Mean-Square Stability Analysis with Distributed Stochastic Uncertainty

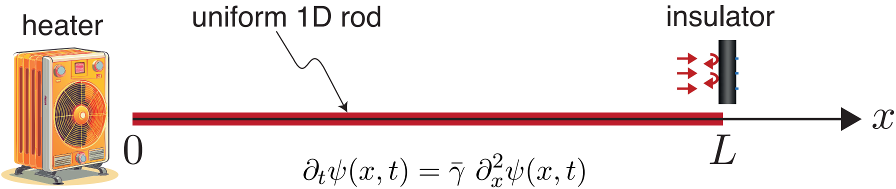
Maurice Filo, ETH Zurich

maurice.filo@bsse.ethz.ch
Swiss Federal Institute of Technology in Zürich (ETHz)
University of California, Santa Barbara (UCSB)

https://maurice-filo.github.io/

MTNS 2024, Cambridge

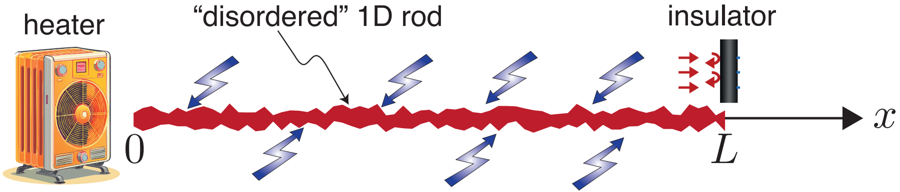
Example 1: Heat Equation in Disordered Material



$$\partial_t \psi(x,t) = \bar{\gamma}\ \partial_x^2 \psi(x,t)$$

$$\text{s.t.} \begin{cases} \psi(0,t) = u(t) \\ \partial_x \psi(L,t) = 0 \end{cases}$$

Example 1: Heat Equation in Disordered Material

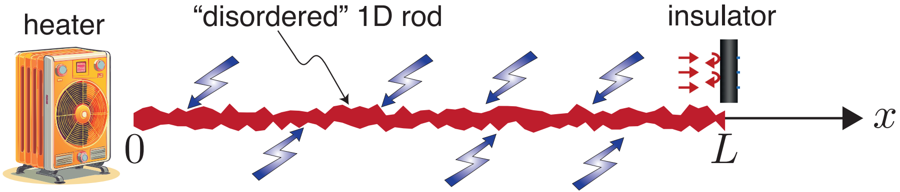


$$\partial_t \psi(x,t) = \partial_x \left[\left(\bar{\gamma} + \gamma(x,t) \right) \partial_x \psi(x,t) \right] + w(x,t)$$

$\gamma(x,t)$ ↑ multiplicative

$w(x,t)$ ↑ additive

Example 1: Heat Equation in Disordered Material

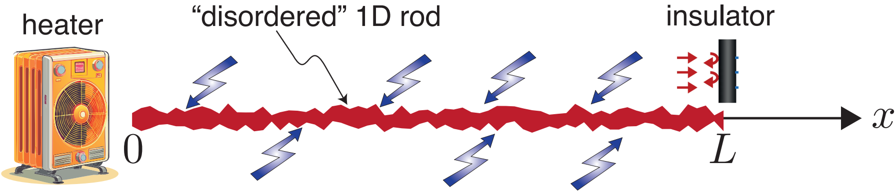


$$\partial_t \psi(x,t) = \partial_x \Big[\Big(\bar{\gamma} + \underbrace{\gamma(x,t)}_{\text{multiplicative}} \Big) \, \partial_x \psi(x,t) \Big] + \underbrace{w(x,t)}_{\text{additive}}$$

$$\mathbb{E}\left[\gamma(x,t)\gamma(\xi,\tau)\right] =: \Gamma(x,\xi)\delta(t-\tau)$$

$$\mathbb{E}\left[w(x,t)w(\xi,\tau)\right] =: W(x,\xi)\delta(t-\tau)$$

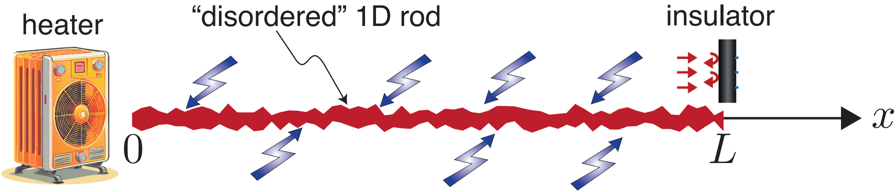
Example 1: Heat Equation in Disordered Material



$$\partial_t \psi(x,t) = \partial_x \left[\left(\bar{\gamma} + \gamma(x,t) \right) \partial_x \psi(x,t) \right] + w(x,t)$$

$$= \bar{\gamma}\ \partial_x^2 \psi(x,t) + \begin{bmatrix} \partial_x & \mathcal{I} \end{bmatrix} \gamma(x,t) \begin{bmatrix} \partial_x \\ \partial_x^2 \end{bmatrix} \psi(x,t) + w(x,t)$$

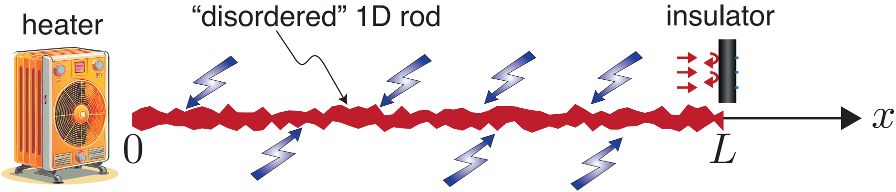
Example 1: Heat Equation in Disordered Material



$$\partial_t \psi(x,t) = \partial_x \left[\left(\bar{\gamma} + \gamma(x,t) \right) \partial_x \psi(x,t) \right] + w(x,t)$$

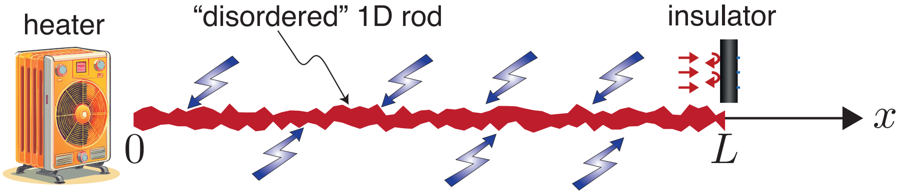
$$= \underbrace{\bar{\gamma}\ \partial_x^2}_{\mathcal{A}} \psi(x,t) + \underbrace{\begin{bmatrix} \partial_x & \mathcal{I} \end{bmatrix}}_{\mathcal{B}} \gamma(x,t) \underbrace{\begin{bmatrix} \partial_x \\ \partial_x^2 \end{bmatrix}}_{\mathcal{C}} \psi(x,t) + w(x,t)$$

Example 1: Heat Equation in Disordered Material



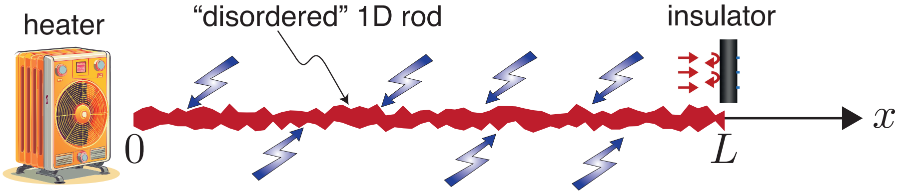
$$\partial_t \psi(x,t) = \mathcal{A}\psi(x,t) + \mathcal{B}\gamma(x,t)\mathcal{C}\psi(x,t) + w(x,t)$$

Example 1: Heat Equation in Disordered Material

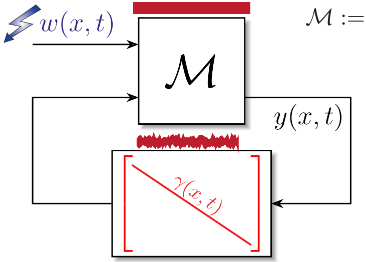


$$\partial_t \psi(x,t) = \mathcal{A}\psi(x,t) + \mathcal{B}\gamma(x,t)\underbrace{\mathcal{C}\psi(x,t)}_{y(x,t)} + w(x,t)$$

Example 1: Heat Equation in Disordered Material



$$\partial_t\psi(x,t) = \mathcal{A}\psi(x,t) + \mathcal{B}\gamma(x,t)\underbrace{\mathcal{C}\psi(x,t)}_{y(x,t)} + w(x,t)$$

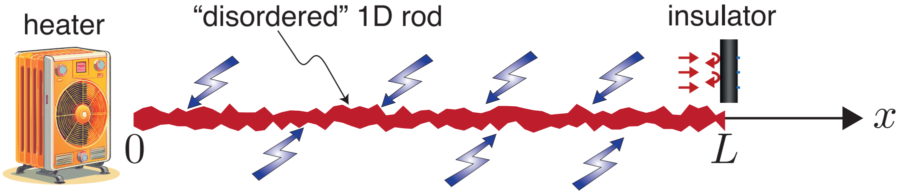


$$\mathcal{M} := \left[\begin{array}{c|c} \mathcal{A} & [\mathcal{B} \quad \mathcal{I}] \\ \hline \mathcal{C} & \end{array}\right]$$

$$\mathbb{E}\left[\gamma(x,t)\gamma^*(\xi,\tau)\right] = \mathbf{\Gamma}(x,\xi)\delta(t-\tau)$$

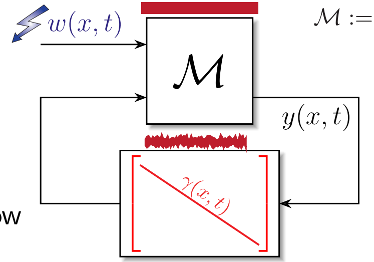
$$\mathbb{E}\left[\mathrm{w}(x,t)\mathrm{w}^*(\xi,\tau)\right] = \mathbf{W}(x,\xi)\delta(t-\tau)$$

Example 1: Heat Equation in Disordered Material



$$\partial_t\psi(x,t) = \mathcal{A}\psi(x,t) + \mathcal{B}\gamma(x,t)\underbrace{\mathcal{C}\psi(x,t)}_{y(x,t)} + w(x,t)$$

$$\mathcal{M} := \left.\begin{array}{c|c} \mathcal{A} & [\mathcal{B} \quad \mathcal{I}] \\ \hline \mathcal{C} & \end{array}\right.$$

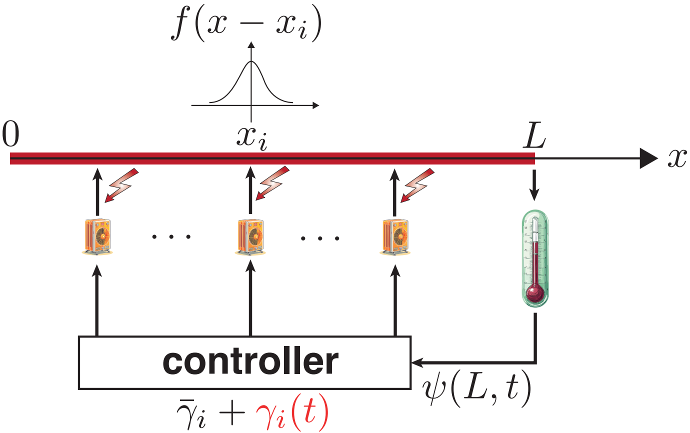


Two main questions:

- Covariance signals remain bounded? Conditions?
- If not, what spatial modes grow the fastest?

$$\mathbb{E}\left[\gamma(x,t)\gamma^*(\xi,\tau)\right] = \mathbf{\Gamma}(x,\xi)\delta(t-\tau)$$
$$\mathbb{E}\left[\mathrm{w}(x,t)\mathrm{w}^*(\xi,\tau)\right] = \mathbf{W}(x,\xi)\delta(t-\tau)$$

Example 2: Feedback Control



$$\partial_t \psi(x,t) = \alpha \partial_x^2 \psi(x,t) - \sum_{i=1}^{n} (\bar{\gamma}_i + \gamma_i(t))\, f(x - x_i)\psi(L,t)$$

$$\gamma(t) := \begin{bmatrix} \gamma_1(t) & \cdots & \gamma_n(t) \end{bmatrix}^T \qquad \mathbb{E}\left[\gamma(t)\gamma^T(\tau)\right] =: \Gamma\delta(t-\tau)$$

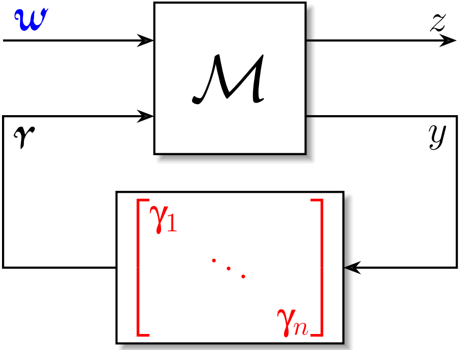
Multiple Settings

- Time of dynamics $\in$ {continuous, discrete}
- Space of nominal dynamics $\in$ {continuous, discrete}
- Space of structured stochastic uncertainty $\in$ {continuous, discrete}

Overview

- Problem Statement: I/O Structured Stochastic Uncertainty
- Stochastic Block Diagrams
- Block Diagram Conversion Between Stochastic Interpretations
- Loop Gain Operator
- Conditions for Mean-Square Stability
- Application to State-Space Realizations
- Final Remarks & Conclusion

Structured Stochastic Uncertainty: Input/Output Approach

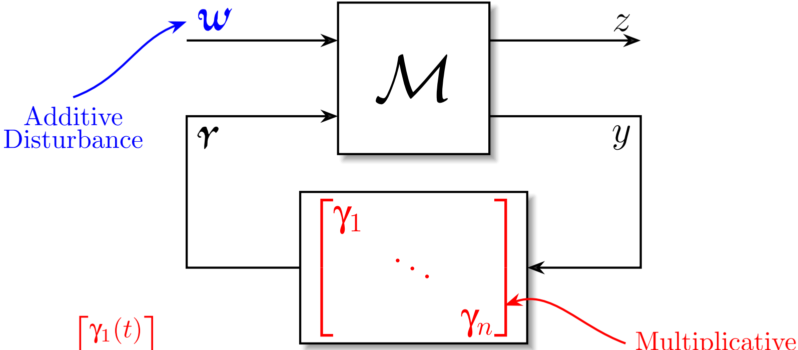


$$\gamma(t) := \begin{bmatrix} \gamma_1(t) \\ \vdots \\ \gamma_n(t) \end{bmatrix}$$

$$\mathbb{E}\left[\gamma(t)\gamma^*(\tau)\right] := \mathbf{\Gamma}\delta(t-\tau)$$

$$\mathbb{E}\left[w(t)w^*(\tau)\right] := \mathbf{W}(t)\delta(t-\tau)$$

Structured Stochastic Uncertainty: Input/Output Approach

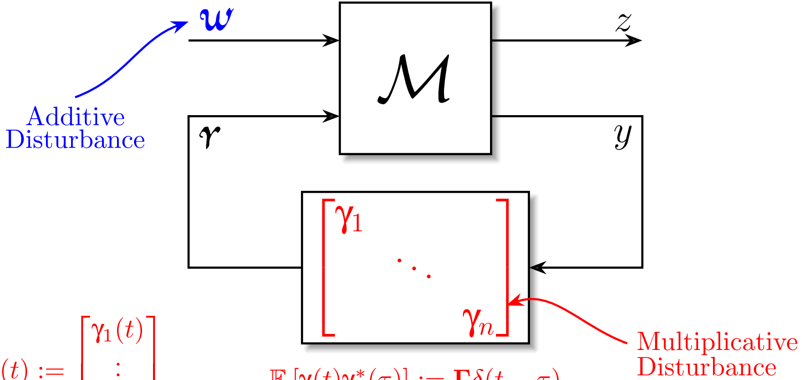


$$\gamma(t) := \begin{bmatrix} \gamma_1(t) \\ \vdots \\ \gamma_n(t) \end{bmatrix}$$

$$\mathbb{E}\left[\gamma(t)\gamma^*(\tau)\right] := \mathbf{\Gamma}\delta(t-\tau)$$

$$\mathbb{E}\left[w(t)w^*(\tau)\right] := \mathbf{W}(t)\delta(t-\tau)$$

Structured Stochastic Uncertainty: Input/Output Approach



$$\gamma(t) := \begin{bmatrix} \gamma_1(t) \\ \vdots \\ \gamma_n(t) \end{bmatrix}$$

$$\mathbb{E}\left[\gamma(t)\gamma^*(\tau)\right] := \mathbf{\Gamma}\delta(t-\tau)$$

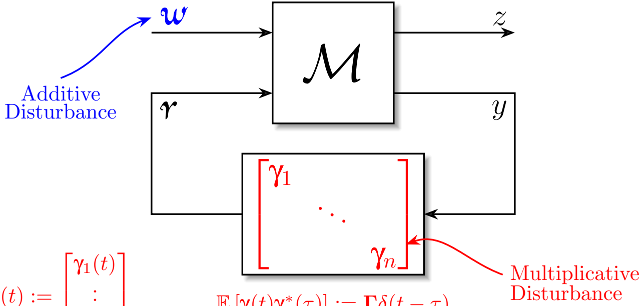
$$\mathbb{E}\left[w(t)w^*(\tau)\right] := \mathbf{W}(t)\delta(t-\tau)$$

$$\begin{bmatrix} z \\ y \end{bmatrix} = \mathcal{M}\begin{bmatrix} w \\ r \end{bmatrix} \iff \begin{bmatrix} z(t) \\ y(t) \end{bmatrix} = \int_0^t M(t-\tau)\begin{bmatrix} w(\tau) \\ r(\tau) \end{bmatrix} d\tau$$

$$r(t) = \mathsf{Diag}\Big(\gamma(t)\Big)y(t).$$

Mean-Square Stability & Structured Stochastic Uncertainty

Goal: What are the conditions of MSS?



$$\gamma(t) := \begin{bmatrix} \gamma_1(t) \\ \vdots \\ \gamma_n(t) \end{bmatrix}$$

$$\mathbb{E}\left[\gamma(t)\gamma^*(\tau)\right] := \mathbf{\Gamma}\delta(t-\tau)$$

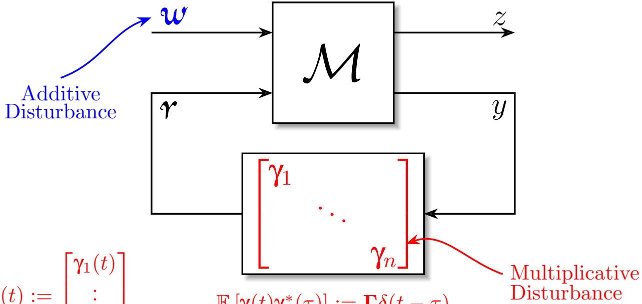
$$\mathbb{E}\left[w(t)w^*(\tau)\right] := \mathbf{W}(t)\delta(t-\tau)$$

$$\begin{bmatrix} z \\ y \end{bmatrix} = \mathcal{M}\begin{bmatrix} w \\ r \end{bmatrix} \iff \begin{bmatrix} z(t) \\ y(t) \end{bmatrix} = \int_0^t M(t-\tau)\begin{bmatrix} w(\tau) \\ r(\tau) \end{bmatrix} d\tau$$

$$r(t) = \mathrm{Diag}\Big(\gamma(t)\Big)y(t).$$

Mean-Square Stability & Structured Stochastic Uncertainty

Goal: What are the conditions of MSS?



$$\gamma(t) := \begin{bmatrix} \gamma_1(t) \\ \vdots \\ \gamma_n(t) \end{bmatrix}$$

$$\mathbb{E}\left[\gamma(t)\gamma^*(\tau)\right] := \mathbf{\Gamma}\delta(t-\tau)$$

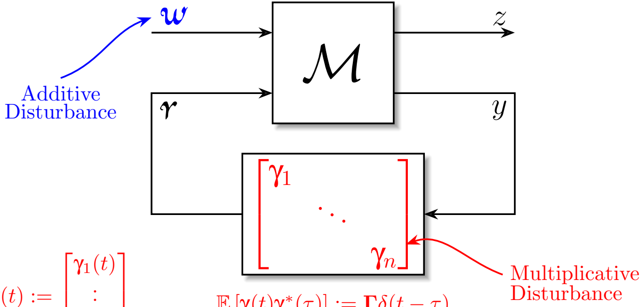
$$\mathbb{E}\left[w(t)w^*(\tau)\right] := \mathbf{W}(t)\delta(t-\tau)$$

$$\begin{bmatrix} z \\ y \end{bmatrix} = \mathcal{M} \begin{bmatrix} w \\ r \end{bmatrix} \iff \begin{bmatrix} z(t) \\ y(t) \end{bmatrix} = \sum_{\tau=0}^{t} M(t-\tau) \begin{bmatrix} w(\tau) \\ r(\tau) \end{bmatrix}$$

$$r(t) = \mathsf{Diag}\Big(\gamma(t)\Big) y(t).$$

Mean-Square Stability & Structured Stochastic Uncertainty

Goal: What are the conditions of MSS?



$$\gamma(t) := \begin{bmatrix} \gamma_1(t) \\ \vdots \\ \gamma_n(t) \end{bmatrix}$$

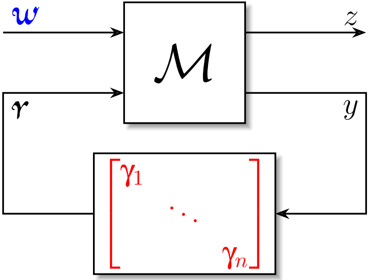
$$\mathbb{E}\left[\gamma(t)\gamma^*(\tau)\right] := \mathbf{\Gamma}\delta(t-\tau)$$

$$\mathbb{E}\left[w(t)w^*(\tau)\right] := \mathbf{W}(t)\delta(t-\tau)$$

$$\begin{bmatrix} z \\ y \end{bmatrix} = \mathcal{M}\begin{bmatrix} w \\ r \end{bmatrix} \iff \begin{bmatrix} z(t) \\ y(t) \end{bmatrix} = \int_0^t M(t-\tau)\begin{bmatrix} w(\tau) \\ r(\tau) \end{bmatrix} d\tau$$

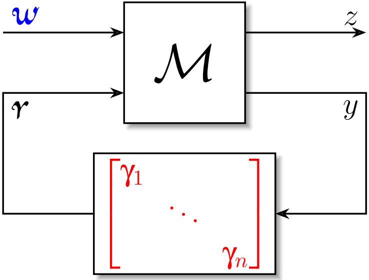
$$r(t) = \mathrm{Diag}\Big(\gamma(t)\Big)y(t).$$

Stochastic Block Diagrams



$$\mathbb{E}\left[\gamma(t)\gamma^*(\tau)\right] = \mathbf{\Gamma}\delta(t-\tau)$$
$$\mathbb{E}\left[w(t)w^*(\tau)\right] = \mathbf{W}(t)\delta(t-\tau)$$

Stochastic Block Diagrams



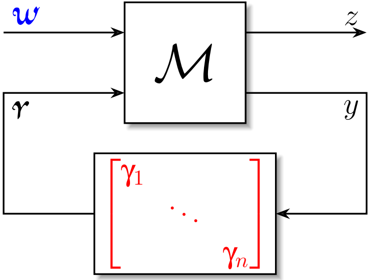
$$\mathbb{E}\left[\boldsymbol{\gamma}(t)\boldsymbol{\gamma}^*(\tau)\right] = \boldsymbol{\Gamma}\delta(t-\tau)$$
$$\mathbb{E}\left[w(t)w^*(\tau)\right] = \mathbf{W}(t)\delta(t-\tau)$$

$$w := \frac{dw}{dt}$$

$$\boldsymbol{\gamma} := \frac{d\gamma}{dt}$$

$$r := \frac{dr}{dt}$$

Stochastic Block Diagrams



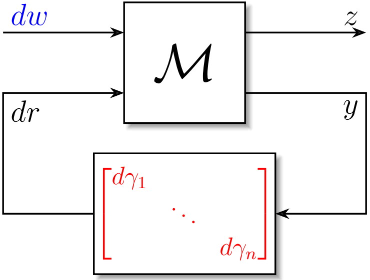
$$\mathbb{E}\left[\boldsymbol{\gamma}(t)\boldsymbol{\gamma}^*(\tau)\right] = \boldsymbol{\Gamma}\delta(t-\tau)$$
$$\mathbb{E}\left[\boldsymbol{w}(t)\boldsymbol{w}^*(\tau)\right] = \mathbf{W}(t)\delta(t-\tau)$$

White Process Representation

$$\boldsymbol{w} := \frac{dw}{dt}$$

$$\boldsymbol{\gamma} := \frac{d\gamma}{dt}$$

$$\boldsymbol{r} := \frac{dr}{dt}$$

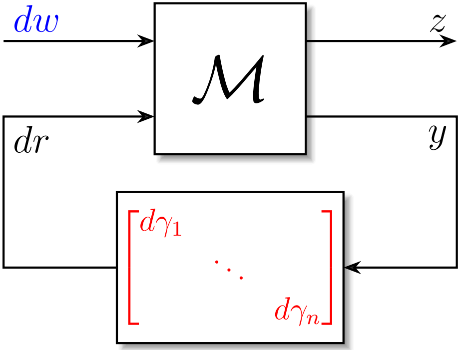


$$\mathbb{E}\left[d\gamma(t)d\gamma^*(t)\right] = \boldsymbol{\Gamma}dt$$
$$\mathbb{E}\left[dw(t)dw^*(t)\right] = \mathbf{W}(t)dt$$

Wiener Process Representation

$$\begin{bmatrix} z \\ y \end{bmatrix} = \mathcal{M}\begin{bmatrix} dw \\ dr \end{bmatrix} \iff \begin{bmatrix} z(t) \\ y(t) \end{bmatrix} = \int_0^t M(t-\tau)\begin{bmatrix} dw(\tau) \\ dr(\tau) \end{bmatrix}$$
$$dr(t) = \mathrm{Diag}\big(d\boldsymbol{\gamma}(t)\big)y(t).$$

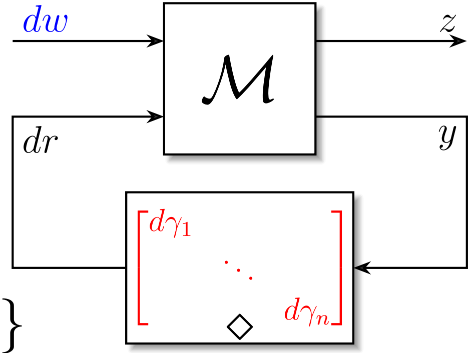
Stochastic Interpretations: Itō & Stratonovich



$$\mathbb{E}\left[d\gamma(t)d\gamma^*(t)\right] = \mathbf{\Gamma}dt$$
$$\mathbb{E}\left[dw(t)dw^*(t)\right] = \mathbf{W}(t)dt$$

$$\begin{bmatrix} z \\ y \end{bmatrix} = \mathcal{M}\begin{bmatrix} dw \\ dr \end{bmatrix} \iff \begin{bmatrix} z(t) \\ y(t) \end{bmatrix} = \int_0^t M(t-\tau)\begin{bmatrix} dw(\tau) \\ dr(\tau) \end{bmatrix}$$
$$dr(t) = \mathrm{Diag}\big(d\boldsymbol{\gamma}(t)\big)y(t).$$

Stochastic Interpretations: Itō & Stratonovich



$$\diamond \in \{\diamond_I, \diamond_S\}$$

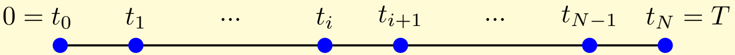
$$\mathbb{E}\left[d\gamma(t)d\gamma^*(t)\right] = \mathbf{\Gamma}dt$$

$$\mathbb{E}\left[dw(t)dw^*(t)\right] = \mathbf{W}(t)dt$$

$$\begin{bmatrix} z \\ y \end{bmatrix} = \mathcal{M}\begin{bmatrix} dw \\ dr \end{bmatrix} \iff \begin{bmatrix} z(t) \\ y(t) \end{bmatrix} = \int_0^t M(t-\tau) \diamond \begin{bmatrix} dw(\tau) \\ dr(\tau) \end{bmatrix}$$

$$dr(t) = \mathrm{Diag}\big(d\boldsymbol{\gamma}(t)\big) \diamond y(t).$$

Recall: Stochastic Integrals



- **Deterministic:**

$$\int_0^T v(t)dt := \lim_{N\to\infty} \sum_{k=0}^{N-1} v(\bar{t}_i)\Big(t_{i+1} - t_i\Big); \quad \forall \bar{t}_i \in [t_i, t_{i+1}]$$

- **Itō:**

$$\int_0^T v(t) \diamond_I dw(t) := \lim_{N\to\infty} \sum_{i=0}^{N-1} v(t_i)\Big(w(t_{i+1}) - w(t_i)\Big)$$

- **Stratonovich:**

$$\int_0^T v(t) \diamond_S dw(t) := \lim_{N\to\infty} \sum_{i=0}^{N-1} v\left(\frac{t_i + t_{i+1}}{2}\right)\Big(w(t_{i+1}) - w(t_i)\Big)$$

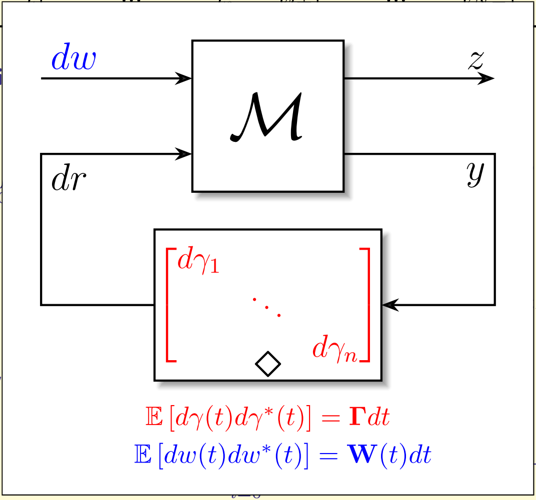
Recall: Stochastic Integrals

$0 = t_0$ t_1 ... t_i t_{i+1} ... t_{N-1} $t_N = T$

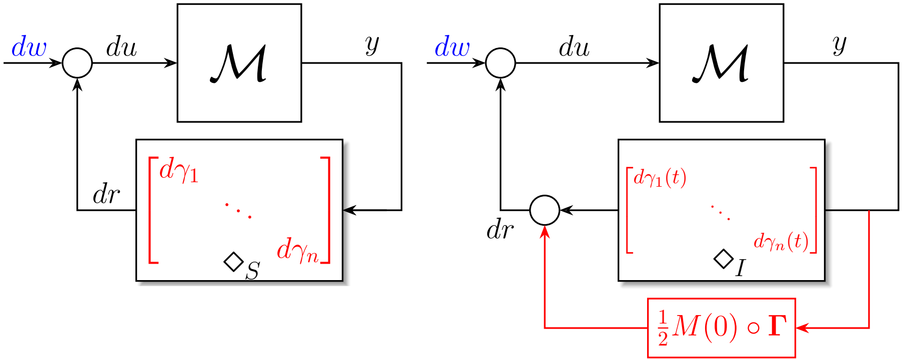
- **Determini**

- **Itō:**

- **Stratonov**



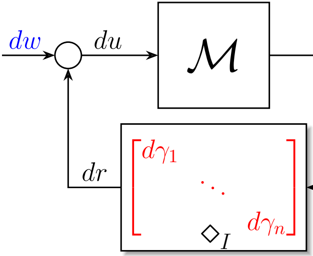
Stratonovich to Ito Conversion



$$\mathbb{E}\left[d\gamma(t)d\gamma^*(t)\right] := \mathbf{\Gamma}dt; \qquad y(t) = \int_0^t M(t-\tau)du(\tau);$$

"$\circ$" is the Hadamard (element-by-element) product

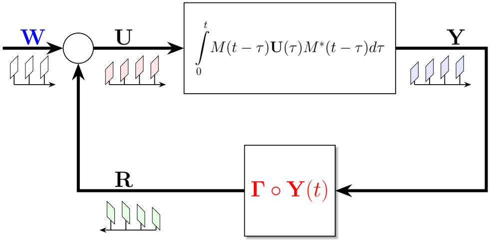
The two stochastic block diagrams are "equivalent in the mean-square sense".



$\mathbb{E}\left[d\gamma(t)d\gamma^*(t)\right] = \mathbf{\Gamma}dt$

$\mathbb{E}\left[dw(t)dw^*(t)\right] = \mathbf{W}(t)dt$

Stochastic Block Diagram

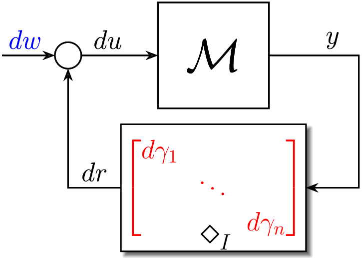


$\mathbb{E}\left[du(t)du^*(t)\right] = \mathbf{U}(t)dt;$ $\quad$ $\mathbb{E}\left[y(t)y^*(t)\right] = \mathbf{Y}(t);$

$\mathbb{E}\left[dr(t)dr^*(t)\right] = \mathbf{R}(t)dt;$ $\quad$ "$\circ$": Hadamard Product;

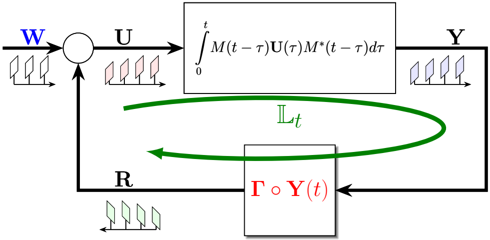
Deterministic Covariance Block Diagram

Loop Gain Operator & Mean-Square Stability



$\mathbb{E}\left[d\gamma(t)d\gamma^*(t)\right] = \mathbf{\Gamma}dt$
$\mathbb{E}\left[dw(t)dw^*(t)\right] = \mathbf{W}(t)dt$

Stochastic Block Diagram

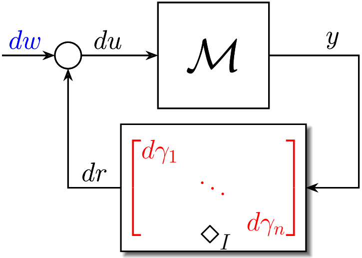


$\mathbb{E}\left[du(t)du^*(t)\right] = \mathbf{U}(t)dt;$ $\mathbb{E}\left[y(t)y^*(t)\right] = \mathbf{Y}(t);$
$\mathbb{E}\left[dr(t)dr^*(t)\right] = \mathbf{R}(t)dt;$ "$\circ$": Hadamard Product;

Deterministic Covariance Block Diagram

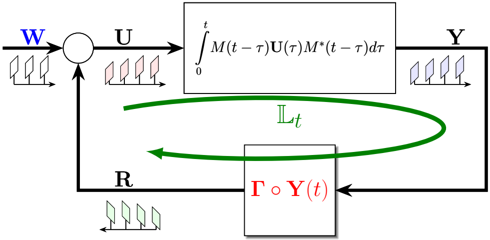
$$\mathbb{L}_t(\mathbf{U}) := \mathbf{\Gamma} \circ \left(\int_0^t M(t-\tau)\mathbf{U}(\tau)M^*(t-\tau)d\tau \right), \qquad \mathbb{L} := \lim_{t\to\infty} \mathbb{L}_t$$

Loop Gain Operator & Mean-Square Stability



$\mathbb{E}\left[d\gamma(t)d\gamma^*(t)\right] = \mathbf{\Gamma}dt$
$\mathbb{E}\left[dw(t)dw^*(t)\right] = \mathbf{W}(t)dt$

Stochastic Block Diagram



$\mathbb{E}\left[du(t)du^*(t)\right] = \mathbf{U}(t)dt;$ $\quad \mathbb{E}\left[y(t)y^*(t)\right] = \mathbf{Y}(t);$
$\mathbb{E}\left[dr(t)dr^*(t)\right] = \mathbf{R}(t)dt;$ $\quad$ "$\circ$": Hadamard Product;

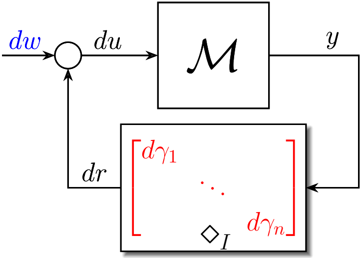
Deterministic Covariance Block Diagram

$$\mathbb{L}_t(\mathbf{U}) := \mathbf{\Gamma} \circ \left(\int_0^t M(t-\tau)\mathbf{U}(\tau)M^*(t-\tau)d\tau\right), \qquad \mathbb{L} := \lim_{t\to\infty} \mathbb{L}_t$$

Necessary & Sufficient Conditions of Mean-Square Stability:

- Forward Block is Stable (Finite H^2-norm)
- Spectral Radius of $\mathbb{L}$ is strictly less than 1, $\rho(\mathbb{L}) < 1$

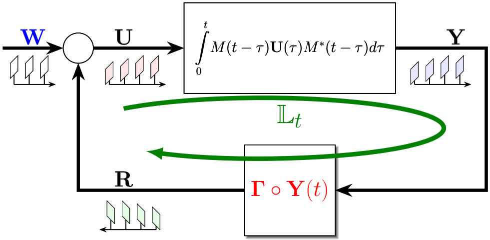
Loop Gain Operator & Mean-Square Stability



$\mathbb{E}\left[d\gamma(t)d\gamma^*(t)\right] = \mathbf{\Gamma}dt$

$\mathbb{E}\left[dw(t)dw^*(t)\right] = \mathbf{W}(t)dt$

Stochastic Block Diagram



$\mathbb{E}\left[du(t)du^*(t)\right] = \mathbf{U}(t)dt;$ $\quad \mathbb{E}\left[y(t)y^*(t)\right] = \mathbf{Y}(t);$

$\mathbb{E}\left[dr(t)dr^*(t)\right] = \mathbf{R}(t)dt;$ $\quad$ "$\circ$": Hadamard Product;

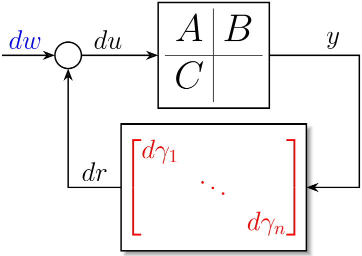
Deterministic Covariance Block Diagram

$$\mathbb{L}_t(\mathbf{U}) := \mathbf{\Gamma} \circ \left(\int_0^t M(t-\tau)\mathbf{U}(\tau)M^*(t-\tau)d\tau \right), \qquad \mathbb{L} := \lim_{t\to\infty} \mathbb{L}_t$$

Two important quantities related to $\mathbb{L}$:

- Spectral Radius: $\rho(\mathbb{L})$
- Worst-Case Covariance: $\mathbb{L}(\hat{\mathbf{U}}) = \rho(\mathbb{L})\hat{\mathbf{U}}$ (Perron-Frobenius "Eigen-matrix")

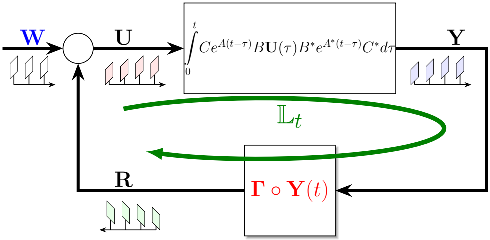
Application to State-Space Realizations



$\mathbb{E}\left[d\gamma(t)d\gamma^*(t)\right] = \mathbf{\Gamma}dt$

$\mathbb{E}\left[dw(t)dw^*(t)\right] = \mathbf{W}(t)dt$

Stochastic Block Diagram



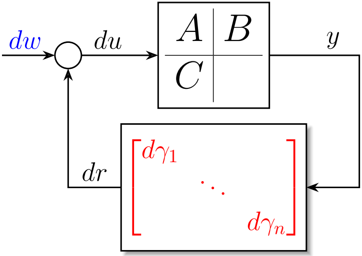
$\mathbb{E}\left[du(t)du^*(t)\right] = \mathbf{U}(t)dt;$ $\quad$ $\mathbb{E}\left[y(t)y^*(t)\right] = \mathbf{Y}(t);$

$\mathbb{E}\left[dr(t)dr^*(t)\right] = \mathbf{R}(t)dt;$ $\quad$ "$\circ$": Hadamard Product;

Deterministic Covariance Block Diagram

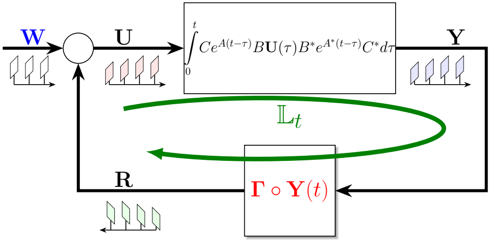
$$\mathbb{L}_t(\mathbf{U}) := \mathbf{\Gamma} \circ \left(\int_0^t Ce^{A(t-\tau)}B\mathbf{U}(\tau)B^*e^{A^*(t-\tau)}C^*d\tau\right), \qquad \mathbb{L} := \lim_{t\to\infty} \mathbb{L}_t$$

Application to State-Space Realizations



$\mathbb{E}\left[d\gamma(t)d\gamma^*(t)\right] = \mathbf{\Gamma} dt$
$\mathbb{E}\left[dw(t)dw^*(t)\right] = \mathbf{W}(t)dt$

Stochastic Block Diagram



$\mathbb{E}\left[du(t)du^*(t)\right] = \mathbf{U}(t)dt;$ $\quad \mathbb{E}\left[y(t)y^*(t)\right] = \mathbf{Y}(t);$
$\mathbb{E}\left[dr(t)dr^*(t)\right] = \mathbf{R}(t)dt;$ $\quad$ "$\circ$": Hadamard Product;

Deterministic Covariance Block Diagram

$$\mathbb{L}_t(\mathbf{U}) := \mathbf{\Gamma} \circ \left(\int_0^t Ce^{A(t-\tau)}B\mathbf{U}(\tau)B^*e^{A^*(t-\tau)}C^*d\tau\right), \qquad \mathbb{L} := \lim_{t\to\infty} \mathbb{L}_t$$

$$\bar{\mathbf{R}} = \mathbb{L}(\bar{\mathbf{U}}) \quad \Leftrightarrow \quad \begin{cases} \bar{\mathbf{R}} = \mathbf{\Gamma} \circ \left(C\bar{\mathbf{X}}C^*\right) \\ 0 = A_k\bar{\mathbf{X}} + \bar{\mathbf{X}}A_k^* + B\bar{\mathbf{U}}B^* \end{cases}$$

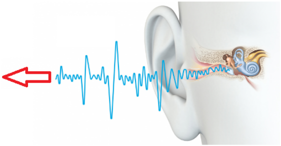
where $A_{\text{Itō}} := A$ and $A_{\text{Strat}} := A + \frac{1}{2}B\Big((CB) \circ \mathbf{\Gamma}\Big)C$

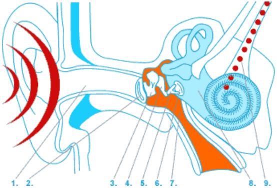
Proofs & An Application

- **Discrete-Time Setting**: Bamieh, B., & Filo, M. (2020). An input–output approach to structured stochastic uncertainty. IEEE Transactions on Automatic Control.
- **Continuous-Time Setting**: Filo, M., & Bamieh, B. (2018). An input-output approach to structured stochastic uncertainty in continuous time. arXiv preprint.
- **Stochastic Block Diagrams**: Filo, M., & Bamieh, B. (2018). A block diagram approach to stochastic calculus with application to multiplicative uncertainty analysis. IEEE CDC.

Proofs & An Application

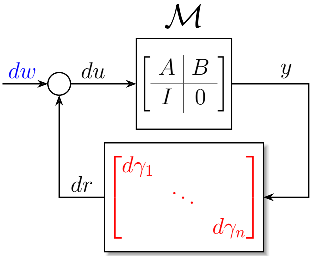
- **Discrete-Time Setting**: Bamieh, B., & Filo, M. (2020). An input–output approach to structured stochastic uncertainty. IEEE Transactions on Automatic Control.
- **Continuous-Time Setting**: Filo, M., & Bamieh, B. (2018). An input-output approach to structured stochastic uncertainty in continuous time. arXiv preprint.
- **Stochastic Block Diagrams**: Filo, M., & Bamieh, B. (2018). A block diagram approach to stochastic calculus with application to multiplicative uncertainty analysis. IEEE CDC.
- **Application:** Stochastic Instabilities in the inner ear!





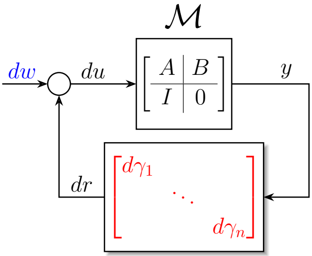
→ Filo, M., & Bamieh, B. (2017). Investigating cochlear instabilities using structured stochastic uncertainty. IEEE CDC
→ Filo, M., & Bamieh, B. (2020). Investigating Instabilities in the Mammalian Cochlea Using a Stochastic Uncertainty Model. IEEE Transactions on Molecular, Biological, and Multi-Scale Communications.

Concluding Remarks & Future Work



SDE: $dy(t) = Ay(t)dt + B\mathrm{Diag}\Big(d\gamma(t)\Big)y(t) + Bdw(t)$

Concluding Remarks & Future Work

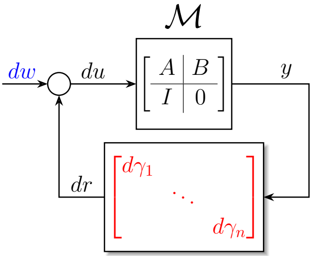


$$\text{SDE: } dy(t) = Ay(t)dt + B\mathrm{Diag}\Big(d\gamma(t)\Big)y(t) + Bdw(t)$$

Extends and unifies the analysis for systems $\mathcal{M}$:

- State space realizations
- Infinite dimensional systems with finite number of multiplicative disturbances
- Systems with delays

Concluding Remarks & Future Work



$$\text{SDE: } dy(t) = Ay(t)dt + B\text{Diag}\Big(d\gamma(t)\Big)y(t) + Bdw(t)$$

Extends and unifies the analysis for systems $\mathcal{M}$:

- State space realizations
- Infinite dimensional systems with finite number of multiplicative disturbances
- Systems with delays

Future Direction: Extend the analysis for

- *Colored* disturbances
- Spatially distributed disturbances with symmetries.