Digital Filters for Real Time Signal Processing

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1 Abstract

Implementation of real time digital filtering algorithms as an alternative to analog filtering methods to conduct signal conditioning of ECG(electrocardiogram) signals. We implement an adaptive 60-Hz interference filter, a hanning filter, two low-pass filters, a high-pass filter for eliminating dc offset, a turning point data reduction algorithm, and band-pass filters for detecting QRS complexes.

2 Important Concepts

2.1 ECG signals

ECG also known as electrocardiogram is the graph of the voltage vs time of the electric activity in our heart. A major problem in the recording of electrocardiograms is the appearance of unwanted 60 Hz interference. This interference appears due to various causes such as magnetic induction, displacement currents in leads or in the body of the patient, and equipment interconnections and imperfections. [2]

2.2 Digital Filters

A digital filter is a system that performs mathematical operations on a sampled, discrete-time signal to reduce or enhance certain aspects of that signal. It possesses various advantages over analog circuit filters. Analog circuit filters are sensitive to interference from the environment while digital filters are resistant to environmental noise. Digital filters are also more portable since they do not possess bulky circuit components.Another advantage of digital filtering methods over analog filtering systems is that they can be easily adapted to fit our needs through changes in code where else analog filters require change in hardware in order to bring about change.

2.3 Discrete time LTI- Systems

Discrete time LTI systems are a class of systems that are linear and time-invariant. They are characterized by their impulse response h[n]. Their input-output relation is given by

$$y[n] = x[n] * h[n]$$

where x[n] is the input signal and y[n] is the output signal. From the convolution property, we also have

$$Y(\omega) = X(\omega)H(\omega)$$

where Y(w), X(w) and H(w) are their respective discrete time fourier transforms. From this, we can see that LTI systems don't add new frequencies to the input system and we can select the desired frequencies using H(w). Hence, LTI systems can be interpreted as frequency selective filters.

2.4 QRS complexes

QRS complex is a group of waves seen on an electrocardiogram, representing ventricular depolarization. The QRS complex is sometimes called the QRS wave. QRS wave in electrocardiogram has been arbitrarily labeled in alphabetical order. The Q wave is a short downwards deflection, the R wave a conspicuous upwards stroke, and the S wave a return to below the level of the base-line.

3 Proposed methods

3.1 Adaptive Filter

Band filters remove all signals in a given frequency range. We wish to remove 60 Hz noise/interference without removing the 60 Hz components of the ECG signal. Thus band filters will not be suitable for our purpose. Instead we shall use an adaptive filter to carry out this process. In this algorithm we shall use signals $e(nTs)$ as our 60 Hz noise estimate. We shall be sampling signal $x(nTs)$ that we receive as input and then increment the estimated noise $e(nTs)$ based on its relation with $x(nTs)$ as specified by function f(nTs).

. Assume $e(nTs) = Asin(\omega nTs)$ (ω=$2\pi 60$ since we know frequency is 60 Hz) Now we shall estimate the value of the $e(nTs + Ts)$ using the following mathematic equations:

$e(nTs - Ts) = Asin\omega(nTs - Ts) = Asin(\omega nTs - \omega Ts)$

$e(nTs + Ts) = Asin\omega(nTs + Ts) = Asin(\omega nTs + \omega Ts)$

Using the trignometric identity $sin(a + b) = 2sin\ a\ cos\ b$,

$e(nTs + Ts) = 2A\ sin\ \omega nTs\ cos\ \omega Ts - A\ sin\ (\omega nTs - \omega Ts)$

Let $N = cos\ \omega Ts = cos(2\pi f/fs)$

$e(nTs + Ts) = 2Ne(nTs) - e(nTs - Ts)$

Next we compute the relation,

$f(nTs + Ts) = (x(nTs + Ts) - e(nTs + Ts)) - (x(nTs) - e(nTs))$.

If $f(nTs + Ts) > 0$, then $e(nTs + Ts)$ is too low and e(nTs + Ts) is incremented,

$e(nTs + Ts) = e(nTs + Ts) + d$

If $f(nTs + Ts) < 0$, then $e(nTs + Ts)$ is too high and $e(nTs + Ts)$ is decremented,

$e(nTs + Ts) = e(nTs + Ts) - d$

After adjustment output, $y(nTs + Ts) = x(nTs + Ts) - e(nTs + Ts)$

We then iterate n in the loop and the current estimate $e(nTs+Ts)$ becomes $e(nTs)$. Repeat this process until the estimation noise signal gets adapted such that $f(nTs+Ts) becomes 0$. At this point noise has been completed eliminated. The rest of the noise signal can be estimated from this value.

Appropriate value of d has to be chosen to prevent significant unlearning when the signal enters QRS regions where it will start to unadapt.

3.2 Hanning Filter

Smoothing signals is often done to produce slow changes in value so that it is easier to see trends in our signals. It is also useful in suppressing noise in our signal. Hanning filters are low-pass filters implemented using a weighted moving average algorithm to smooth out signals and suppress noise. We give the weighted average of three successive inputs as output. The averaging tends to suppress high frequencies and hence, the filter acts as a low-pass filter. The z-transform equation is

$$H(z) = \frac{1}{4}[1 + 2z^{-1} + z^{-2}]$$

and it is implemented with difference equation

$$y(nT) = \frac{1}{4}[x(nT) + 2x(nT - T) + x(nT - 2T)]$$

3.3 Low-pass Filter

Low pass filters are filters that pass signals with a lower frequency than a selected cutoff frequency and attenuates signals with higher frequencies than the cutoff frequency. ECG signals often face high frequency interference like EMG noise which can be removed with the help of low-pass filters. We implemented a low pass filters with transfer function

$$H(z) = \frac{1 - z^{-4}}{1 - z^{-1}}$$

and difference equation

$$y(nT) = y(nT - T) + x(nT) - x(nT - 4T).$$

The transfer function will give us a first-order filter with four equally spaced zeros on the unit circle and a cancelling pole on the zero at z = 1. At the uncancelled zeros, the output of the filter drops to zero. The maximal gain will

occur at dc and will be equal to 4. However, this filter will have a poorly defined cutoff and sidelobes with high magnitude.
This can be improved by implementing another low-pass filter instead, with transfer function

$$H(z) = \frac{(1 - z^{-4})^2}{(1 - z^{-1})^2}$$

and difference equation

$$y(nT) = 2y(nT - T) - y(nT - 2T) + x(nT) - 2x(nT - 4T) + x(nT - 8T)$$

The transfer function will give us a second-order filter which will have side lobes of lower magnitude and gain equal to 16.

3.4 High-pass Filter for elimination of dc offset

Generally, we use analog high-pass filters to remove dc offset from an ECG signal. Here, we remove dc offset by a digital high-pass filter which is implemented as a low-pass signal subtracted from the original signal. The offset can be imagined as a signal with frequency zero. A low pass filter with very low cutoff frequency will be able to filter out just the dc offset which can be subtracted from the input signal to eliminate the offset. The transfer function of the low-pass filter used is

$$H(z) = \frac{1 - z^{-256}}{1 - z^{-1}}$$

and difference equation is

$$y(nT) = y(nT - T) + x(nT) - x(nT - 256T)$$

The gain of the low-pass filter is 256, so we attenuate it by 256 before subtracting from the input. The actual output of the high-pass filter after removing the dc offset is

$$p(nT) = x(nT) - y(nT)/256$$

Its operation can be best described by considering its response to a step function. The difference x(nT) - x(nT - 256T) is equal to the step magnitude for each of the 256 sample times for which x(nT) and x(nT - 256T) are on opposite sides of of the step. y(nT)/256 is thus incremented by 1/256 times step magnitude for each of those sample time until it equals step magnitude. It will remain constant then onwards as x(nT) - x(nT-256T) will be 0. The value of y(nT)/256 is then subtracted from input which gives us a ramp output.

3.5 Turning point data reduction

We will use turning point data reduction to halve the sample rate from 200 to 100 Hz in order to decrease the amount of data stored. At 200Hz sample frequency, ECG signals are over sampled as compared to the Nyquist rate. If we just sample every alternate sample, it would not violate sampling theorem

but could still result in loss of amplitude. So we make use of an algorithm in order to choose and retain the right points needed to prevent loss of amplitude. [1]
Let the first three samples be X_0, X_1, X_2
If $(X_1 - X_0)(X_2 - X_1) < 0$, X_1 is stored else X_2 is stored. This is because if the condition is valid then X_1 is either the highest or the lowest sample of the three(since either $x_1 - x_0$ or $x_2 - x_1$ is negative). If $(X_1 - X_0)(X_2 - X_1) > 0$ then X_2 is either the highest or the lowest(since $X_1 - X_0$ and $X_2 - X_1$ are both positive or negative). If $(X_1 - X_0)(X_2 - X_1) = 0$ then either $X_2 = X_1$ in which case it doesn't matter which we choose or $X_1 = X_0$ in which case we choose X_2 because it is more significant since we already have X_0. This process is repeated for the whole signal by which the sample rate is halved.

3.6 Band-Pass filters for QRS Detection

Analog QRS detectors generally band-pass the signal which produces a ringing output for QRS complex. We can carry out this process digitally using special band-pass filters which have equally spaced zeros on the unit circle and a pair of complex conjugate poles also on the unit circle with the transfer function of form

$$H(z) = \frac{1 - z^{-m}}{1 - 2cos(\theta)z^{-1} + z^{-2}}$$

The coefficient 2cos(θ) will have an integer value only for θ = 0,60,90,120 and 180. Poles a 0 and 180 will correspond to low and high-pass filters. The others will correspond to band-pass filters. We implemented a 2nd order band-pass filter with transfer function

$$H(z) = \frac{(1 - z^{-m})^2}{(1 - z^{-1} + z^{-2})^2}$$

and difference equation
y(nT) = 2y(nT - T) - 3y(nT - 2T) + 2y(nT - 3T) - y(nT - 4T) + x(nT) - 2x(nT - 12T) + x(nT - 24T)

The gain of the filter will be 48. This filter will give us ringing output for QRS complexes which can be used to identify the presence and location of QRS waves. We can also further use analogous methods to trigger a monostable impulse for each QRS complex.

4 Simulations/Plots

Following are the plots obtained of various filters implemented by us. Figure 1 represents Adaptive filter, Figure 2 represents Hanning filter, Figure 3 represents First-order Low-pass filter, Figure 4 represents Second-order Low-pass filter , Figure 5 represents High-pass filter, Figure 6 represents our implementation of turning point algorithm and Figure 7 represents QRS detection using Band-pass filter.

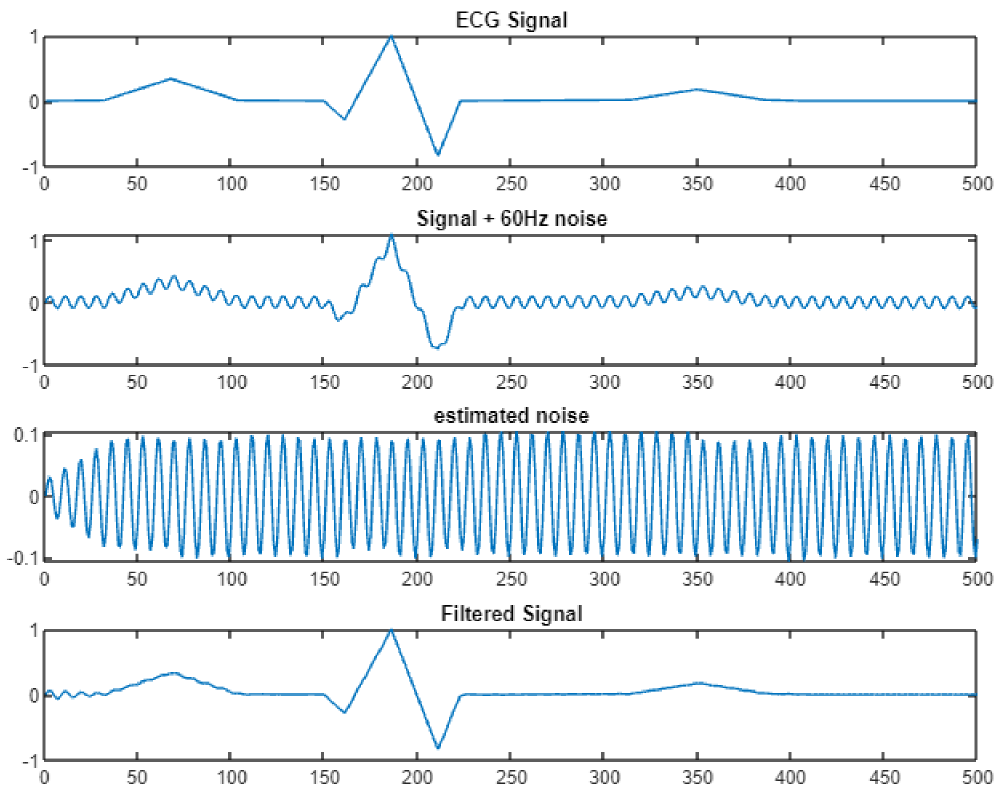

Figure 1: Adaptive Filter

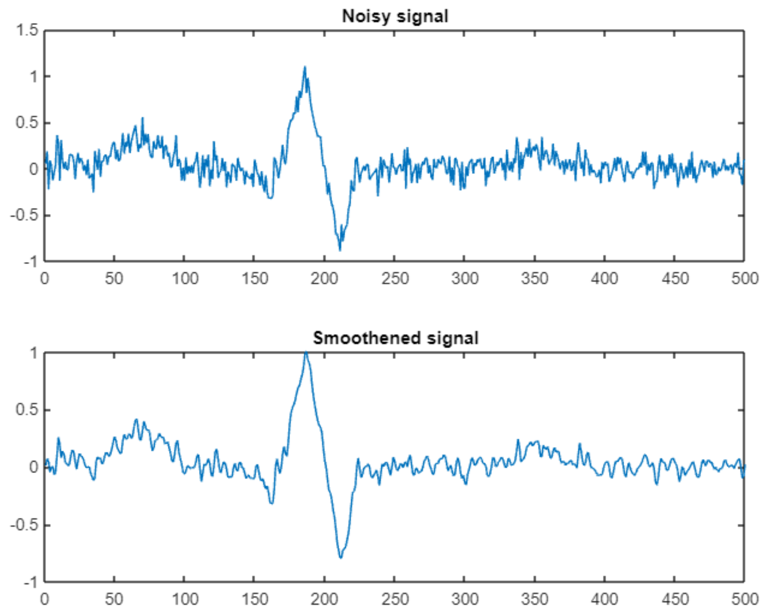

Figure 2: Hanning Filter

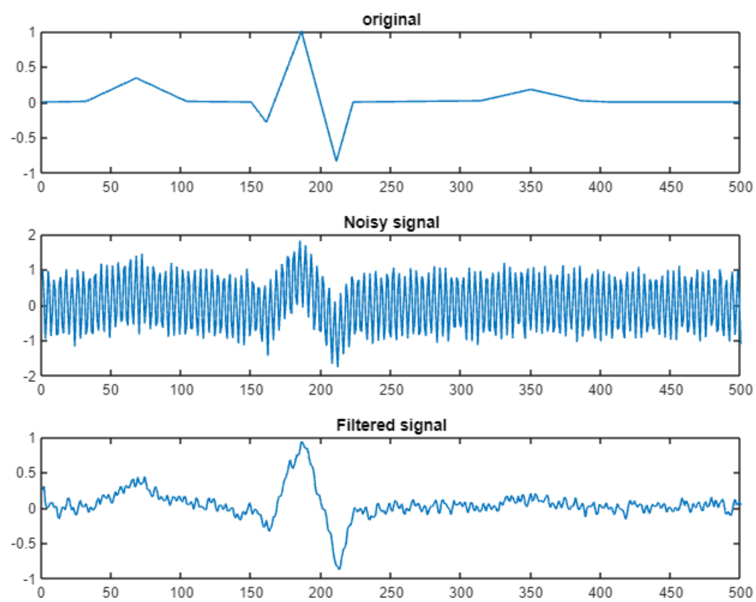

Figure 3: 1st Order Low-Pass Filter

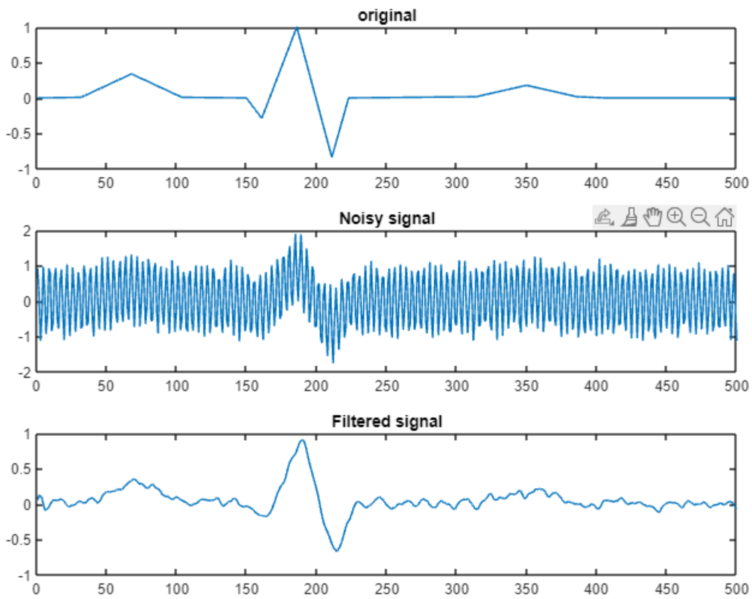

Figure 4: 2nd Order Low-Pass Filter

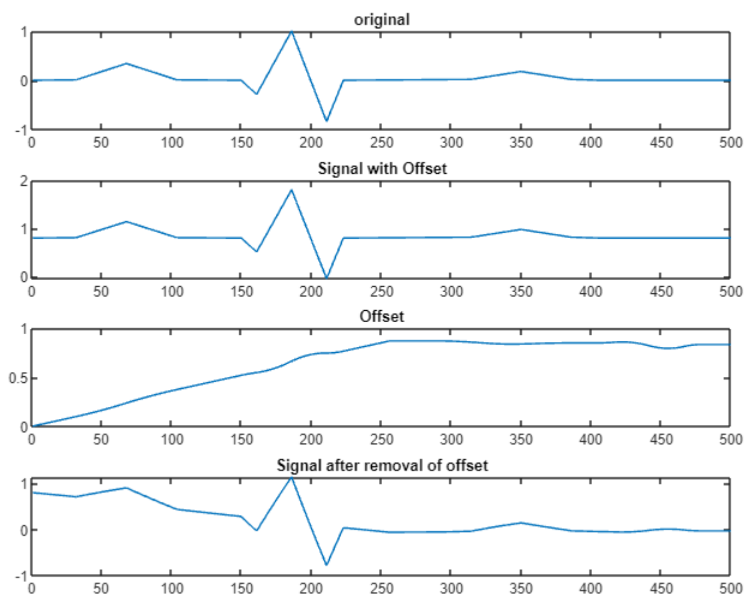

Figure 5: Elimination of Offset using High-Pass Filter

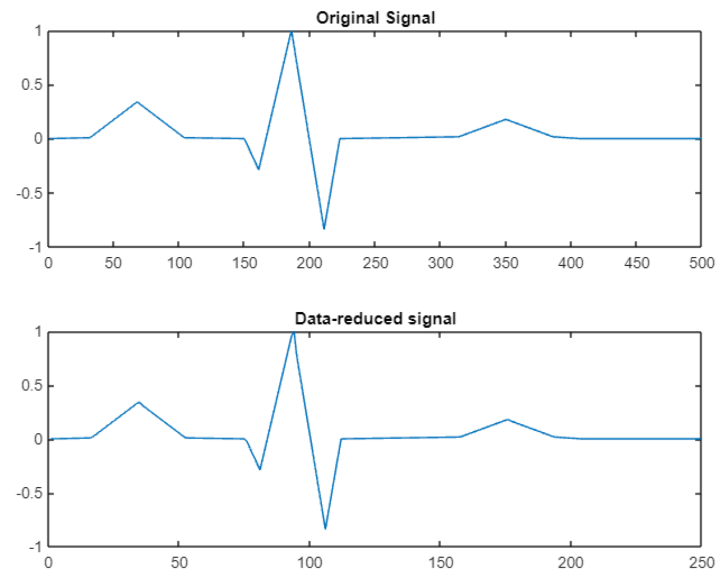

Figure 6: Turning Point Data Reduction

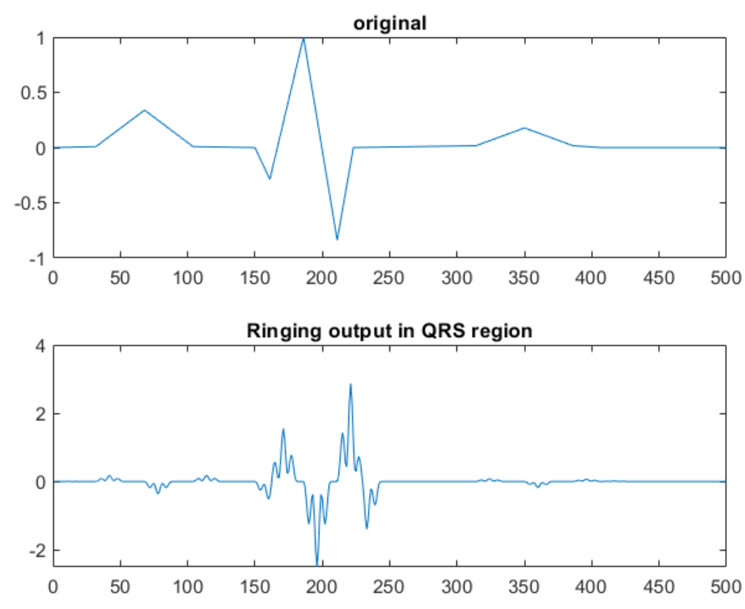

Figure 7: QRS Detection using Band-Pass Filter

5 Conclusion

It is practical to replace many of the ECG filtering processes that typically require circuits with digital filtering algorithms which can accomplish many of the filtering tasks required in an electrocardiograph instrument in an efficient, easy and quicker way.

References

[1] Dr Monisha Chakraborty Muzaffar Saba Anjum. Ecg data compression using turning point algorithm. *IJIRMPS*, 2(6):42, 2014.

[2] B. Widrow, J.R. Glover, J.M. McCool, J. Kaunitz, C.S. Williams, R.H. Hearn, J.R. Zeidler, Jr. Eugene Dong, and R.C. Goodlin. Adaptive noise cancelling: Principles and applications. *Proceedings of the IEEE*, 63(12):1692–1716, 1975.