# Probability and Distributions

random process: it's Known what outcomes could happen, but not wich particular will happen.

$P(A)$ : probability of event A.

* $0 \leq P(A) \leq 1$

the probability of an outcome is the proportion of times the outcome would occur if we observed the random process an infinite number of times.

Law of large numbers: as more observations are collected the proportion of occurrences with a particular outcome converges to the probability of that outcome.

disjoint events: events can not happen at the same time. (mutually exclusive)

union of disjoint events: $P(A \cup B) = P(A) + P(B)$

union of non-disjoint events: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Sample space: is a collection of all possible outcomes of a trial.

probability distribution: lists all possible outcomes in the sample space and the probabilities with which they occur.

* the events listed must be disjoint
* probabilities between 0 and 1
* the probabilities must total 1

Complementary events.

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independence: two processes are independent if knowing the outcome of one provides no useful information about the outcome of the other.

$$P(A|B) = P(A)$$

$$P(A \wedge B) = P(A) \cdot P(B)$$

Examples:

$P(A) = 0.362$

$P(B) = 0.138$

$P(A \wedge B) = 0.036 \neq 0 \rightarrow$ not disj!

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.464$

$P(\text{no } A \cap \text{no } B) = 1 - P(A \cup B) = 0.536$

$P(A \cap B) \neq P(A) \cdot P(B)$ → not independent

at least 1 in 5 randomly selected people agree

$P(A) = 0.362$

$S = \{1, 2, 3, 4, 5\}$

⇒ $S = \{0, \text{at least } 1\}$

$P(\text{at least } 1) = 1 - P(\text{none agree})$

$= 1 - 0.638^5$

$= 0.894$

$P(\text{disagree})$

$= 1 - P(\text{agree})$

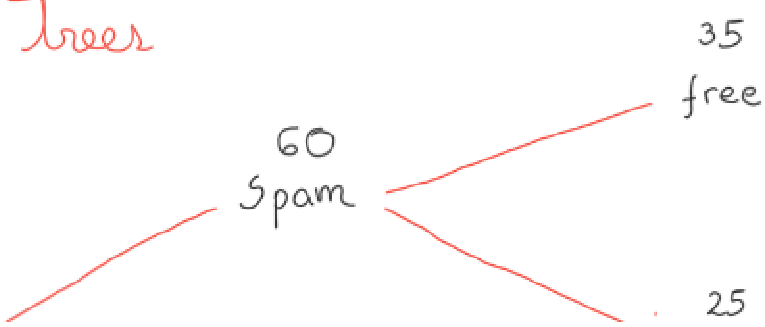
$= 1 - 0.362$

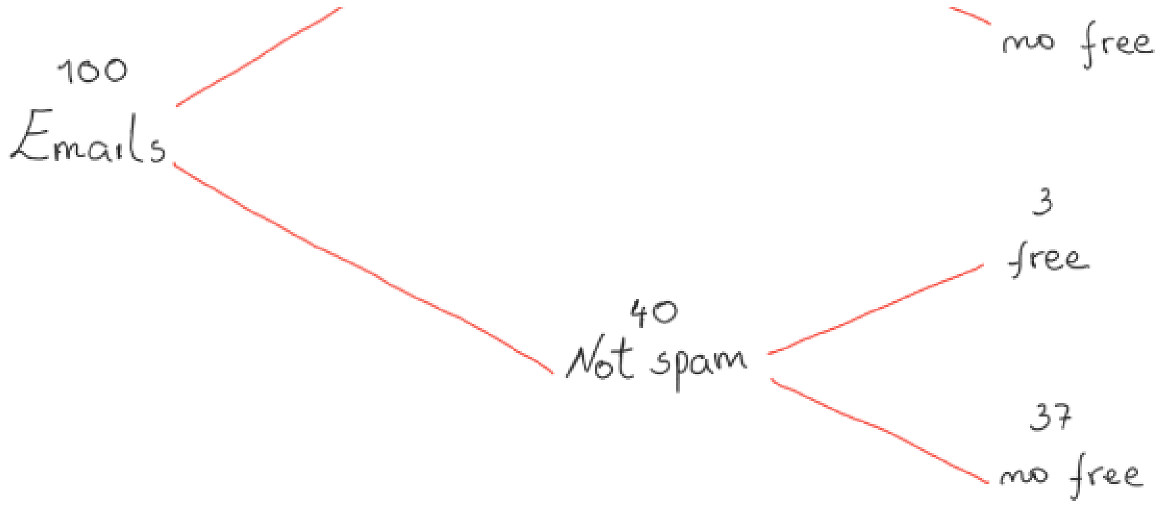
$= 0.638$

Baye's theorem: $P(B|A) = P(A \cap B) / P(A)$

probability of A given B.

Probability trees






$$P(\text{spam} \mid \text{free}) = \frac{35}{35+3}$$ → Joint probabilities
→ marginal " "

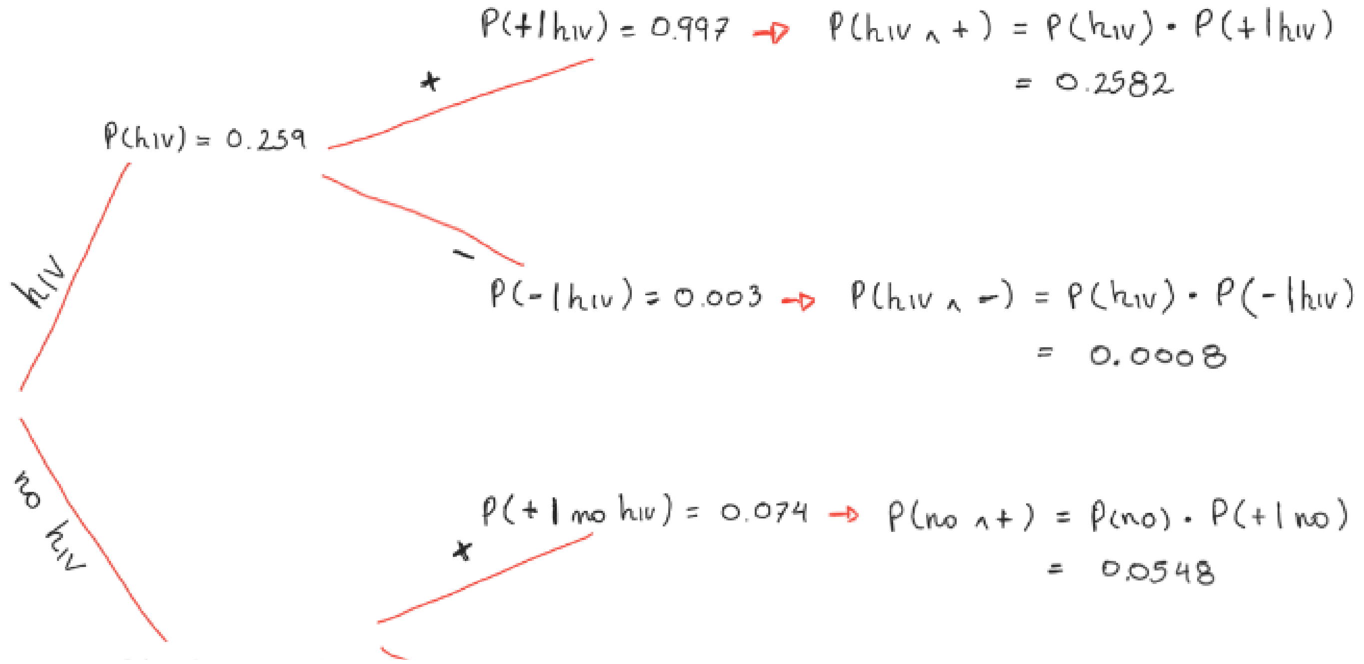
Just counting!

Now:

$$P(hiv) = 0.259$$

$$P(+ \mid hiv) = 0.997$$

$$P(- \mid \text{no } hiv) = 0.926$$



$P(\text{no hiv}) = 0.741$

$P(-\mid \text{no hiv}) = 0.926 \rightarrow P(\text{no} \wedge -) = P(\text{no}) \cdot P(-\mid \text{no})$
$= 0.6862$

$$P(\text{hiv} \mid +) = \frac{P(\text{hiv} \wedge +)}{P(+)} = \frac{0.2582}{0.2582 + 0.0548} \rightarrow \text{tested } +$$

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Posterior probability: $P(\text{hypothesis} \mid \text{data})$

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