Statisitical Inference Course Project

Me

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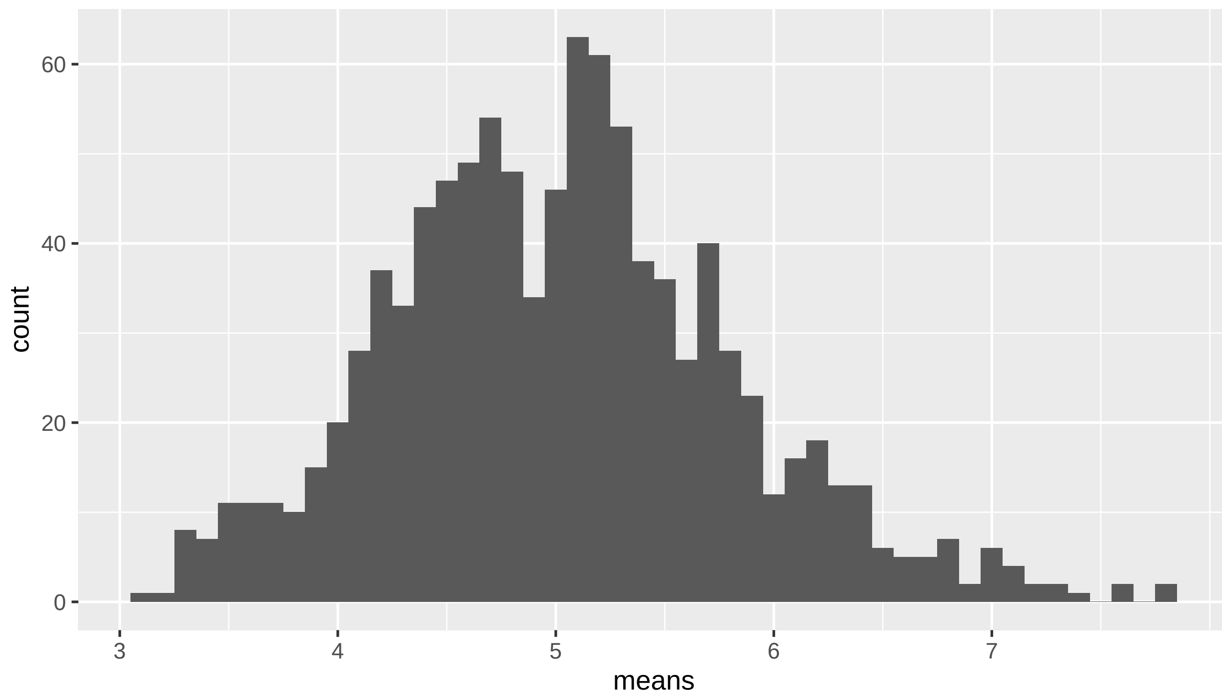
Overview

This project will investigate the exponential distribution in R and compare it with the Central Limit Theorem. The exponential distribution can be simulated in R with rexp(n, lambda) where lambda is the rate parameter. The mean of exponential distribution is 1/lambda and the standard deviation is also 1/lambda. Set lambda = 0.2 for all of the simulations. Investigate the distribution of averages of 40 exponentials. Note that I will need to do a thousand simulations.

Simulations

```r
# load necessary libraries
library(ggplot2)
# set constants
lambda <- 0.2 # lambda for rexp
n <- 40 # number of exponentials
numberOfSimulations <- 1000 # number of tests
# set the seed to create reproducability
set.seed(111)
# run the test resulting in n x numberOfSimulations matrix
exponentialDistributions <- matrix(data=rexp(n * numberOfSimulations, lambda), nrow=numberOfSimulations)
exponentialDistributionMeans <- data.frame(means=apply(exponentialDistributions, 1, mean))
```



Sample Mean versus Theoretical Mean

The expected mean μ of an exponential distribution of rate λ is

$\mu = \frac{1}{\lambda}$

```
mu <- 1/lambda
mu
```

```
## [1] 5
```

Let $\bar{X}$ be the average sample mean of 1000 simulations of 40 randomly sampled exponential distributions.

```
meanOfMeans <- mean(exponentialDistributionMeans$means)
meanOfMeans
```

```
## [1] 5.02562
```

As you can see the expected mean and the avarage sample mean are very close

Sample Variance versus Theoretical Variance

The expected standard deviation σ of an exponential distribution of rate λ is

$\sigma = \frac{1/\lambda}{\sqrt{n}}$

The e

```
sd <- 1/lambda/sqrt(n)
sd
```

```
## [1] 0.7905694
```

The variance Var of standard deviation σ is

$Var = \sigma^2$

```
Var <- sd^2
Var
```

```
## [1] 0.625
```

Let Var_x be the variance of the average sample mean of 1000 simulations of 40 randomly sampled exponential distribution, and σ_x the corresponding standard deviation.

```
sd_x <- sd(exponentialDistributionMeans$means)
sd_x
```

```
## [1] 0.7944996
```

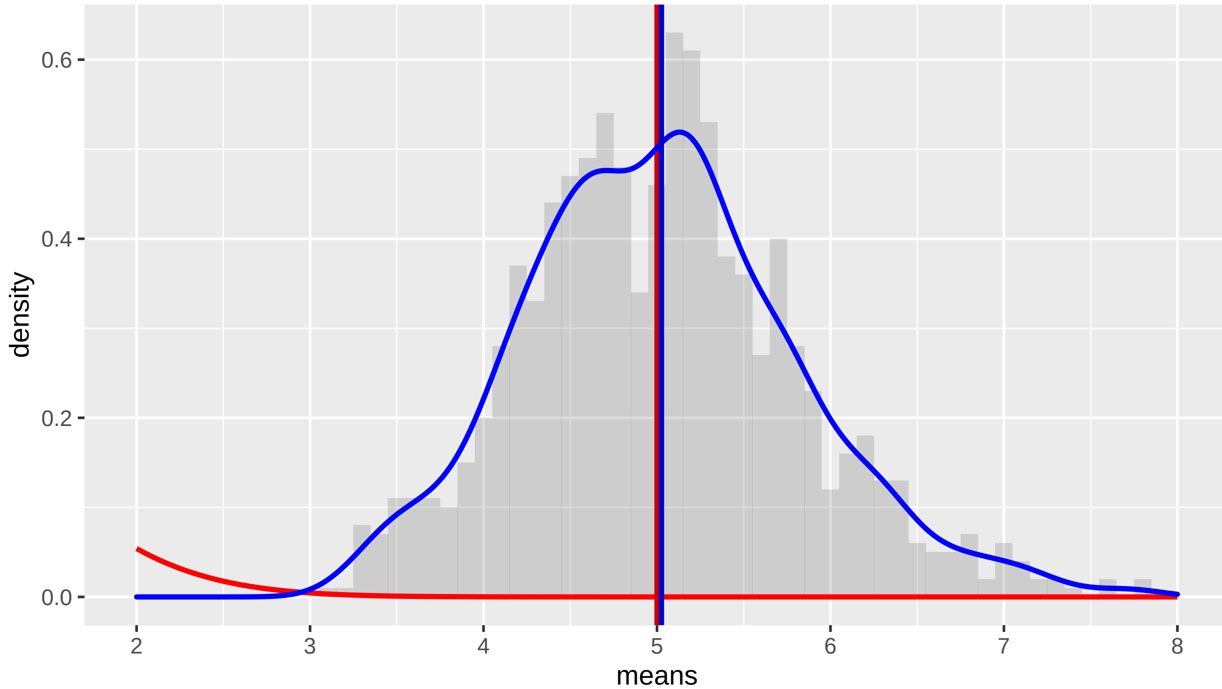
```
Var_x <- var(exponentialDistributionMeans$means)
Var_x
```

```
## [1] 0.6312296
```

As you can see the standard deviations are very close Since variance is the square of the standard deviations, minor differences will be enhanced, but are still pretty close.

Distribution

Comparing the population means & standard deviation with a normal distribution of the expected values. Added lines for the calculated and expected means



As you can see from the graph, the calculated distribution of means of random sampled exponential distributions overlaps the normal distribution with the expected values based on the given lambda