POLITECNICO DI TORINO

Computer aided simulations and performance evaluation

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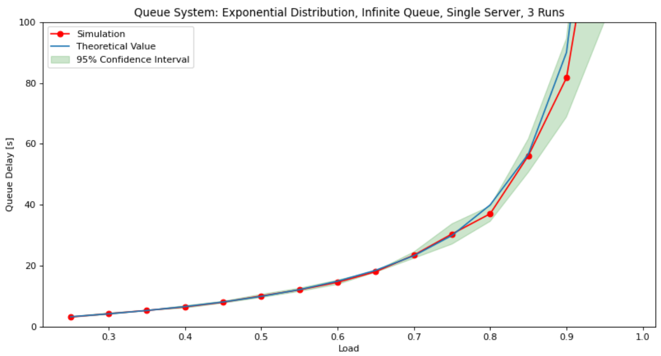
Contents

Queue Systems

These experiments are performed to understand the behaviour of a queue system with a given number of servers (one or more), a capacity waiting line (finite or infinite) a FIFO (First In First Out) serving discipline and inter-arrivals time that follow a Poisson process.

1.1 A single-Server queuing system

1.1.1 Exponential Distributed Service Time

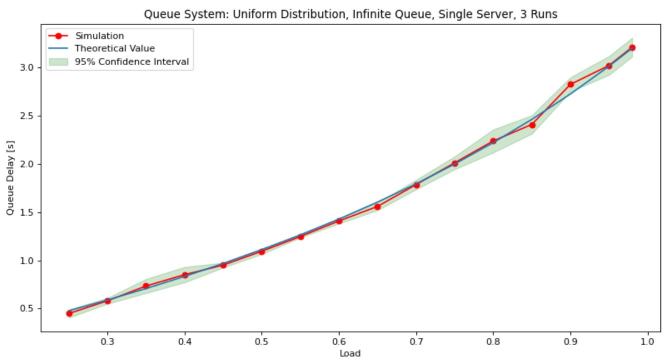


It is possible to see that the results obtained with simulation are close to the theoretical formula. The theoretical value is calculated as:

$$theorical\_queue\_time = \frac{1}{\mu - \lambda} \tag{1.1}$$

where $\lambda = \frac{1}{ARRIVAL}$ and $\mu = \frac{1}{SERVICE}$

1.1.2 Uniform Distributed Service Time

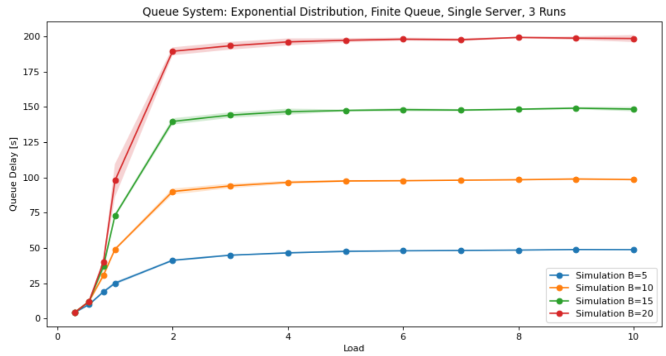


$$general\_th\_queue\_time = E(S) \times (1 + ro\frac{1 + C_s^2}{2 \times (1 - ro)}) \tag{1.2}$$

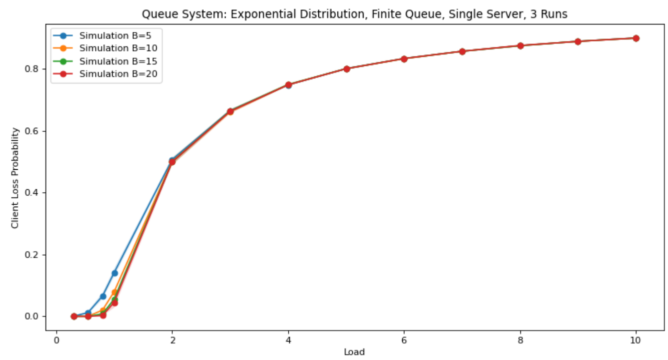
where $ro = \frac{\lambda}{\mu} = \frac{LOAD}{2}$, $C_s^2 = \frac{Var(S)}{E^2(s)}$ is the *Coefficient of Variation* (ratio between the Variance and the Mean squared) and it depends on the service time distribution.
This is the general formula indeed can be applied also to a service time with Exponential distribution ($C_s^2 = 1$); in the case of Uniform distribution the $C_s^2 = \frac{1}{3}$.
It is possible to see that the average waiting time is lower with respect to the Exponential distribution. Therefore to improve the performance of a queue system a possible way is to assign to a job a service time that is uniformly distributed (not always possible to decide a priori).

1.2 Finite Capacity of the waiting line

1.2.1 Exponential Distributed Service Time

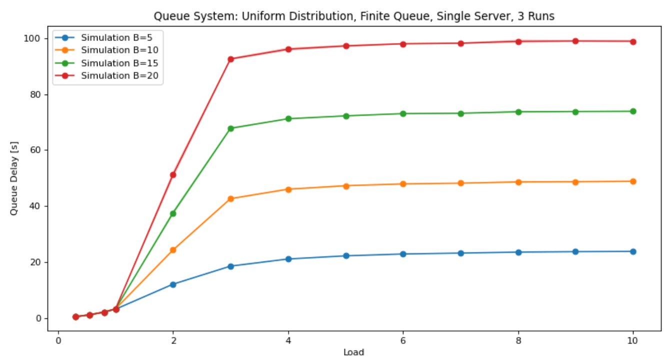


It is possible to see that the average waiting time converges for any value of B to a specific value. This value is $queue\_time = B \times SERVICE$, where B is the maximum value of allowed client in the system and SERVICE is the average time spent in a server.
It is worth to mention that with an unlimited system length capacity the average waiting time would diverge for any $LOAD \geq 1$.

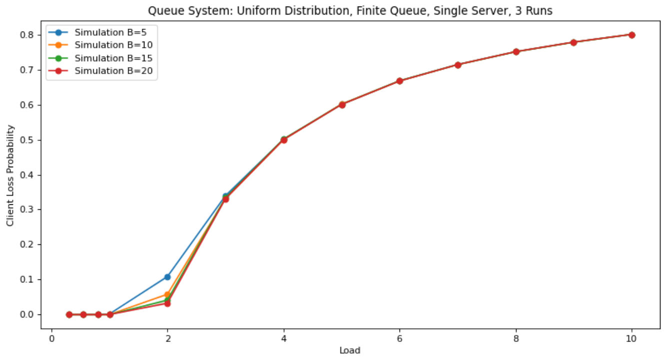


The probability of loosing a customer tends to 1 as the load increases.

1.2.2 Uniform Distributed Service Time



Also for the Uniform distribution service time, the average waiting time converges for any value of B to a specific value. This value is $queue\_time = B \times \frac{SERVICE}{2}$ since the mean value of a Uniform distribution (U(A,B)) is $E(x) = \frac{(B-A)}{2}$; in this case A is 0 and B is the SERVICE time.
With an unlimited queue length the average waiting time would diverge for any $LOAD \geq 2$.



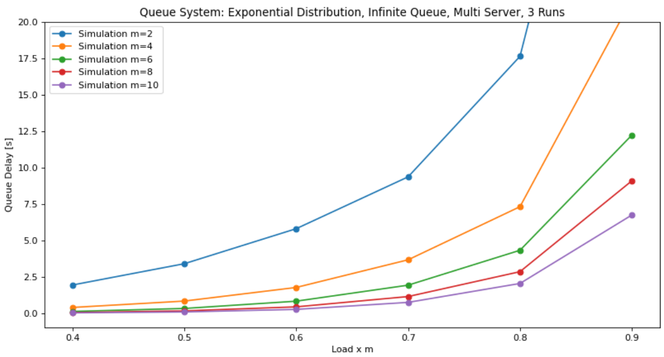
For the Uniform distribution service time the probability of loss a customer tends to 1 but in a slower way with respect to the Exponential distribution.

1.3 Multiple-Server queuing system

1.3.1 Random Policy

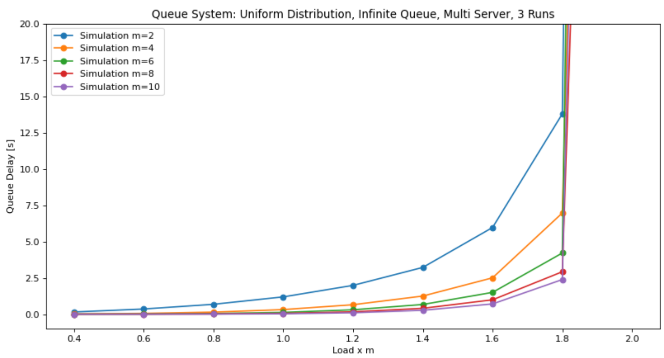
With the Random policy when a new customer arrives he chooses a random server among the ones that are not busy.

Exponential Distributed Service Time



It is possible to notice that the average waiting time, for a given load, decreases with increasing the number of available servers

Uniform Distributed Service Time

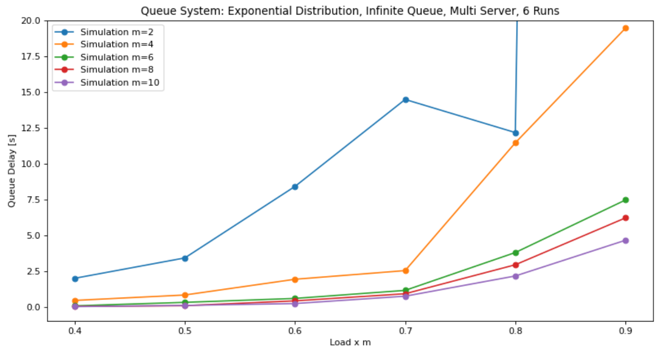


The same happens for the Uniform Distribution.

1.3.2 Fastest Policy

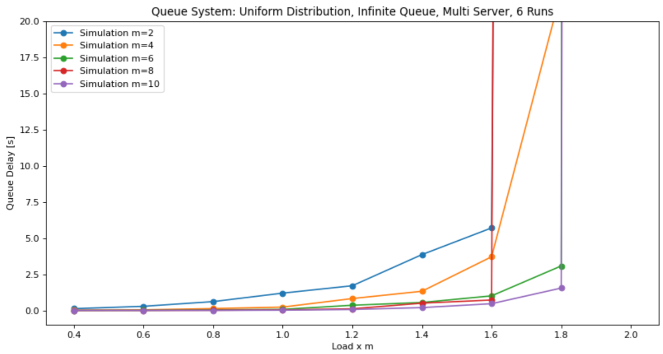
With the Fastest policy when a new customer arrives he chooses the fastest server among the ones that are not busy.

Exponential Distributed Service Time



This policy seems to work not very well (with respect to the Random policy) with a low number of services while it performs slightly better if the number of services increases.

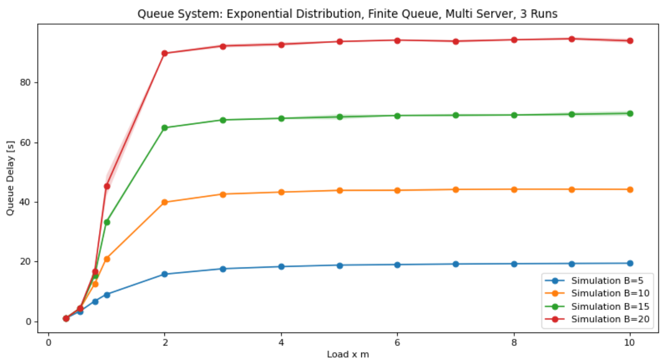
Uniform Distributed Service Time



Also for the Uniform distributions the performance are better even if for some values it seems to diverge earlier (m=2, m=8).

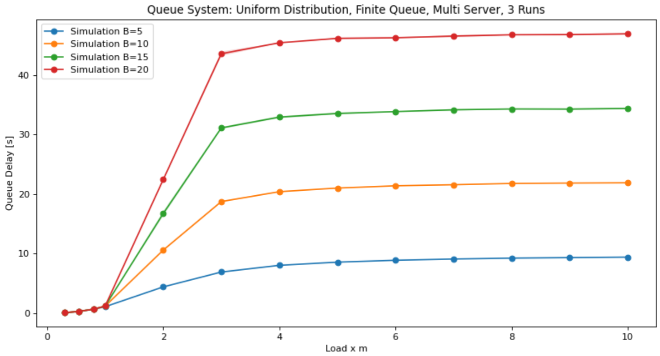
1.3.3 Finite Queue

Exponential Distributed Service Time



Also for multi server, in the case of a finite queue system, the average waiting time converges to a defined value. This value is $queue\_time = B \times \frac{SERVICE}{m}$ (where m is 2 in the graph above), so it is better than a single server system.

Uniform Distributed Service Time



Similar for the Uniform distribution, it converges to $queue\_time = B \times \frac{SERVICE}{2 \times m}$.