**Derivation of Factor-Composite Correlation**

Assume the following model as an example, where three latent factors load on three items with measurement error defined for each item:

X2

X1

Let aij denote the loading of factor i on item j. The goal is to find the correlation between F1 and an composite score C = x1 + x2. In a first step, we try to find the covariance between F1 and C:

Cov(F1, C)

= Cov(F1, x1 + x2)

= Cov(F1, x1) + Cov(F1, x2)

= Cov(F1, a11\*F1 + a21\*F2 + e1) + Cov(F1, a12\*F1 + a22\*F2 + e2)

= Cov(F1, a11\*F1) + Cov(F1, a21\*F2) + Cov(F1, a12\*F1) + Cov(F1, a22\*F2)

= a11\*cov(F1,F1) + a21\*cov(F1,F2) + a12\*cov(F1,F1) + a22\*cov(F1,F2)

for standardized latent variables:

= a11 + a21\*r12 + a12 + a22\*r12

= a11 + a12 + a21\*r12+ a22\*r12

Can be rewritten as

This formula generalizes to more indicators and factors. The resulting sum is still a covariance, not a correlation. The factor variance was already standardized, so we must still divide by the standard deviation of the composite C. This can be calculated by either calculated from the raw data or from the empirical (or model implied) covariance matrix by summing up the variances and covariances of the items that belong to a scale and taking the square root.