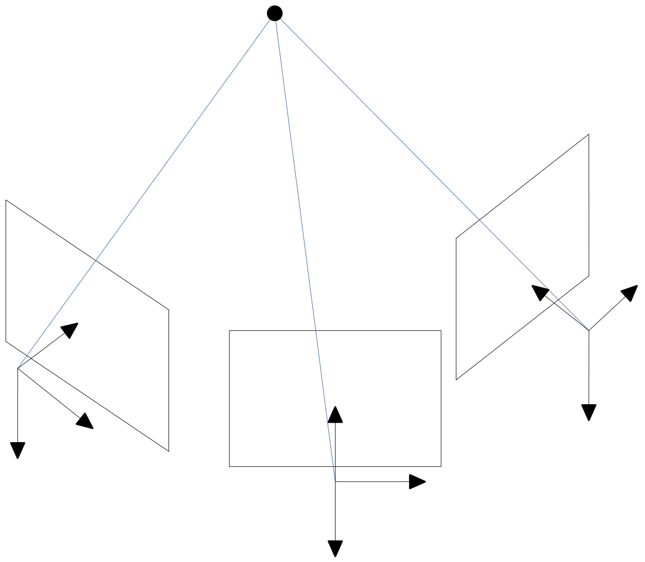
Reconstruction of One Point from many Images



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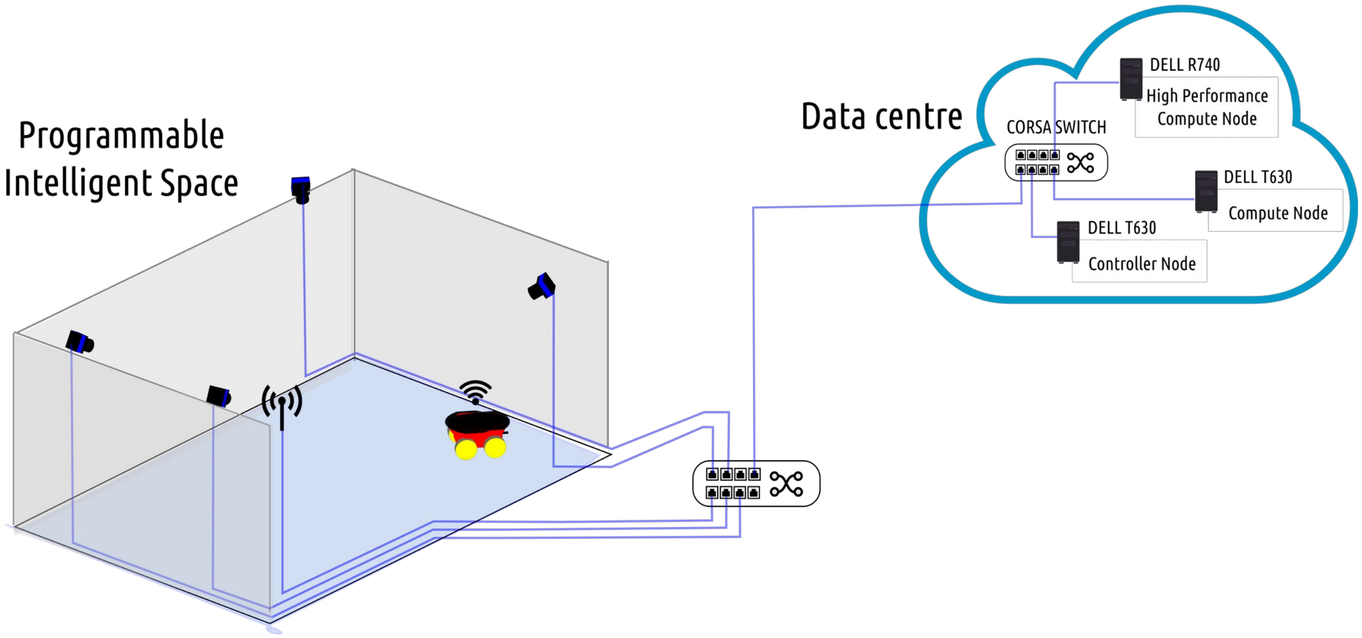
Intelligent Space – Four Cameras



General Projection Matrix

For each camera i.

$$\lambda_i \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} = \begin{bmatrix} fs_x & fs_\theta & o_x \\ 0 & fs_y & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} R_i & T_i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\lambda_i \tilde{\mathbf{m}}_i = \mathbf{K}_i \; \Pi \; [\mathbf{R}_i, \mathbf{T}_i] \; \tilde{\mathbf{M}}$$

General Projection Matrix

For each camera i.

The 3D cartesian point: $\mathbf{M} = [x,\ y,\ z]^T$

$$\lambda_i \tilde{\mathbf{m}}_i = \mathbf{K}_i \ \Pi \ [\mathbf{R}_i, \mathbf{T}_i] \ \tilde{\mathbf{M}}$$

$$\lambda_i \tilde{\mathbf{m}}_i = \mathbf{K}_i(\mathbf{R}_i \mathbf{M} + \mathbf{T}_i)$$

$$\lambda_i \mathbf{K}_i^{-1} \widetilde{\mathbf{m}}_i = \mathbf{R}_i \mathbf{M} + \mathbf{T}_i$$

$$\lambda_i \mathbf{R}_i^{-1} \mathbf{K}_i^{-1} \widetilde{\mathbf{m}}_i = \mathbf{M} + \mathbf{R}_i^{-1} \mathbf{T}_i$$

$$\lambda_i (\mathbf{K}_i \mathbf{R}_i)^{-1} \tilde{\mathbf{m}}_i - \mathbf{M} = \mathbf{R}_i^{-1} \mathbf{T}_i$$

General Projection Matrix

For each camera i.

$$\lambda_i(\mathbf{K}_i\mathbf{R}_i)^{-1}\tilde{\mathbf{m}}_i - \mathbf{M} = \mathbf{R}_i^{-1}\mathbf{T}_i \quad\Longrightarrow\quad \underbrace{\begin{bmatrix} -\mathbf{I} & (\mathbf{K}_i\mathbf{R}_i)^{-1}\tilde{\mathbf{m}}_i \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ \lambda_i \end{bmatrix}}_{\text{3x1}} = \underbrace{\mathbf{R}_i^{-1}\mathbf{T}_i}_{\text{3x1}}$$

$$\mathbf{W}_i = (\mathbf{K}_i\mathbf{R}_i)^{-1}\,\widetilde{\mathbf{m}}_i \quad\Longrightarrow\quad \mathbf{W}_i = (\mathbf{K}_i\mathbf{R}_i)^{-1}_{3\times 3} \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix}_{3\times 1}$$

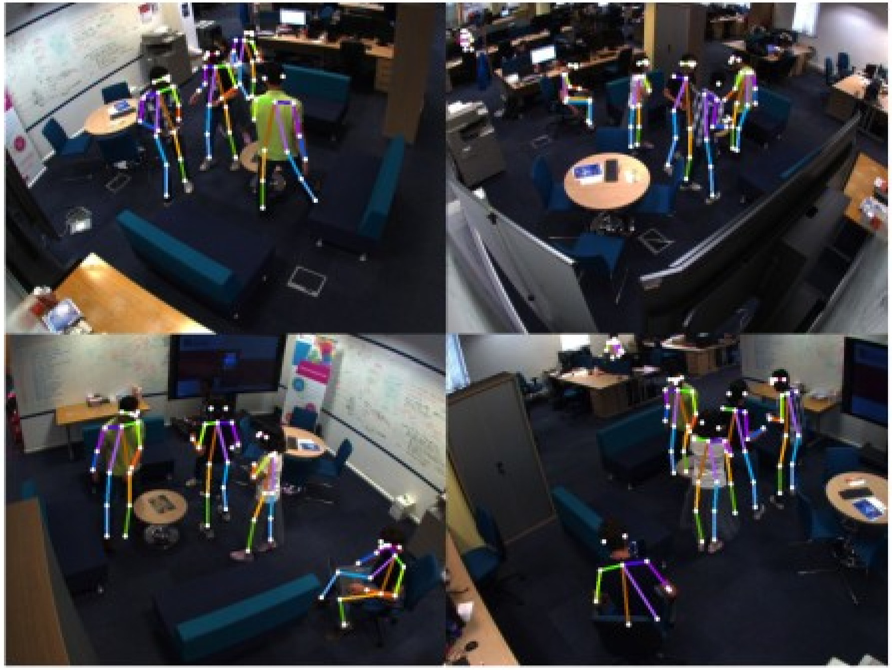
Joining the equations of each camera

$$\underbrace{\begin{bmatrix} -\mathbf{I} & \mathbf{W}_1 & \mathbf{0}_{3\times n-1} & \\ \vdots & \vdots & \vdots & \vdots \\ -\mathbf{I} & \mathbf{0}_{3\times i-1} & \mathbf{W}_i & \mathbf{0}_{3\times n-i} \\ \vdots & \vdots & \vdots & \vdots \\ -\mathbf{I} & & \mathbf{0}_{3\times n-1} & \mathbf{W}_n \end{bmatrix}}_{\mathbf{A}_{3n \times 3+n}} \underbrace{\begin{bmatrix} x \\ y \\ z \\ \lambda_1 \\ \vdots \\ \lambda_i \\ \vdots \\ \lambda_n \end{bmatrix}}_{3+n \times 1} = \underbrace{\begin{bmatrix} \mathbf{R}_1^{-1}\mathbf{T}_1 \\ \vdots \\ \mathbf{R}_i^{-1}\mathbf{T}_i \\ \vdots \\ \mathbf{R}_n^{-1}\mathbf{T}_n \end{bmatrix}}_{3n \times 1}$$

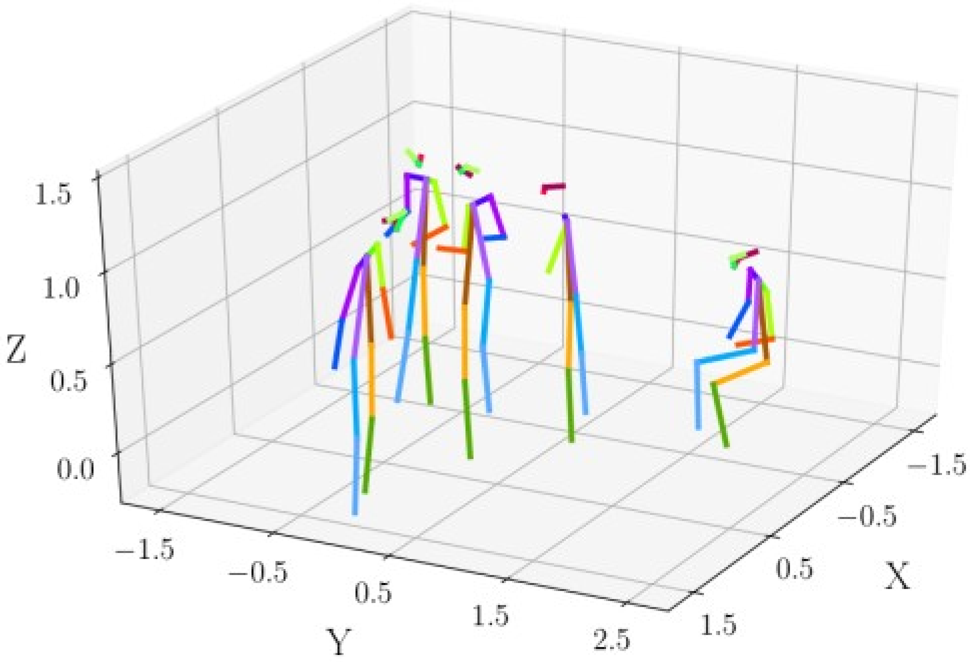
with $\mathbf{W}_i = (\mathbf{K}_i\mathbf{R}_i)^{-1}\,\tilde{\mathbf{m}}_i$

Solve the equation using the pseudo-inverse of **A.**

Example

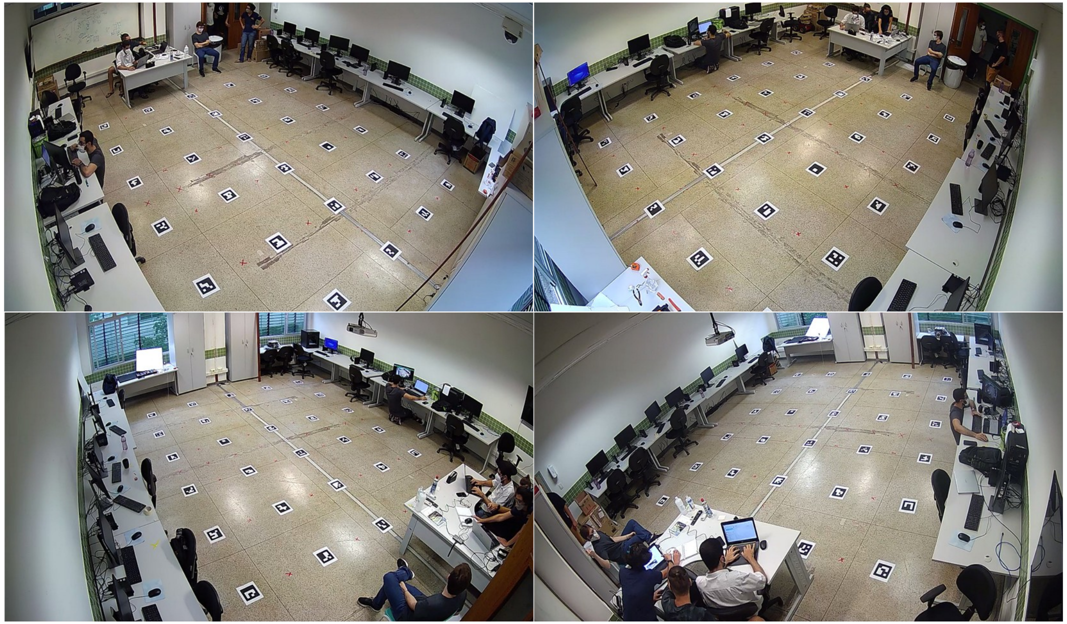


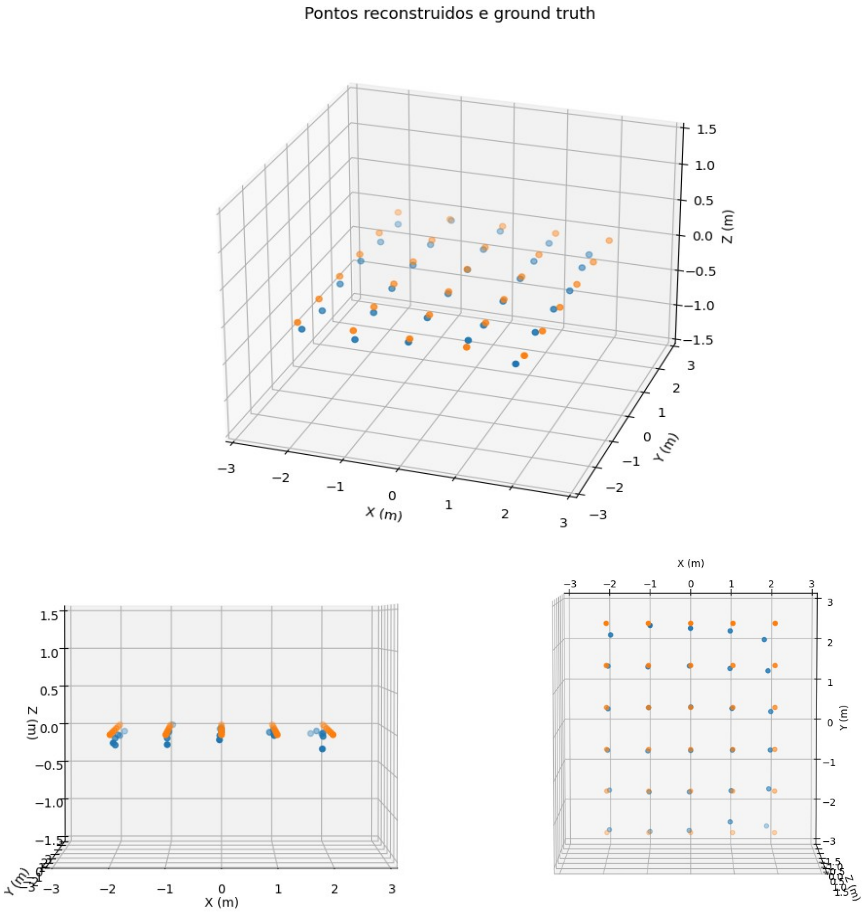
(a)



(b)

Example





Another approach

$$\lambda_1 \widetilde{\mathbf{m}}_1 = \mathbf{P}_1 \widetilde{\mathbf{M}} \rightarrow \mathbf{P}_1 \widetilde{\mathbf{M}} - \lambda_1 \widetilde{\mathbf{m}}_1 = 0$$

$$\lambda_2 \widetilde{\mathbf{m}}_2 = \mathbf{P}_2 \widetilde{\mathbf{M}} \rightarrow \mathbf{P}_2 \widetilde{\mathbf{M}} - \lambda_2 \widetilde{\mathbf{m}}_2 = 0$$

$$\vdots$$

$$\lambda_n \widetilde{\mathbf{m}}_n = \mathbf{P}_n \widetilde{\mathbf{M}} \rightarrow \mathbf{P}_n \widetilde{\mathbf{M}} - \lambda_n \widetilde{\mathbf{m}}_n = 0$$

with $\mathbf{P}_i = \mathbf{K}_i \Pi [\mathbf{R}_i, \mathbf{T}_i]$

$$\underbrace{\begin{bmatrix} \mathbf{P}_1 & -\widetilde{\mathbf{m}}_1 & 0 & \cdots & 0 \\ \mathbf{P}_2 & 0 & -\widetilde{\mathbf{m}}_2 & \cdots & 0 \\ \vdots & \vdots & & & \vdots \\ \mathbf{P}_n & 0 & \cdots & 0 & -\widetilde{\mathbf{m}}_n \end{bmatrix}}_{\mathbf{B}_{3n \times 4+n}} \underbrace{\begin{bmatrix} \widetilde{\mathbf{M}} \\ \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_n \end{bmatrix}}_{4+n \times 1} = \mathbf{0}_{3n \times 1}$$

Solve the equation using the SVD of **B.**

Use the first 4 elements of the last column of matrix $\mathbf{V}$ as the estimation of $\widetilde{\mathbf{M}}$.

Credits

- QUEIROZ, F. M. ; PICORETI, R. ; SANTOS, C. C. ; RAMPINELLI, M. ; VASSALLO, R. F. . Estimating Tridimensional Coordinates of Skeleton Joints in a Multicamera System. XIV Workshop de Visão Computacional, 2018, Ilhéus - BA.
- Jan Erik Solem.
  Programming Computer Vision with Python.
  Published by O'Reilly Media, ISBN: 978-1-449-31654-9