Errata of Algorithm 2 in the Paper "A Novel Incremental Principal Component Analysis and Its Application for Face Recognition"

Corrected by: Deep C. Patel (1401010), Maunil R. Vyas(1401007), Shreyas G. Patel(1401025)

Algorithm 2—Proposed SVDU-IPCA Algorithm: Given the original data $X_1 = [x_1, x_2, \ldots, x_m]$ and the rank-k eigen-decomposition of $\Sigma_1 = P^{\mathrm{T}}P$, for the newly added data $[x_{m+1}, x_{m+2}, \ldots, x_{m+r}]$, do the following.

1) Compute the matrix Σ_2 and Σ_3 according to (2).
2) Obtain the best rank-k approximation of $P_{l\times m}$, which is

$$\tilde{P}^{(1)} = \begin{bmatrix} I_k \\ 0 \end{bmatrix}_{l\times k} \Lambda_k V_k^{\mathrm{T}}.$$

Error 1:

Here, there should be only V_k instead of Transpose (V_k).

3) Compute Q_1 according to (3) and Q_3 as the square root of $\Sigma_3 - Q_1^{\mathrm{T}}Q_1$ in (5).
4) Obtain the QR decomposition $\left(I_{(l+r)\times(l+r)} - \begin{bmatrix} I_k \\ 0 \end{bmatrix}[I_k\ 0]\right)\begin{bmatrix} Q_1 \\ Q_3 \end{bmatrix} = JK$, i.e.,

$$\begin{bmatrix} 0_{k\times k} & \\ & I \end{bmatrix}_{(l+r)\times(l+r)} \begin{bmatrix} Q_1 \\ Q_3 \end{bmatrix} = JK.$$

Error 2:

We should not take Q_3 as square root of $\Sigma_3 - Q_1^{\mathrm{T}}Q_1$, instead find Eigen decomposition of $\Sigma_3 - Q_1^{\mathrm{T}}Q_1$ as R*θ*R' and obtain Q_3 = R*sqrt(θ)*R'.

5) Obtain the SVD of the smaller matrix

$$\begin{bmatrix} \Lambda_k & [I_k\ 0]\begin{bmatrix} Q_1 \\ Q_3 \end{bmatrix} \\ 0 & K \end{bmatrix} = \hat{U}\hat{\Lambda}\hat{V}^{\mathrm{T}}.$$

Important Caution 1:

Here,

$$\Lambda_k = (\mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_k))^{1/2}$$

6) Obtain the best rank-k approximation of $\begin{bmatrix} P & Q_1 \\ Q_2 & Q_3 \end{bmatrix}$, which is

$$\begin{bmatrix} P & Q_1 \\ Q_2 & Q_3 \end{bmatrix} = \left([I_k\ J]\hat{U}\right)\hat{\Lambda}\left(\begin{bmatrix} V_k & 0 \\ 0 & I \end{bmatrix}\hat{V}\right)^{\mathrm{T}}.$$

Error 3:

Writing as $[I_k J]$ will give dimension error as dimensions of I_k and J would not match. So, append zeros below I_k to match the dimensions and then compute like $\begin{bmatrix} I_k & J \\ 0 & \end{bmatrix}$

7) Obtain the best rank-k approximation of

$$\Sigma = \begin{bmatrix} P & Q_1 \\ Q_2 & Q_3 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} P & Q_1 \\ Q_2 & Q_3 \end{bmatrix}.$$