

Exercise 4

$$C_2 = \overbrace{12\ 31}^a \cdot \overbrace{16\ 22}^b \quad (k=2)$$

rec Mult
 $n = 4 = 2^2 = 2k$

$$a = \overbrace{12}^{a_1} \cdot 100 + \overbrace{31}^{a_0}$$

$$b = \overbrace{16}^{b_1} \cdot 100 + \overbrace{22}^{b_0}$$

$$c_{11} = \overbrace{12}^{a_1} \cdot \overbrace{16}^{b_1} \quad (k=1)$$

$$a'_1 = \overbrace{1}^{a_1} \cdot 10 + \overbrace{2}^{a_0}$$

$$b'_1 = \overbrace{1}^{b_1} \cdot 10 + \overbrace{6}^{b_0}$$

$$c'_{11} = 1 \cdot 1 = 1 \quad (k=0)$$

$$c'_{00} = 2 \cdot 6 = 12 \quad (k=0)$$

$$= 1 \cdot 100 + \left[(1+2) \cdot (1+6) - 1 - 12 \right] \cdot 10 + 12$$

\uparrow
 $3 \cdot 7 = 21 \quad k=0$

$$= 1 \cdot 100 + (21 - 1 - 12) \cdot 10 + 12$$

$$= 100 + 80 + 12$$

$$= 192$$

$$c_{00} = \overbrace{31}^{a_0} \cdot \overbrace{22}^{b_0} \quad (k=1)$$

$$a'_1 = 3 \cdot 10 + 1$$

$$b'_1 = 2 \cdot 10 + 2$$

$$c'_{11} = 3 \cdot 2 = 6 \quad (k=0)$$

$$c'_{00} = 1 \cdot 2 = 2 \quad (k=0)$$

$$= 6 \cdot 100 + \left[(3+1) \cdot (2+2) - 6 - 2 \right] \cdot 10 + 2$$

\uparrow
 $4 \cdot 4 = 16 \quad (k=0)$

$$= 6 \cdot 100 + (16 - 6 - 2) \cdot 10 + 2$$

$$= 600 + 80 + 2$$

$$= 682$$

$$C_4 = 192 \cdot 10000 + \left[(12+31) \cdot (16+22) - 192 - 682 \right] \cdot 100 + 682$$

\uparrow
 $43 \cdot 38 \quad (k=1)$

$$a = 4 \cdot 10 + 3$$

$$b = 3 \cdot 10 + 8$$

$$c_{11} = 4 \cdot 3 = 12 \quad (k=0)$$

$$c_{00} = 3 \cdot 8 = 24 \quad (k=0)$$

$$a \cdot b = 12 \cdot 100 + \left[(4+3) \cdot (3+8) - 12 - 24 \right] \cdot 10 + 24$$

$$\downarrow$$

$$07 \cdot 11 \quad (k=1)$$

$$a' = 0 \cdot 10 + 7$$

$$b' = 1 \cdot 10 + 1$$

$$c_{11}' = 0 \cdot 1 = 0$$

$$c_{00}' = 7 \cdot 1 = 7$$

$$a' \cdot b' = 0 \cdot 100 + \left[(0+7) \cdot (1+1) - 0 - 7 \right] \cdot 10 + 7$$

$$\downarrow$$

$$7 \cdot 2 = 14 \quad (k=0)$$

$$= 0 + 20 + 7$$

$$= 27$$

$$= 12 \cdot 100 + [27 - 12 - 24] \cdot 10 + 24$$

$$= 1200 + 410 + 24$$

$$= 1634$$

$$C = 152 \cdot 10000 + [1634 - 152 - 682] \cdot 100 + 682$$

$$= 1520000 + 76000 + 682$$

$$= 1596682$$