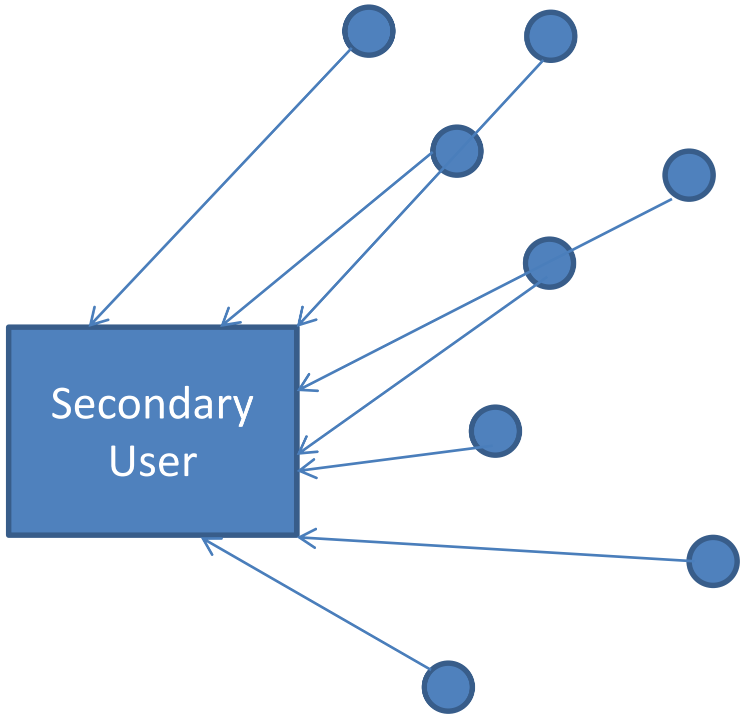
PRP Project

A Nonparametric Approach for Spectrum Sensing Using Bootstrap Technique

Outline

- Introduction to problem statement
- System model
- Hypothesis Testing
- Introducing bootstrap based method
- Explanation of algorithms for generating graph
- Graph

System Model



Primary User

- Here we want to sense presence of primary user by receiving y(t) from primary user.
- But y(t) may be only noise (Gaussian noise/non Gaussian) or it may be information signal and noise.
- Two Hypothesis

$$\mathcal{H}_0 \quad : \quad \mathbf{y}(t) = \mathbf{n}(t),$$
$$\mathcal{H}_1 \quad : \quad \mathbf{y}(t) = \mathbf{h}s(t) + \mathbf{n}(t), \;\; t = 1, 2, ..., L,$$

Hypothesis testing

$$\hat{T} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma,$$

where $$\hat{T} = \beta_1 - \frac{1}{M-1}\sum_{i=2}^{M}\beta_i.$$

Where ϒ is the test threshold to ensure a target false alarm probability which is define as under

$$P_f = \Pr(\hat{T} > \gamma | \mathcal{H}_0).$$

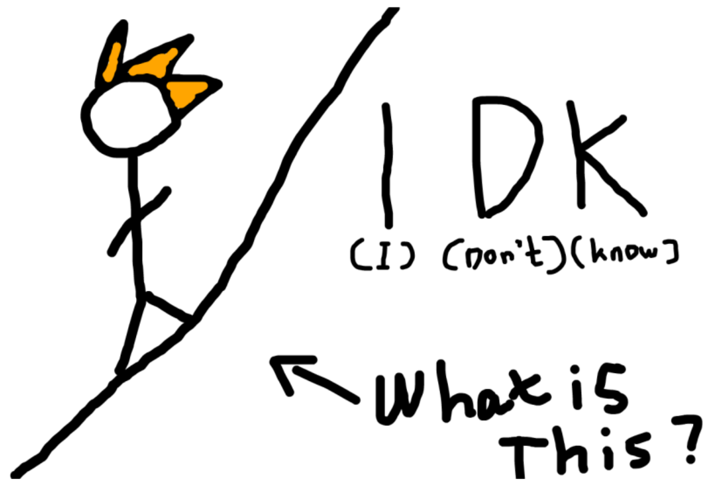
So, false alarm probability is with us which dependence on how much you want for you application or analysis and according to that you decide what you want and we can find ϒ. For that we have one method called **<u>Bootstrap method</u>**

Why bootstrap?

- The bootstrap technique is an attractive tool for estimating parameter or testing hypothesis when conventional methods are no longer valid.
- In other word, other method make some assumptions on Gaussianity and large sample size, which are inapplicable to this case.
- Where as bootstrap distribution is free of this things and works in small samples.

What is Monte Carlo Simulation?

- Monte Carlo simulation, or probability simulation, is a **technique used to understand the impact of risk and uncertainty** in financial, project management, cost, and other forecasting models.
- **Forecasting models**
  - Any model plans ahead for the future and for that some assumptions on that are required.
  - Based on past data or past experience you can draw an estimate
- **Estimating Ranges of Values**



What Monte Carlo Simulation can Tell You?

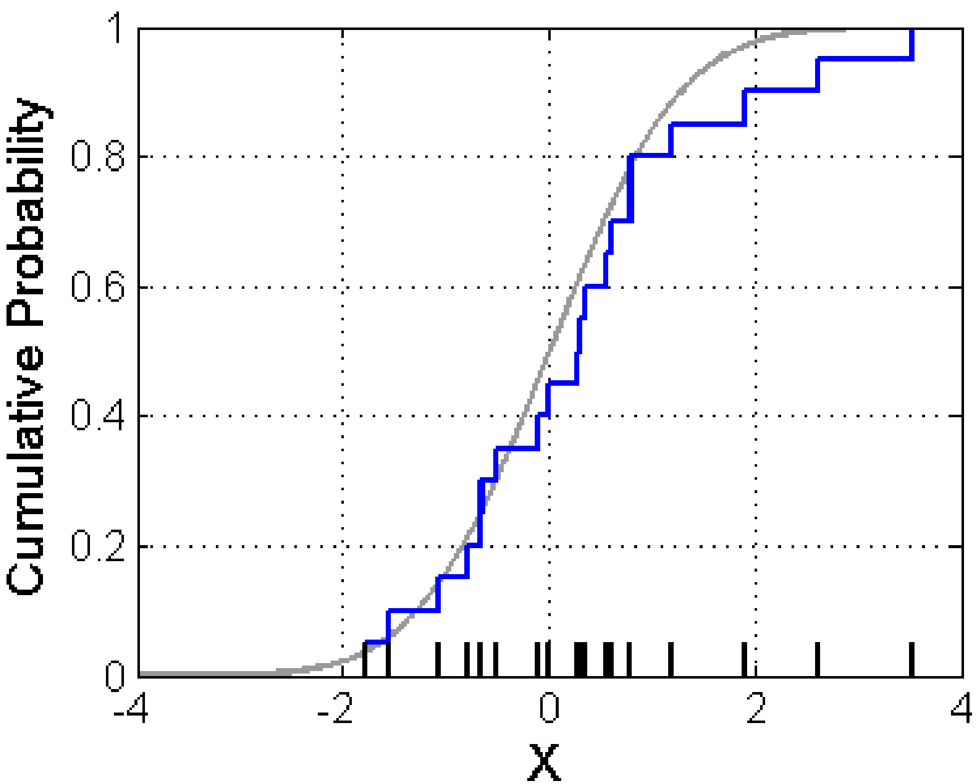
- When you have a range of values as a result, you are beginning to understand the risk and uncertainty in the model. The key feature of a Monte Carlo simulation is that it can tell you – based on how you create the ranges of estimates – how *likely* the resulting outcomes are.

How it works?

- A random value is selected for each of the tasks, based on the range of estimates. The model is calculated based on this random value.
- **The result of the model is recorded, and the process is repeated.**
- A typical Monte Carlo simulation calculates the model hundreds or thousands of times, **each time using different randomly-selected values.**
- When the simulation is complete, we have a large number of results from the model, each based on random input values.
- **These results are used to describe the likelihood, or probability, of reaching various results in the model.**

Method

- The basis bootstrap approach uses **Monte Carlo Sampling** to generate an **empirical estimate** of ɼ's sampling distribution.
- Empirical estimation estimates the CDF of random variable by assigning equal probability to each observation in sample.
- Let try to understand with example.



Understand bootstrap Method which Example

- We will illustrate the concept of sampling distributions with a simple example. Figure 1 shows three pool balls, each with a number on it. Two of the balls are selected randomly (with replacement) and the average of their numbers is computed.

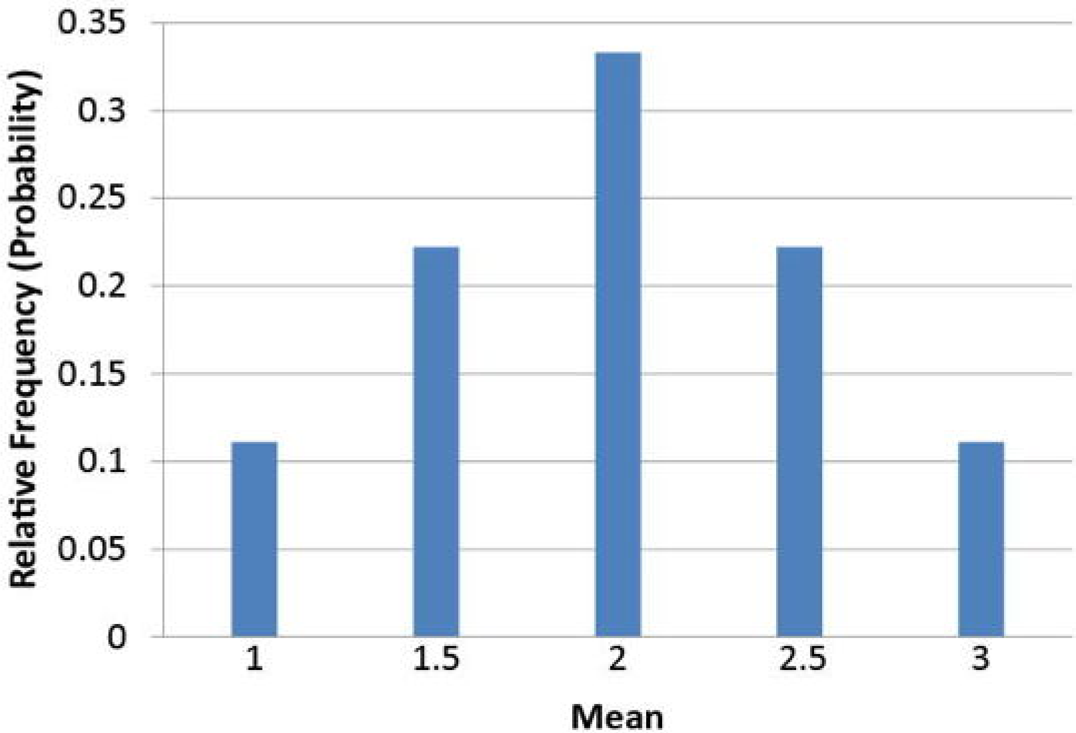
Figure 1. The pool balls.

Notice that all the means are either 1.0, 1.5, 2.0, 2.5, or 3.0. The frequencies of these means are shown in Table 2. The relative frequencies are equal to the frequencies divided by nine because there are nine possible outcomes.

Outcome	Ball 1	Ball 2	Mean
1	1	1	1.0
2	1	2	1.5
3	1	3	2.0
4	2	1	1.5
5	2	2	2.0
6	2	3	2.5
7	3	1	2.0
8	3	2	2.5
9	3	3	3.0

Mean	Frequency	Relative Frequency
1.0	1	0.111
1.5	2	0.222
2.0	3	0.333
2.5	2	0.222
3.0	1	0.111

Relative frequency (Probability)



Most General Procedure of Bootstrap

THE BOOTSTRAP PRINCIPLE

1) Given an i.i.d data set $\chi = [x_1, x_2, \ldots, x_L]$.
2) Draw a bootstrap sample set $\chi^* = [x_1^*, x_2^*, \ldots, x_L^*]$ via resampling χ with replacement. An example can be : $\chi^* = [x_1, x_1, \ldots, x_8]$.
3) Compute the bootstrap statistic $\hat{\theta}^*$ from χ^*.
4) Repeat 2) and 3) B times to obtain a set of bootstrap statistic $\{\hat{\theta}^*(b), b = 1, 2, \ldots, B\}$.
5) Estimate the statistical properties of $\hat{\theta}$ from $\hat{\theta}^*(b)$.

Cont.

- Here, Step 2 and 3 are part of **Monte-Carlo procedure**

Than, what should be value of B?

- Again, let me clear that B is number of time this Monte-Carlo procedure repeated.
- Practical size of B depends on the tests to be run on the data.
- Typical value :- $B \leq 1000$

Bootstrap for hypothesis testing

Hypothesis for Bootstrap

$$\mathcal{H}_0 : \vartheta \leq \vartheta_0$$
$$\mathcal{H}_1 : \vartheta > \vartheta_0$$

Test statistics for bootstrap

$$\hat{T}_b = \hat{\vartheta} - \vartheta_0.$$

Two guidelines for bootstrap hypothesis testing

First Guideline: Resample $\hat{\theta}^* - \hat{\theta}$, not $\hat{\theta}^* - \theta_0$.

Second Guideline: $\Pr^*(|\hat{\theta}^* - \hat{\theta}|/\hat{\sigma}^* > \hat{t}) = .05$

Cont.

$$\alpha = \frac{1}{B}\sum_{b=1}^{B} I[(\hat{\vartheta}^*(b) - \hat{\vartheta}) > \gamma_b],$$

where $I[\cdot]$ denotes the indicator function.

Where α = false alarm probability in spectrum sensing (Given)
B = Number of time experiment repeated
ϒb = Threshold

Application to the proposed detector

Hypothesis for application

$$
\begin{aligned}
\mathcal{H}_0 \quad &: \quad T = 0, \\
\mathcal{H}_1 \quad &: \quad T > 0,
\end{aligned}
$$

Test statistics for application

$$T = \lambda_1 - \frac{1}{M-1}\sum_{i=2}^{M} \lambda_i$$

Why require bias correction

- As we know signal don’t have DC component but our noise has DC component.
- Now if we have large number of samples, this DC component has negligible effect, but here we take less samples for bootstrap and as we resample and repeat samples this DC component repeats and has considerable effect so we require bias correction.

Bootstrap Bias Correction

Input: $\mathbf{Y} = [\mathbf{y}(1), \mathbf{y}(2), \ldots, \mathbf{y}(L)]$.

1) Compute the sample eigenvalues β_i, $i = 1, 2, \ldots M$.

2) Draw a bootstrap sample set $\mathbf{Y}^*$ from $\mathbf{Y}$.

3) Compute the bootstrapped sample eigenvalues: β_i^*, $i = 1, 2, \ldots M$.

4) Repeat 2) and 3) B_1 times to obtain

$$Bias(\beta_i) = \frac{1}{B_1} \sum_{b=1}^{B_1} \beta_i^*(b) - \beta_i, \; i = 1, 2, \ldots, M.$$

5) Compute the bias reduced sample eigenvalue $\hat{\beta}_i = \beta_i - Bias(\beta_i)$, $i = 1, 2, \ldots, M$.

Output: The ordered bias reduced sample eigenvalue $\hat{\beta}_1 > \hat{\beta}_2 > \ldots > \hat{\beta_M}$.

Detection Procedure Using Bootstrap

Input: $\mathbf{Y} = [\mathbf{y}(1), \mathbf{y}(2), \ldots, \mathbf{y}(L)]$.
Target false alarm probability α.

1) Compute the bias corrected sample eigenvalues using Table III and obtain the test statistic

$$\hat{T} = \hat{\beta}_1 - \frac{1}{M-1}\sum_{i=2}^{M}\hat{\beta}_i.$$

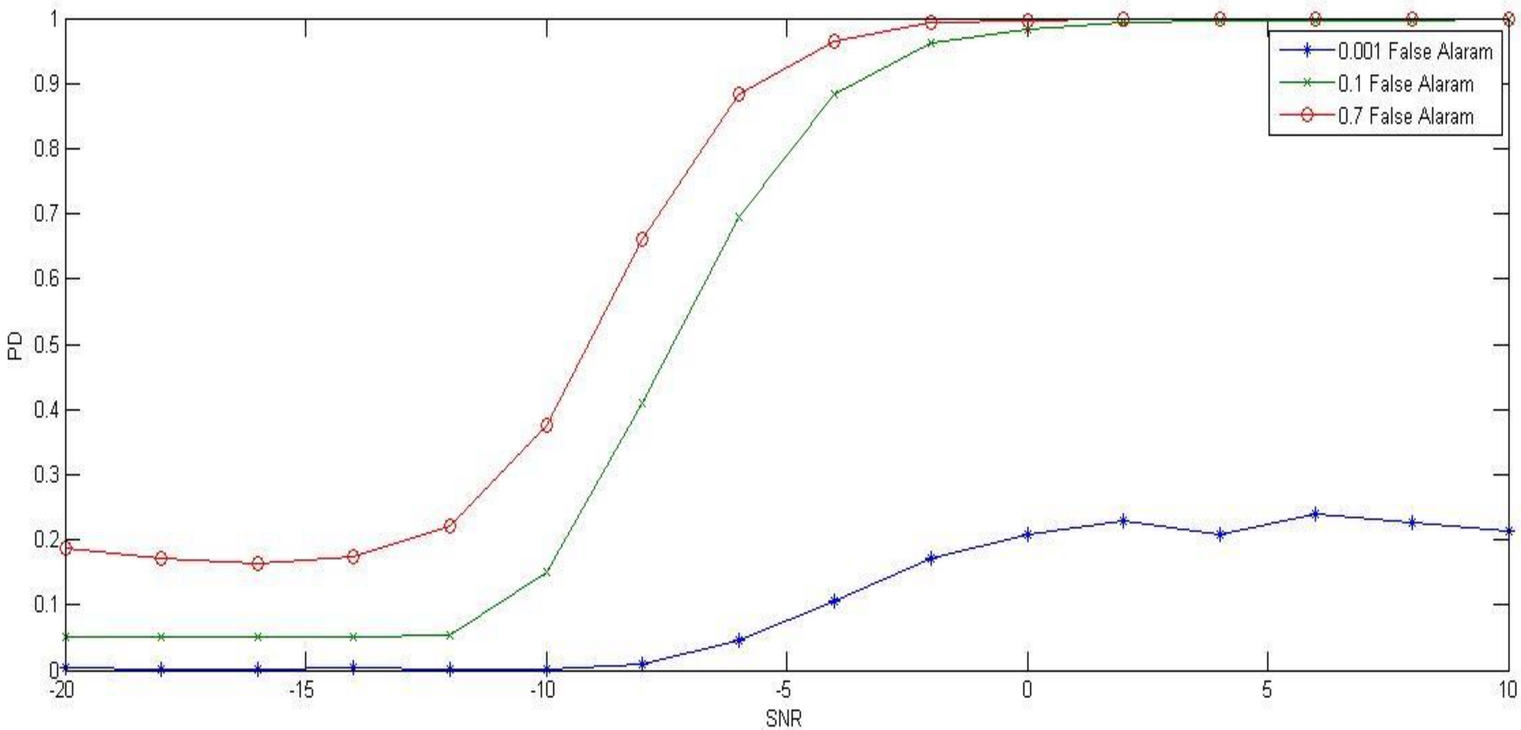
2) Draw a bootstrap sample set $\mathbf{Y}^*$ from $\mathbf{Y}$.
3) Compute the bias corrected bootstrap test statistic

$$\hat{T}^* = \hat{\beta}_1^* - \frac{1}{M-1}\sum_{i=2}^{M}\hat{\beta}_i^*.$$

4) Repeat 2) and 3) B times. Ranking the bootstrap statistics as $(\hat{T}^*(1) - \hat{T}) \leq \ldots \leq (\hat{T}^*(k) - \hat{T}) \leq \ldots \leq (\hat{T}^*(B) - \hat{T})$
5) From the ordered statistics, choose the index k by $\alpha = 1 - k/B$. The test threshold is obtained as $\gamma = \hat{T}^*(k) - \hat{T}$.

Output: Hypothesis testing $\hat{T} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma$.

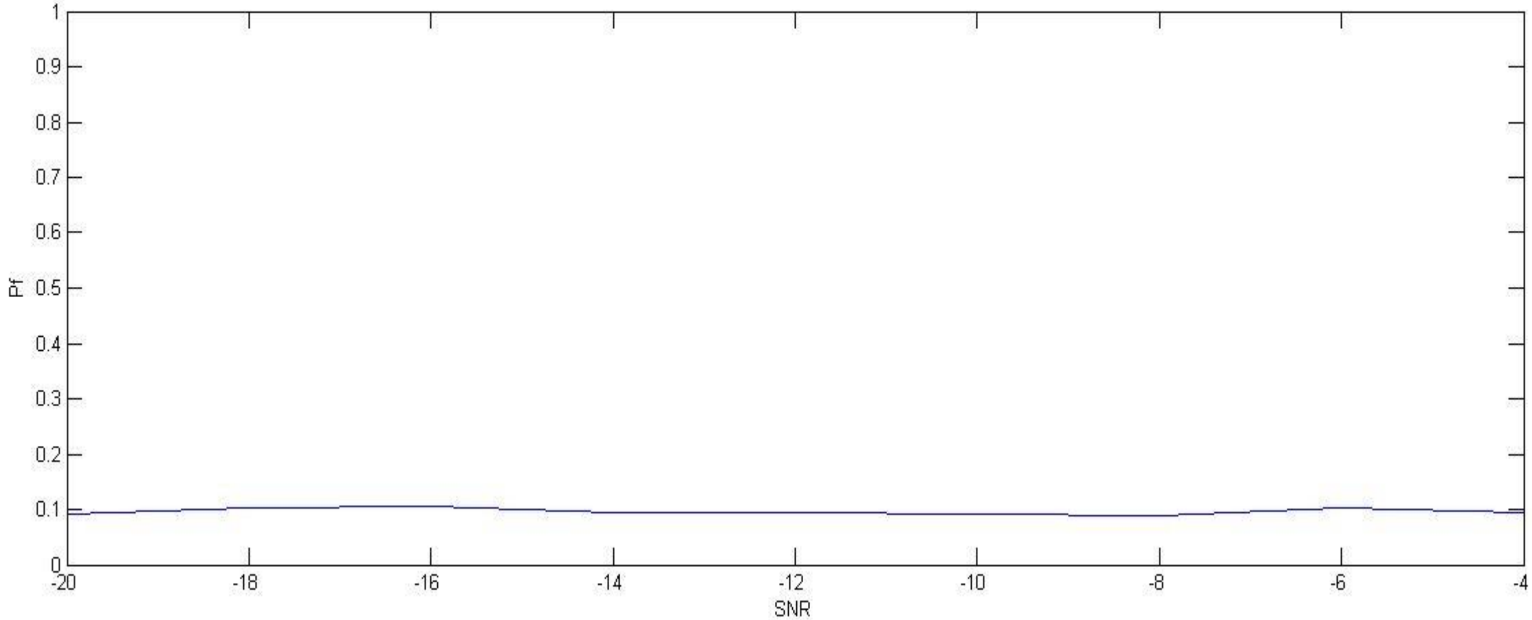
Result 1



M=4 – 4 Rayleigh fading channel
Primary signal model as Gaussian distributed
Sample Size L = 100
Laplacian Noise

False alarm probability alpha = 0.1
B1 = 30
B = 300

Result 2



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References

- A Nonparametric Approach for Spectrum Sensing Using Bootstrap Techniques.pdf
- Monte Carlo Simulation.pdf
- Two guideline for hypothesis of bootstrap method.pdf