lqr-after-trim

December 22, 2023

[1]:
```python
import rosbag
import numpy as np
import matplotlib.pyplot as plt
from scipy.signal import butter, filtfilt
```

1 System Equations

1.0.1 Linearizing $\ddot{x}$ and $\ddot{y}$

The dynamics of the drone in the x-direction can be described by the following equation:

$$m\ddot{x} = \text{thrust} \cdot \sin(\theta)$$

Assuming the drone is in a near-hover state, we can equate the thrust to the weight of the drone, i.e., thrust $= m \cdot g$. Substituting this into the equation, we get:

$$\ddot{x} = g \cdot \sin(\theta)$$

To simplify the model for small angles, we linearize $\sin(\theta)$ around 0. This leads to the following approximation:

$$\ddot{x} \approx g \cdot \theta$$

1.0.2 Linearizing Thrust

The thrust in the context of vertical dynamics can be expressed as:

$$\text{thrust} = m \cdot \frac{\ddot{z} + g}{\cos(\theta) \cdot \cos(\phi)}$$

For small angles, we approximate $\cos(\theta)$ and $\cos(\phi)$ as 1. Hence, the equation simplifies to:

$$\text{thrust} \approx m \cdot (\ddot{z} + g)$$

From this approximation, we derive the vertical acceleration:

$$\ddot{z} = \frac{\mathrm{thrust} - \mathrm{m} \cdot \mathrm{g}}{m}$$

These linearizations facilitate the analysis and control design for the drone in its near-hover state.

1.0.3 System Matrices with States x, y, z, $\dot{x}$, $\dot{y}$, and $\dot{z}$

For the given system states, the matrices A, B, and C are defined as follows:

Matrix A (State Transition Matrix):

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Matrix B (Control Matrix):

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ g & 0 & 0 \\ 0 & g & 0 \\ 0 & 0 & \frac{1}{m} \end{bmatrix}$$

Matrix C (Output Matrix):

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

These matrices represent the linearized model of the drone's motion, with A describing the system dynamics, B showing how control inputs affect the state, and C representing the measured outputs.

2 LQR Matrices

For the implementation of LQR (Linear Quadratic Regulator), specific matrices are used to define system dynamics and measurement models.

Matrix Q (State-Cost Matrix):

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Matrix R (Control-Cost Matrix):

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 100 \end{bmatrix}$$

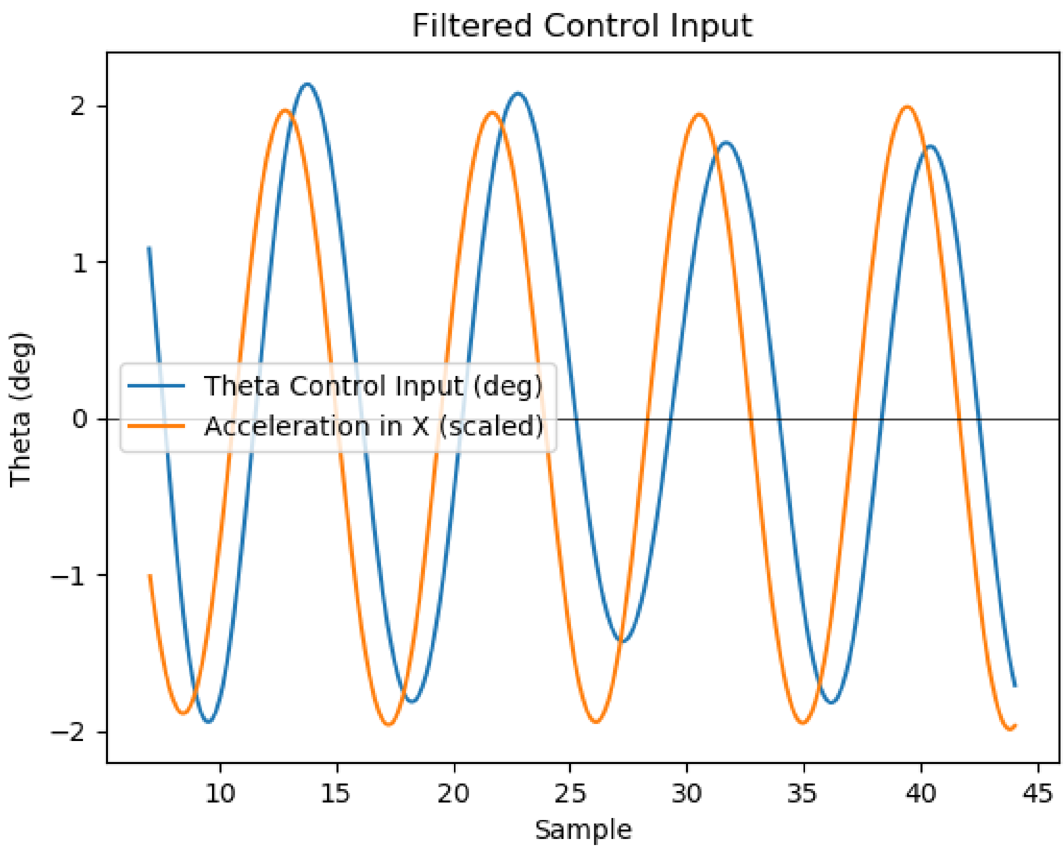
[2]:
```
bag = rosbag.Bag('/home/miguel/catkin_ws/src/crazyflie/crazyflie_controller/src/
↪data/LQR_after_trim.bag')

position_optitrack = []
desired_position = []
vel_optitrack = []
desired_vel = []
control_input = []

for topic, msg, t in bag.read_messages(topics=['position_Optitrack',␣
↪'vel_Optitrack', 'desired_position', 'desired_vel', 'control_input']):

    if topic == 'position_Optitrack':
        position_optitrack.append((msg.x, msg.y, msg.z))

    if topic == 'vel_Optitrack':
        vel_optitrack.append((msg.x, msg.y, msg.z))

    if topic == 'desired_position':
        desired_position.append((msg.x, msg.y, msg.z))

    if topic == 'desired_vel':
        desired_vel.append((msg.x, msg.y, msg.z))

    if topic == 'control_input':
        control_input.append((msg.x, msg.y, msg.z))
bag.close()

position_optitrack = np.array(position_optitrack)
vel_optitrack = np.array(vel_optitrack)
desired_position = np.array(desired_position)
desired_vel = np.array(desired_vel)
control_input = np.array(control_input)
```

[3]:
```python
time = []
initial_time = 0
Ts = 1/30

for i in range(len(position_optitrack)):
    time.append(initial_time)
    initial_time+=Ts
```

3 X accel vs angle

[4]:
```python
############### Finding the Acceleration ##################
# Filter parameters
N = 5  # Filter order
Wn = 0.01  # Cutoff frequency (as a fraction of the Nyquist frequency)
b, a = butter(N, Wn, 'low')

# Filter the velocity
vel_optitrack_ = np.array(vel_optitrack)[:, 0]
filtered_velocity = filtfilt(b, a, vel_optitrack_.squeeze())

# Numerical differentiation to find acceleration
dt = np.diff(time)  # Time intervals
acceleration = np.diff(filtered_velocity) / dt  # Numerical derivative


############## Filtering the Control Input ###############
control_input_ = np.array(control_input)[:, 0]
# Apply the filter
filtered_control_input = filtfilt(b, a, control_input_.squeeze())

# Convert to degrees
filtered_control_input_deg = np.rad2deg(filtered_control_input)

# Plotting
max_t = 18
min_t = 7
plt.plot(time[min_t*30:-max_t*30], filtered_control_input_deg[min_t*30:
 ↪-max_t*30], label='Theta Control Input (deg)')
plt.plot(time[(1+min_t*30):-max_t*30], [x*5 for x in acceleration[min_t*30:
 ↪-max_t*30]], label='Acceleration in X (scaled)')

plt.axhline(y=0, color='k', linewidth=0.5)
plt.xlabel('Sample')
plt.ylabel('Theta (deg)')
plt.title('Filtered Control Input')
plt.legend()
```

```
plt.show()
```



[5]:
```
np.mean(filtered_control_input_deg[min_t*30:-max_t*30])
```

[5]: 0.06758932785106668

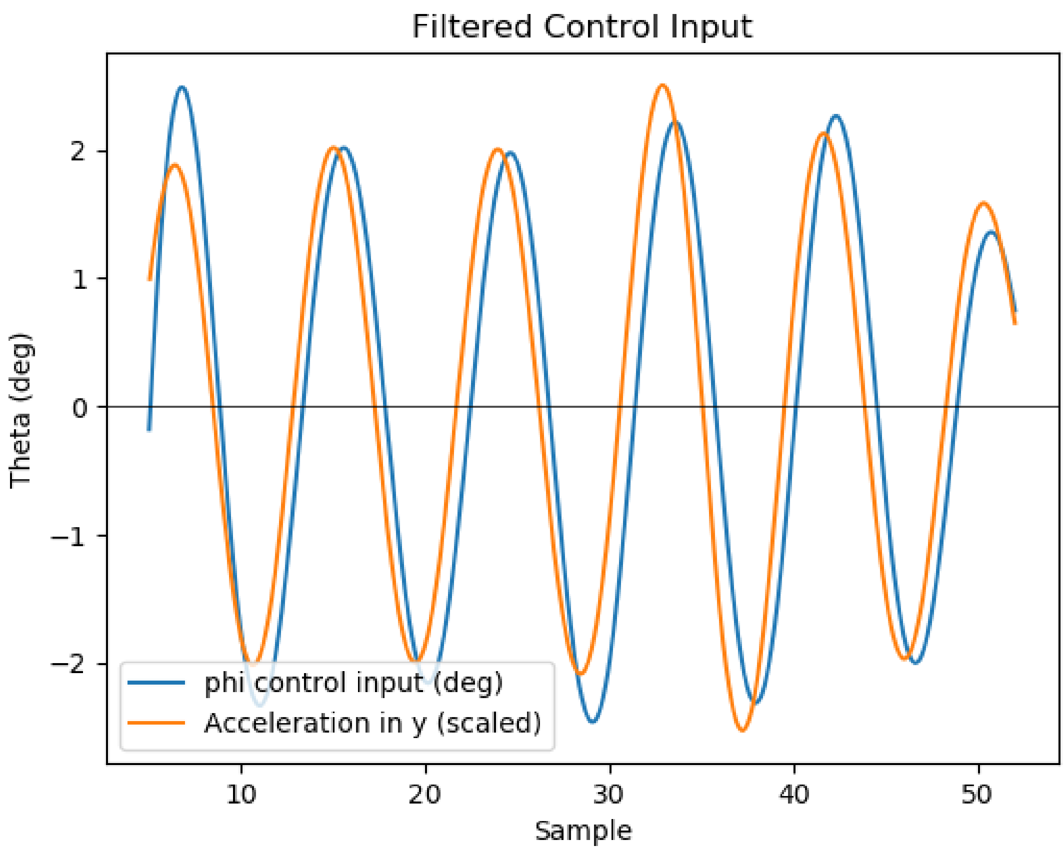
[6]:
```
3.48-1.57
```

[6]: 1.91

4 Y accel vs angle

[7]:
```
############### Finding the Acceleration ##################
# Filter parameters
N = 5  # Filter order
Wn = 0.01  # Cutoff frequency (as a fraction of the Nyquist frequency)
b, a = butter(N, Wn, 'low')

# Filter the velocity
```

```
vel_optitrack_ = np.array(vel_optitrack)[:, 1]
filtered_velocity = filtfilt(b, a, vel_optitrack_.squeeze())

# Numerical differentiation to find acceleration
dt = np.diff(time)  # Time intervals
acceleration = np.diff(filtered_velocity) / dt  # Numerical derivative


############## Filtering the Control Input ###############
control_input_ = np.array(control_input)[:, 1]
# Apply the filter
filtered_control_input = filtfilt(b, a, control_input_.squeeze())

# Convert to degrees
filtered_control_input_deg = np.rad2deg(filtered_control_input)

# Plotting
max_t = 10
min_t = 5

plt.plot(time[min_t*30:-max_t*30], filtered_control_input_deg[min_t*30:
 ↪-max_t*30], label='phi control input (deg)')
plt.plot(time[(1+min_t*30):-max_t*30], [x*5 for x in acceleration[min_t*30:
 ↪-max_t*30]], label='Acceleration in y (scaled)')
plt.axhline(y=0, color='k', linewidth=0.5)
plt.xlabel('Sample')
plt.ylabel('Theta (deg)')
plt.title('Filtered Control Input')
plt.legend()
plt.show()
```

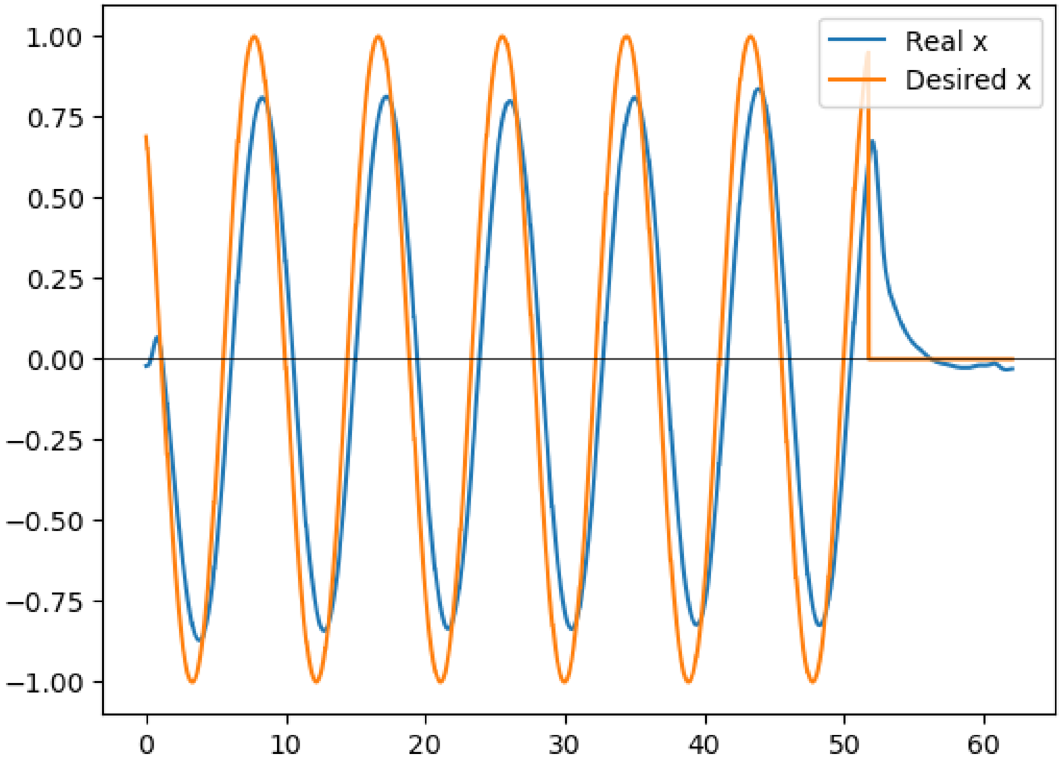


[8]:
```
np.mean(filtered_control_input_deg[min_t*30:-max_t*30])
```

[8]: 0.007033753609597994

5 X

[9]:
```
plt.plot(time, [x[0] for x in position_optitrack], label='Real x')
plt.plot(time, [x[0] for x in desired_position], label='Desired x')
plt.axhline(y=0, color='k', linewidth=0.5)
plt.legend()
plt.show()
```



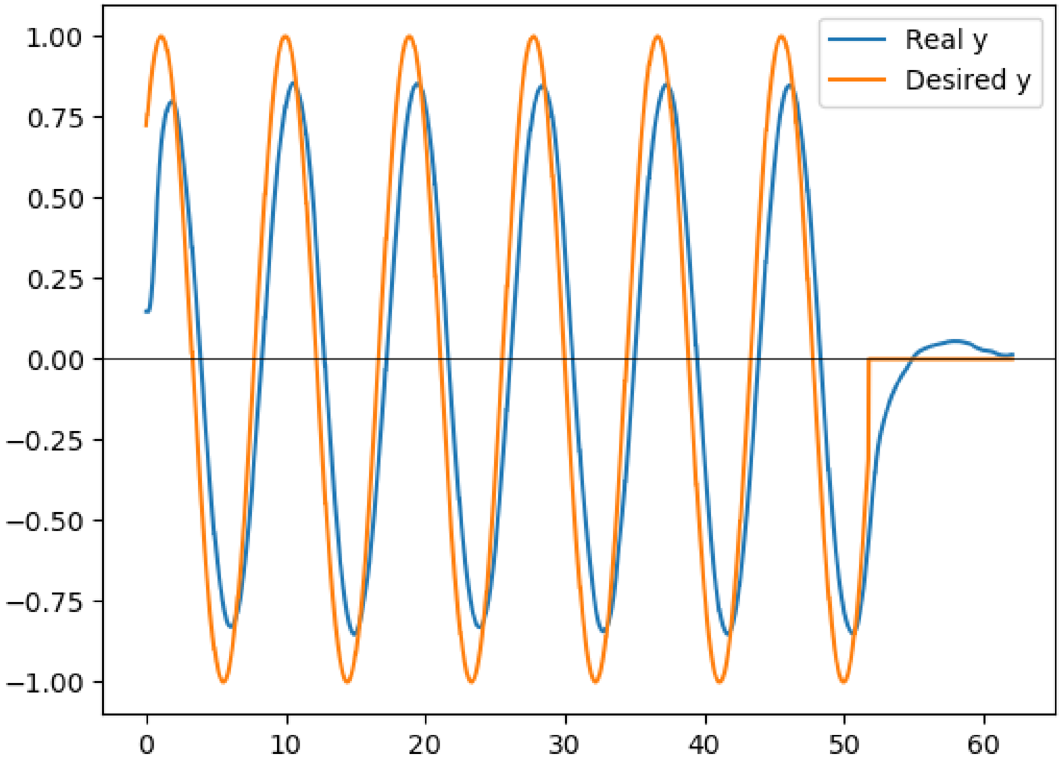
5.0.1 X RMSE

[10]:
```
x_square_error = (desired_position[:, 0] - position_optitrack[:, 0])**2
x_mse = np.sqrt(np.mean(x_square_error))
x_mse
```

[10]: 0.27236870442811356

6 Y

[11]:
```
plt.plot(time, [x[1] for x in position_optitrack], label='Real y')
plt.plot(time, [x[1] for x in desired_position], label='Desired y')
plt.axhline(y=0, color='k', linewidth=0.5)
plt.legend()
plt.show()
```



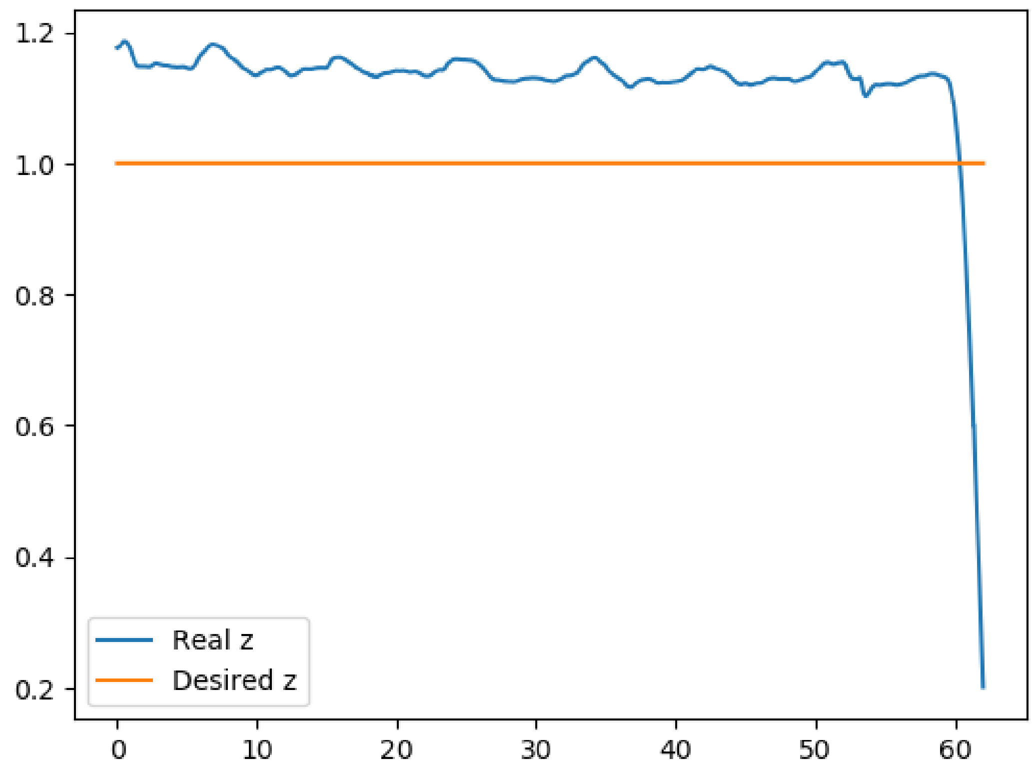
6.0.1 Y MSE

[12]:
```
y_square_error = (desired_position[:, 1] - position_optitrack[:, 1])**2
y_mse = np.sqrt(np.mean(y_square_error))
y_mse
```

[12]: 0.27217777365117096

7 Z

[13]:
```
plt.plot(time, [x[2] for x in position_optitrack], label='Real z')
plt.plot(time, [x[2] for x in desired_position], label='Desired z')

plt.legend()
plt.show()
```



acc max used = 13.72

7.0.1 Mean z position

[14]:
```
np.mean(position_optitrack[:-5*30, 2])
```

[14]: 1.139577220324601

7.1 Mean thrust

[15]:
```
np.mean(np.array(control_input)[:-5*30, 2])/0.032 # Should be equal the gravity
```

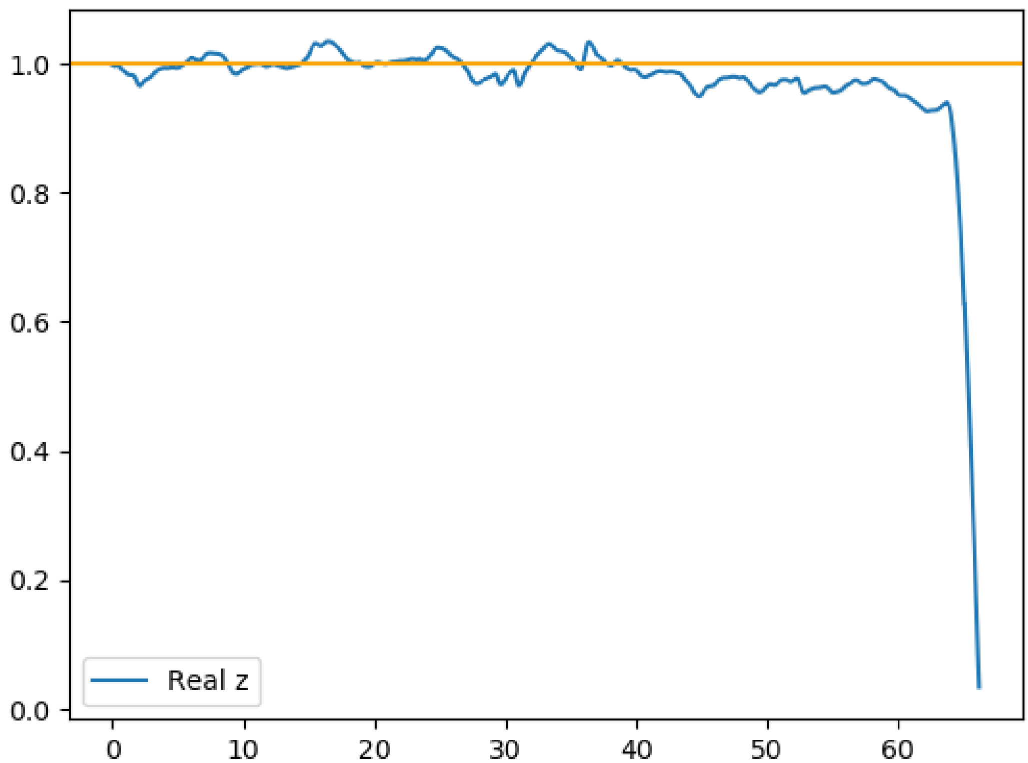
[15]: 10.224950438361851

[16]:
```
(10.224950438361851/9.81)*13.72 # Correcting the a_max
```

[16]: 14.300338431633495

7.1.1 Results

[17]:
```
bag = rosbag.Bag('/home/miguel/catkin_ws/src/crazyflie/crazyflie_controller/src/
↪data/LQR_after_amax_trim.bag')

position_optitrack = []

for topic, msg, t in bag.read_messages(topics=['position_Optitrack']):
    position_optitrack.append((msg.x, msg.y, msg.z))

bag.close()

position_optitrack = np.array(position_optitrack)
z_position = position_optitrack[:, 2]

time = []
initial_time = 0
Ts = 1/30

for i in range(len(z_position)):
    time.append(initial_time)
    initial_time+=Ts

plt.plot(time, [x for x in z_position], label='Real z')
plt.axhline(y=1, color='orange', linewidth=1.5)
plt.legend()
plt.show()
```



8 Control Effort

8.1 Theta

[18]:
```
def control_effort(u):
    effort = 0
    for i in range(len(u) - 1):
        effort += u[i+1]-u[i]

    return effort

control_effort(np.array(control_input)[:, 0])
```

[18]: -0.14555874056117118

8.2 phi

[19]:
```
control_effort(np.array(control_input)[:, 1])
```

[19]: -0.26934239089097406

8.3 Thrust

[20]:
```
control_effort(np.array(control_input)[:, 2])
```

[20]: 0.15622823482693715

9 Conclusion

The LQR (Linear Quadratic Regulator) displayed impressive results, underscoring the significance of accurately estimating the drone's maximum achievable acceleration to avoid errors in the steady-state regime. In terms of positioning, this controller is much quicker compared to the H infinity controller, although it falls behind in trajectory tracking performance.