



Differential evolution optimization combined with chaotic sequences for image contrast enhancement

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ABSTRACT

Evolutionary Algorithms (EAs) are stochastic and robust meta-heuristics of evolutionary computation field useful to solve optimization problems in image processing applications. Recently, as special mechanism to avoid being trapped in local minimum, the ergodicity property of chaotic sequences has been used in various designs of EAs. Three differential evolution approaches based on chaotic sequences using logistic equation for image enhancement process are proposed in this paper. Differential evolution is a simple yet powerful evolutionary optimization algorithm that has been successfully used in solving continuous problems. The proposed chaotic differential evolution schemes have fast convergence rate but also maintain the diversity of the population so as to escape from local optima. In this paper, the image contrast enhancement is approached as a constrained non-linear optimization problem. The objective of the proposed chaotic differential evolution schemes is to maximize the fitness criterion in order to enhance the contrast and detail in the image by adapting the parameters using a contrast enhancement technique. The proposed chaotic differential evolution schemes are compared with classical differential evolution to two testing images. Simulation results on three images show that the application of chaotic sequences instead of random sequences is a possible strategy to improve the performance of classical differential evolution optimization algorithm.

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1. Introduction

The goal of enhancement techniques is neighborhood of to process an image so that the result is more suitable than the original image for specific applications or set of objectives [1]. A vast literature has emerged recently on image enhancement using linear techniques or nonlinear techniques [2–4].

The problem of enhancing contrast of images enjoys much attention and spans a wide gamut of applications. The existing image enhancement methods can be categorized into four classes; (i) contrast enhancement, (ii) edge enhancement, (iii) noise reductions, and (iv) edge restorations. Among those techniques, this paper is focused on contrast enhancement.

Many images, such as medical images, remote sensing images, electron microscopy images and even our real-life photographic pictures, suffer from poor contrast. Therefore, it is necessary to enhance the contrast of such images before further processing or analysis can be conducted.

Various image enhancement algorithms have been proposed [1–4]. Histogram equalization [5] and contrast manipulations [6] are well-known methods for enhancing the contrast of a given image. Recently, several meta-heuristic methods, mainly evolutionary algorithms, have been devised for image processing applications [7–16], including image enhancement problems [7,17,18].

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In this paper, the image contrast enhancement is approached as a constrained nonlinear optimization problem. In this context, this paper introduces an image enhancement technique driven by three differential evolution approaches based on chaotic sequences using logistic equation [19,20].

The traditional differential evolution (DE) developed by Storn and Price [21,22] is one of the most superior evolutionary algorithms. The DE algorithm presents simple structure, convergence speed, versatility, and robustness. In the context of population diversity to improve the performance of DE, the use of chaotic sequences combined with DE can be useful [23–26].

The objective of the proposed chaotic DE scheme is to maximize the objective fitness criterion in order to enhance the contrast and detail in the image by adapting the parameters using a local enhancement technique. The proposed chaotic DE schemes have been compared with classical DE algorithm to enhance two testing images.

The remainder of this paper is organized as follows. In Section 2, the fundamentals of image enhancement problem are introduced, while the concepts of traditional and chaotic DE approaches are described in Section 3. Simulation results are presented in Section 4, while the conclusion is presented in Section 5.

2. Description of image contrast enhancement problem

Image enhancement is a technology to improve the presentation of an image, including increase the contrast and sharpen the features, which improves their visual quality for human eyes. Image enhancement methods selectively emphasize or inhibit certain information in an image to strengthen the usability of the image. Image enhancement does not increase the intrinsic information in the original image, but is beneficial to further image applications, such as facilitating image segmentation, or enforcing the capability of human or machine recognition systems to recognize and interpret useful information in the image [27].

Recently, with the growing quality in image acquisition, improved image enhancement technologies are needed for many applications. In this context, extensive research has been conducted on local image enhancement algorithms [1–6]. Each of these algorithms can be classified into two types of image enhancement methods [28]: indirect image enhancement methods and direct image enhancement methods. The algorithms based on indirect image contrast enhancement methods enhance the image without measuring the contrast. The algorithms called direct local contrast enhancement methods establish a criterion of contrast measure and enhance the images by improving the contrast measurement directly [29].

In this paper, the traditional DE and three chaotic DE approaches are applied to adapt the gray-level intensity transformation in the image using a direct image enhancement approach. A key step in the direct image enhancement approach is the establishment of a suitable image contrast measure.

One of the requirements of the DE algorithms based image enhancement is to choose a criterion that is related to a fitness function. The objective function (fitness function) shown in Eq.(1)[17,30] is adopted in this paper for an enhancement criterion:

$$F(Z) = \log(\log(E(I(Z)))) \cdot \frac{ne(I(Z))}{PH \cdot PV} \cdot H(I(Z)) \quad (1)$$

and

$$g(i,j) = \left[d \cdot \frac{PH}{\sigma(i,j) + b} \right] \cdot [f(i,j) - c \cdot m(i,j)] + m(i,j)^a \quad (2)$$

where $m(i,j)$ and $\sigma(i,j)$ are the gray-level mean and standard deviation, respectively; $F(Z)$ is the fitness function (maximization problem), $I(Z)$ denotes the original image I with the transformation T in each pixel at location (i,j) applied according to Eq. (2); a , b , c , and d are the parameters defined over the real positive numbers and they are the same for the whole image. The parameters a , b , c , and d are the respective parameters that given by the individual of differential evolution, $Z = (a \ b \ c \ d)$. Furthermore, $E(I(Z))$ is the intensity of the edges detected with a Sobel edge detector that is applied to the transformed image $I(Z)$, ne is the number of edge pixels as detected with the Sobel edge detector, PH and PV are the number of pixels in the horizontal and vertical direction of the image, respectively; $E(I)$ is the sum of intensities of the edges included in the enhanced image, lastly, $H(I(Z))$ measures the entropy of the image $I(Z)$ [10,17].

The proposed DE approaches are to determine the solution that maximizes $F(Z)$. To achieve these objectives we need to (i) increase the relative number of pixels in the edges of the image; (ii) increase the overall intensity of edges; (iii) transform the histogram of the image to one that approximates a uniform distribution by maximizing the entropic measure [17].

3. Optimization using differential evolution

Evolutionary computation paradigms, derived from biological adaptation paradigms, are stochastic population based methods that have proven to be powerful and robust techniques to solve optimization problems for complex systems. Classical evolutionary computation is comprised of techniques called evolutionary algorithms involving evolutionary programming, evolution strategies, genetic algorithms, and genetic programming.

The advantages of EAs include global search capability, effective constraints handling capacity, reliable performance and minimum information requirement, make it a potential choice for solving image processing problems [7–18].

DE is one of the most prominent new generation evolutionary algorithms, proposed by Storn and Price [21,22], to exhibit consistent and reliable performance in nonlinear optimization applications, including problems in various fields such as control and synchronization of chaotic systems [31], parameter identification [32,33], reliability-redundancy optimization [34], economic dispatch of electric power systems [35], inverse method [36], and others. The advantages of DE over other evolutionary algorithms, like simple and compact structure, few control parameters, high convergence characteristics, have made it a popular stochastic optimizer for some global optimization problems over continuous spaces.

While DE shares similarities with other EAs, it differs significantly in the sense that distance and direction information from the current population is used to guide the search process. DE uses real coding of floating point numbers. DE technique combines simple arithmetic operators with the classical events of crossover, mutation and selection individual solution. The key idea behind DE is a scheme for generating trial parameter vectors. Mutation and crossover are used to generate new vectors (trial vectors), and selection then determines which of the vectors will survive the next generation.

In this section, we describe two optimization approaches based on DE. The first one is the traditional DE which is presented in Section 3.1. The second one is a DE approach based on chaotic sequences which is described in Section 3.2.

3.1. Traditional DE strategy

A DE algorithm is a stochastic parallel direct search optimization method that is fast and reasonably robust. Price and Storn [21,22] proposed 10 different strategies for DE based on the individual being perturbed, the number of individuals used in the mutation process and the type of crossover used. Each strategy generates trial vectors by adding the weighted difference between other randomly selected members of the population. The general convention used above is $\text{DE}/\alpha/\beta/\gamma$. DE stands for differential evolution, α represents a string denoting the vector to be perturbed, β is the number of difference vectors considered for perturbation of α , and γ stands for the type of crossover being used (*exp*: exponential; *bin*: binomial).

The strategy implemented here was the $\text{DE}/\text{rand}/1/\text{bin}$, meaning that the target vector is randomly selected, and only one difference vector is used. The *bin* acronym indicates that the recombination is controlled by a *binomial* decision rule. The optimization procedure of $\text{DE}/\text{rand}/1/\text{bin}$ is given by the following steps and procedures [34]:

Step 1: Determination of parameters setting

The user must choose the key parameters that control DE, i.e., population size, boundary constraints of optimization variables, mutation factor (f_m), crossover rate (CR), and the stopping criterion (t_{max}).

Step 2: Initialization of the initial population of individuals

Initialize the generation's counter, $t = 0$, and also initialize a population of individuals (solution vectors) $x(t)$ with random values generated according to a uniform probability distribution in the n -dimensional problem space.

Step 3: Evaluation of individuals

Evaluate the fitness value of each individual.

Step 4: Mutation operation (or differential operation)

Mutation generally refers to an operation that adds a uniform random variable to one or more vector parameters. The DE relies upon the population itself to supply increments of the appropriate magnitude and orientation. Mutate individuals according to the following equation:

$$z_i(t+1) = x_{i,1}(t) + f_m \cdot [x_{i,2}(t) - x_{i,3}(t)] \quad (3)$$

In the above equations, $i = 1, 2, \dots, N$ is the individual's index of population; $x_i(t) = [x_{i,1}(t), x_{i,2}(t), \dots, x_{i,n}(t)]^T$ stands for the position of the i -th individual of population of real-valued n -dimensional vectors; $z_i(t) = [z_{i,1}(t), z_{i,2}(t), \dots, z_{i,n}(t)]^T$ stands for the position of the i -th individual of a *mutant vector*; $f_m > 0$ is a real parameter, called *mutation factor*, which controls the amplification of the difference between two individuals so as to avoid search stagnation. The mutation operation randomly select the target vector $x_{i,1}(t)$, with $i \neq i_3$. Then, two individuals $x_{i,2}(t)$ and $x_{i,3}(t)$ are randomly selected with $i_1 \neq i_2 \neq i_3 \neq i$, and the difference vector $x_{i,2} - x_{i,3}$ is calculated.

Step 5: Crossover (recombination) operation

Following the mutation operation, crossover is applied in the population. For each mutant vector, $z_i(t+1)$, an index $rnbr(i) \in \{1, 2, \dots, n\}$ is randomly chosen using a uniform distribution, and a *trial vector*, $u_i(t+1) = [u_{i,1}(t+1), u_{i,2}(t+1), \dots, u_{i,n}(t+1)]^T$, is generated with

$$u_{ij}(t+1) = \begin{cases} z_{ij}(t+1) & \text{if } randb(j) \leq CR \text{ or } j = rnbr(i), \\ x_{ij}(t) & \text{otherwise,} \end{cases} \quad (4)$$

where $j = 1, 2, \dots, n$ is a parameter index; $x_{ij}(t)$ stands for the i -th individual of j -th real-valued vector; $z_{ij}(t)$ stands for the i -th individual of j -th real-valued vector of a *mutant vector*; $u_{ij}(t)$ stands for the i -th individual of j -th real-valued vector after

crossover operation; $randb(j)$ is the j -th evaluation of a uniform random number generation in the range $[0, 1]$; CR is a crossover rate in the range $[0, 1]$.

Step 6: Selection operation

To decide whether or not the vector $u_i(t+1)$ should be a member of the population comprising the next generation, it is compared to the corresponding vector $x_i(t)$. Thus, if F denotes the objective function under minimization, then

$$x_i(t+1) = \begin{cases} u_i(t+1), & \text{if } F(u_i(t+1)) > F(x_i(t)), \\ x_i(t), & \text{otherwise.} \end{cases} \quad (5)$$

Step 7: Verification of the stopping criterion

Update the generation number using $t = t + 1$. Proceed to Step 3 until a stopping criterion is met, usually a maximum number of iterations (generations), t_{max} .

3.2. Chaotic DE approaches using logistic equation

DE has essential three control parameters: the population number, the mutation factor and the crossover rate. All of these parameters effect the speed and robustness of the search with various degrees. The difficulty in the use of DE arises in that the choice of these is mainly based on empirical evidence and practical experience [37–39]. DE's parameters usually are constant throughout the entire search process. However, it is difficult to properly set control parameters in DE.

The application of chaotic sequences in mutation factor design is a powerful strategy to diversify the DE population and improve DE's performance in preventing premature convergence to local minima. Recently, some applications of chaotic sequences in DE design have been investigated in the literature [23–26,40]. The application of chaotic sequences can be a good alternative to provide the search diversity in stochastic optimization procedures. Due to the ergodicity property, chaos can be used to enrich the searching behavior and to avoid being trapped into local optimum in optimization problems.

In this paper, to enrich the searching behavior and to avoid being trapped into local optimum, chaotic dynamics is incorporated into the DE. In this context, three chaotic DE approaches are proposed. Proposed different chaotic DE approaches have used the well-known logistic equation [41,42], which exhibits the sensitive dependence on initial conditions, for determining the mutation factor. The logistic equation is defined as follows.

$$y(k) = \mu \cdot y(k-1) \cdot [1 - y(k-1)] \quad (6)$$

where k is the sample, and μ is a control parameter, $0 < \mu \leq 4$. The behavior of the system of (6) is greatly changed with the variation of μ . The value of determines whether y stabilizes at a constant size, oscillates between a limited sequence of sizes, or behaves chaotically in an unpredictable pattern. A very small difference in the initial value of y causes substantial differences in its long-time behavior. Eq. (6) is deterministic, displaying chaotic dynamics when $\mu = 4$ and $y(1) \notin \{0, 0.25, 0.50, 0.75, 1\}$. In this work, $y(k)$ is distributed in the range $[0, 1]$ provided the initial $y(1) \notin \{0, 0.25, 0.50, 0.75, 1\}$.

The proposed three chaotic DE (CDE) approaches in combination of chaotic sequences are described as follows.

Table 1

Convergence results of $F(Z)$ (30 runs) for the Cameraman image using DE and CDE approaches.

Optimization method	$F(Z)$			
	Maximum (best)	Mean	Minimum (worst)	Standard deviation
DE	234.6990	223.3126	210.8841	7.2417
CDE1	302.6270	232.0065	182.0696	36.4928
CDE2	279.2091	222.7967	182.4815	29.5555
CDE3	265.7112	228.1997	215.8346	15.0172

Table 2

Best result (30 runs) for the Cameraman image using DE and CDE approaches.

Parameter	DE	CDE1	CDE2	CDE3
$F(Z)$	234.6990	302.6270	279.2091	265.7112
a	2.22×10^{-16}	0.1756	0.0409	0.5341
b	0.3008	0.0413	0.0977	0.0286
c	1.0966	1.0606	1.0634	1.0085
d	5.9619	1.8559	4.1014	3.1964
$E(I(Z))$	6.1089×10^4	6.8396×10^4	9.1159×10^4	13.6660×10^4
$ne(I(Z))$	3432	4241	3974	4035
$H(I(Z))$	7.2099	7.4914	7.2989	6.7446

CDE1 approach: The parameter f_m of (3) is modified by the formula (7) through the following equation:

$$z_i(t+1) = x_{i,r_1}(t) + y(k) \cdot [x_{i,r_2}(t) - x_{i,r_3}(t)] \quad (7)$$

CDE2 approach: The parameter f_m of (3) is modified by the formula (8) through the following equation:

$$z_i(t+1) = x_{i,r_1}(t) + y(k) \cdot |\exp(-t/t_{\max})| \cdot [x_{i,r_2}(t) - x_{i,r_3}(t)] \quad (8)$$

CDE3 approach: The parameter f_m of (3) is modified by the formula (9) through the following equation:

$$\tilde{z}_i(t+1) = x_{i,r_1}(t) + (y(k) \cdot 0.8|\sin(6t)| + 0.4) \cdot [x_{i,r_2}(t) - x_{i,r_3}(t)] \quad (9)$$

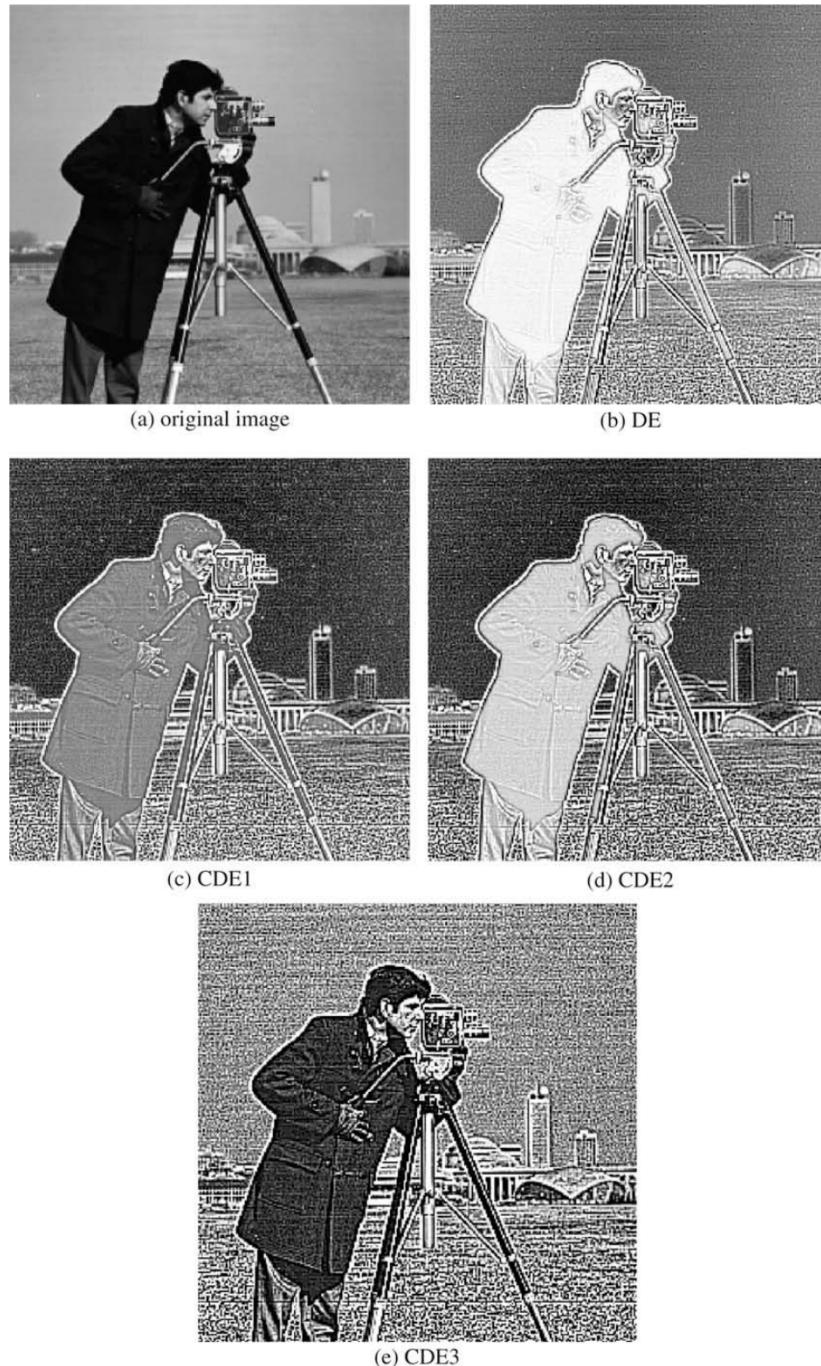


Fig. 1. Enhancements results of Cameraman image: (a) original image, (b) DE, (c) CDE1, (d) CDE2, and (e) CDE3.

4. Simulation results

The optimization problem considered in this paper is to solve the enhancement problem using traditional DE and CDE approaches. Our objective is to maximize the number of pixels in the edges, increase the overall intensity of the edges, and increase the measure of the entropy. Each individual x of population in DE and CDE approaches uses the variables (a b c d), where the boundaries are [0,20] for each variable.

Each optimization method was implemented in Matlab (MathWorks). All the programs were run on a 1.73 GHz Intel Pentium dual-core processor with 2 GB of Random Access Memory. In each case study, 30 independent runs were made for each of the optimization methods involving 30 different initial trial solutions for each optimization method. In this paper, the DE and CDE approaches are adopted using 500 cost function evaluations in each run.

The difficulty in the use of DE arises in that the choice of control parameters. In this case study, CR was set to 0.8, the population size M was 10 and the stopping criterion t_{max} was 50 generations for the traditional DE and CDE approaches. Traditional DE uses $f_m = 0.5$.

In order to evaluate the enhancement method based on DE approaches, two images were evaluated. They are the Cameraman (size: 256 × 256 pixels) and Pout (size: 291 × 240 pixels). The fitness function $F(Z)$ (maximization problem) is given by Eq. (1).

The results of statistical comparison obtained for the Cameraman are summarized in Table 1 with best results in bold font. It is important to note that in terms of mean and maximum of $F(Z)$, the simulation results show that CDE1 performed significantly better than the other tested approaches.

Table 2 and Fig. 1 show the best result (see best value in Table 1) of each tested DE approach. The results of statistical comparison obtained for the Pout are summarized in Table 3, which shows that the CDE2 presented the best result in terms of maximum and mean $F(Z)$. Table 4 and Fig. 2 show the best result (see best value in Table 3 with bold font) of each tested DE approach.

5. Conclusion and further research

The performance of traditional DE is sensitive to the choice of control parameters including the design of mutation factor. In this paper, three DE approaches based on chaotic sequences using logistic equation to adapt the mutation factor for image enhancement process are proposed. The objective of the proposed chaotic DE approaches is to maximize the fitness criterion in order to enhance the contrast and detail in the image by adapting the parameters using a local enhancement technique.

Both the traditional DE and the CDE approaches were successfully applied to enhance the contrast and detail in two case studies (Cameraman and Pout images).

The CDE approaches can be used as promising optimization methods in image processing field. Specifically, the authors are interested in extending the CDE so that it can deal with multiobjective optimization for the image enhancement, segmentation, feature extraction, and classification tasks.

Table 3

Convergence results of $F(Z)$ (30 runs) for the Pout image using DE and CDE approaches.

Optimization method	$F(Z)$			
	Maximum (best)	Mean	Minimum (worst)	Standard deviation
DE	146.6014	132.8575	120.1629	10.1544
CDE1	149.6836	136.3391	122.2417	8.5532
CDE2	150.2066	137.2928	104.4839	14.2109
CDE3	149.2232	132.5218	119.1340	9.4119

Table 4

Best result (30 runs) for the Pout image using DE and CDE approaches.

Parameter	DE	CDE1	CDE2	CDE3
$F(Z)$	146.6014	149.6836	150.2066	149.2232
a	2.22×10^{-16}	0.0574	0.2326	0.3548
b	0.0572	0.0347	0.0319	0.0284
c	1.0806	1.0378	1.0182	1.0155
d	2.7052	3.5196	4.3204	4.1625
$E(I(Z))$	3.7771×10^4	6.9768×10^4	8.9101×10^4	9.3550×10^4
$ne(I(Z))$	2286	2191	2162	2164
$H(I(Z))$	7.3449	7.6409	7.7011	7.6303

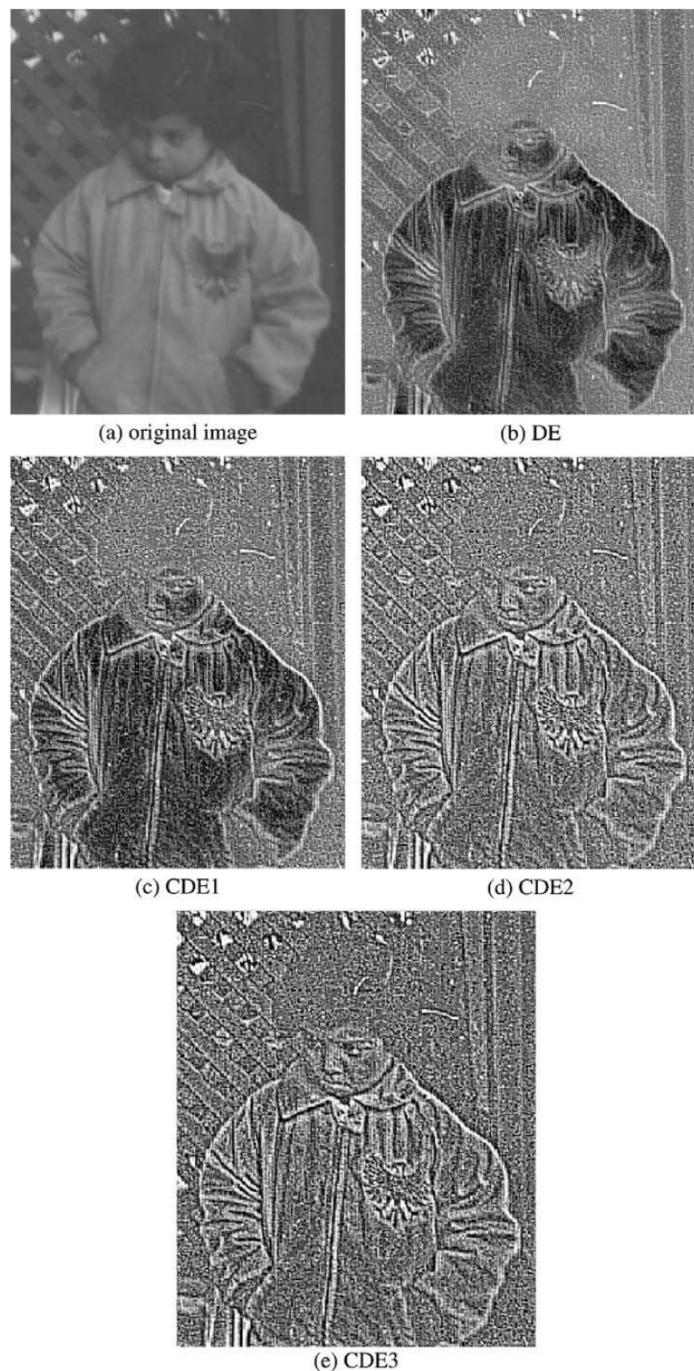


Fig. 2. Enhancements results of Pout image: (a) original image, (b) DE, (c) CDE1, (d) CDE2, and (e) CDE3.

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