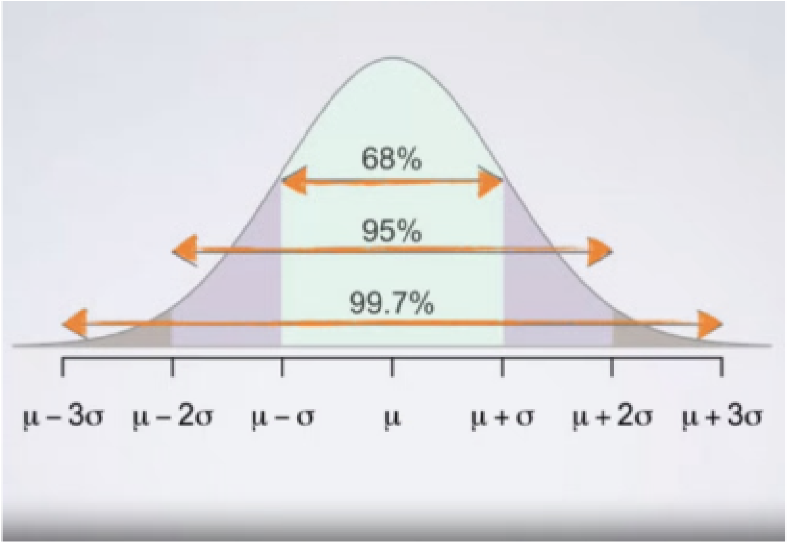
Normal distribution

- unimodal
- symmetric

$N(\mu, \sigma)$

mean

standard deviation



example:

$\mu = 110$
min = 65
max = 155

$110 \pm (3 \times 15) = (65, 155)$

example 2: P (1800 SAT) SAT scores ~ $N(\mu = 1500, \sigma = 300)$

J (24 ACT) ACT scores ~ $N(\mu = 21, \sigma = 5)$

P: $\frac{1800 - 1500}{300} = 1$ SD   J: $\frac{24 - 21}{5} = 0.6$ SD

Standardizing with Z scores

it's the number of standard deviations it falls above or below the mean.

Z = observation − mean

SD

Percentile : is the percentage of observations that fall below a given data point.

code R :

```
> pnorm(-1, μ=0, sd=1)
```

example :

$\mu = 1500$ $\Rightarrow$ $Z = \frac{1800 - \mu}{SD} = 1$

$sd = 300$

$\Rightarrow P(Z < 1) = 0.8413$

example 2 :

$\mu = 1500$

$sd = 300$

$per = 90\%$ $\Rightarrow$ $Z = 1.28 = \frac{x - 1500}{300}$

$x = 1884$

code R :

```
> qnorm(0.90, 1500, 300)
```

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example :

$\mu = 45$

$\sigma = 3.2$ $\quad Z = \frac{50 - 45}{3.2} = 1.56 \Rightarrow per = 0.9406$

$x = 50$ $\quad \% (x > 50) = 5.93\%$

example 2:

$\mu = 77$

$\sigma = 5$

x , per = 0.20

$z = -0.85 = \dfrac{x - 77}{5}$

$x = 72.75$

Binomial Distribution

bernoulli random variable: only two possible outcomes

the binomial distribution describes the probability of having exactly k successes in n independent bernoulli trials with probability of success p.

\# of scenarios × P(single scenario)

$p^k (1-p)^{(n-k)}$

number of ways to choose k successes in n trials.

probability of success to the power of number of successes.

probability of failure to the power of number of failures.

$= \dfrac{n!}{k!(n-k)!}$; $P(k \text{ successes in } n \text{ trials}) = \binom{n}{k} p^k (1-p)^{n-k}$

R code: > choose (n, k)

binomial conditions

Binomial conditions

1. the trials must be independent
2. the number of trials, *n*, must be fixed
3. each trial outcome must be classified as a success or a failure
4. the probability of success, *p*, must be the same for each trial

example:

$p(S) = 0.13$
$n = 10$
$K = 8$

$$p = \frac{10!}{8!\,(10-8)!}\, 0.13^{8}\, (0.87)^{2} = 2,78 \times 10^{-6}$$

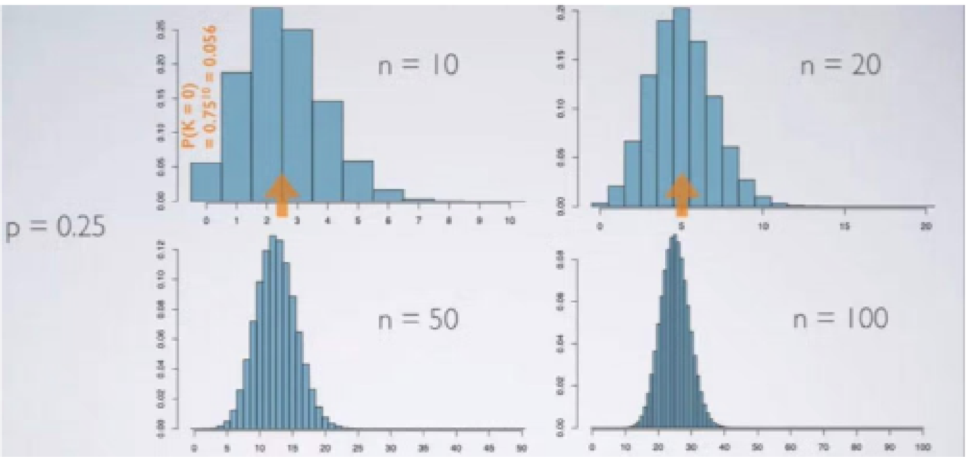
code R:

```
> dbinom(8, size = 10, p = 0.13)
```

expected value (mean) of binomial distribution: $\mu = np$

standard deviation " " : $\sigma = \sqrt{np(1-p)}$

Normal approximation to binomial

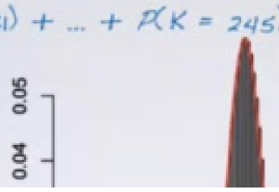


example:

$P(K \geq 70) = ?$

$= P(K = 70) + P(K = 71) + \ldots + P(K = 245)$

(1) $n = 245$, fixed

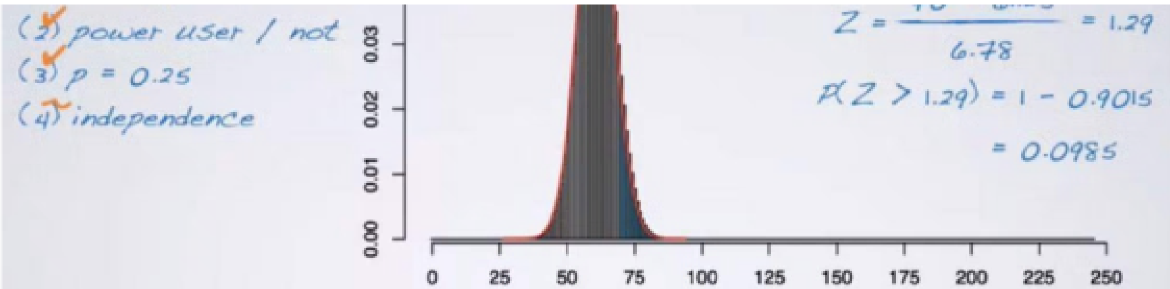


$N(\text{mean}, SD)$

$\text{mean} = 245 \times 0.25 = 61.25$

$SD = \sqrt{245 \times 0.25 \times 0.75} = 6.78$

$70 - 61.25$



code R: Sum(dbinom(70:245, size = 245, p = 0.25)) = 0.113

Success - failure condition

a b-d with at least 10 expected successes and 10 expected failures closely follows a n-d.

$np \geq 10$ and $n(1-p) \geq 10$