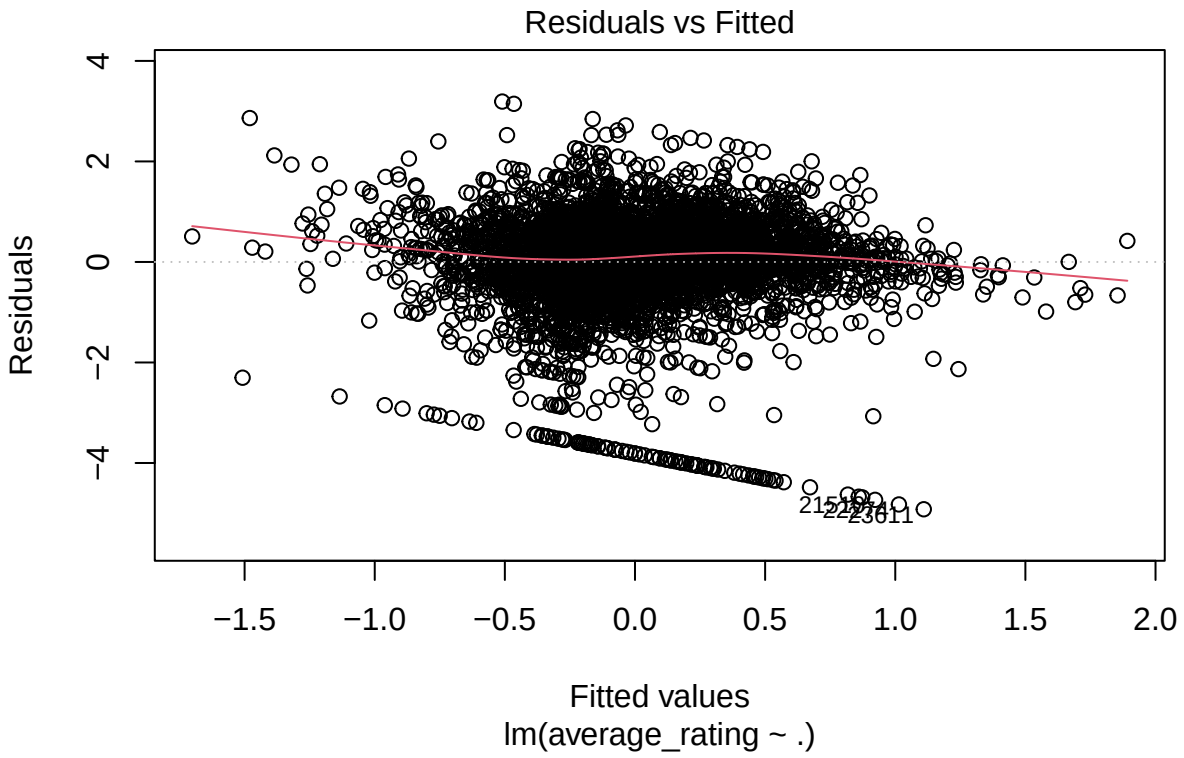
```
lm_game=lm(data=sub_data, average_rating ~ .)

res_sub_data=matrix(NA, 1, ncol(sub_data)+2)
res_sub_data[1,]=c(lm_game$coefficients, summary(lm_game)$r.squared,
                max((lm_game$coef["playingtime"]-lm_game$coef["minage"])/(lm_game$coef["minplayers"]+lm_

colnames(res_sub_data)=c(colnames(sub_data), "R2" , "theta_hat")
res_sub_data[,"yearpublished"]-1
```

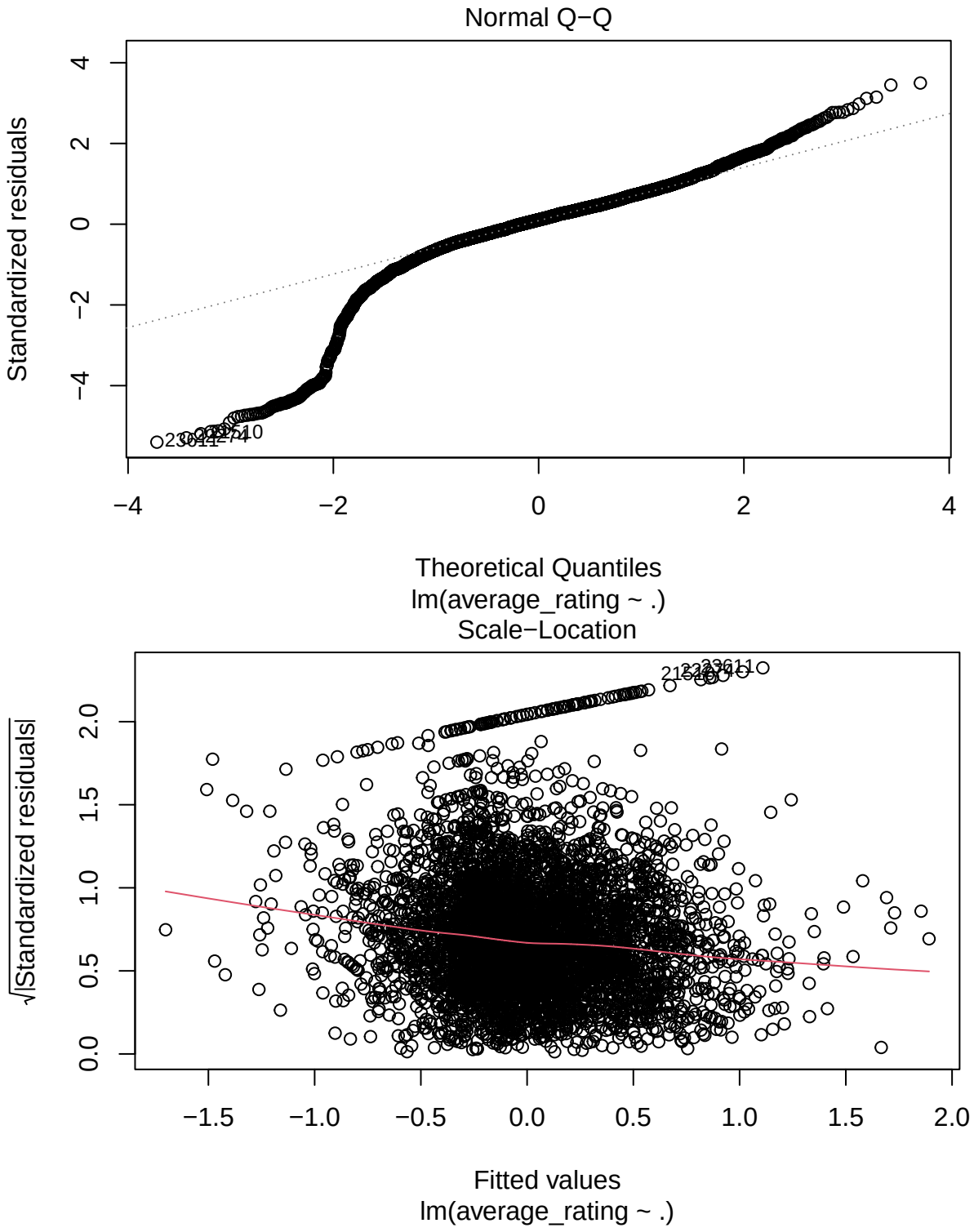
```
## yearpublished
##    -0.9916079
```

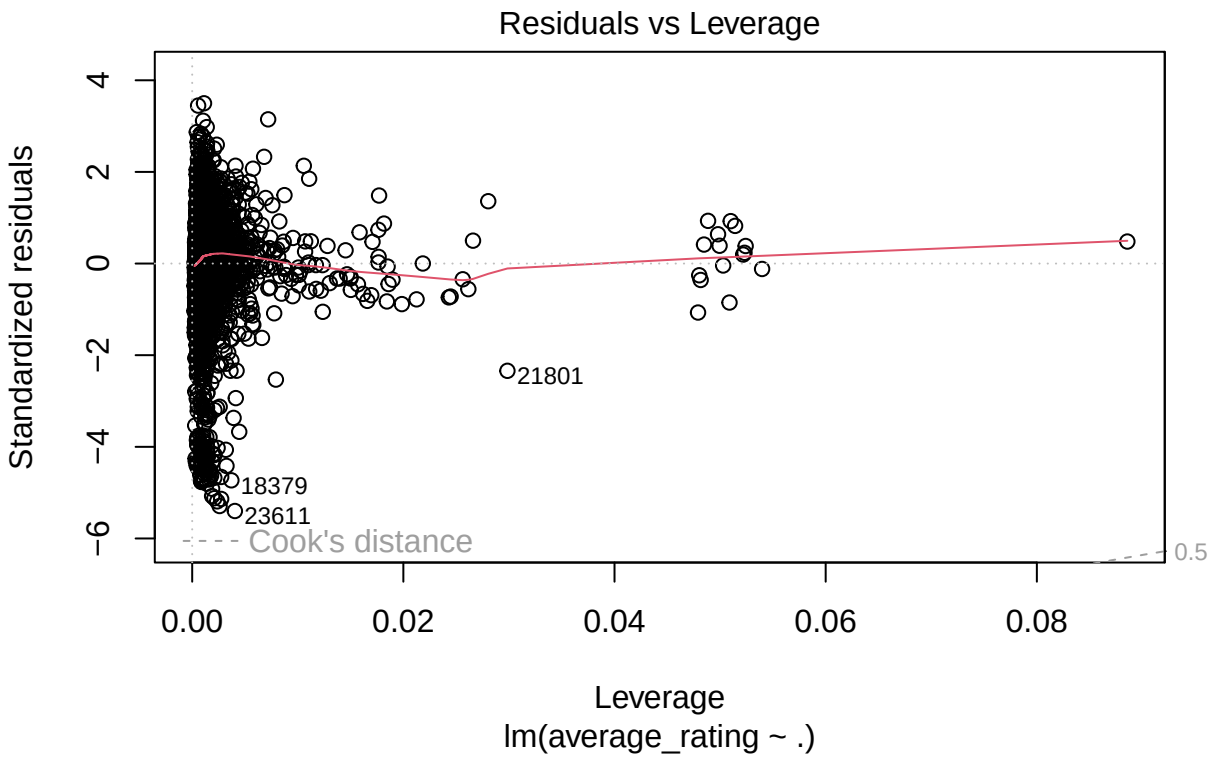
COMMENT

1.6 Point g)

```
plot(lm_game)
```




Normal Q−Q
Standardized residuals
Theoretical Quantiles
lm(average_rating ~ .)
Scale−Location
√|Standardized residuals|
Fitted values
lm(average_rating ~ .)



We can notice some features from this plots, useful for the choice of which bootstrap would be a sensitive choice. First of all the assumption of constant variance is not realistic in this case. Also the assumption of the errors distributed as a normal has problems, since we have a long tailed distribution. From this quick analysis of the model we can choose the bootstrap method more suited and explain why the others are not. 1. shuffle the errors -> this method relies on assumption that the chosen model fits the data well and the residuals have constant variance 2. Parametric bootstrap for logistic regression ->we are assuming that we know the distribution of the population which we don't 3. Resample the line of the two datasets -> this approach violates the assumption of constant (fixed) design matrix. however this applies perfectly to our case

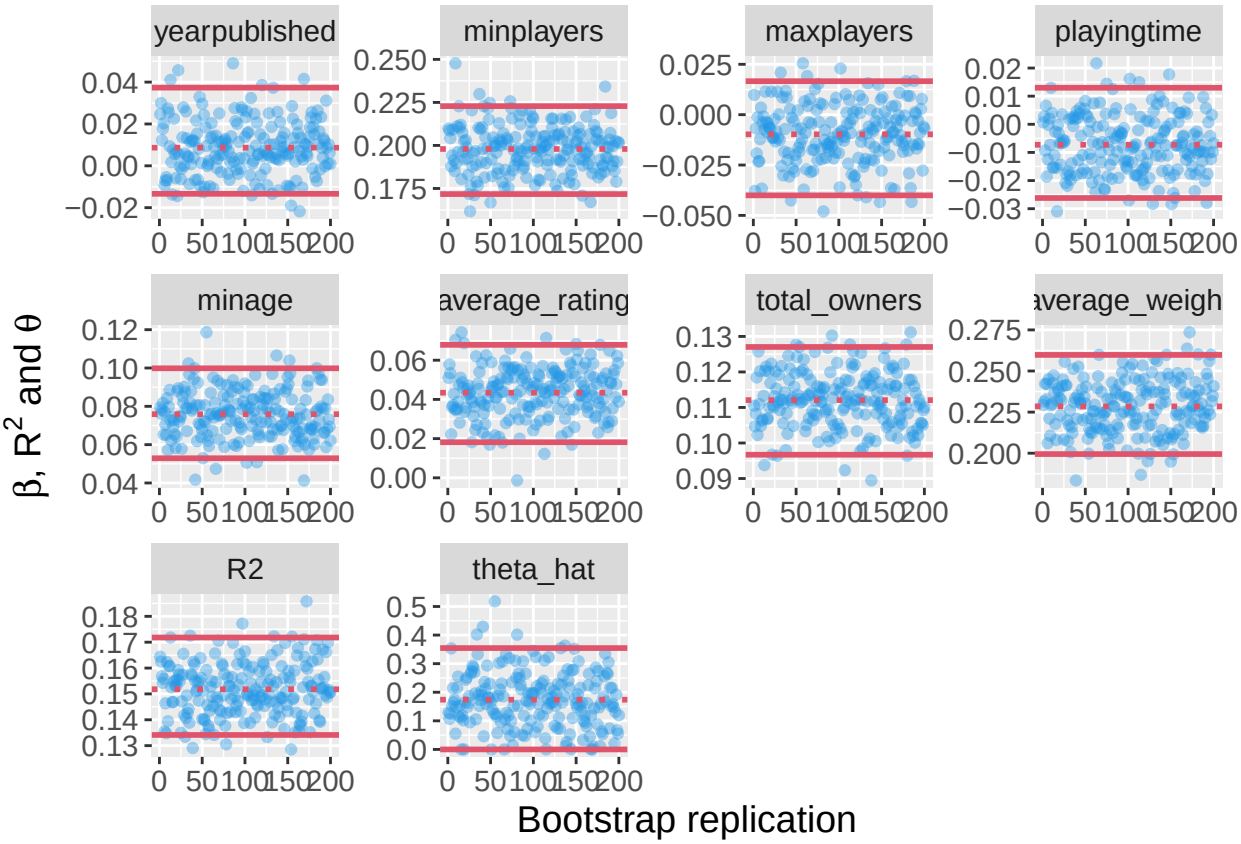
```
boot_reg_game <- function(data, B=200){

  n0 <- nrow(data)

  res <- replicate(B,{
    ind   <- sample(1:n0, n0, TRUE)
    Dstar <- data[ind,]
    Mstar <- lm(data=Dstar, average_rating ~ .)

    c(Mstar$coef, summary(Mstar)$r.squared, max((Mstar$coef["playingtime"]-Mstar$coef["minage"])/(Mstar$
  })

  t(res)
}

res_game_boot=boot_reg_game(data=sub_data)
colnames(res_game_boot)=c(colnames(sub_data), "R2" , "theta_hat")
```

Compute the estimates

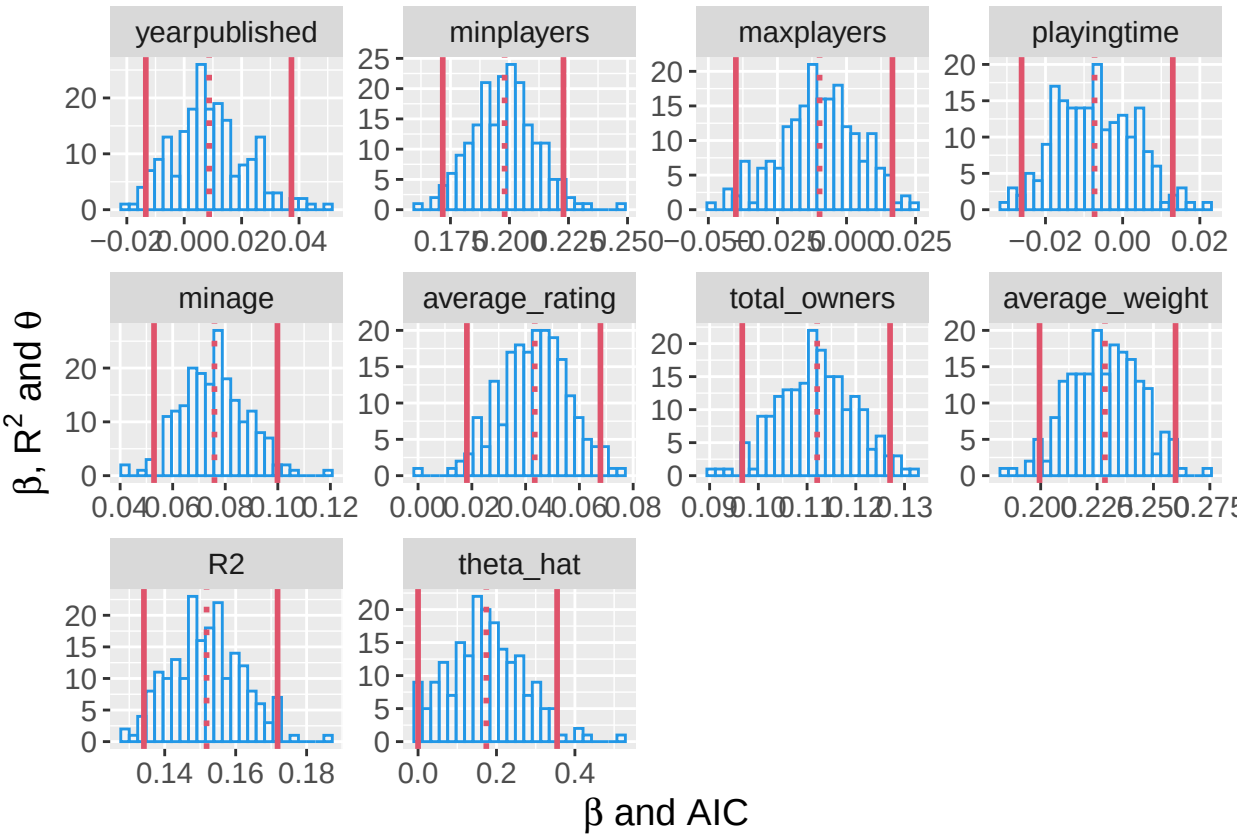
```
melted <- reshape2::melt(res_game_boot)
melted <- melted %>% group_by(Var2) %>% mutate( boot_sd = sd(value),
                                                boot_lowerbound = quantile(value, .025), boot_upperbound = quan
                                                boot_mean = mean(value), bias=boot_mean-res_sub_data[,as.chara
```

Plot the results

```
ggplot( melted )+
  geom_point(aes(x=Var1,y=value),alpha=.4,col=4)+
  geom_hline(aes(yintercept = boot_upperbound) ,col=2, lwd = 1)+
  geom_hline(aes(yintercept = boot_lowerbound) ,col=2, lwd = 1)+
  geom_hline(aes(yintercept = boot_mean) ,col=2, lwd = 1, lty=3)+
  facet_wrap(~Var2,scales = "free")+
  xlab("Bootstrap replication")+
  ylab(TeX("$\\beta$, $R^2$ and \\theta")) +
  theme(text = element_text(size=14), legend.position = "none")
```




```
ggplot( melted )+
  geom_histogram(aes(x=value),alpha=.4,col=4,fill="white",bins=25)+
  geom_vline(aes(xintercept = boot_upperbound) ,col=2, lwd = 1)+
  geom_vline(aes(xintercept = boot_lowerbound) ,col=2, lwd = 1)+
  geom_vline(aes(xintercept = boot_mean) ,col=2, lwd = 1, lty=3)+
  facet_wrap(~Var2,scales = "free")+
  xlab(TeX("$\\beta$ and AIC"))+
  ylab(TeX("$\\beta$, $R^2$ and \\theta")) +
  theme(text = element_text(size=14), legend.position = "none")
```



1.7 Point h)

Jackknife approach

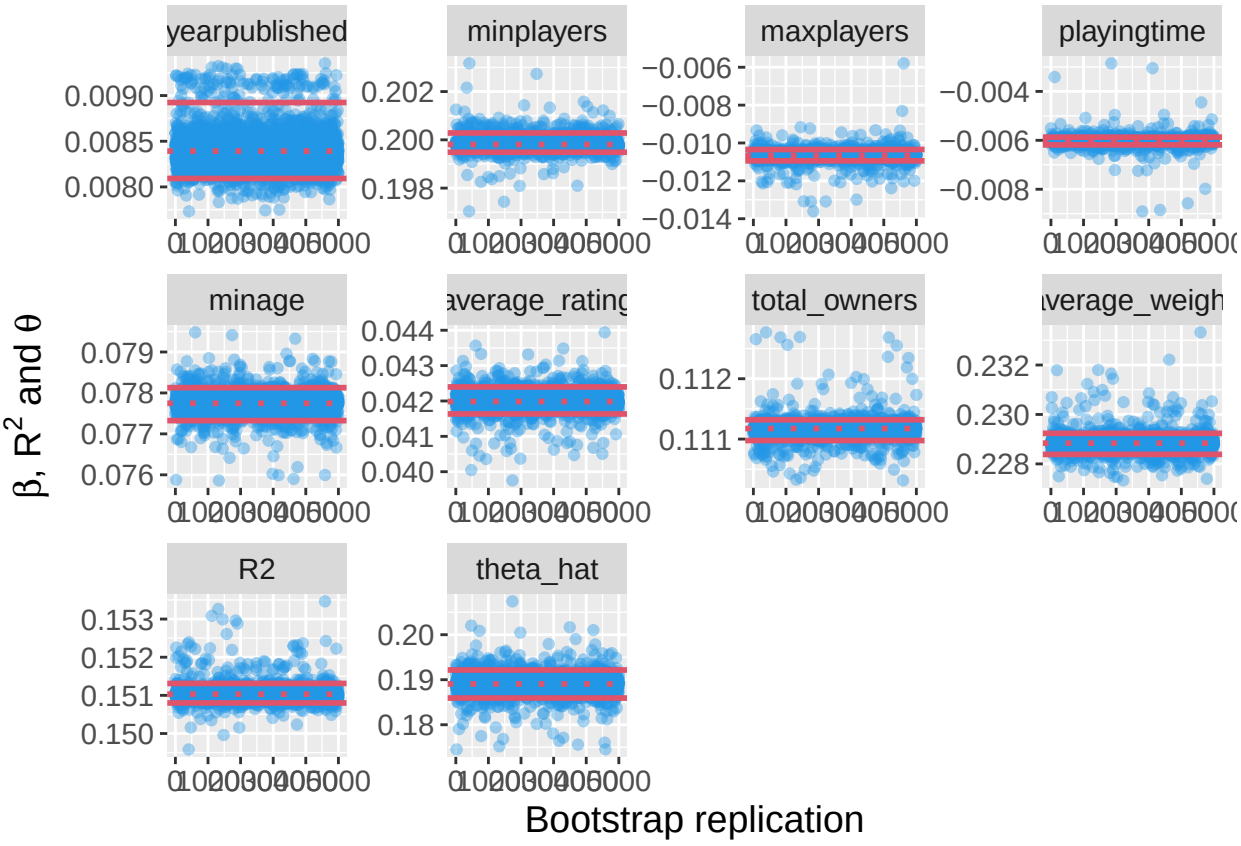
```
jack_reg_game = function(data, B){

  n0 <- nrow(data)

  res=matrix(NA, n0, ncol(data)+2)
for(i in 1:n0){
  Mjack=lm(data=data[-i,], average_rating ~ .)
  res[i,]=c(Mjack$coef, summary(Mjack)$r.squared, max((Mjack$coef["playingtime"]-Mjack$coef["minage"])/
}



  return(res)
}

res_game_jack=jack_reg_game(sub_data, 10)
colnames(res_game_jack)=c(colnames(sub_data), "R2" , "theta_hat")
```

Compute the jackknife estimate, bias-corrected estimate and standard error

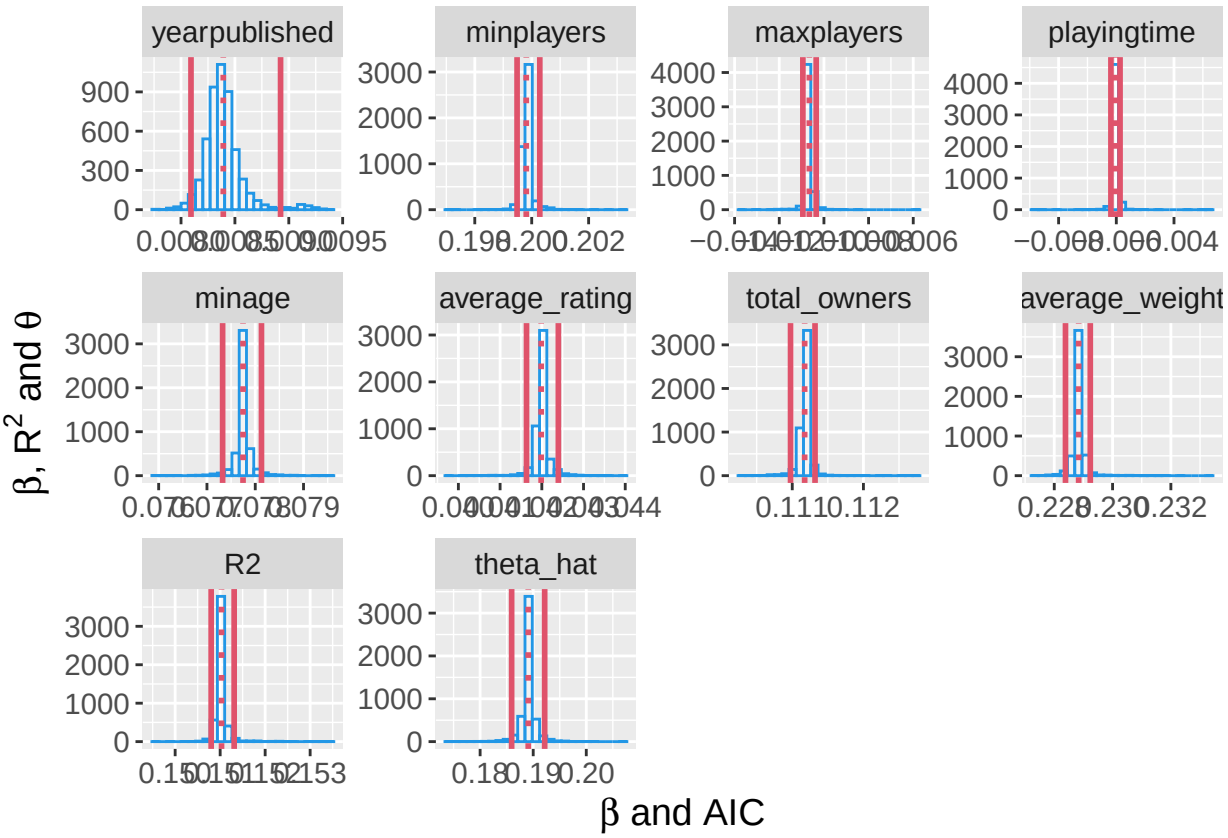
```
melted <- reshape2::melt(res_game_jack)
melted <- melted %>% group_by(Var2) %>% mutate( boot_sd = sd(value),
                                                boot_lowerbound = quantile(value, .025), boot_upperbound = quar
                                                boot_mean = mean(value), bias=boot_mean-res_sub_data[,as.chara
```

Plot the results

```
ggplot( melted )+
  geom_point(aes(x=Var1,y=value),alpha=.4,col=4)+
  geom_hline(aes(yintercept = boot_upperbound) ,col=2, lwd = 1)+
  geom_hline(aes(yintercept = boot_lowerbound) ,col=2, lwd = 1)+
  geom_hline(aes(yintercept = boot_mean) ,col=2, lwd = 1, lty=3)+
  facet_wrap(~Var2,scales = "free")+
  xlab("Bootstrap replication")+
  ylab(TeX("$\\beta$, $R^2$ and \\theta")) +
  theme(text = element_text(size=14), legend.position = "none")
```




```
ggplot( melted )+
  geom_histogram(aes(x=value),alpha=.4,col=4,fill="white",bins=25)+
  geom_vline(aes(xintercept = boot_upperbound) ,col=2, lwd = 1)+
  geom_vline(aes(xintercept = boot_lowerbound) ,col=2, lwd = 1)+
  geom_vline(aes(xintercept = boot_mean) ,col=2, lwd = 1, lty=3)+
  facet_wrap(~Var2,scales = "free")+
  xlab(TeX("$\\beta$ and AIC"))+
  ylab(TeX("$\\beta$, $R^2$ and \\theta")) +
  theme(text = element_text(size=14), legend.position = "none")
```



2 Exercise 2 - We need some music!

2.1 Point a)

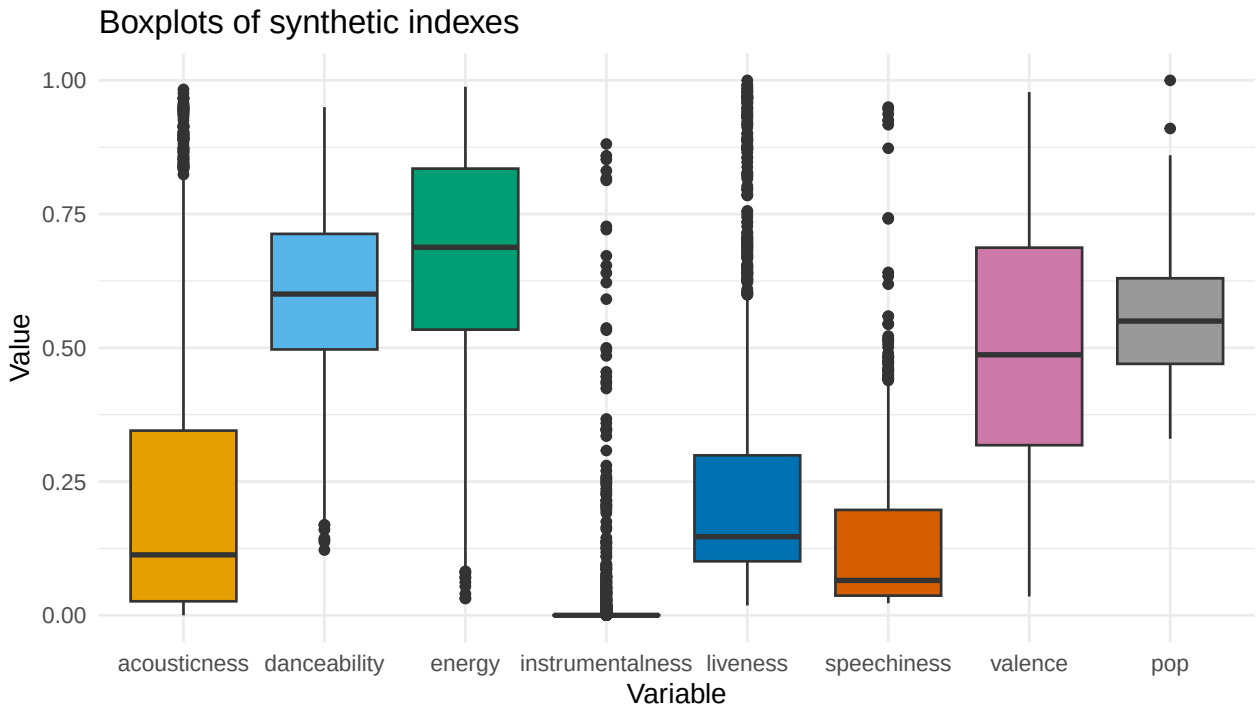
The dataset we're working with is a nested from a bigger version comprehending 2072 observations for 18 variables providing data of a Spotify tracks sample. Data are provided by the Spotify for Developers API (source: https://developer.spotify.com/documentation/web-api/reference/get-audio-features).

	acousticness	danceability	energy	instrumentalness	liveness	speechiness
	Min. :0.0000063	Min. :0.1220	Min. :0.0316	Min. :0.0000000	Min. :0.0184	Min. :0.02250
	1st Qu.:0.0262000	1st Qu.:0.4970	1st Qu.:0.5340	1st Qu.:0.0000000	1st Qu.:0.1010	1st Qu.:0.03678
	Median :0.1130000	Median :0.6005	Median :0.6880	Median :0.0000013	Median :0.1470	Median :0.06520
	Mean :0.2224369	Mean :0.5968	Mean :0.6678	Mean :0.0141941	Mean :0.2267	Mean :0.12895
	3rd Qu.:0.3452500	3rd Qu.:0.7130	3rd Qu.:0.8350	3rd Qu.:0.0001810	3rd Qu.:0.2990	3rd Qu.:0.19700
	Max. :0.9830000	Max. :0.9500	Max. :0.9880	Max. :0.8810000	Max. :1.0000	Max. :0.95000

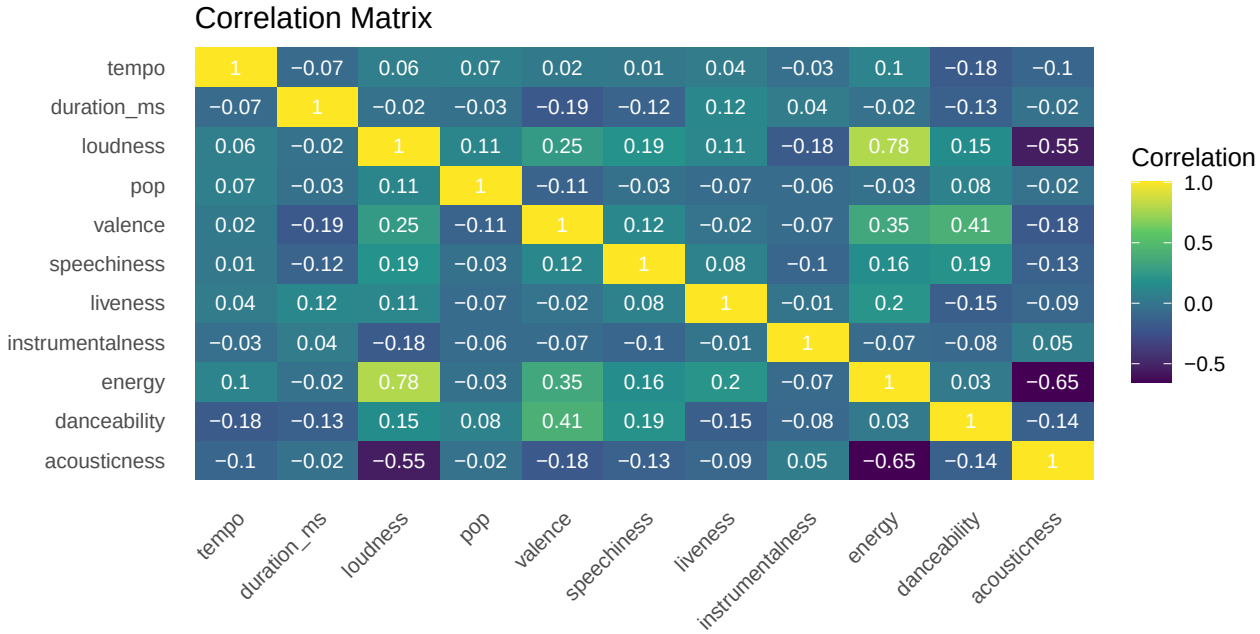
	valence	pop	loudness	duration_ms	tempo	n
	Min. :0.0352	Min. :0.3300	Min. :-24.383	Min. : 31360	Min. : 46.17	Min. : 11.00
	1st Qu.:0.3180	1st Qu.:0.4700	1st Qu.: -8.141	1st Qu.:200723	1st Qu.: 95.28	1st Qu.: 44.00
	Median :0.4870	Median :0.5500	Median : -6.285	Median :234886	Median :117.96	Median :102.00
	Mean :0.5019	Mean :0.5545	Mean : -6.798	Mean :241978	Mean :121.07	Mean : 88.41
	3rd Qu.:0.6873	3rd Qu.:0.6300	3rd Qu.: -4.773	3rd Qu.:273056	3rd Qu.:141.91	3rd Qu.:126.00
	Max. :0.9780	Max. :1.0000	Max. : -0.564	Max. :768640	Max. :220.07	Max. :142.00

Note that factors and characters are excluded from the summary. The first 8 variables are all synthetic float indexes from 0 to 1 describing one feature of the track, sometime based on musical features like path,

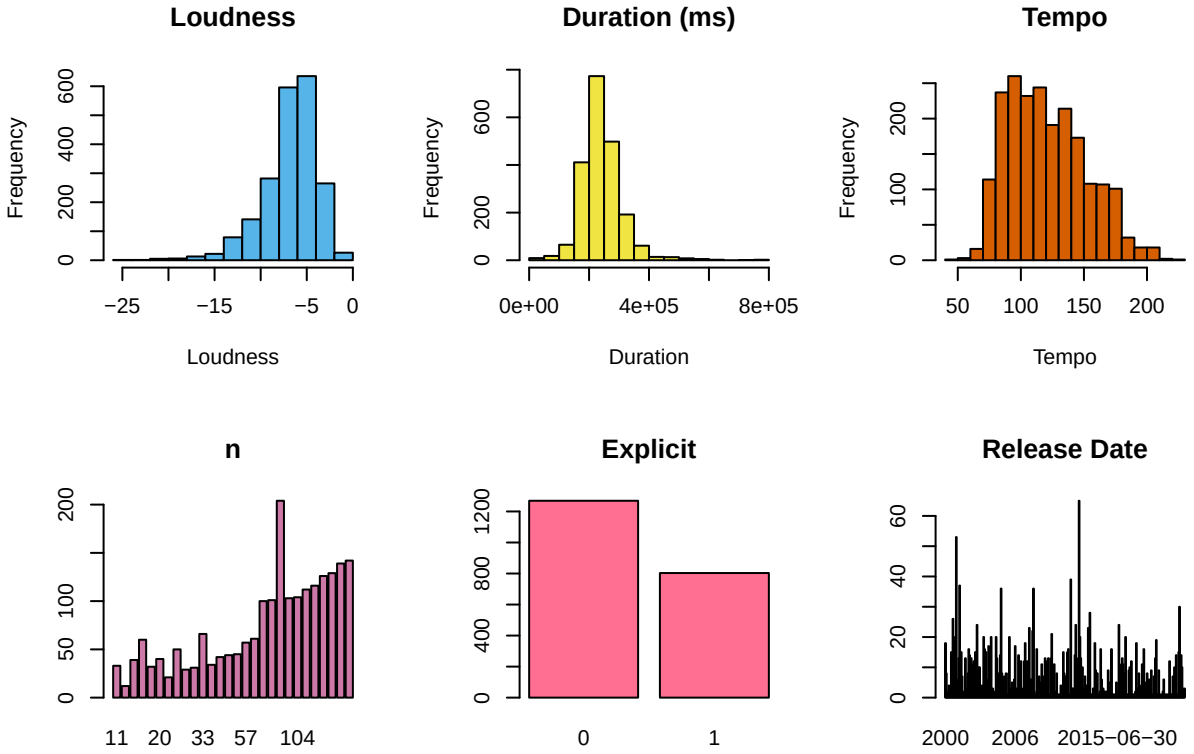
tempo, beat, stability, strenght, regularity, and sometime based on elements which may come out during the analysis (background noise for liveness, sound for acousticness). These ones measure for example danceability or energy as well as liveness or acousticness. Then there are three numerical variables which are measures of duration, loudness and the tempo of the song and a factor which tells if the track is explicit (0) or not (1). Four qualitative variables then tell us the original author, the artist who actually published the track, and an ID of the track itself. Variable n is eventually the numer of songs by the same artist in this dataset.



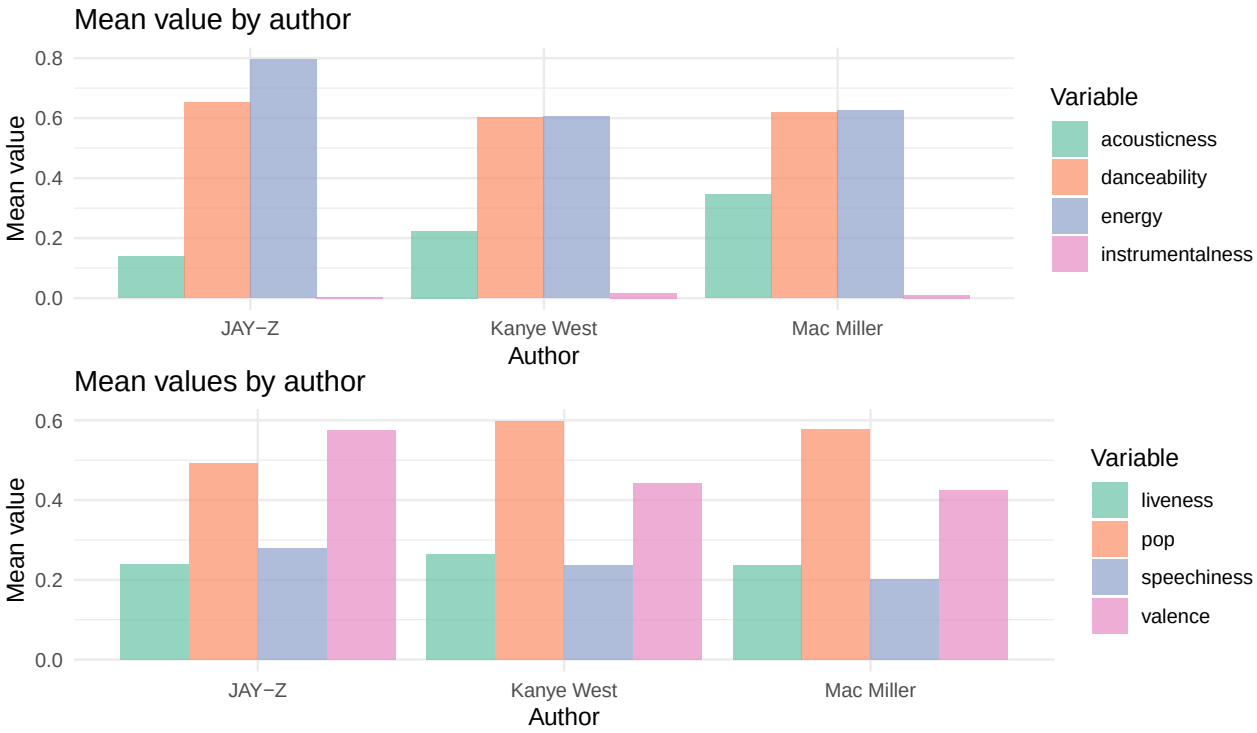
From these boxplots we can have an idea of the distributions of each feature among the tracks. For example we can say that most songs are likely to be non live, acoustic or spoken. Danceability, energy and popularity are distributed a little above 0.5 and the first two present a fatter left tail and viceversa for the third. The variable valence seems to be the most symmetric around the middle value, with equally weighted tails.



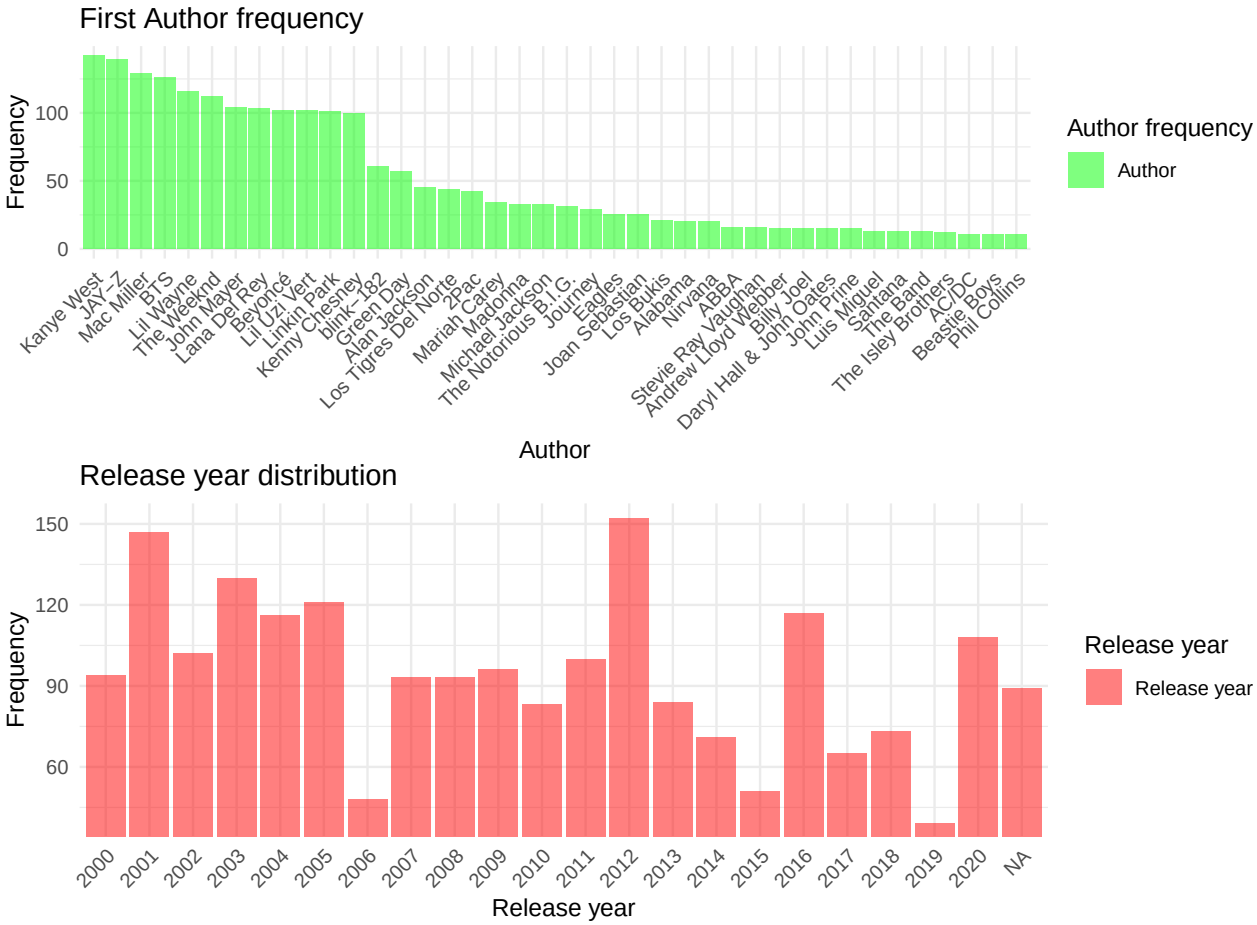
The correlation matrix plotted on a heatmap shows that most variables are almost uncorrelated with each other. The variable loudness is strongly positive related with energy and negative related with acousticness, which perfectly makes sense as well as the fact that energy and acousticness are negative related. Another reasonable result is that valence (negativity or positivity of the song) has some correlation with danceability and energy.



The histograms show the distribution for numerical variables.



Histograms above show the mean value of our indexes for the top three artists (based on number of songs in the data) to have a qualitative idea of at least the three authors that contributed the most to the data.



We plotted the frequencies of authors in the dataset and frequencies of release per year, to have an idea of how the two most relevant qualitative variables are distributed, aggregating tracks among their author and over the years of release.

2.2 Point b)

2.3 Point c)

2.4 Point d)

2.5 Point e)

2.6 Point f)

2.7 Point g)