MA7008 – Financial Mathematics

Case Study Report 2019/20

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1 INTRODUCTION

The five companies that I decided to study to develop a portfolio are Ggreggs ,Campari , First Derivatives , UBM Development and Experian not all the company are operating on the London stocks market exchange. We will further analyse them individually to give a brief description of the core business and a short profile. The analysis period is going from 03/01/2017 to the 30/12/2019 it's almost three years interval. The full period is not three years(365days*3) due to the fact that the company are operating in different markets with different days of holidays. This make sure that all the companies have the same number of observations rows for the time period.

The data set is sourced from the website Yahoo Finance, from where we downloaded all the csv files.

The first 5 rows of Greggs columns are displayed in figure 1. We will clean the data sets and preserve just the Date as index and GRG.L that is the Adjusted closed price. This will be done for all assets.

And all the Adjusted closed prices merged in a single flat file shown in figure 1.

GRG.L

Date	Open	High	Low	Close	GRG.L	Volume
03/01/2017	980.5	981.5	966	979.5	890.862915	186340
04/01/2017	968	985	968	981.5	892.681946	201047
05/01/2017	975	995	975	984	894.955688	211124
06/01/2017	991.5	991.5	970.200012	980	891.317627	185476
09/01/2017	975.5	990.299988	975.5	985	895.865234	193063

Stocks Finance

Date	GRG.L	CPR.MI	FDP.L	UBS.VI	EXPN.L
2017-01-03	890.8629	4.4649	2135.4373	27.2282	1503.4973
2017-01-04	892.6819	4.4842	2149.1130	27.4008	1497.8094
2017-01-05	894.9557	4.4939	2124.6914	27.4008	1487.3816
2017-01-09	895.8652	4.4456	2124.6914	26.7622	1496.8615
2017-01-10	883.1321	4.4890	2166.6968	26.8398	1496.8615

Figure 1

1.1 Companies Assets Description

Greggs

Greggs PLC is the operator of the largest bakery chain in the United Kingdom, specializing in sandwiches, savouries , and other bakery-related products. Greggs' stores operate under two names, Greggs, which focus on takeaway sales, and Bakers Oven, which offer in-store dining. The company's Bakers Oven units feature in-store bakeries. The Greggs units rely on large regional bakeries that serve clusters of retail outlets. The company's stores are located throughout the United Kingdom. Greggs also operates two stores in Belgium, units that represent the beginning of the company's expansion campaign into mainland Europe.

For the first three decades of its existence, Greggs was a small, local business. The business was founded by John Gregg during the 1930s, when he opened a small bakery in the Tyne suburb of Newcastle, England. There the business stood, mostly unchanged for the next 30 years, as John Gregg served nearby residents his selection of breads, rolls, cakes, and related items. Greggs did not begin to assume the stature of an industry giant until John Gregg died unexpectedly in 1964. His son, Ian Gregg, take over the family business. Ironically, it was its founder's death that gave Greggs new life. In 1984, the £37-million-in-sales Greggs completed its initial public offering (IPO) of stock, finding a wealth of investors willing to pay 135 pence per share for a stake in the company's future. A decade after its IPO, Greggs stood as a towering chain, with the creation of new divisions spawning clusters of new shops. By 1994--a significant year in the company's history--Greggs consisted of more than 500 stores operating in seven regional divisions. The rest is history until the most recent years.

Campari

Long a single-product company, Davide Campari-Milano S.p.A. has transformed itself into one of the world's top ten spirits and beverages specialists. The company is famed for its Campari Bitters, which remains a world leader in its category after nearly 150 years.

Yet, through a series of acquisitions in the 1990s and 2000s, Campari has built a strong stable of internationally recognized brands, including Cinzano, SKYY Vodka, Cynar, Ouzo 12, Gold Cup, and Gregson's. Other brands in the group's stable include winemakers Sella & Mosca and Zedda Piras, both acquired in 2002, and Biancosarti. The company also produces a line of soft drinks, including Lemonsoda, Oransoda, Pelmosoda, and the nonalcoholic aperitif Crodino.
In addition to its own brands, Campari holds manufacturing and distribution licenses for a number of brands, including Glenfiddich, Grant's, Henkell Trocken, Lipton Ice Tea, and Jägermeister. Spirits represent nearly 65 percent of the company's sales, which topped EUR 660 million ($650 million) in 2002. Soft drinks contributed more than 19 percent, and wines added nearly 15 percent to sales. Campari distributes to more than 190 countries; Italy, however, remains the group's single largest market, at more than 50 percent of sales. Davide Campari-Milano has been controlled by the Garavoglia family since the 1970s; the family reduced its holding to 51 percent after listing the company on the Milan stock exchange in 2001. As said previously this company is not operating in the London stocks exchange. In the analysis we will still be considering a single market index.

First Derivatives

Founded in 1996, FD initially focused on the capital markets sector providing mission-critical support for core systems. The experience gained working for some of the world’s leading investment banks led to the formation of a software business, utilising the kdb+ database from Kx Systems as the foundation for applications such as market surveillance and liquidity management. The acquisition of Kx Systems in 2014 and the rebranding of our software as Kx technology was the springboard to target other markets, including digital marketing, the industrial internet of things, utilities, retail, telecoms and automotive. FD is a global technology provider with 20 years of experience working with some of the world’s largest finance, technology, retail, pharma, manufacturing and energy institutions.
The Group’s Kx technology, incorporating the kdb+ time-series database, is a leader in high-performance, in-memory computing, streaming analytics and operational intelligence.

The Company's software is also used in a range of industries including telecoms, digital marketing, pharma and utilities to help organisations solve their most demanding data management and analytics challenges.

The Company holds a niche market position in terms of domain knowledge of capital market asset classes (equities, fixed income, foreign exchange, commodities, etc), as well as expertise in leading financial services systems. This combination of domain knowledge and technical expertise in leading financial services technologies has motivated the Company to invest in developing its own product suite. First Derivatives is a publicly held company, trading on the London Stock Exchange (LSE FDP.L) and Irish Stock Exchange (IEX:FDP.I .The Company has continued to expand its service offering and now has operational bases in Europe, North America, Asia and Australia to service its global client base. It is recognized as one ofthe fastest-growing capital markets service providers in the world. FD now employs over 2,400 employees worldwide.

UBM Development

UBM Development creates real estate for Europe's top cities. The strategic focus is on the Residential and Office asset classes in major European metropolitan areas like Vienna, Berlin, Munich or Prague. With over 145 years of history, UBM is a one-stop provider for the entire development value chain from initial planning all the way to marketing. The company's shares are listed in the Prime Market of the Vienna Stock Exchange, the segment with the highest transparency requirements. A syndicate comprising the industrialists Ortner and Strauss holds an investment of roughly 39% as the core shareholder. The Executive Committee, an expert board of 20 UBM managers, has invested €5m in UBM shares. The headquarters of UBM Development AG are located in Vienna. Together with its subsidiaries, the company has also established an efficient network in Central Europe. Strong local integration and wide-ranging market expertise support the timely identification and utilisation of market opportunities which, in turn, protect the future pipeline. UBM's interdisciplinary team develops ideas for new projects and innovative usage concepts, and also brings wide-ranging interests and users together offering a complete range of services required for this integrated approach from a single hand:

market analysis, project development, planning and project management, financing, rental and asset management. With this know-how, they cover all phases of the real estate value chain internally. In addition to real estate development, UBM is also active as a hotel lessee. The subsidiary UBM hotels Management GmbH was founded in 2016 to bundle the operational know-how for hotel management and to combine all UBM hotel leasing operations.

As Campari the UBM Development is not listed in the FTSE 100. The fact that the company is operating in almost all Europe will balance the absence of the Austrian market index given the fact that the FTSE 100 can be considered as a general index for the whole Europe.

Expenian

Experian plc is an Irish-domiciled multinational consumer credit reporting company.

Experian collects and aggregates information on over one billion people and businesses including 235 million individual U.S. consumers and more than 25 million U.S. businesses. Based in Dublin, Ireland, the company operates in 37 countries with offices in Brazil, the United Kingdom, the United States. The company employs approximately 17,000 people and reported revenue for 2018 of US$4.6 billion. It is listed on the London Stock Exchange and is a constituent of the FTSE 100 Experian is a partner in the U.K. government's Verify ID system and USPS Address Validation. It is one of the "Big Three" credit-reporting agencies, alongside TransUnion and Equifax. In addition to its credit services, Experian also sells decision analytic and marketing assistance to businesses, including individual fingerprinting and targeting. Its consumer services include online access to credit history and products meant to protect from fraud and identity theft. Like all credit reporting agencies, the company is required by U.S. law to provide consumers with one free credit report every year.

1.2 Expected Return and Volatility(Standard Deviation)

In order to value an asset, we must develop a mathematical description of how the underlying asset behaves. In this chapter we will use our real stocks market data and performs some basic statistical tests. When we operate as a well-informed investor , one of the first step is to know the expected return and standard deviation of each single asset. Further we will need examine the intertemporal relation between risk and expected returns.

The price of an asset is, of course, a measure of investors' confidence, and, as such, is strongly dependent on news, rumours, speculation, and so on. Although an oversimplification, it is reasonable to assume that the market responds instantaneously to external influences, and hence: we derive that " *the current asset price reflects all past information".* Under this hypothesis, if we want to predict the asset price at some future time, knowing the complete history of the asset price gives no advantage over just knowing its current price ,there is no edge to be gained from 'reading the charts'.
In figure 2 we plot all the normalized to a starting point of 100 of the daily shares prices from the beginning of January 2017 to the end of December 2019.

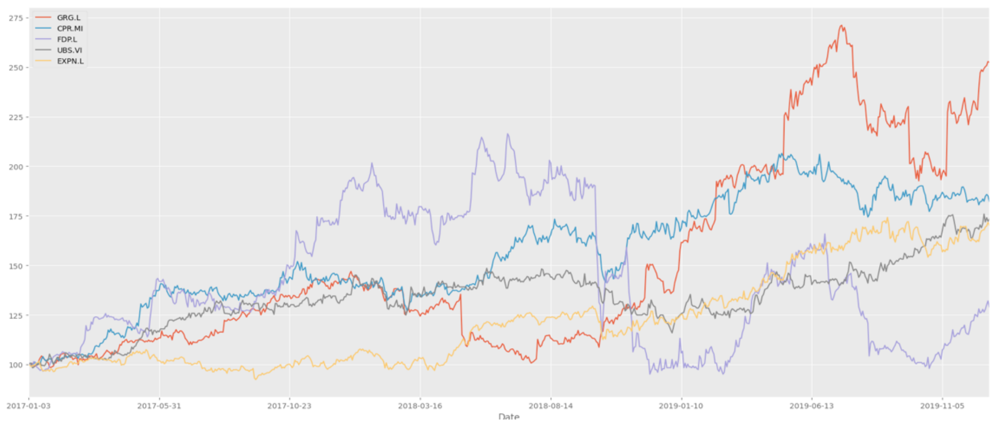


Figure 2 Daily shares Prices

A much more clear picture we will be available when calculating the daily returns. The daily return $R(t_0, t_1)$ is expressed in the formula above.

$$R(t_0, t_1) = \frac{P_{t_1} - P_{t_0}}{P_{t_0}}$$

Where $R(t_0, t_1)$ is the daily return and P_{t1} is the closed price at time t_1 P_{t0} is the closed price at time t_0
The daily return distributions $R(t_0, t_1)$ are the daily percentage change in price used to calculate the Average Returns for all the stocks, there are several method to calculate the average, using the arithmetic average of the returns means to sum up all the returns and divide them by the number of return period of times. Let R_t denote the simple daily return of an asset. The continuously compounded daily return r_t is defined as:

$$r_t = \ln(1 + R_t) = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

Where ln() is the natural log function. This is to be contrasted with $R(t_0, t_1)$; which is the simple growth rate in prices between days t + 1 and t without any compounding.

From the above formula we derive that since ln (x/y)= ln(x) − ln(y) it follows that

$$\begin{aligned} r_t &= \ln\left(\frac{P_t}{P_{t-1}}\right) \\ &= \ln(P_t) - \ln(P_{t-1}) \\ &= p_t - p_{t-1}, \end{aligned}$$

where $p_t = \ln(P_t)$,the continuously compounded monthly return, r_t can be computed simply by taking the first difference of the natural logarithms of daily prices. We will calculate first the daily and then the monthly continuously compounded monthly returns. The daily and monthly percentage change compounded returns data set are reported in figure 3.In Figure 3.1 the monthly moving period average candle box it's reported.

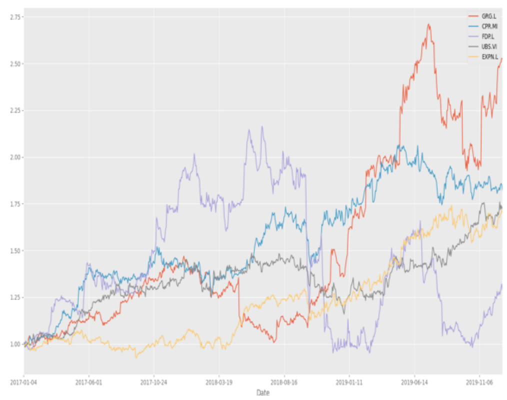
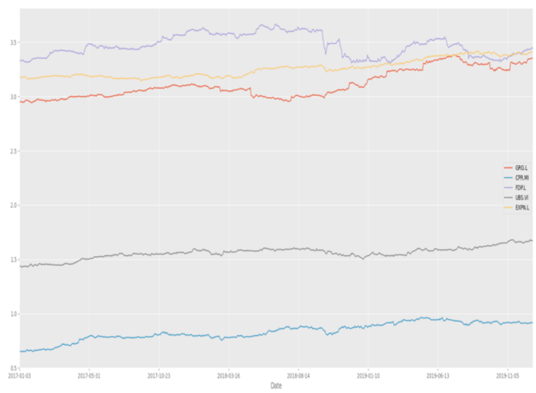


Figure 3 Daily Compounded Returns



Monthly Compounded Returns

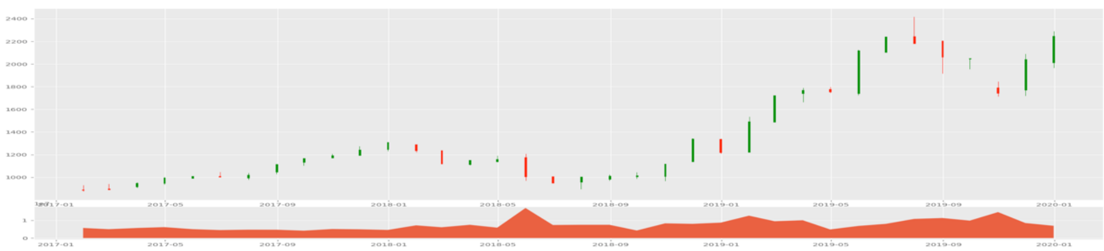


Figure 3.1 Monthly Candle Box plot of Greggs.

After the Returns we do need to calculate the st.d that is expressing the risk of each individual stock.

The standard deviation is calculated by the squared root of the summary of the difference of each returns at time t and the mean return elevated to the potency of 2. The formula it' s the above.

$$SD= \sqrt{\frac{1}{(N-1)} \sum_{i=1}^{N}(R_i - \bar{R})^2},$$

The standard deviation is the squared root of the variance. After all the means (arithmetic average) and the standard deviations have been calculated we will go further with the correlations between the assets returns. The monthly returns and standard deviations are presented in the above figure 4.

	GRG.L	CPR.MI	FDP.L	UBS.VI	EXPN.L	FTSE
Mean	0.02667349	0.0170964	0.00618857	0.01570014	0.01623869	0.002344
StD	0.086005	0.053655	0.136014	0.05134	0.042428	4.211542

After the calculations of the return we do need to annualize them. This will help to confront it with the annual risk free rate. The annual risk free rate is annualised so to have a fare comparison we will proceed with the annualization of the monthly assets returns. The result with the annual free risk rate in UK in the years from 2015 to 2020 in figure 4

Simple Return	0.02667349	0.0170964	0.00618857	0.01570014	0.01623869
Annualized Return	0.37147582	0.22559056	0.07684343	0.2055526	0.21324548

$$R_A = (1 + R)^{12} - 1$$

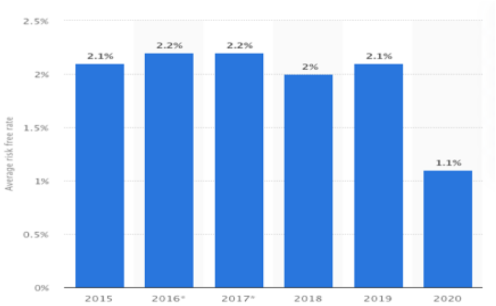
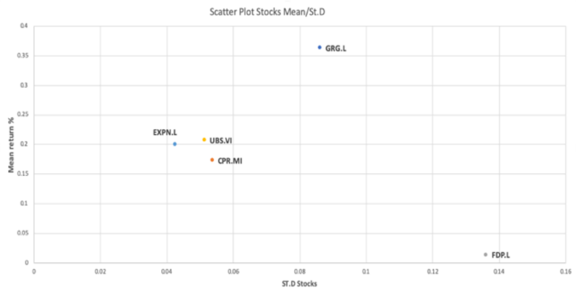


Figure 4 Risk free rate Uk



1.3 Assets Returns Correlations

The correlation coefficient ranges from −1 to 1. A value of 1 implies that a linear equation describes the relationship between X and Y perfectly, with all data points lying on a line for which Y increases as X increases.

A value of −1 implies that all data points lie on a line for which Y decreases as X increases. A value of 0 implies that there is no linear correlation between the variables. The correlation and specifically the Pearson's correlation coefficient it's giving an suggestion on the possible linear relationships possible in between the assets. The single coeff is calculated from the formula :

$$r = r_{xy} = \frac{\mathrm{Cov}\,(x, y)}{S_x \times S_y}$$

Where the Cov(x,y) is the covariance in between the assets x and y , the Sx is the st.d of the asset x and the Sy the st.d of the asset y. The Correlation coefficient are presented in figure 5 with the hitmap of the correlation values.

Correlation	Using Excel Data Analysis				
	GRG.L	CPR.MI	FDP.L	UBS.VI	EXPN.L
GRG.L	1				
GRG.L	0.11646518	1			
GRG.L	-0.1568919	0.30952296	1		
GRG.L	0.0090293	0.23637646	0.18401205	1	
GRG.L	0.18077028	0.27920469	0.27959042	0.15290011	1

Correlation using Matrix Algebra					
	GRG.L	CPR.MI	FDP.L	UBS.VI	EXPN.L
GRG.L	1	0.11646518	-0.1568919	0.0090293	0.18077028
CPR.MI	0.11646518	1	0.30952296	0.23637646	0.27920469
FDP.L	-0.1568919	0.30952296	1	0.18401205	0.27959042
UBS.VI	0.0090293	0.23637646	0.18401205	1	0.15290011
EXPN.L	0.18077028	0.27920469	0.27959042	0.15290011	1

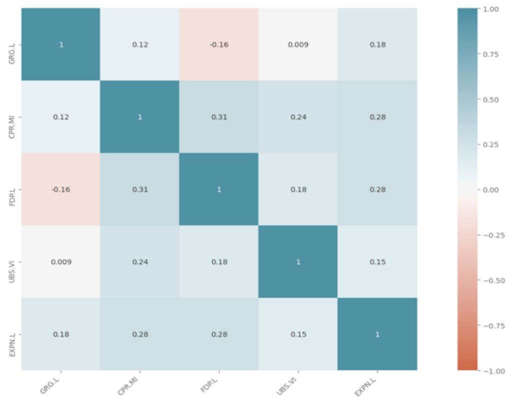


Figure 5 Stocks Correlation

The porpoise of a portfolio is to minimize the risk using the diversification, this means not just investing the whole amount in one single features but spreading the 100% of the capital on different assent in different proportions and knowing that all the assets move in the same direction will neutralize the effect of the diversification. In all our stocks the correlation is not extremely high or low.

Further Analysis

The single distributions of the returns for each asset should be the most possible close to the normality this will help in the optimization of the prediction. We will not use the fitted distributions but the empirical one. In figure 5 all the fitted normal distribution showing us that overall all the distribution are normally distributed. With different values of variance.

$$\log\left(\frac{S(t_{i+1})}{S(t_i)}\right) = \log\left(1 + \frac{S(t_{i+1}) - S(t_i)}{S(t_i)}\right) \approx \frac{S(t_{i+1}) - S(t_i)}{S(t_i)}$$

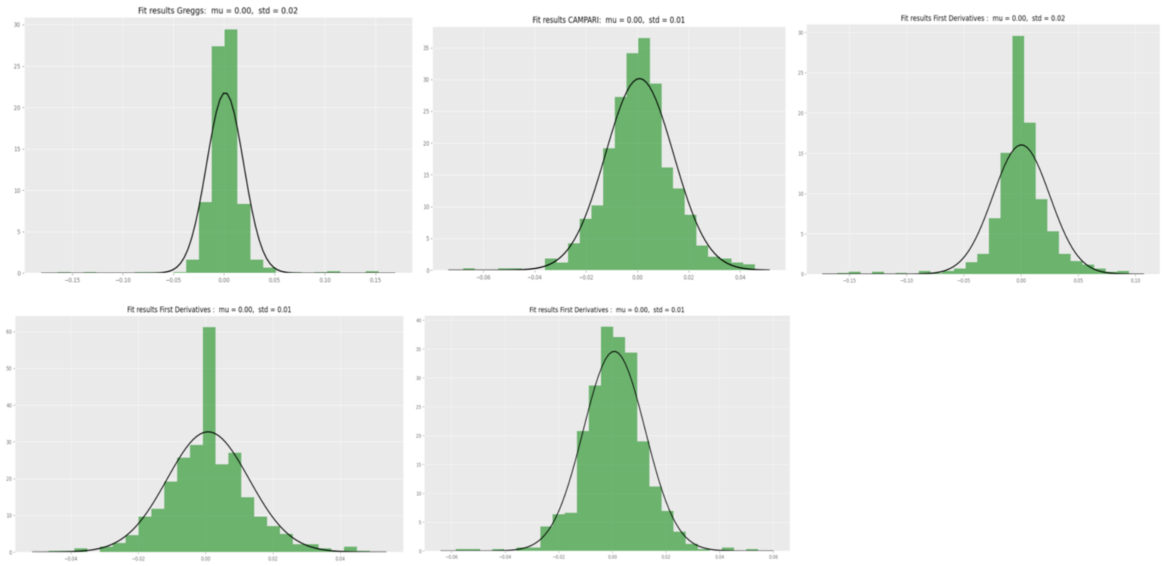


Figure 5 Fitted normal distributions

2 Quadratic Programming Problem

If we denote that the amount of wealthy that an investor is willing to invest as x_i and the assets i (i= GRG,CMP,FDP.UBS.EXPN) and that if all the wealthy is invested in the five assets in the way that $x_{GRG} + x_{CMP} + x_{FDP} + x_{UBS} + x_{EXPN} = 1$. The portfolio return $R_{p,x}$ will be the random variable :

$$R_{p,x} = x_{GRG} R_{GRG} + x_{CMP} R_{CMP} + x_{FDP} R_{FDP} + x_{UBS} R_{UBS} + x_{EXPN} R_{EXPN}$$

So the portfolio is built using the x- weight x_{GRG}, x_{CMP} , x_{FDP} , x_{UBS} , x_{EXPN} . The portfolio expected return is :

$$u_{p,x} = E[R_{p,x}] = x_{GRG} R_{GRG} + x_{CMP} R_{CMP} + x_{FDP} R_{FDP} + x_{UBS} R_{UBS} + x_{EXPN} R_{EXPN}$$

and the variance of the portfolio is

$$\sigma^2_{p,x} = \mathrm{var}(R_{p,x}) \qquad \sigma_P^2 = \sum_{i=1}^{N} X_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j=1 \neq i}^{N} X_i X_j \sigma_{ij}$$

$= (x^2_{GRG} * \sigma^2_{GRG}) + (x^2_{CMP} * \sigma^2_{CMP}) + (x^2_{FDP} * \sigma^2_{FDP}) + (x^2_{UBS} * \sigma^2_{UBS}) + (x^2_{EXPN} * \sigma^2_{EXPN}) +$

$+ 2(x_{GRG} * x_{CMP} * cov_{GRG\ CMP}) + 2(x_{GRG} * x_{FDP} * cov_{GRG\ FDP}) + 2(x_{GRG} * x_{UBS} * cov_{GRG\ UBS})$

$+2(x_{GRG} * x_{EXPN} * cov_{GRG\ EXPN}) + 2(x_{CMP} * x_{FDP} * cov_{CMP\ FDP}) + 2(x_{CMP} * x_{UBS} * cov_{CMP\ UBS})$

$+ 2(x_{CMP} * x_{EXPN} * cov_{CMP\ EXPN}) + 2(x_{FDP} * x_{UBS} * corv_{FDP\ UBS}) + 2(x_{FDP} * x_{EXPN} * cov_{FDP\ EXPN})$

$+ 2(x_{UBS} * x_{EXPN} * cov_{UBS\ EXPN})$

As we can see the variance of the portfolio depends on five variance terms and ten covariance terms. Those as much as the number of assets will increase the terms of the calculation becomes complex. Just with five stocks the calculation become heavy, we will show the simplified algebra using the matrix notation.

For celerity and simplification we will use the example of a three assets portfolio matrix notation but we will apply the calculus on the five stocks of our portfolio.

2.1 Portfolio Characteristics Using Matrix Notation

Starting with a 3 X 1 column vectors containing the assets returns and portfolio weights.

$$\mathbf{R} = \begin{pmatrix} R_A \\ R_B \\ R_C \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x_A \\ x_B \\ x_C \end{pmatrix}$$

Since each element in R is a single random variable we call R a random vector. The probability distribution of the random vector R is the joint distribution of the elements R.

The returns as we saw are approximately normally distributed. The Distribution is characterized by the means , variance and covariance of the returns. The 3 X 1 (our case 5X1) vector of the expected portfolio returns is :

$$E[\mathbf{R}] = E\left[\begin{pmatrix} R_A \\ R_B \\ R_C \end{pmatrix}\right] = \begin{pmatrix} E[R_A] \\ E[R_B] \\ E[R_C] \end{pmatrix} = \begin{pmatrix} \mu_A \\ \mu_B \\ \mu_C \end{pmatrix} = \boldsymbol{\mu}$$

And the 3X3 (5X5 our case) covariance matrix of returns.

$$\mathrm{var}(\mathbf{R}) = \begin{pmatrix} \mathrm{var}(R_A) & \mathrm{cov}(R_A,R_B) & \mathrm{cov}(R_A,R_C) \\ \mathrm{cov}(R_B,R_A) & \mathrm{var}(R_B) & \mathrm{cov}(R_B,R_C) \\ \mathrm{cov}(R_C,R_A) & \mathrm{cov}(R_C,R_B) & \mathrm{var}(R_C) \end{pmatrix} = \begin{pmatrix} \sigma_A^2 & \sigma_{AB} & \sigma_{AC} \\ \sigma_{AB} & \sigma_B^2 & \sigma_{BC} \\ \sigma_{AC} & \sigma_{BC} & \sigma_C^2 \end{pmatrix} = \mathbf{\Sigma}.$$

It's worth to note that on the main diagonal we do have the variance value. The covariance matrix is symmetric (so that $\sum = \sum'$, where $\sum'$ denote the transpose of $\sum$) since cov (R_A,R_B)= cov(R_B,R_A). We to present the return vector , st.deviation , variance and the covariance matrix calculated from the monthly log returns of the assets in the figure 6 above using excel covariance function in data analysis. So using the matrix notation we will be able to calculate for any portfolio the returns and so the expected returns with the relative variance using the covariance matrix. The formulation is reported above.

Covar Using Excel Data Analysis

	GRG.L	CPR.MI	FDP.L	UBS.VI	EXPN.L
GRG.L	0.00739687				
CPR.MI	0.00053744	0.00287883			
FDP.L	-0.0018353	0.00225884	0.01849979		
UBS.VI	3.99E-05	0.00065113	0.00128494	0.00263577	
EXPN.L	0.00065964	0.0006356	0.00161346	0.00033305	0.00180014

Covar using matrix Algebra

	GRG.L	CPR.MI	FDP.L	UBS.VI	EXPN.L
GRG.L	0.00739687	0.00053744	-0.0018353	3.99E-05	0.00065964
CPR.MI	0.00053744	0.00287883	0.00225884	0.00065113	0.0006356
FDP.L	-0.0018353	0.00225884	0.01849979	0.00128494	0.00161346
UBS.VI	3.99E-05	0.00065113	0.00128494	0.00263577	0.00033305
EXPN.L	0.00065964	0.0006356	0.00161346	0.00033305	0.00180014

Asset Stats

	Mean	StD	Var
GRG.L	0.37147582	0.08600508	0.00739687
CPR.MI	0.22559056	0.05365477	0.00287883
FDP.L	0.07684343	0.13601395	0.01849979
UBS.VI	0.2055526	0.05133971	0.00263577
EXPN.L	0.21324548	0.04242802	0.00180014

Figure 6

Return of a portfolio :

$$R_{p,x} = \mathbf{x}'\mathbf{R} = (x_A, x_B, x_C) \cdot \begin{pmatrix} R_A \\ R_B \\ R_C \end{pmatrix} = x_A R_A + x_B R_B + x_C R_C.$$

Similarly the expected return on the portfolio :

$$\mu_{p,x} = E[\mathbf{x}'\mathbf{R}] = \mathbf{x}'E[\mathbf{R}] = \mathbf{x}'\boldsymbol{\mu} = (x_A, x_B, x_C) \cdot \begin{pmatrix} \mu_A \\ \mu_B \\ \mu_C \end{pmatrix} = x_A\mu_A + x_B\mu_B + x_C\mu_C.$$

And finally the variance :

$$\sigma_{p,x}^2 = \mathrm{var}(\mathbf{x}'\mathbf{R}) = \mathbf{x}'\Sigma\mathbf{x} = (x_A, x_B, x_C) \cdot \begin{pmatrix} \sigma_A^2 & \sigma_{AB} & \sigma_{AC} \\ \sigma_{AB} & \sigma_B^2 & \sigma_{BC} \\ \sigma_{AC} & \sigma_{BC} & \sigma_C^2 \end{pmatrix} \begin{pmatrix} x_A \\ x_B \\ x_C \end{pmatrix} = x_A^2\sigma_A^2 + x_B^2\sigma_B^2 + x_C^2\sigma_C^2 + 2x_Ax_B\sigma_{AB} + 2x_Ax_C\sigma_{AC} + 2x_Bx_C\sigma_{BC}$$

The condition that the portfolio weights sum to one is :

$$\mathbf{x}'\mathbf{1} = (x_A, x_B, x_C) \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = x_A + x_B + x_C = 1$$

2.2 Definition Quadratic Problem

If we assume that the investor chose portfolios to maximise expected returns subject to a defined level of risk ,or, equivalently ,to minimize risk subject to a target expected return.

All those portfolios lie above the minimum variance portfolio. This framework developed by Harry Markowitz, assume that investor wish to find portfolios that have the best expected return-risk trade-off. As we said previously the efficient portfolios can be defined in two equivalent way. Investor can try to minimize portfolio expected returns for a given level of risk(variance).

The Markowitz defined the constrained maximization problem to find an efficient portfolio as

$$\max_{\mathbf{x}} \mu_p = \mathbf{x}'\boldsymbol{\mu} \text{ s.t.}$$
$$\sigma_p^2 = \mathbf{x}'\Sigma\mathbf{x} = \sigma_{p,0}^2 \text{ and } \mathbf{x}'\mathbf{1} = 1$$

Where u_p is the expected portfolio return and o^2 the variance.

The equivalent representation is given when the investor is trying to minimize the risk(variance) subject to a defined level of return.

$$\min_{\mathbf{x}} \sigma_{p,x}^2 = \mathbf{x}'\Sigma\mathbf{x} \text{ s.t.}$$
$$\mu_p = \mathbf{x}'\boldsymbol{\mu} = \mu_{p,0}, \text{ and } \mathbf{x}'\mathbf{1} = 1$$

$$\sigma_P^2 = \sum_{i=1}^{N} X_i^2\sigma_i^2 + \sum_{i=1}^{N}\sum_{j=1\neq i}^{N} X_iX_j\sigma_{ij}$$

To find efficient portfolios of risky assets in practice, the dual problem is most often solved.

This is partially due to computational conveniences and partly due to investors being more willing to specify target expected returns rather than target risk levels. The efficient portfolio frontier is a graph of μ_p versus o_p for the set of efficient portfolios generated by solving the minimization problem for all possible target expected return levels u_p above the expected return on the global minimum variance portfolio.

In the next chapter we will build the efficient frontier solving the quadratic problem given a fixed expected returns.

Solving the constrained minimization problem we need to form the Lagrangian function :

$$L(x, \lambda_1, \lambda_2) = \mathbf{x}'\mathbf{\Sigma}\mathbf{x} + \lambda_1(\mathbf{x}'\boldsymbol{\mu} - \mu_{p,0}) + \lambda_2(\mathbf{x}'\mathbf{1} - 1)$$

Given the two constrains ($x' \mu = \mu_{p,0}$ and $x' 1 = 1$) there are two Lagrange multipliers λ_1 and λ_2.

In the case of three assets the first order conditions for a minimum are the linear equations

$$\begin{aligned}
\frac{\partial L(\mathbf{x}, \lambda_1, \lambda_2)}{\partial \mathbf{x}} &= 2\mathbf{\Sigma}\mathbf{x} + \lambda_1 \boldsymbol{\mu} + \lambda_2 \mathbf{1} = \mathbf{0}, \\
\frac{\partial L(\mathbf{x}, \lambda_1, \lambda_2)}{\partial \lambda_1} &= \mathbf{x}'\boldsymbol{\mu} - \mu_{p,0} = 0, \\
\frac{\partial L(\mathbf{x}, \lambda_1, \lambda_2)}{\partial \lambda_2} &= \mathbf{x}'\mathbf{1} - 1 = 0.
\end{aligned}$$

The FOCs consist of five(our case seven) linear equations in five(our case seven $x_{GRG} + x_{CMP} + x_{FDP} + x_{UBS} + x_{EXPN}$, λ_1 and λ_2). In the case of the three The matrix algebra of the equations are reported above.

$$\begin{pmatrix} 2\mathbf{\Sigma} & \boldsymbol{\mu} & \mathbf{1} \\ \boldsymbol{\mu}' & 0 & 0 \\ \mathbf{1}' & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mu_{p,0} \\ 1 \end{pmatrix}$$

$$\mathbf{A}\mathbf{z}_x = \mathbf{b}_0,$$

Where

$$\mathbf{A} = \begin{pmatrix} 2\mathbf{\Sigma} & \boldsymbol{\mu} & \mathbf{1} \\ \boldsymbol{\mu}' & 0 & 0 \\ \mathbf{1}' & 0 & 0 \end{pmatrix}, \ \mathbf{z}_x = \begin{pmatrix} \mathbf{x} \\ \lambda_1 \\ \lambda_2 \end{pmatrix} \text{ and } \mathbf{b}_0 = \begin{pmatrix} \mathbf{0} \\ \mu_{p,0} \\ 1 \end{pmatrix}$$

The solution for z_x is

$$Z_x = A^{-1} b_0$$

The first three element(first five in our case) are the portfolio weights x = ($x_{GRG} + x_{CMP} + x_{FDP} + x_{UBS} + x_{EXPN}$)' for the minimum variance portfolio with expected return $\mu_{p,x} = \mu_{p,0.}$ If $\mu_{p,0}$ is greater than or equal to the expected return on the global minimum variance portfolio then x is an efficient portfolio.
The problem is defined quadratic given to the fact
In the next chapter we will set up an excel spreadsheet to solve the minimization problem and built the efficient frontier.

3 Efficient Frontier (Simple Returns)

In this chapter we will estimate the variance and covariance of multiple stocks. To make sense of this we do need to develop the correlation matrix as well. We have already done this in the chapter 1. We will use the correlation matrix to calculate the portfolio variance. The variance and covariance matrix is a 5 x 5 matrix , this matrix convey the variance of a single stock and the covariance of stock one and the other four stocks.
The formula to create a variance covariance matrix is :

$$\Sigma_{k\,x\,k} = \left(\frac{1}{n}\right) X^T X$$

Where k is the number of stocks in the portfolio, n the number of observation , X the n x k excess return matrix and X^T the transpose of X. Simply, we first calculate the n x k excess return matrix, then multiply this matrix by its own transpose matrix, resulting in a k x k matrix. Then each element of the k x k matrix is divided by n ,the number of observations. The resulting k x k matrix is the covariance matrix. We could calculate each single matrix involved and then develop the calculations , but the we will do it using the Excel Covariance function on our monthly stocks returns.
The resulting k x k matrix where k is the number of stocks is presented above.

		GRG.L	CPR.MI	FDP.L	UBS.VI	EXPN.L	FTSE
28/02/2017		0.006154	0.028682	0.010513	0.047329	0.043513	0.023129
31/03/2017		0.064347	0.129451	0.172653	-0.021378	0.019225	0.008192
30/04/2017		0.050141	-0.000916	-0.039448	0.058154	0.018863	-0.016250
31/05/2017		0.012939	0.159223	0.168432	0.067582	-0.025024	0.043879
30/06/2017		-0.008299	-0.008883	-0.078557	0.061243	-0.0134	-0.027566

Figure 7 First 5 rows of Monthly Returns

From monthly return we do have the covariance 5 x 5 matrix.

Covar Using Excel Data Analysis

	GRG.L	CPR.MI	FDP.L	UBS.VI	EXPN.L
GRG.L	0.00739687				
CPR.MI	0.00053744	0.00287883			
FDP.L	-0.0018353	0.00225884	0.01849979		
UBS.VI	3.99E-05	0.00065113	0.00128494	0.00263577	
EXPN.L	0.00065964	0.0006356	0.00161346	0.00033305	0.00180014

Covar using matrix Algebra

	GRG.L	CPR.MI	FDP.L	UBS.VI	EXPN.L
GRG.L	0.00739687	0.00053744	-0.0018353	3.99E-05	0.00065964
CPR.MI	0.00053744	0.00287883	0.00225884	0.00065113	0.0006356
FDP.L	-0.0018353	0.00225884	0.01849979	0.00128494	0.00161346
UBS.VI	3.99E-05	0.00065113	0.00128494	0.00263577	0.00033305
EXPN.L	0.00065964	0.0006356	0.00161346	0.00033305	0.00180014

Figure 7 Covariance Matrix

From the covariance matrix we know the covariance between any two stocks , the matrix suggest that the covariance between Greggs and Campari is 0.00053744. As we noticed previously the covariance between Greggs and Greggs is 0.00739687 defined as the variance of Greggs. From the covariance matrix we can derive the correlation knowing the formula of the correlation we do know that the correlation between Greggs and Expedian is

$$r = r_{xy} = \frac{\text{Cov}(x, y)}{S_x \times S_y}$$

From our data the cov(GREGGS,EXPEDIAN) is 0.00065964 and S_{GREGGS} = 0.08600508 and $S_{EXPEDIAN}$ = 0.0424280 are the standard deviation of Greggs and Expedian respectively. Developing the formula we do have 0.1807703.

Compute the global minimum variance portfolio m(MVP) by solving

$$\min_{\mathbf{m}} \ \sigma^2_{p,m} = \mathbf{m}'\Sigma\mathbf{m} \text{ s.t. } \mathbf{m}'\mathbf{1} = 1$$

and compute $\mu_{p,m}$ = m ' μ and o = m' Σm.

This the first efficient portfolio is the global minimum variance portfolio

Once that the MVP has been calculated ,we will proceed with the calculation of the all the possible x_i portfolios, knowing that the linear combination of all the possible five weights from the MVP to the MRP(max return portfolio) are the efficient portfolios. The efficient frontier of portfolios are those

portfolios with expected return greater than the expected return on the global minimum variance portfolio. The first step is to compute the efficient portfolios x_i (i=1,2,3,4..n) with target expected return greater or equal to the defined expected return of the MVP . That is, solve

$$\underset{\mathbf{x}}{} \begin{aligned} \mu_p &= \mathbf{x}'\mu \text{ s.t.} \\ \sigma_p^2 &= \mathbf{x}'\Sigma\mathbf{x} = \sigma_{p,0}^2 \text{ and } \mathbf{x}'\mathbf{1} = 1 \end{aligned}$$

with $\mu_{p,} = x'\mu$ and $o^2 = x'\,\Sigma x$ and $x^{'}1 = 1$.

From the previous calculations we already have the covariance matrix of the monthly return and the correlation matrix. We already have the monthly annualised average , variance and standard deviation of the returns. We will calculate the weight vector of all the possible efficient portfolios using the solve function of Excel starting from a weight vector of 0.2% for each assets and calculate the mean, variance , standard deviation and sharp ratio.

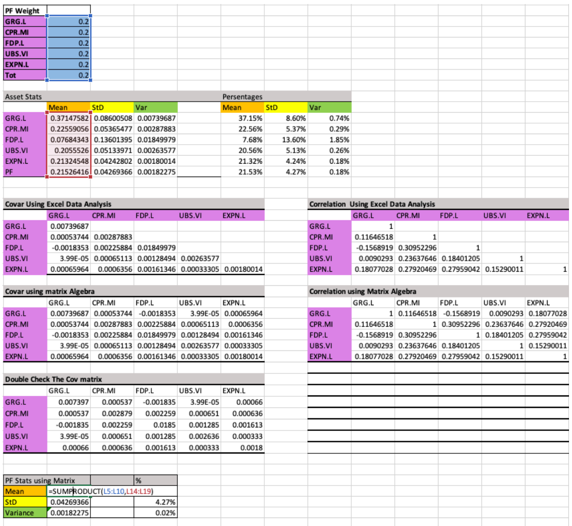
From the equally diversified portfolio we will pass to the global MVP , than the FMVP the first minimum variance portfolio and the SMVP second one. The first and the second minimum variance portfolio are portfolios that have greater expected returns compared to the MVP but with higher variances. Those portfolios are still efficient but are not minimising the variance as the MVP. If an investor is willing to gain more return in exchange of an higher variance will be more incline to invest in those portfolios. The calculation of the expected return for the equally distribute portfolio are rported in the above figure 8. The SUMPRODUCT function has been use for the dot product of the two vectors.

$$R_{p,x} = \mathbf{x}'\mathbf{R} = (x_A, x_B, x_C) \cdot \begin{pmatrix} R_A \\ R_B \\ R_C \end{pmatrix} = x_A R_A + x_B R_B + x_C R_C.$$

The variance

$$\sigma^2_{p,x} = \mathrm{var}(\mathbf{x}'\mathbf{R}) = \mathbf{x}'\Sigma\mathbf{x} = (x_A, x_B, x_C)\cdot\begin{pmatrix}\sigma^2_A & \sigma_{AB} & \sigma_{AC}\\ \sigma_{AB} & \sigma^2_B & \sigma_{BC}\\ \sigma_{AC} & \sigma_{BC} & \sigma^2_C\end{pmatrix}\begin{pmatrix}x_A\\ x_B\\ x_C\end{pmatrix}$$

The MatrixMoltiplication function has been used to solve the variance matrix product between the covariance and the weights. In figure 8 the deployment of the matrix product for the equally diversified portfolio has been show.



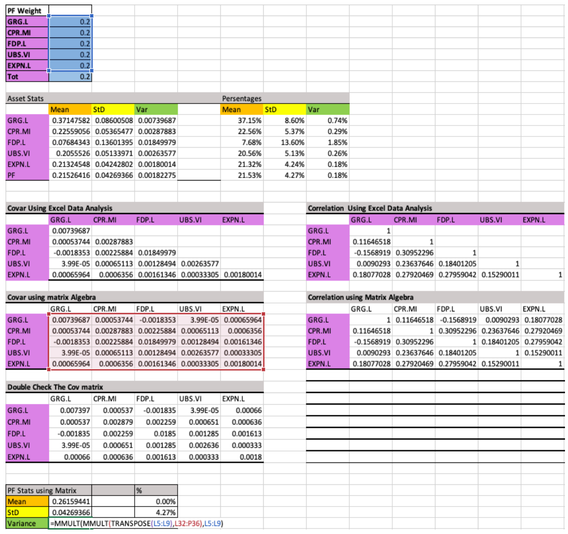


Figure 8 The return dot Product and the Variance Matrix Product

Starting from the expected return and variance of the equally weighted portfolio and using the solve function in the data analysis tool pack of Microsoft we do find the MVP and then the FMVP and SMVP. Once those portfolio have been calculated all the other possible portfolios along the linear combinations are derived. The calculated weights for the minimum variance portfolio the first efficient portfolio and the second one are reported in figure 9.

Min Variance				
PF Weights	Weight	PF Stats using Matrix		
GRG.L	0.0809955	Mean	0.2142125	21.42%
CPR.MI	0.1968055	StD	0.0322961	3.23%
FDP.L	-0.0189379	Var	0.001043	0.10%
UBS.VI	0.2992815	rF	0.021	
EXPN.L	0.4418553	Sharpe	5.9825333	
Total	1			

FMVP			
PF Weights	PF	PF Stats usin	PF Weights
GRG.L	0.1060003	Mean	0.2204621
CPR.MI	0.1816848	StD	0.0324191
FDP.L	-0.0282354	Var	0.001051
UBS.VI	0.304868		
EXPN.L	0.4356823	rF	0.021
Total	1	Sharpe	6.152609

SMVP			
PF Weights	PF	PF Stats usin	PF Weights
GRG.L	0.1364483	Mean	0.2258502
CPR.MI	0.2067928	StD	0.0328078
FDP.L	-0.035032	Var	0.0010764
UBS.VI	0.2816956		
EXPN.L	0.4100953	rF	0.021
Total	1	Sharpe	6.2439422

Figure 9 MVP,FMVP,SMVP

We will develop a number of portfolios with $\mu_p > u_{mp}$. All the efficient portfolio are plotted in figure 10.

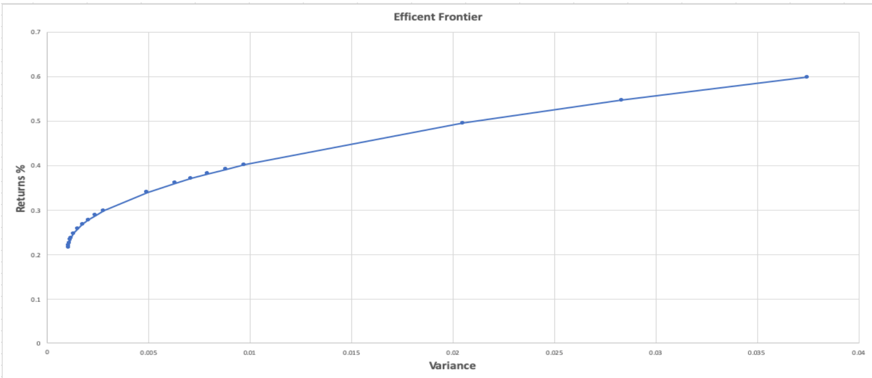


Figure 10 Efficient portfolios

In the next figure the efficient frontier calculated using the simple returns of the five stocks. Plus the Tangent portfolio that is the portfolio that maximise the CML capital market line.

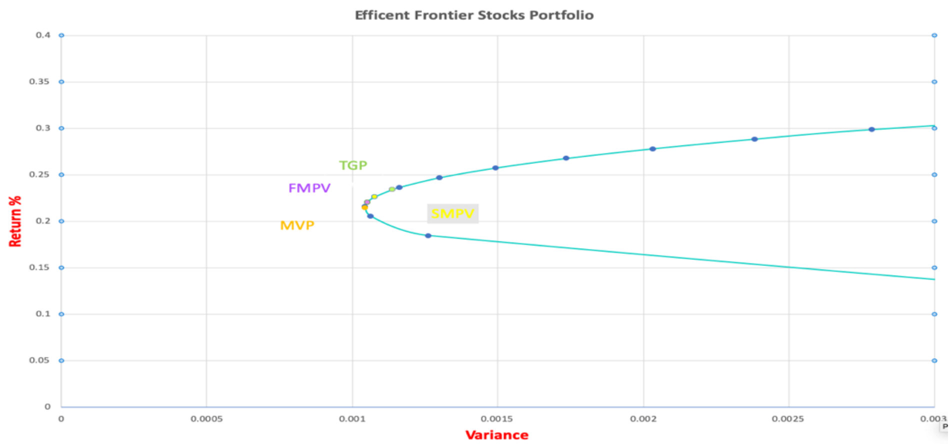


Figure 10 Efficient portfolios with the most important Porfolios.

4 Single Index Model

In the previous chapter we developed the mean variance theory by Markowitz to determining the efficient portfolio, from which the investor can determine optimum portfolio. This kind of implementations is hard in practice given the amount and type of input data needed to perform the portfolio analysis specially when the number of stocks is high. Simplification of the implementational process led to the index model. The index model facilitate the determination of the efficient frontier in comparison to the mean-variance theory. The index model shows how the security returns are sensitive to various factor. In the single index model the returns are assumed to depend on one index in our case the FTSE100.

The single index model explains the returns of a security using one factor, market model in the form of

$$\hat{R}_i = \hat{\alpha}_i + \hat{\beta}_i R_m$$

Where $\hat{a}_i$ can be broken in two components, a_i that represent the expected value of a and e_i[1] the random element of a_i. We can rewrite

$$\hat{R}_i = \hat{\alpha}_i + \hat{\beta}_i R_m + e_i$$

Where , $Cov(e_i, R_m) = 0$[2], $Cov(e_j, e_i) = 0$[3] and $E(e_i) = 0$.

The returns of a security are a linear function of a market factor.

The fact that the returns are approximated to a normal distribution imply that the error its 0.

When using the single index model, where the weight are equal; the risk of the portfolio can be shown as:

$$\sigma_p = \beta_p \sigma_m = \sigma_m \left[\sum_{i=1}^{N} X_i \beta_i \right]$$

As defined by Sharpe the $ß_i$ is the slope term in the simple linear regression function where the rate of returns on a market index is the independent variable and the stocks' returns the independent variable. From the formula of the risk in the single index model we notice that o'$_m$ the risk of the market does not change given different stock, $ß_i$ is the measure of his contribution to total risk of the portfolio. $ß_i$ is the measure of the systematics risk of the stock. The unsystematic risk can be diminished by the diversification , $ß_i$ is used as the measure of the security's risk in selecting portfolios. The betas coeff are important for understanding risk-return relationships.

Based on the single index model we do need to calculate the $E[r_i]$ for each single stock as a linear regression line. To do so we do need to calculate the alphas and betas for each stock, running a linear regression function in excel using as a dependent variable the security monthly returns we have calculated and as independent variables the monthly Index returns of the FTSE100.

We do run the regression on all the variables and the resulting alphas and betas are exposed in figure 11.

Stocks	Alpha	Beta	E[r]		Market Risk Free Rate	
GRG.L	0.02554263	0.482410786	=K11+L11*O11		0.0284956	
CPR.MI	0.01588114	0.518418085	0.03065377		Market Variance	
FDP.L	0.0011979	2.128968422	0.06186414		0.0008867	
UBS.VI	0.01413571	0.667371522	0.03315286			
EXPN.L	0.01459302	0.702025367	0.03459765			

Figure 11 E[r] From the Single Index Model

To double check the betas we do know that the betas are equal to the Cov(Stok,Market)/ Variance Market. From the Variance covariance matrix of all the stocks and the market presented in figure 12 we can calculate the beta of Greggs. The cov($x_{Greggs}x_{marke}$) is 0.00045968 the variance of the market is on the diagonal is 0.0008867. The beta 0.482410786 the fraction 0.00045968/0.0008867.

	GRG.L	CPR.MI	FDP.L	UBS.VI	EXPN.L	FTSE
GRG.L	0.00739687	0.00739687	0.0073969	0.0073969	0.0073969	0.0073969
CPR.MI	0.00053744	0.00287883				
FDP.L	-0.0018353	0.00225884	0.0184998			
UBS.VI	3.9869E-05	0.00065113	0.0012849	0.0026358		
EXPN.L	0.00065964	0.0006356	0.0016135	0.0003331	0.0018001	
FTSE	0.00042775	0.00045968	0.0018878	0.0005918	0.0006225	0.0008867

Figure 12 Variance and Covariance Stocks/Variance

Once the alphas and the betas are calculated and the market stocks expected returns derived as well we can build as before the efficient frontier. This efficient frontier will be more close to the real expected returns. We do use the solve function of excel to resolve the optimization problem and calculate the minimum variance portfolio(MVP), the one with the highs return and the one tangent to the CML the portfolio that maximise the sharp ratio(Returns Portfolio- Risk free rate/ St.deviation of the portfolio) the sharp ratio give us the ratio of the mean/variance relationship given a risk free rate of 0.021(2.1%), derided by the average risk return free of the Bank of England. The sharp ratio is the slope of the CML.

4.1 Efficient Frontier Market Return

Once all the components have been calculated the efficient frontier is derived. The MVP, the MRP(max return portfolio) and the TGP(max sharp ratio) of the new efficient frontier are shown in figure 13.

Minimum Variance				Maximum Return				Max Sharp Ratio			
GRG.L	0.08319334	Mean	0.03516677	GRG.L	0.4408772	Mean	0.09010234	GRG.L	0.10862928	Mean	0.03907408
CPR.MI	0.21799967	St.D	0.10395657	CPR.MI	-1.7627736	St.D	0.77459667	CPR.MI	0.07711336	St,D	0.10395657
FDP.L	0.0494936	Variance	0.01080697	FDP.L	1.63722544	Variance	0.60000001	FDP.L	0.16242325	Variance	0.01080697
UBS.VI	0.21521782	rf	0.021	UBS.VI	-1.2754304	rf	0.021	UBS.VI	0.10921959	rf	0.021
EXPN.L	0.43409558	SharpRatio	0.13627582	EXPN.L	1.96010131	Sharp Ratio	0.12671992	EXPN.L	0.54261451	ShrpRatio	0.17386184

Figure 13 MVP, TGP and Maximum return

The MVP and the Max Sharp Ratio portfolio(TGP) with all the others possible efficient combination of the weights stocks are plotted on figure 14.

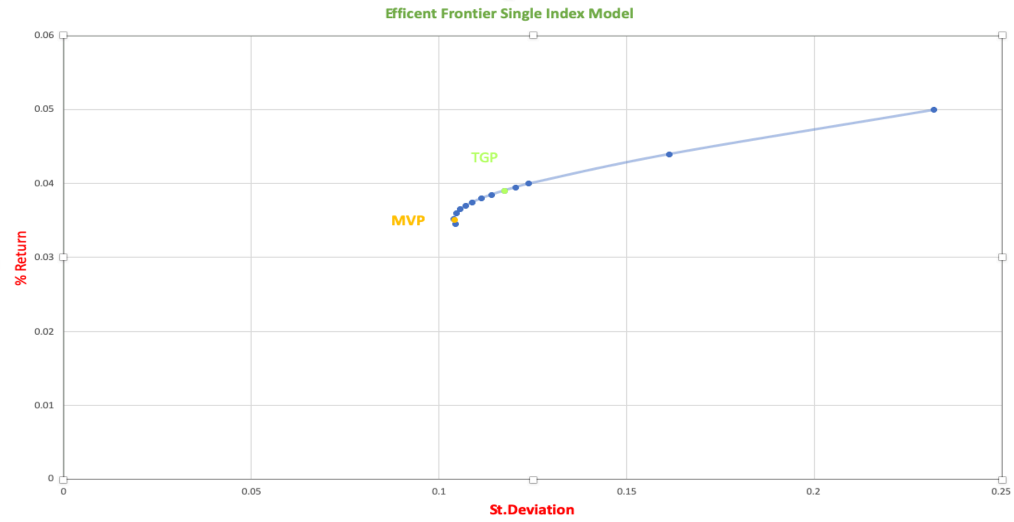


Figure 14 Market Efficient Frontier

The Betas and Alphas calculated with the regression of excel are presented in the figure above. As shown are the same calculated from the variance covariance matrix .

GRG.L	*Coefficients*	CPR.MI	*Coefficients*	FDP.L	*Coefficients*	UBS.VI	*Coefficients*	EXPN.L	*Coefficients*
Intercept	*0.0255426*	*Intercept*	*0.0158811*	*Intercept*	*0.0011979*	*Intercept*	*0.0141357*	*Intercept*	*0.014593*
FTSE	*0.4824108*	*FTSE*	*0.5184181*	*FTSE*	*2.1289684*	*FTSE*	*0.6673715*	*FTSE*	*0.7020254*

The next step is to build a portfolio where the total wealthy is invested with proportion in stocks free rate and market stocks using the weights of the tangent portfolio. That is the optimal portfolio.

5 Capital Market Line (CML)

The capital market line is a graphical representation of all the portfolios that optimally combine risk and return. CML is a theoretical concept that gives optimal combinations of a risk-free asset and the market portfolio. The CML is superior to Efficient Frontier in the sense that it combines the risky assets with the risk-free asset. CML is the tangent line drawn from the risk free point to the feasible region for risky assets. This line show s the relation between R_p and σ_p for efficient portfolios (risky assets plus the risk free asset). Starting from the TGP portfolio that represent the market portfolio so called because every rational investor should hold their risky asset in those proportion as the weights in the market portfolio. Based on the risk level that an investor is willing to take, it will combine the market portfolio of risky asset with risk free rate asset.

The return of the tangent portfolio is the Market Return expected and the St.Deviation is the minimum market risk expected that maximise the return for that portfolio.

The CML is shown in figure 15.

The slope of the capital market line is the Shar Ratio of the Market Portfolio.

If we draw a line from risk free rate of return which is tangent to the efficient frontier we get the CML.

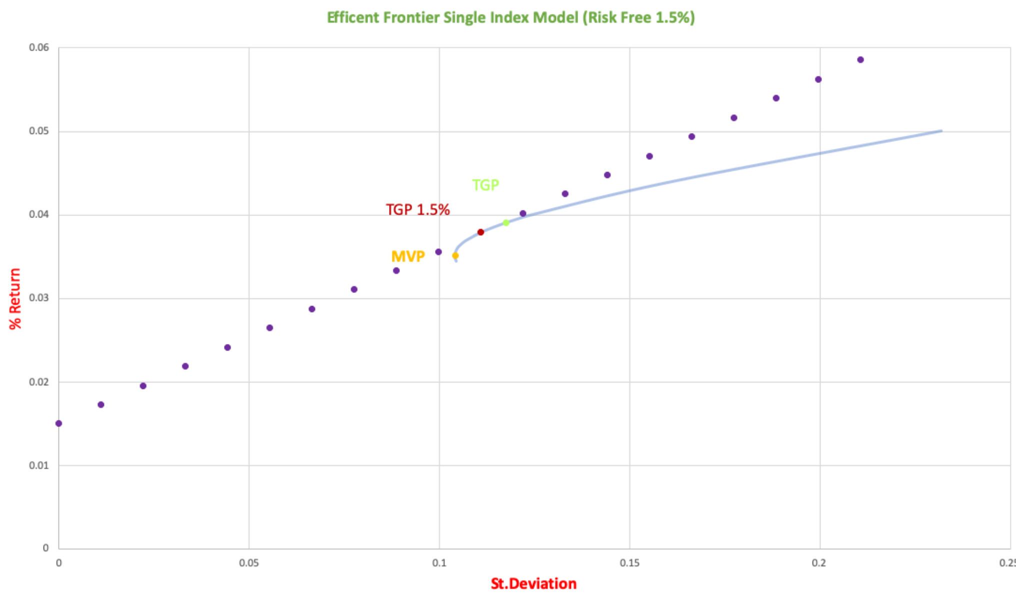


Figure 15 Capital Market Line (1.5% Risk free Rate)

Starting from the sharp ratio of the tangent portfolio:

Sharp Ratio 0.2050244 = ((0.03907408 – 0.015) / 0.11742056) that is expected return of the tangent portfolio of the risk market stocks minus the risk free rate divided by the standard deviation of the market tangent portfolio. The equation of the capital market line is shown above.

$R_p = r_f + (\text{Sharp Ratio}) * \sigma_{pt}$

The capital market line is defined in the equation above. Substituting the two variable or one of them we will be able to get all the information needed to take decision about the proportions. If we assign different weights to the proportion of wealthy an investor decide to invest in risk free rate market and in the market portfolio we derive the single point 9 return and standard deviation of the CML. From the formulas above we calculate the point of the CML

$R_p = w_{rf} * r_f + (1 - w_{rf}) * R_{pt}$ $\sigma_p = (1 - w_{rf}) * \sigma_{pt}$

The w_{rf} is the proportion of wealthy an investor decide to invest in risk free rate market and the 1- w_{rf} the proportion in the market portfolio.

The r_f is the risk free rate of 1.5% ant the R_{pt} 0.03907408 the return of the tangent portfolio(market portfolio= max sharp ratio).

For the ST.deviation 1- w_{rf} is the proportion invested in the market portfolio and σ_{pt} = 0.117420562 the standard deviation of the tangent market portfolio.

The first five point calculated are shown above:

% of Risk Free Rate	Return Portf	St.d Portfolio
1	0.015	0
0.9	0.017291165	0.01108058
0.8	0.01958233	0.02216115
0.7	0.021873495	0.03324173
0.6	0.02416466	0.04432231

We have defined the equation of the capital market line :

$R_p = r_f + (\text{Sharp Ratio}) * \sigma_p$

And then derived the component that define it :

$R_p = w_{rf} * 0.015 + (1 - w_{rf}) * (R_{pt} = 0.03907)$ $\sigma_p = (1 - w_{rf}) * (\sigma_{pt} = 0.117420562)$

So given the risk free market return of 1.5% and the equation line of the CML and the component of it as the return of the tangent market portfolio and the standard deviation of the tangent portfolio the only component missing are the weights (w_{rf} and 1 - w_{rf}).

Different proportion of the weights will give us different expected return and standard deviation. At this point the decision of the weight once knowing the CML is about the aversion to the risk an investor is propense too.

An investor with higher aversion to the risk will decide to invest in portfolios on the CML that have a lower level of risk(St.Deviation) than the market portfolio and so lower expected return, this means taking less risk, holding risk free asset also . At the contrary an investor willing to risk more will borrowing to increase the leverage. Leverage is the strategy of using borrowed money to increase return on an investment. If the return on the total value invested in the security (your own cash plus borrowed funds) is higher than the interest you pay on the borrowed funds, you can make significant profit as shown in figure 16.

The sharp portfolio stats of the risk free rate of 1.5% are present in the picture above

Max Sharp Ratio				
GRG.L	0.10862928		Mean	0.03907408
CPR.MI	0.07711336		St,D	0.11742056
FDP.L	0.16242325		Variance	0.01378759
UBS.VI	0.10921959		rf	0.015
EXPN.L	0.54261451		ShrpRatio	0.2050244
	1			

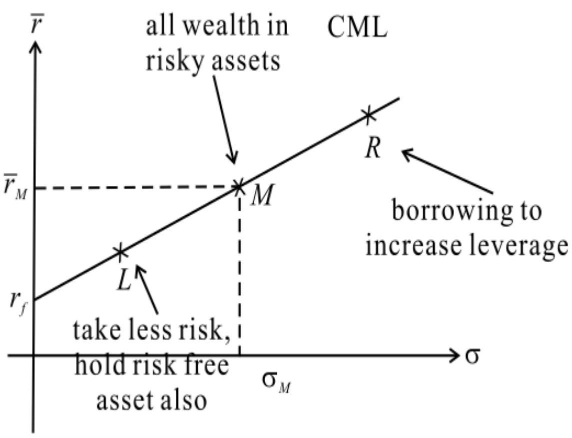


Figure 16 Capital Market Line Strategy

Summary of findings

As said the capital market line will guide an investor in the decision of borrowing money to invest instead of taking less risk and invest an higher proportion in risk free assets.
The decision is all to the investor, as said if is risk averse it will go for higher proportion of wealthy invested in risk free market assets granting a more secure lower return compared to an higher proportion of the market portfolio borrowing money expecting a far less secure so more risky higher return.
The possible portfolio with different strategy have been showed in the picture 17 where two portfolios one RiskAvPort (risk averse portfolio) with higher secure lower return and NRiskAvport (non risk averse portfolio) with possible less secure higher return.
The sharp portfolio stats of the risk free rate of 1.5% are present in the picture above

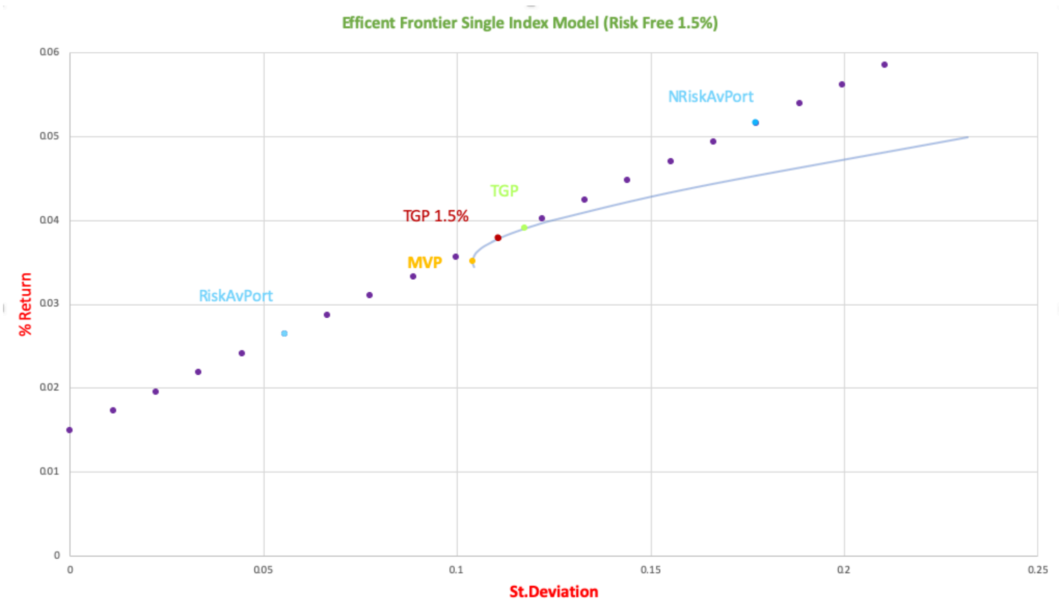


Figure 17 RiskAvPort and NRiskAvPort on the CML

As show from the figure the two portfolio RiskAvPort and NRiskAvPort ar ploted on the CML. The two portfolios have been calculated from the previous formula R_p and σ_p assigning different weights to the risk free assets and market portfolio.
The two portfolio weights for the risk free assets have been assigned as a 0.5 (50%) for the risk free assets and 0.5 (1 – 0.5, 50%) to the market portfolio keeping the assets

tangent portfolio weights for the RiskAvPort and -0.6 (-60%) to the weight for the risk free assets NRiskAvPor so 1.6 to (1 –(-0.6) =1.6,160%) is the market weight, meaning that the investor should borrow money for the 60 % of the total wealthy.

	% of Risk Free Rate	Return Portf	St.d Portfolio
RiskAvPort	0.5	0.026455826	0.055402886
NRiskAverPort	-0.6	0.051658642	0.177289234

The returns and standard deviation of the two portfolios (RiskAvPort and NRiskAvPort) are calculated.

If an investor would like to achieve an expected return of the 3% he can Easley derive the standard deviation and decide if the level of risk is acceptable.

The standard deviation and the w_{rf} have been derived as 0.074 and 0.33

	% of Risk Free Rate	Return Portf	St.d Portfolio
RiskAvPort	0.5	0.026455826	0.055402886
NRiskAverPort	-0.6	0.051658642	0.177289234
3 % Expected Return	0.33	0.030350806	0.074239867

The 3% expected return is plotted in the figure 18 .

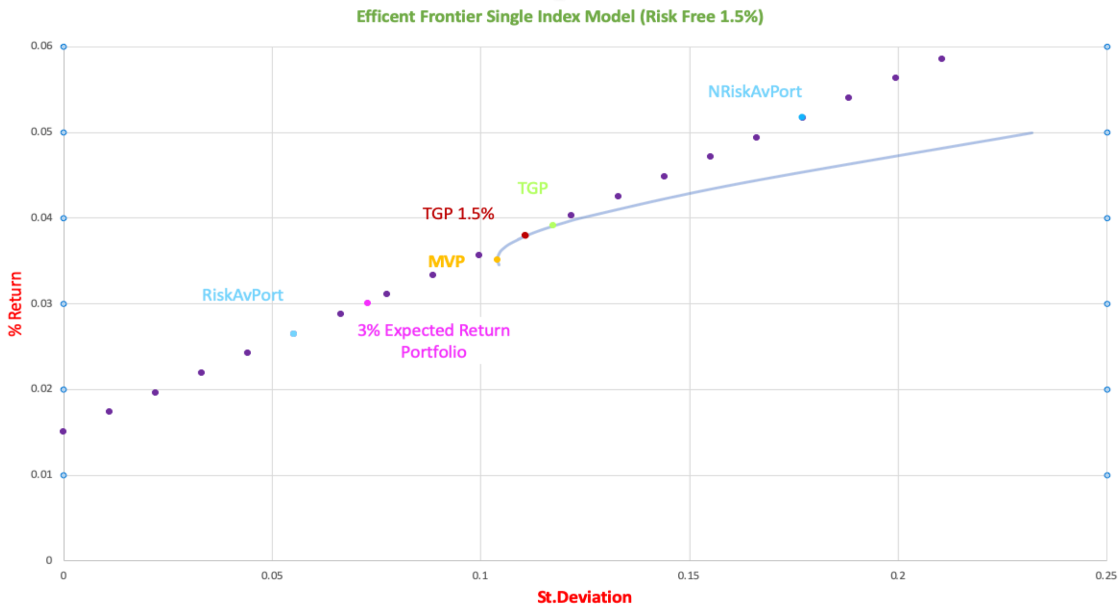


Figure 18 3% Expected Return Portfolio on the CML