Case 1: Interest rate swaps and liability hedging

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1 The Term Structure of Interest Rates (20 points)

During the first week of the course, we discussed the financial position of defined benefit pension funds. Typically, we use the nominal funding ratio to measure the healthiness of a pension fund. The nominal funding ratio FR at time t is defined as:

$$FR_t = \frac{A_t}{L_t},$$

where A_t represents the market value of all assets and L_t the value of the nominal liabilities. The value of the nominal liabilities is determined by the expected cash-flows and the discount rate:

$$L_t = \sum_{j=0}^{N} \frac{CF_{t+j}}{(1 + y_{t:t+j})^j}.$$

There are many ways to construct the term structure of discount rates. Before 2007 pension funds were allowed to use a fixed rate of 4%. As of the beginning of 2007 the regulation changed. From that moment pension funds had to use market interest rates rates for the valuation of nominal liabilities. Because of the lack of supply of long term bonds, market swap rates were chosen as basis for the zero nominal term structure.

The Excel-file provides the closing quotes of nominal interest rate swap rates in the Euro swap market on February 8, 2021 (tab Question 1, column B). Based on these swap rates an interest rate curve of nominal "risk-free" zero rates and the forward curve needs to be constructed. For the purpose of this exercise you can assume that the floating rate of the swaps is 1-year risk free rate. Fixed and floating payments take place once per year.

(a) Verify that the nominal zero term structure (with annual compounding) for years 1-5 is given by the numbers in column E of the Excel-file (again tab Question 1). Then construct the curve for maturities 6-20 years. Provide a table which shows your results for maturities 1, 5, 10, 15 and 20 years. Hint: as you know from the lectures a nominal interest rate swap can be decomposed in a par fixed coupon bond and a par floating rate bond. The swap rate is the coupon of the fixed leg in the swap.

In order to be able to discount expected cash-flows for maturities longer than 20 years, we need to make additional assumptions. We assume in this exercise that the forward rate between maturities 19 and 20, can be extrapolated to all maturities beyond 20 years. To be specific: the forward rate is assumed to be constant after 20 years.

(b) Verify that the annually compounded forward rate between maturities 19 and 20 equals 0.703%. Use this forward rate to construct the nominal zero curve for maturities 21 years until 60 years. Provide a table which shows your results for maturities 21, 40 and 60 years.

The regulator in the Netherlands changed the methodology for calculating the discount curve several times. In 2012, for instance, the regulator introduced the so-called Ultimate Forward Rate (UFR)-methodology (also used in the world of insurance companies). In this methodology, the forward rate in the long run ultimately converges to a value of 4.2%. Speed of convergence can be defined in many different ways, but let us assume that **per annum forward rates** can be determined as follows:

$$f_{t:t+20:t+20+h} = 4.2\% + (f_{t:t+19:t+20} - 4.2\%)B(h),$$

where $B(h)$ is defined as:

$$B(h) = \frac{1 - e^{-0.5h}}{0.5h}.$$

Note that the forward rates between year 1 and year 20 are not affected by this methodology.

(c) Using the UFR-methodology construct the UFR <u>forward</u> curve. Provide a table which shows your results for maturities 1, 10, 30 and 50 years.

(d) Construct the UFR nominal zero curve using the forward curve from the previous question and plot both the UFR nominal zero curve and the market zero curve (questions (a) and (b)) in one graph.

(e) Describe in words the impact of the methodology change on the financial position of pension funds and give a possible reason for the change towards this methodology in 2012.

2 Liability hedging (20 points)

In the Excel-file (tab Question 2, column B) you find a nominal cash-flow pattern from pension fund Hollandia. These cash-flows are the future expected payments of the pension fund. Pension fund Hollandia is in reasonably healthy shape: the nominal funding ratio of the fund is 115%. You can find the zero curve in column C of tab Question 2 in the Excel file. The present value of all future expected cash-flows equals EUR 22,773,412,004.

(a) Verify that the modified duration and the modified DV01 of the nominal liabilities are given by 20.14 years and EUR 45.86 mln, respectively. Report the contribution of the 10-year, 20-year and 30-year cash-flow to these metrics in a table.

(b) Shock the zero curve with -50bp, recalculate the value of the liabilities using the zero rates after the shock and show that the difference between the new and the old value of the liabilities can be (almost) fully explained by modified DV01 and modified convexity.

Suppose we want to match the interest rate sensitivity of the **balance sheet** by means of a 30-years swap (receive fixed, pay floating) and cash (cash amount equal to the value of the assets). Assume that cash is invested in a 1-year bond with a yield equal to 1-year Euribor (i.e., the 1-year zero rate). As in Exercise 1, assume that the floating rate of the swaps is 1-year Euribor. Fixed and floating payments take place once per year.

(c) First, show that the modified DV01 of EUR 1 notional in the 30-years swap is 0.2726% by using a 30-years swap rate of 0.3756%. Then, calculate the amount of DV01 you would like to have in the 30-year swap contract taking into account the position in the 1-year bond (which provides a bit of interest rate sensitivity). Finally, calculate the notional amount that is needed to meet the objective of the interest rate risk hedging strategy.

Column D of tab Question 2 shows for each maturity the change I'd like you to apply to the zero curve (in basis points).

(d) Show that the match with the 30-years swap is rather ineffective for this particular change of the zero curve. Hint: calculate the change in the value of the liabilities and the change in the value of the assets. You can assume that the first cash flow (with maturity 0) is still part of the liabilities after the change interest rates. If you were not able to solve question (c), please use a notional amount of EUR 20,000,000,000 in the 30-years swap.

Let us go back to the initial situation (before the curve change). Now you have the opportunity to add two more swaps to the hedge portfolio. However, there are only a limited number of maturities you can choose from: 10 years, 20 years, 40 years and 50 years.

(e) First motivate the choice you would make and then calculate the notionals of the swaps that are needed to match the interest rate sensitivity of the funding ratio and assess the effectiveness of your new hedging portfolio after the shock in the zero curve as given in column D.

3 Euribor-OIS (only for Finance, 10 points)

In the Excel file (tab Exercise 3) you'll find zero curves bootstrapped from Euribor-swaps (column B) and OIS-swaps (column C) as of September 30, 2020. Notice that the Euribor-curve is the same as we used in the previous exercise.

Suppose I have a nominal interest rate swap in my portfolio (receive fixed, pay floating) with notional 500,000,000 Euros, swap rate 4%, floating based on 1-year Euribor, and a remaining time to maturity of 30 years.

(a) Calculate the value of the swap using the zero curve that is bootstrapped from **Euribor-swaps** for discounting. Assume that we calculate this value just after the yearly coupon payments took place.

(b) Calculate the **discounting DV01** of the interest rate swap. This means: shock the Euribor-based zero curve with 1bp and use the shocked curve to discount future expected cash-flows of the interest rate swap. Note: you need to take into account all future payments of the floating leg.

Next step is to change the discounting curve to the zero curve that is bootstrapped from OIS-swaps (column C in the Excel-sheet).

(c) Calculate the value of the swap using the zero curve that is bootstrapped from OIS-swaps for discounting and relate your answer to the result of (b).

Suppose that we want to trade a new 30-years swap on September 30, 2020 with notional EUR 500,000,000 and Euribor as basis for the floating rate. This swap is fully collateralized with safe assets on a daily basis. This implies that the correct discounting curve is the zero curve that is bootstrapped from OIS-swaps.

(d) Calculate the fair swap rate of the new 30-years swap.

4 Tracking error in Vasicek-model (only for QRM, 10 points)

Purpose of this exercise is to get a feeling of what **tracking error** means in a simple nominal matching portfolio. Suppose we live in a Vasicek world:

$$dr_t = -\kappa(r_t - \theta)dt + \sigma dW_t,$$

where W is a Brownian Motion under the real world probability measure. Assume that the estimate of κ equals 0.35, the estimate of θ is the 1-years zero rate from Exercise 1 and the estimate of σ is 0.5%. Note that under the risk neutral probability measure we have an additional parameter λ. See slides for underlying details. You can also assume that the value of the short rate at the start equals the 1-years zero rate.

(a) Calculate the value of λ that matches the model price to the market price of the 20-years zero coupon bond. The market price in this case is the price that follows from the 20-years zero rate that is given in Exercise 2 (0.23%).

From now on we keep the model parameters constant.

(b) Use the parameters to construct the complete term structure of nominal interest rates and use this term structure to value the cash-flows given in Exercise 2. Note that the term structure differs from the term structure in Exercise 2 and therefore leads to a different value of the liabilities.

(c) Simulate 2,000 scenarios of the short rate under the Vasicek model on a horizon of one year. For this exercise a monthly simulation frequency is sufficient but you can also use the distributional properties of the short rate at a horizon of one year.

We assume that the matching portfolio consists of cash and the 30-years swap of which the notional was calculated in the second exercise (part (c)). You can assume that the cash amount at the start equals the value of the nominal liabilities as calculated in (b) and earns the 1-year zero rate of Exercise 2.

(d) Value the nominal liabilities and the matching portfolio after one year in each of the 2,000 scenarios. Calculate the average return difference and the standard deviation of relative returns (also known as tracking error).

Requirements

- Groups of max three students
- Deadline for this case is Friday February 26 at 23:59 CET
- Final reports should be uploaded on Canvas
- Reports should be written in English. This means that you also should use English decimal notation in the text and in graphs
- There is no maximum to the number of words or pages but please try to be concise
- It is not allowed to just give answers to the questions. It should be clear to the reader how you have come to the answer
- Don't refer to cells of your own spreadsheets, I don't plan to read spreadsheets or other kind of computer source code
- Figures and graphs should have captions such that they can be read independently from the text
- Use page numbers
- Formulas are part of the sentence
- Put **names** and **VU student numbers** of all group members on the front page.

And last but not least...ENJOY!!