Problem Formulation: Function Space Approach

$$\begin{aligned} \underset{x,u}{\text{minimize}} \quad & J(x,u) = \frac{1}{2}\int_0^T x^*(t)Qx(t) + u^*(t)Ru(t) \quad dt \\ \text{subject to} \quad & \dot{x}(t) = f\big(x(t),u(t)\big); \quad x(0) = x_0 \end{aligned}$$

Define:

$$z := \begin{bmatrix} x \\ u \end{bmatrix}; \qquad x = \mathcal{H}(u)$$

Then:

$$\begin{aligned} \underset{z}{\text{minimize}} \quad & J(z) = \frac{1}{2}\langle z, Hz\rangle \qquad\qquad H := \begin{bmatrix} Q & 0 \\ 0 & R \end{bmatrix} \\ \text{subject to} \quad & x = \mathcal{H}(u) \end{aligned}$$

Unconstrained Optimization: $\mathcal{J}(u) := J(\mathcal{H}(u),u) = \frac{1}{2}\left\langle \begin{bmatrix} \mathcal{H}(u) \\ u \end{bmatrix}, H \begin{bmatrix} \mathcal{H}(u) \\ u \end{bmatrix} \right\rangle$

- First Order Method: Gradient Descent $\longrightarrow$ Cheap but Slow Convergence
- Second Order Method: Newton $\longrightarrow$ Fast Convergence but Expensive