

Lab 5 – Discrete-time FT and LTI systems

Objectives: In this lab we will

- numerically compute DTFT of signals
 - study a few discrete-time LTI systems
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5.1 Discrete-time Fourier transform (DTFT)

The DTFT of any discrete-time signal $x[n]$ is given as

$$X(e^{j\omega}) = \sum_{-\infty}^{\infty} x[n]e^{-j\omega n}$$

(a) Write a **matlab function** of the form `X = DT_Fourier(x, N0, w)` that takes as inputs

- x , a discrete-time signal of finite duration (assume that the signal is zero elsewhere)
- N_0 , location of the sample $x[0]$ in the given input signal x , note that N_0 is a positive integer between 1 and `length(x)`
- ω , a vector of frequencies at which to compute the DTFT (though frequency is a continuous variable in DTFT we can evaluate it at only finite set of points)

The function should return X , a complex vector corresponding to the DTFT computed at the frequencies in ω . Write the function using **at most** one for-loop.

(b) Write a **matlab script** which calls the `DT_Fourier()` function. Set $\omega = -10:0.01:10$ for computing the DTFT. Your script should compute DTFT for each of the following discrete-time signals:

1. unit impulse $\delta[n]$
2. shifted unit impulse $\delta[n + 3]$
3. rectangular pulse from -3 to 3
4. finite duration sinusoid $\sin(\omega_0 n)$ with $\omega_0 = \frac{\pi}{4}$ for -200 to 200

Appropriately choose inputs x and N_0 for each of the signals. For each signal, plot the DTFT spectrum (i.e. magnitude, phase, real, imaginary parts) in a 2x2 figure. Compare your plots with the analytical answers worked out in class.

(c) In the same script, compute DTFT for the signal $a^n u[n]$. Restrict your signal ($n = 0$ to 100) for finite computations. Set $\omega = -10:0.01:10$. In a 2x2 figure, plot two time domain signals (corresponding to $a = b$ and $a = -b$, values of b are given below) in the top panels and their DTFT magnitude spectrum in the bottom panels. Do this for $b = 0.01, 0.5, 0.99$ and note your observations as b changes.

5.2 Discrete-time filters

An order-M moving average filter is a discrete-time LTI system with input $x[n]$ and output $y[n]$ relation given by

$$y[n] = \frac{1}{M} \sum_{m=0}^{M-1} x[n-m]$$

- What is the impulse response $h[n]$ of this LTI system?
- Look up the matlab convolution command `conv()`. You must implement the above filter using this command (do not write any for-loops)
- In a **matlab script** generate the sine wave $s[n] = 5 \sin(\omega_0 n)$ and its noisy version given by $x[n] = s[n] + v[n]$. Let $\omega_0 = \frac{\pi}{200}$ and generate this for $n = 0$ to 1000. Here $v[n]$ is the Gaussian noise which you can generate using the command `randn()`. In one panel of a 2x2 figure plot $s[n]$ and $x[n]$ on top of each other.
- Filter the signal $x[n]$ with a moving average filter using the command `conv()` and set the shape parameter to 'full'. Do this for $M = 5, 21, 51$. For each M , plot the original signal $s[n]$ and the filtered signal $y[n]$ on top of each other in the remaining panels.
- What are your observations as M is changed? What are the trade-offs?
- Use your function `DT_Fourier()` to compute the DTFT of the noisy and the filtered signals for different M . Set $\omega = -10:0.01:10$. Plot the magnitude spectrum in a 2x2 figure. Note your observations.
- A simple digital differentiator has input output relation given by

$$y[n] = x[n] - x[n-1]$$

Repeat steps (a), (c), (d) & (f) for this filter and note your observations.

- In terms of frequency selectivity, what is the nature of all the above implemented filters?