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# Fluid flow measurements by means of vibration monitoring

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## Abstract

The achievement of accurate fluid flow measurements is fundamental whenever the control and the monitoring of certain physical quantities governing an industrial process are required. In that case, non-intrusive devices are preferable, but these are often more sophisticated and expensive than those which are more common (such as nozzles, diaphragms, Coriolis flowmeters and so on). In this paper, a novel, non-intrusive, simple and inexpensive methodology is presented to measure the fluid flow rate (in a turbulent regime) whose physical principle is based on the acquisition of transversal vibrational signals induced by the fluid itself onto the pipe walls it is flowing through. Such a principle of operation would permit the use of micro-accelerometers capable of acquiring and transmitting the signals, even by means of wireless technology, to a control room for the monitoring of the process under control. A possible application (whose feasibility will be investigated by the authors in a further study) of this introduced technology is related to the employment of a net of micro-accelerometers to be installed on pipeline networks of aqueducts. This apparatus could lead to the faster and easier detection and location of possible leaks of fluid affecting the pipeline network with more affordable costs. The authors, who have previously proven the linear dependency of the acceleration harmonics amplitude on the flow rate, here discuss an experimental analysis of this functional relation with the variation in the physical properties of the pipe in terms of its diameter and constituent material, to find the eventual limits to the practical application of the measurement methodology.

Keywords: flowmeter, acceleration measurement, micro-accelerometer, signal processing, laser Doppler vibrometer

(Some figures may appear in colour only in the online journal)

## 1. Introduction

Fluid flow measurements are extensively used in industrial and civil fields [1]. An accurate and precise flow rate measurement is crucial for the proper characterisation of the physical parameters involved in certain processes. In addition, the achievement of accurate fluid flow measurements is fundamental whenever the control and the monitoring of certain physical quantities governing an industrial process are required [2]. The technical and scientific literature provides descriptions of many examples of fluid flow meters, which are

based on different physical properties, [1]. The most important characteristic of a given flow meter, which enables the user to define its suitability for usage in a certain context, is the physical effect which it is based on. Depending on this physical principle and technology, it is possible to state its adequacy for a given application.

Furthermore, the operating conditions (such as flow temperature, pressure, average flow speed, turbulence conditions and so on) influence the choice of the most appropriate flow rate meter device. Recent developments in sensing technology have introduced many sensors that are able to

guarantee high performance for both static and dynamic measurements, with enclosed signal processing units that are able to perform post-processing computations on the acquired digitalised signals [3]–[5]. Although the recently introduced sensors are characterised by high performance in terms of accuracy, precision and dynamic response, there is the need for non-intrusive devices which show a relative insensitivity to the flow type (in particular to its magnetic properties) and do not require their direct installation along the pipe under analysis. Coriolis, ultrasonic and magnetic flow meters (which represent the most considered choices in industrial contexts) could suffer from inaccuracies due to flow electric and magnetic properties which can prejudice the measurements [6]. In addition, flow meters based on the Coriolis effect and ultrasonic devices are sensitive to the installation modalities, [7, 8].

The inaccurate or ineffective installation of these devices can lead to erroneous measurements. Therefore, the need for more accurate measurements and devices whose performance are uninfluenced by the physical properties of the fluid under investigation and by installation modalities, has encouraged some efforts aimed at introducing non-intrusive, low-cost and more reliable instrumentation. On the basis of the studies reported in [5, 9–11], where it is proven that it is possible to estimate the fluid flow rate by means of the measurement of flexural vibrations affecting pipelines, the authors experimentally prove the existence of a linear relation between the fluid flow rate and the amplitude of the frequency spectra of the vibration signals acquired on the pipes. This relation persists as long as the flow is turbulent and it varies as the pipe material and/or its diameter change (although the frequency spectrum remains unperturbed).

This paper further develops this relation by taking into consideration pipe material and size. By means of a detailed, qualitative and quantitative analysis of the influence of pipe size and material on the relation between the pipe transversal acceleration amplitudes and the flow rates through it, the authors set the purpose of introducing an innovative method for monitoring the fluid flow rate through pipes for the detection of possible water leaks in pipeline networks. This task (whose feasibility is currently under investigation by the authors of this paper) can be accomplished by means of a smart network of micro-accelerometers installed at several points along the pipelines under monitoring. This apparatus may meet the approval of the operators in a water/gas supply context, since it provides a more affordable way for the detection of leaks, especially for those pipelines belonging to worn networks.

Once the main purpose of the paper and the most peculiar developments implied by the introduced technique have been described, the authors provide, in the following sections, an overview of the theoretical basis of the study presented here, a description of the measurement procedures performed in order to assess the relation existing between the fluid flow rate in the pipe and its vibrational and physical properties, a description of the experimental set-up and, finally, a presentation and in-depth discussion of the test results.

**Table 1.** Nomenclature.

Symbol	Quantity
$x$	Axial direction
$r$	Radial direction
$g$	gravity
$A$	Cross sectional area of the beam/duct
$\gamma$	Specific weight of the beam/duct
$\dot{p}(x)$	Pressure fluctuation in axial direction

## 2. Theoretical background

The authors in [11], describe and theoretically prove how the flow rates in pipes could be linearly related to their flexural vibration amplitudes. By considering the differential equation of motion for the transverse vibration of a beam, as given in [12], and the relation between the pressure fluctuations and the rate of change of the momentum along the length of the beam (from structural mechanics), [13], it can be written:

$$\frac{\partial^2 r}{\partial t^2} = -\frac{g}{A\gamma}\dot{p}(x) \quad (1)$$

where the meanings of the symbols are reported in table 1

Bird *et al* [14] points out that the flow turbulence is significantly responsible for the flexural vibrations of the pipe walls. In addition, in [15], the relation existing between the shear stress  $\tau_w$  at the pipe wall and the pressure gradient  $\dot{p}(x)$  for a circular cross section pipe of radius  $r_w$  is given. This relation is given by (2).

$$\tau_w = -\frac{r_w}{2}\dot{p}(x) \quad (2)$$

In the same reference [15], it is shown how the turbulent shear stress can be related to the time average of product of the bi-dimensional velocity fluctuations (namely Reynolds stresses),  $\overline{u'v'}$ , by invoking Navier–Stokes equations, leading to (3).

$$\tau_w = -\rho\overline{u'v'} \quad (3)$$

In (3),  $u'$  and  $v'$  are the velocity fluctuations in the axial and transversal directions, respectively. From the last two equations, then, a relation between the pressure fluctuation in the axial direction and the Reynolds stresses comes out as  $\dot{p}(x) \propto \overline{u'v'}$ . The author of reference [16] clarifies the correlation existing between pipe vibrations and flow rates. Then, by means of the definition of the turbulence intensity,  $I$ , it is possible to explain the linear proportionality between the averaged flow velocity,  $\overline{U}$ , and the oscillatory components of the velocity. In fact, for turbulent and dynamically similar flows [14] it is:

$$I = \frac{\sqrt{\overline{u'^2} + \overline{v'^2} + \overline{w'^2}}}{\overline{U}} \approx \text{const} \quad (4)$$

Equation (4) leads to (5), (for one-dimensional flows in the x-direction).

$$\overline{U} \propto u' \quad (5)$$

Based on the previous theoretical background, the authors define in [11] the experimental relation existing between the

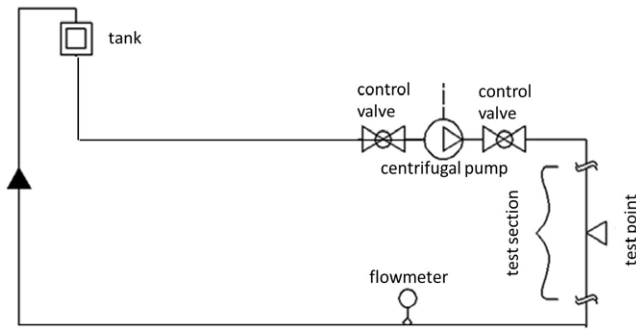


Figure 1. Hydraulic circuit layout.

flow rate and the amplitude of the vibrational oscillations due to the axial turbulence intensity caused by the fluid flow itself.

$$\dot{Q} = A\bar{U} \propto u' \propto \frac{dp}{dx} \propto \tau_w \propto \frac{\partial^2 r_w}{\partial t^2} \quad (6)$$

In (6), the term  $\frac{\partial^2 r_w}{\partial t^2}$  is the acceleration affecting the pipe wall in the radial direction. In this paper, the authors have performed a series of experimental tests with the purpose of first confirming this relation, and then analysing the influence of the physical characteristics of the pipe (in terms of the dimensions and materials) on the sensitivity of the stated relation.

With this aim, this paper is devoted to the investigation of such influences on the functional relation between the fluid flow rate and the vibrational oscillations amplitude.

### 3. Measurement methodology

A suitable open loop hydraulic system was set up. For the measurement of the wall pipe radial vibrations, a laser Doppler vibrometer (LDV) was employed. The LDV has the main advantage of non-intrusive and contactless measurements without distorting the dynamics of the structures under analysis, besides a very large bandwidth useful for investigating not well-known dynamic phenomena [17]. Each tested pipe belongs to a comprehensive hydraulic system, supplied by an electro-pump running at a fixed rotational speed. The hydraulic circuit employed in the experiments carries several modifications from the original plant described in [11] in order to allow the removal and the replacement of pipes with different materials and diameters.

#### 3.1. Description of the experimental set-up

The hydraulic circuit, as shown in figure 1, consists of a centrifugal electro-pump (50 Hz ac supply). The pump has a nominal power of 3 kW @ 2815 rpm, a maximum flow rate of  $7.5 \text{ l s}^{-1}$  and a manometric head ranging from 19.8 to 39.8 m. In order to vary the fluid flow rate, a manual control valve was installed downstream from the pump.

The pipe under test is installed on appropriate supports, aimed at attenuating and decreasing the flexural oscillation of the pipe itself mainly due to the vibrations transmitted by the pump and not properly related to the fluid flow rate.

Table 2. Experimental tests summary. The fluid flow rates for each pipe material and size is in  $\text{l s}^{-1}$ .

Material	$\Phi$	Test 1	Test 2	Test 3	Test 4	Test 5
PVC	$\Phi_{in,1}$	4.8	4.5	4.1	3.7	3.1
PVC	$\Phi_{in,2}$	5.2	4.8	4.5	4.1	3.3
PVC	$\Phi_{in,3}$	5.3	5.0	4.8	4.5	4.3
Steel	$\Phi_{in,1}$	4.8	4.5	4.1	3.7	3.4
Steel	$\Phi_{in,2}$	5.2	4.9	4.6	4.3	4.0
Steel	$\Phi_{in,3}$	5.3	5.0	4.7	4.3	4.0

Note: Reynolds number range  $\approx 9500$ – $22\,700$ .

Table 2 specifies the fluid flow rates tested for each pipe material and inner diameter.

The vibrometer laser head was located on the pipe under test in order to detect the transverse vibration signal transmitted from the flow to the pipe wall.

#### 3.2. Measurement devices description

The measurement transducers used in the experimental tests are summarised as:

- Polytec Portable Digital Vibrometer (PDV 100), with a dynamic range from 0 kHz to 22 kHz.
- Ultrasonic Flowmeter TTFM10B-HH-NG BIMATIK manufactured (linearity error  $\pm 0.5\%$ , repeatability  $\pm 0.2\%$  and accuracy  $\pm 1\%$ ).

The measurements by means of the LDV are characterised by a full scale range of  $\pm 20 \text{ mm s}^{-1}$  and a sensitivity factor of  $0.2 \text{ V mm}^{-1} \text{ s}^{-1}$ . Signals from the LDV are proportional to the velocity of vibration and the correspondent acceleration signals are obtained by deriving the velocity signals by means of smoothing and differentiation by means of the continuous wavelet technique (Morlet Wavelet) [18]. The post-processing of the acquired signals was implemented in a MATLAB environment. After a few runs of the system performed in order to suitably choose the acquisition parameters, the sample rate was set at 6 kHz for an acquisition time of 20 s.

#### 3.3. Test procedure

In order to establish how the diameter and material of the pipe influence the amplitude of vibrations, the tests were performed on six different pipes, as shown in figure 2.

The repeatability of the procedure was verified by replicating each acquisition five times.

### 4. Results

In this paragraph, the authors report and analyse the results of the experimental tests, thus confirming the linear relation between the flow rate and amplitude vibrations taken at the same frequency, as obtained in [11] with a different experimental set-up. Once the pipe diameter and material were chosen, several tests were carried out by varying the fluid flow rate. As cited above, five replications were carried out for each

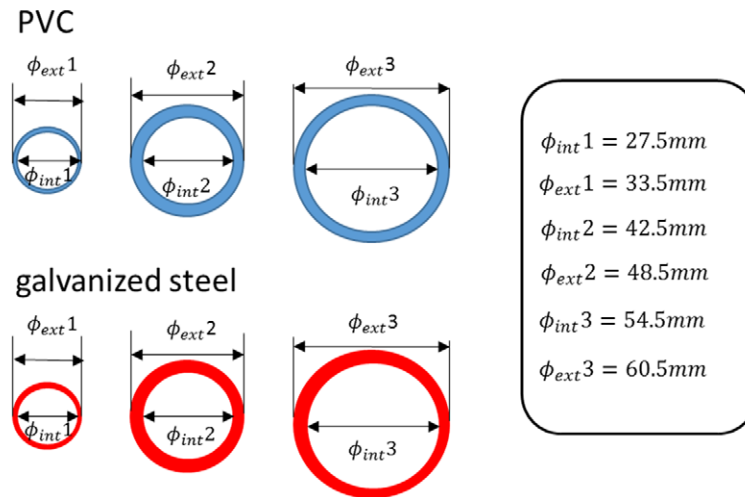


Figure 2. Pipes under test with different materials and diameters.

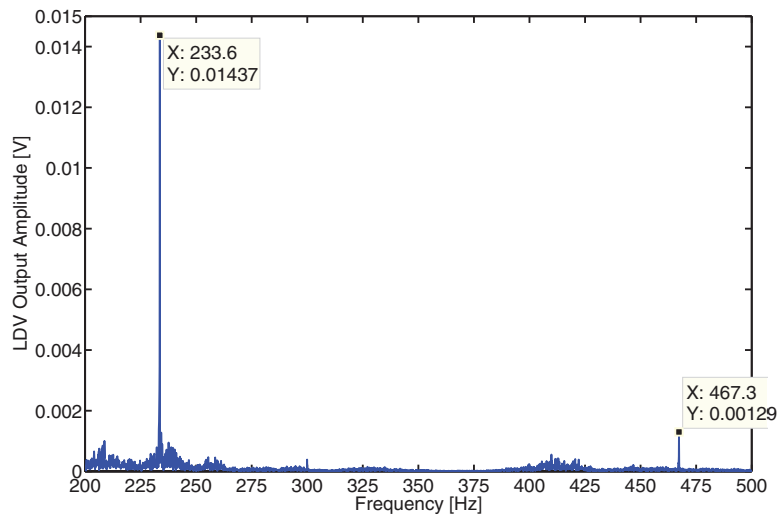


Figure 3. LDV output signal amplitude spectrum for a steel pipe with an inner diameter of 27.5 mm and a water flow rate of  $4.8 \text{ l s}^{-1}$ . Two of the most significant harmonics (in the range 0–3 kHz) are plotted.

tested flow rate in order to make the forthcoming statistical considerations more robust (assessment of the repeatability of the measurement conditions).

As pointed out in [11], there exists a linear relation between the amplitude of the whole acceleration spectrum and the fluid flow rate in the pipe.

In order to extrapolate the linear trend between the acceleration spectrum and the fluid flow rate, the first harmonic frequency was considered. The linear relations were obtained by the least squares fit of the experimental data for each test performed.

Afterwards, a series of tests for steel and PVC pipes was carried out for different diameters.

#### 4.1. Tests for galvanised steel pipes

In this subsection, the results obtained from the measurements performed on galvanised steel pipes of different diameters are provided. An example of the acceleration spectrum amplitude obtained for a test performed on a steel pipe with an inner

diameter of 27.5 mm and for a water flow rate of  $4.8 \text{ l s}^{-1}$  is shown in figure 3, where only the first and second harmonics are considered.

The spectrum in figure 3 refers to the LDV output signal, which is proportional to the vibration velocity of the analysed pipe. Since the vibration acceleration is the derivative of the velocity signal, the location of the most significant harmonics remains unchanged, as the derivation operation affects the harmonic amplitudes but not the frequencies.

For the spectrum in figure 3, the authors consider the harmonic at 234 Hz as the most significant one. The harmonic analysis was carried out for each flow rate for a given pipe. The use of the Power Spectral Density of the LDV output (proportional to vibrational velocity), makes the detection of the phenomenon characteristic frequencies easier. The analysis has shown that the content of the frequency spectrum is almost the same, and the harmonic amplitude at the chosen frequency linearly varies with the flow rate. Hereafter, the plots of these spectra for all the investigated diameters

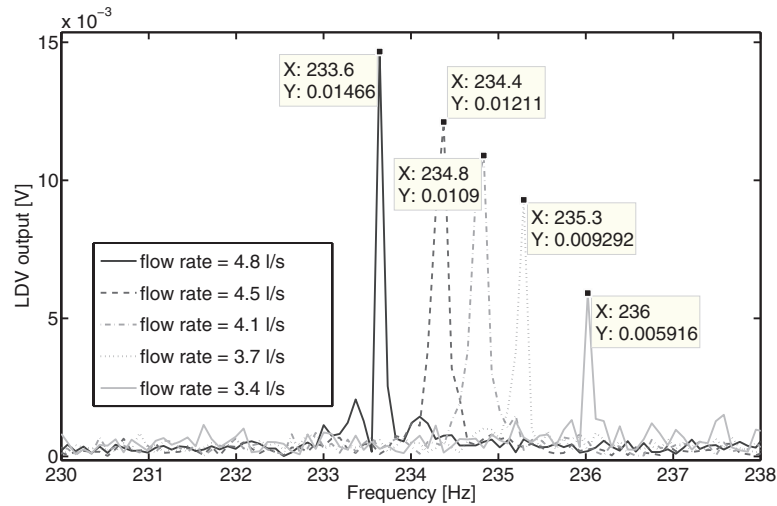


Figure 4. Spectrum analysis for a galvanised steel pipe with  $\Phi_{\text{int}} = 27.5$  mm at several tested water flow rates.

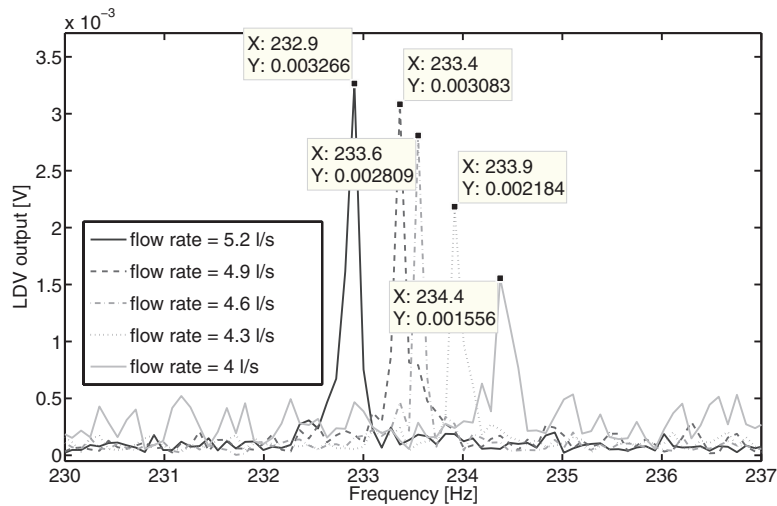


Figure 5. Spectrum analysis for a galvanised steel pipe with  $\Phi_{\text{int}} = 42.5$  mm at several tested water flow rates.

are shown. Figures 4–6 represent the LDV output signal frequency spectra (for all tested flow rates) for the different steel pipe internal diameters.

For a given pipe (characterised by fixed material and inner diameter), the most significant harmonic considered remains almost unchanged as the fluid flow rate changes (the tested fluid flow rate ranges are indicated in each figure). Very few deviations among each harmonic can be noticed because of the noise affecting all the spectra.

Figures 7–9 show a series of linear interpolations aimed at proving the linear relationship existing between the fluid flow rate and the harmonic amplitudes of the pipe transverse acceleration for a steel pipe characterised by three different inner diameters, as indicated in the figures. Error bars are not reported because they are too small in the figure's used scale. Indeed, besides the laser accuracy, the evaluated repeatability errors are negligible in terms of output (mV) once almost the same flow rate during the repetitions is guaranteed.

Figure 10 allows the evaluation of the influence of the pipe's inner diameter on the slope of the interpolating curves. An increase in the pipe's inner diameter causes a decrease in

the interpolating curve slope. A physical explanation of such behaviour will be provided in the Conclusion section.

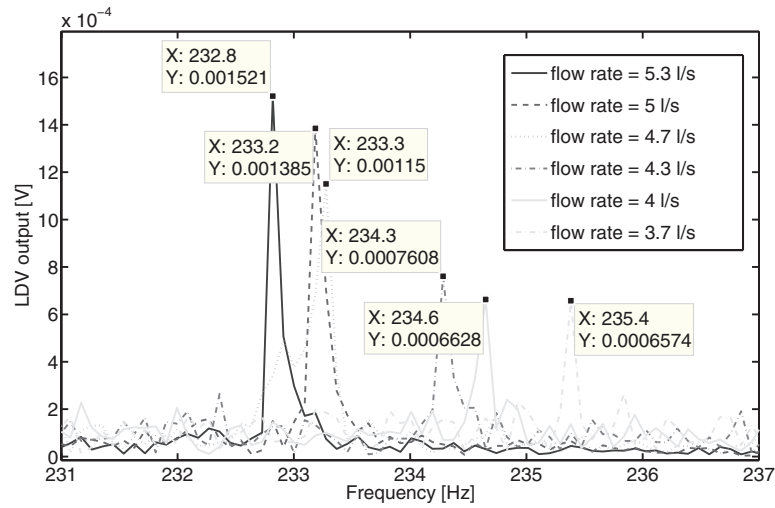
Figures 11–18 refer to the results of the experimental tests performed on PVC pipes.

## 5. Discussion of the results

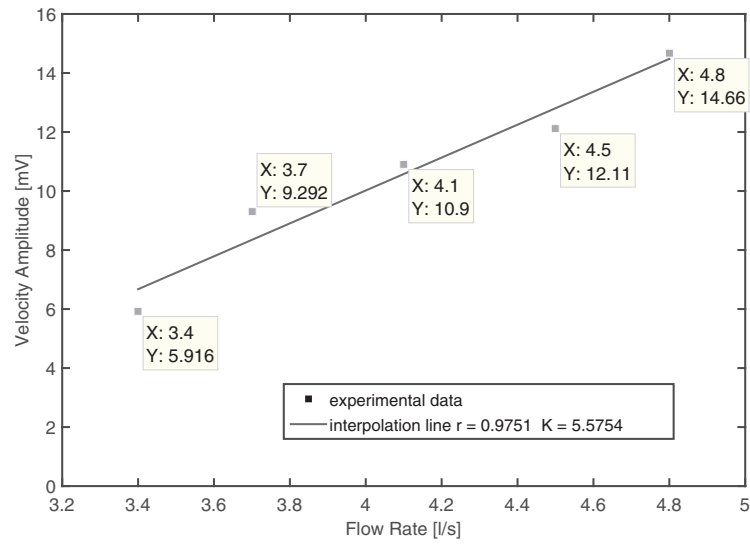
### 5.1. Influence of pipe inner diameter

From figures 10 and 18 it can be noted how increasing the pipe diameter determines the decrease in the sensitivity for both materials considered. This behaviour is justified by (1), showing that an increase in the cross section area implies a decrease in the pipe wall acceleration at a fixed flow rate ( $\dot{p}(x) = \text{const}$ ). Because of the noise, not filtered in the present experimentations, it might be difficult to distinguish the acceleration peak at the lowest flow rates (see, for example, figures 5, 12 and 13). Therefore, it might be necessary to consider the filtering of the signals when the signal-to-noise ratio results are too low (at the smallest diameters for steel pipes and the highest ones for PVC pipes).

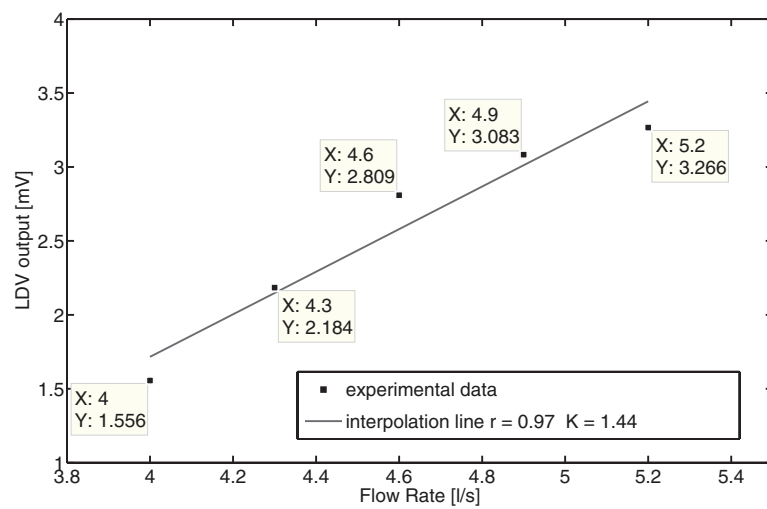




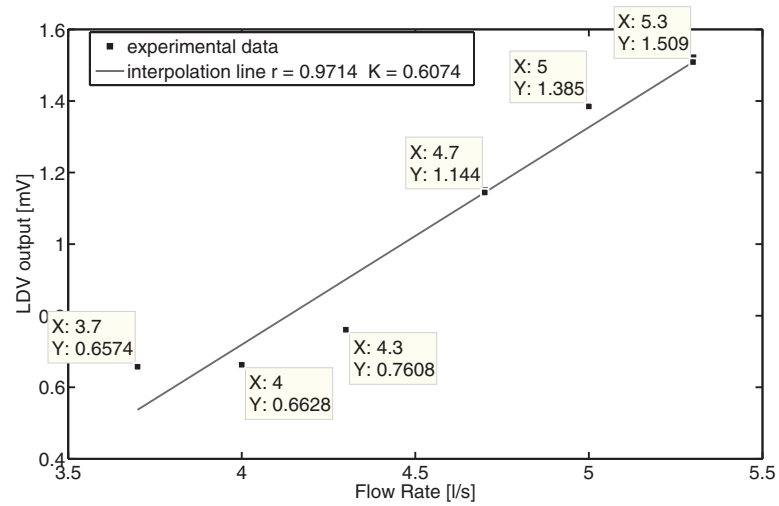
**Figure 6.** Spectrum analysis for a galvanised steel pipe with  $\Phi_{\text{int}} = 54.5$  mm at several tested water flow rates.



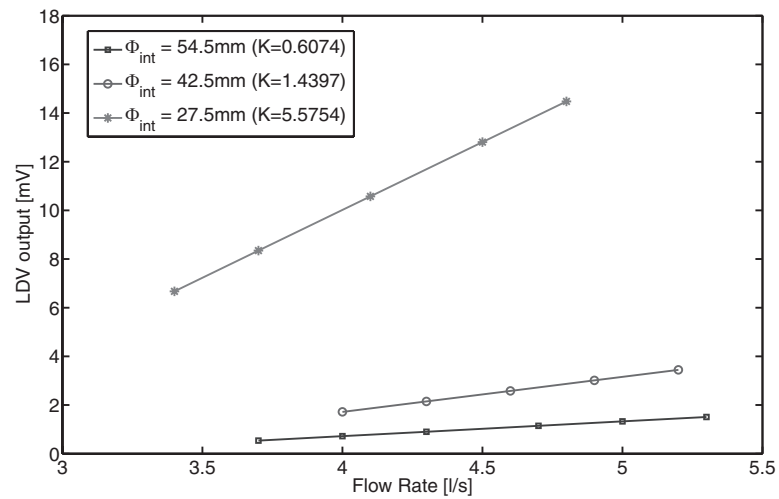
**Figure 7.** Linear fit of the LDV harmonics at 235 Hz for each tested flow rate in the case of a steel pipe with  $\Phi_{\text{int}} = 27.5$  mm. The term  $r$  refers to the correlation coefficient of the interpolation and  $K$  is the sensitivity of the function.



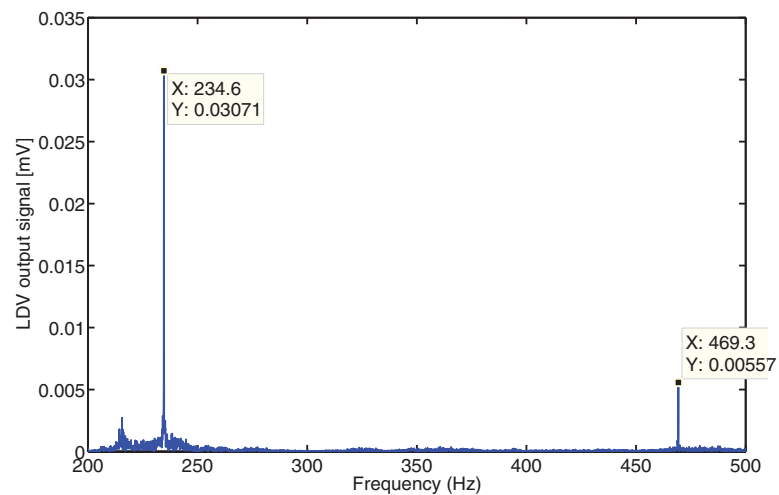
**Figure 8.** Linear fit of the LDV harmonics at 235 Hz for each tested flow rate in the case of a steel pipe with  $\Phi_{\text{int}} = 42.5$  mm.



**Figure 9.** Linear fit of the LDV harmonics at 235 Hz for each tested flow rate in the case of a steel pipe with  $\Phi_{\text{int}} = 54.5$  mm.

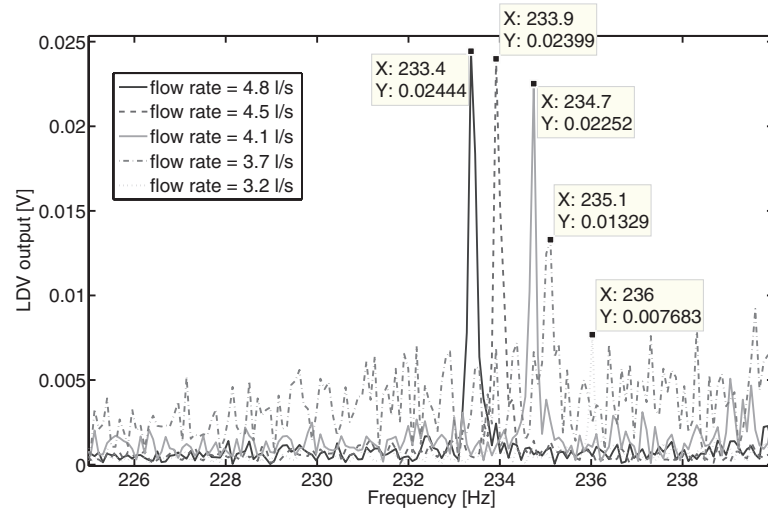


**Figure 10.** Comparison for the linear fit of figures 7–9 in the case of a steel pipe. The term  $K$  indicates the slope of each line and, therefore, expresses the proportionality coefficient between the fluid flow rate and the vibrational acceleration amplitudes of the tested pipes.

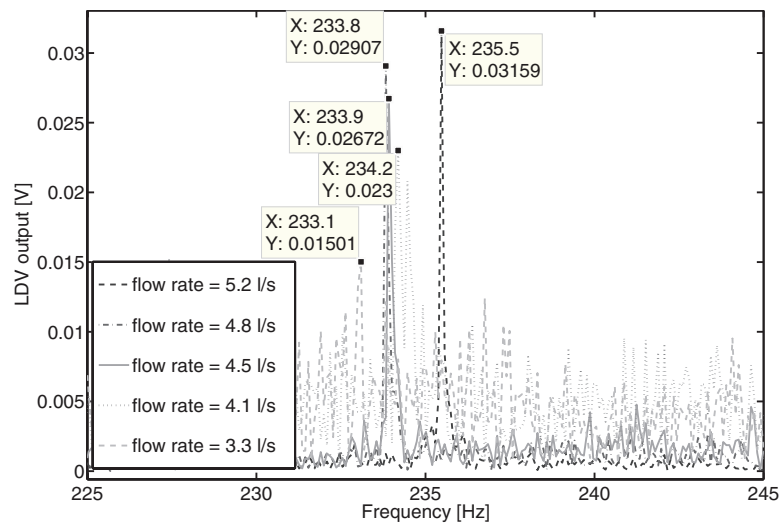


**Figure 11.** LDV output signal amplitude spectrum for a PVC pipe with an inner diameter of 42.5 mm and a water flow rate of  $5.3 \text{ l s}^{-1}$ . The most significant harmonics are plotted.

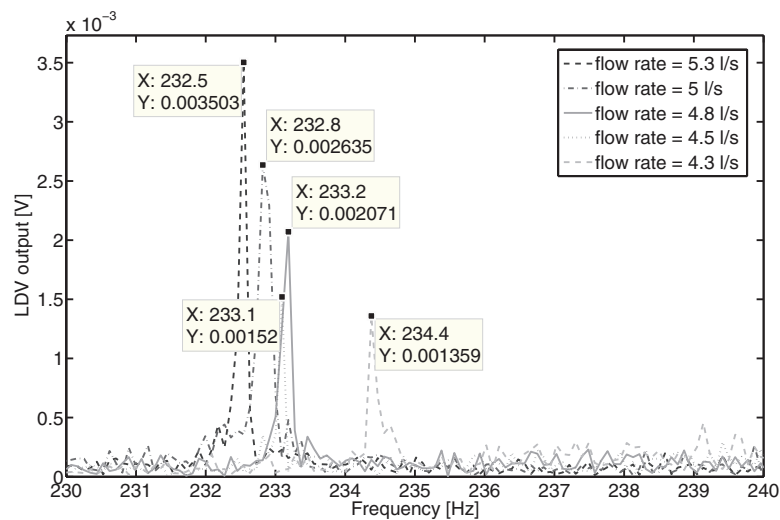




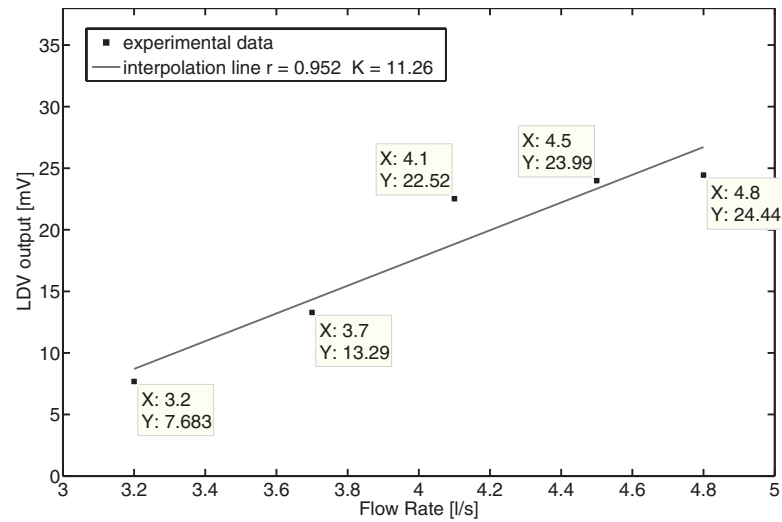
**Figure 12.** Spectrum analysis for a PVC pipe with  $\Phi_{\text{int}} = 27.5$  mm at several tested water flow rates.



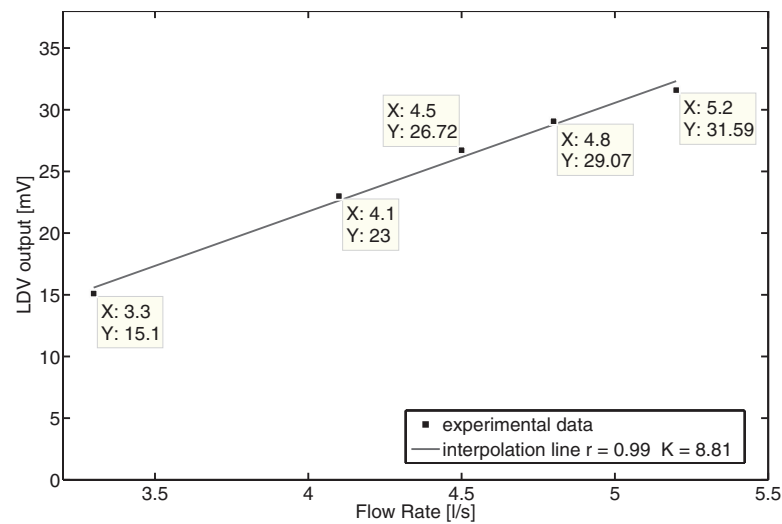
**Figure 13.** Spectrum analysis for a PVC pipe with  $\Phi_{\text{int}} = 42.5$  mm at several tested water flow rates.



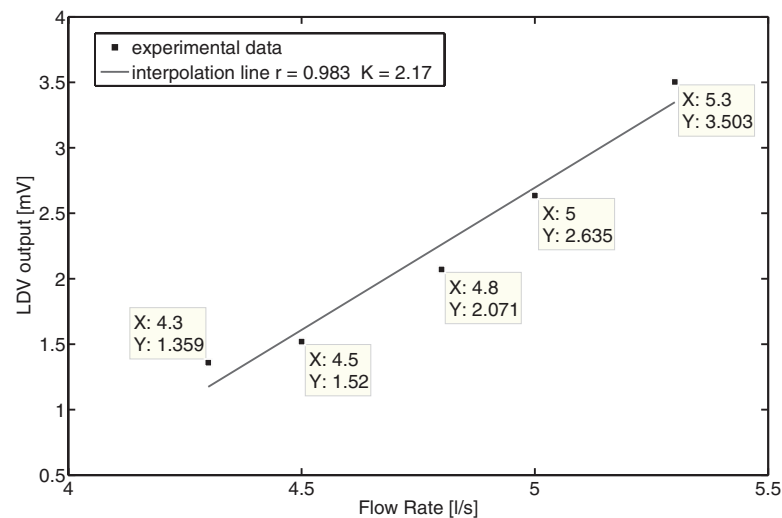
**Figure 14.** Spectrum analysis for a PVC pipe with  $\Phi_{\text{int}} = 54.5$  mm at several tested water flow rates.



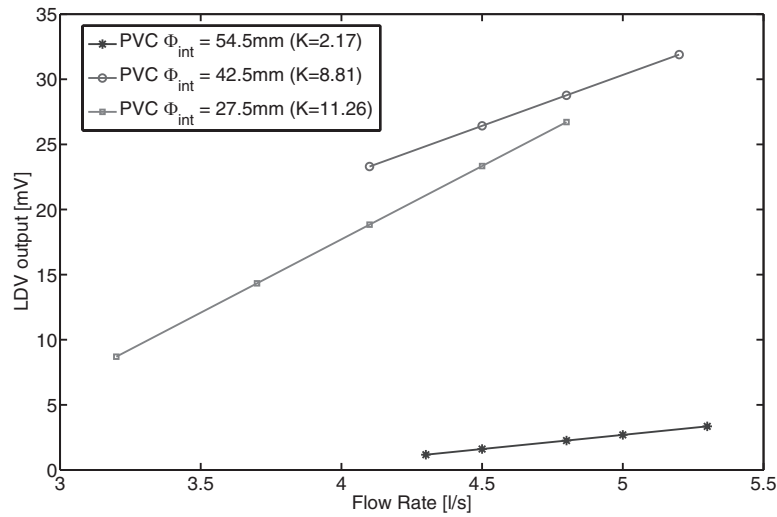
**Figure 15.** Linear fit of the LDV harmonics at 235 Hz for each tested flow rate in the case of a PVC pipe with  $\Phi_{\text{int}} = 27.5$  mm.



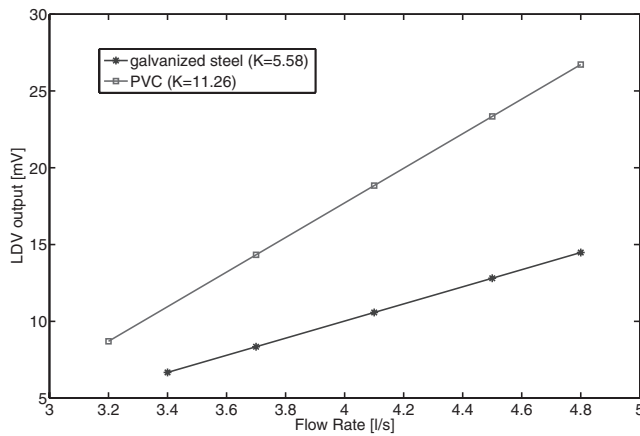
**Figure 16.** Linear fit of the LDV harmonics at 235 Hz for each tested flow rate in the case of a PVC pipe with  $\Phi_{\text{int}} = 42.5$  mm.



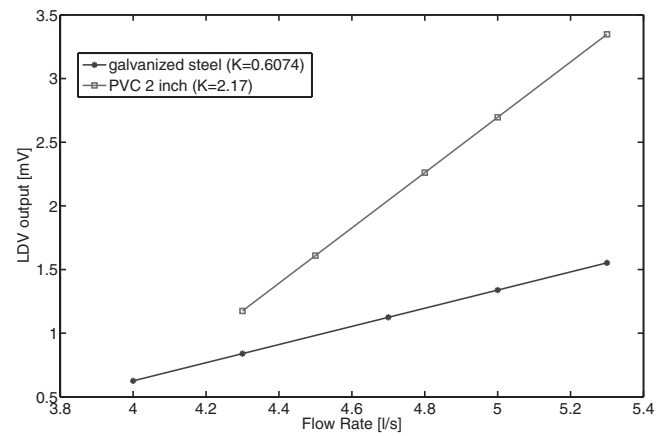
**Figure 17.** Linear fit of the LDV harmonics at 235 Hz for each tested flow rate in the case of a PVC pipe with  $\Phi_{\text{int}} = 54.5$  mm.



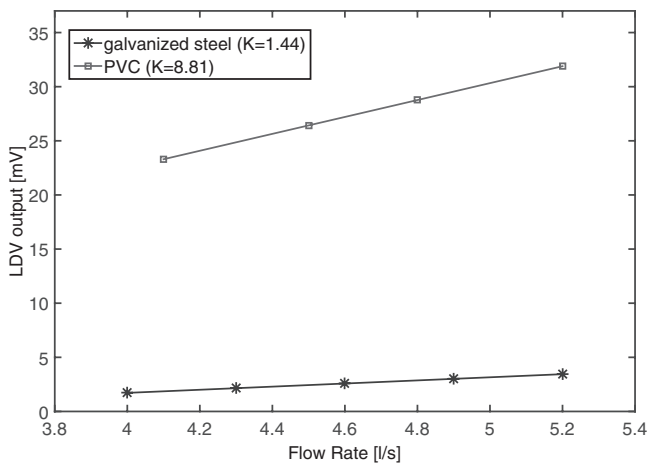
**Figure 18.** Comparison for the linear fit of figures 15–17 in the case of a PVC pipe.



**Figure 19.** Influence of pipe material on the linear fitting sensitivity for the pipes considered with an inner diameter of 27.5 mm.



**Figure 21.** Influence of pipe material on the linear fitting sensitivity for the pipes considered with an inner diameter of 54.5 mm.



**Figure 20.** Influence of pipe material on linear fitting sensitivity for the pipes considered with inner diameter of 42.5 mm.

### 5.2. Influence of the pipe material

The same comparison was made for the pipes of different materials considered (PVC and galvanised steel) and carried out for each specific inner diameter tested. From

figures 19–21 it is possible to notice that the sensitivity is greater for PVC pipes than for steel, even if it is not so evident for smaller diameters. Further, the vibrational peaks are always greater for PVC than for steel pipes with the same diameter: this result is due to the different specific weight of the materials considered, as it can be easily deductible from (1), where the quantity is at the denominator of the right-hand side term.

## 6. Conclusion

The purpose of this paper is to validate the theoretical model, according to which a linear relation between the fluid flow through a pipe and the amplitude of the transversal vibrational oscillations of the pipe wall due to flow turbulence exists, even when either the diameter or the material of the pipe varies. In a previous paper, [11], the same authors experimentally demonstrated the effectiveness of such a relation for a pipe of a given diameter and material. Stating that a linear relation between the fluid flow rate and the amplitude of the vibrational oscillations of the pipe wall exists, the authors performed further tests in order to establish how the diameter and the material

of the pipe under analysis could influence the sensitivity of this relation. For this purpose, two different materials (galvanised steel and PVC) and three pipe diameters of 27.5 mm, 42.5 mm and 54.5 mm were investigated. The experimental study allowed several comparisons between the least-squared regression straight lines representing the peaks of the acceleration amplitude at the same frequency versus the flow rates for each considered pipe (characterised by a specified diameter and material). In the first instance, the tests confirmed the increasing trend exhibited by the peak of the acceleration as the fluid flow rate increases. Once the pipe material was selected (preliminary steel), pipe diameters of 27.5 mm, 42.5 mm and 54.5 mm was considered. These tests indicate that an increase in pipe inner diameter results in a decrease in the line slope. Thus, for a given pipe material, an increase in the inner diameter implies a lower sensitivity of the linear relation between the flow rate and acceleration peak in the same range of the flow rate tested. Thus, the different material of the pipe highlights a major sensitivity of the transfer function for PVC pipes.

The robustness of the highlighted linear relationship with the variation in the pipe characteristics (in terms of size and constituent material) suggests an easier methodology for monitoring fluid flows through pipeline networks by means of the acquisition of vibrational signals of pipe walls through the use of micro-sensors commercially available at very affordable costs.

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