Sequential Monte Carlo Methods

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Suppose that, for some random variable $\boldsymbol{X}$ with probability density function p, we wish to compute the expectation

$$\mathbb{E}[f(\boldsymbol{X})] = \int f(\boldsymbol{x})p(\boldsymbol{x})\mathrm{d}\boldsymbol{x} \tag{1}$$

for some function f. In many cases, f and p are such that (1) cannot be computed analytically, therefore we must look for suitable approximations. Consider a random sample $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_m$ drawn from p, and compute the sample mean

$$\bar{f}_m \coloneqq \frac{1}{m}\sum_{i=1}^{m} f(\boldsymbol{x}_i). \tag{2}$$

From the weak law of large numbers, it follows that

$$\lim_{m\to\infty} \mathbb{P}(|\bar{f}_m - \mathbb{E}[f]| > \epsilon) = 0 \tag{3}$$

for any $\epsilon > 0$. This result forms the basis of sampling/monte carlo methods, which approximate integrals of the form (1) by a discrete sum of samples drawn from p. In this section we cover sampling methods which are most relevant to sequential monte carlo (SMC) methods, howver a general overview of sampling methods can be found in [1].

References

[1] Christopher M. Bishop. *Pattern Recognition and Machine Learning (Information Science and Statistics)*. Berlin, Heidelberg: Springer-Verlag, 2006. ISBN: 0387310738.