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# Depletion-Based Stock Reduction Analysis: A catch-based method for determining sustainable yields for data-poor fish stocks

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ABSTRACT

We describe a method for determining reasonable yield and management reference points for data-poor fisheries in cases where approximate catches are known from the beginning of exploitation. The method, called Depletion-Based Stock Reduction Analysis (DB-SRA), merges stochastic Stock-Reduction Analysis with Depletion-Corrected Average Catch. Data requirements include estimates of historical annual catches, approximate natural mortality rate and age at maturity. A production function is specified based on general fishery knowledge of the relative location of maximum productivity and the relationship of MSY fishing rate to the natural mortality rate. This leaves unfished biomass as the only unknown parameter, which can be estimated given a designated relative depletion level near the end of the time series. The method produces probability distributions of management reference points concerning yield and biomass. Uncertainties in natural mortality, stock dynamics, optimal harvest rates, and recent stock status are incorporated using Monte Carlo exploration. Comparison of model outputs to data-rich stock assessments suggests that the method is effective for estimating sustainable yields for data-poor stocks.



## 1. Introduction

Management of "data-rich" stocks is often based on complex assessments that incorporate a variety of data sources and that provide estimates of stock status, various management targets or reference points, and sustainable yield. Yet there remain very many fish stocks for which data are too limited to support these data-rich approaches. The focus of assessment for "data-poor" stocks is often restricted to estimates of sustainable yield because insufficient data are available to estimate the full suite of reference points. Common approaches for data-poor stocks include defining a proxy value for sustainable catch, e.g. recent average catch, which may be precautionarily reduced to account for uncertainty. Advice regarding the extent of these reductions has been based on qualitative descriptions of stock status, such as "above $B_{msy}$" or "overfished" (Restrepo et al., 1998).

In this paper we describe Depletion-Based Stock Reduction Analysis (DB-SRA), a new data-poor approach that combines components of Depletion-Corrected Average Catch (DCAC, described by MacCall, 2009) and the modeling framework commonly referred to as stock reduction analysis (SRA, Kimura et al., 1984). Our approach is a modification of stochastic SRA (Walters et al., 2006) and similarly uses Monte Carlo exploration to derive probability distributions of stock attributes and management reference points. In their application of stochastic SRA to white sturgeon, Walters et al. identified a distribution of population trajectories that were consistent with an estimate of current absolute abundance. Since absolute abundance estimates are generally lacking for data-poor stocks, we replace this model input with an assumed distribution of relative abundance in a recent year, and evaluate the sustainable yield conditional on values drawn from that distribution. Hilborn (2001) has demonstrated the robustness of this approach when used in traditional SRA. We also use a very simple model of stock dynamics, requiring relatively little biological knowledge and allowing parameterization with direct interpretability for fishery management (Forrest et al., 2008; Martell et al., 2008). As with stochastic SRA, Monte Carlo simulations generate a set of plausible trajectories, resulting in distributions of estimated biomass (current and unfished) and other reference points such as maximum sustainable yield (MSY), the spawning biomass which produces MSY ($B_{msy}$), and the catch resulting from fishing the current stock at the MSY fishing rate ($C_{Fmsy}$), which we also refer to as the overfishing limit (OFL; NMFS, 2009). To illustrate model outputs, we present details of an application to cowcod (*Sebastes levis*) in southern California. To evaluate model performance, we use data from 31 assessed stocks of northeast Pacific groundfish off the west coast of the United States (Table 1) and compare results from these nominally data-rich age-structured assessment models with the Depletion-Based SRA described below.

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**Table 1**
Stock assessments for 31 stocks of Pacific groundfish used to determine performance of Depletion-Based Stock Reduction Analysis. Assessment documents are available from the Pacific Fishery Management Council's website (www.pcouncil.org).

| Group | Common name, region | Scientific name | Species code | $M$ [1/yr] | Age at maturity | Assessment year |
|---|---|---|---|---|---|---|
| Flatfish | Arrowtooth flounder | *Atheresthes stomias* | ARTH | 0.166 | 8 | 2007 |
| Rockfish | Black Rockfish, North | *Sebastes melanops* | BLCK_N | 0.24 | 10 | 2007 |
| Rockfish | Black Rockfish, South | *Sebastes melanops* | BLCK_S | 0.24 | 7 | 2007 |
| Rockfish | Blackgill Rockfish | *Sebastes melanostomus* | BLGL | 0.04 | 20 | 2005 |
| Rockfish | Blue Rockfish | *Sebastes mystinus* | BLUR | 0.1 | 6 | 2007 |
| Rockfish | Bocaccio | *Sebastes paucispinis* | BCAC | 0.15 | 3 | 2009 |
| Roundfish | Cabezon, CA North | *Scorpaenichthys marmoratus* | CBZN_CA_N | 0.25 | 7 | 2009 |
| Roundfish | Cabezon, CA South | *Scorpaenichthys marmoratus* | CBZN_CA_S | 0.25 | 7 | 2009 |
| Roundfish | Cabezon, OR | *Scorpaenichthys marmoratus* | CBZN_OR | 0.25 | 7 | 2009 |
| Rockfish | Canary Rockfish | *Sebastes pinniger* | CNRY | 0.08 | 8 | 2009 |
| Rockfish | Chilipepper | *Sebastes goodei* | CLPR | 0.16 | 8 | 2007 |
| Rockfish | Cowcod | *Sebastes levis* | CWCD | 0.055 | 11 | 2009 |
| Rockfish | Darkblotched Rockfish | *Sebastes crameri* | DBRK | 0.07 | 7 | 2009 |
| Flatfish | Dover sole | *Microstomus pacificus* | DOVR | 0.09 | 7 | 2005 |
| Flatfish | English sole | *Parophrys vetulus* | EGLS | 0.26 | 4 | 2007 |
| Rockfish | Gopher Rockfish | *Sebastes carnatus* | GPHR | 0.2 | 4 | 2005 |
| Roundfish | Lingcod, North | *Ophiodon elongatus* | LCOD_N | 0.18 | 4 | 2009 |
| Roundfish | Lingcod, South | *Ophiodon elongatus* | LCOD_S | 0.18 | 4 | 2009 |
| Elasmobranch | Longnose skate | *Raja rhina* | LSKT | 0.2 | 16 | 2007 |
| Rockfish | Longspine Thornyhead | *Sebastolobus altivelis* | LSPN | 0.06 | 13 | 2005 |
| Rockfish | Pacific Ocean Perch | *Sebastes alutus* | POP | 0.05 | 8 | 2009 |
| Flatfish | Petrale sole | *Eopsetta jordani* | PTRL | 0.2 | 7 | 2009 |
| Roundfish | Sablefish | *Anoplopoma fimbria* | SABL | 0.07 | 6 | 2008 |
| Rockfish | Shortspine Thornyhead | *Sebastolobus alascanus* | SSPN | 0.05 | 10 | 2005 |
| Rockfish | Splitnose Rockfish | *Sebastes diploproa* | SNOS | 0.048 | 8 | 2009 |
| Flatfish | Starry Flounder, North | *Platichthys stellatus* | STRY_N | 0.3 | 3 | 2005 |
| Flatfish | Starry Flounder, South | *Platichthys stellatus* | STRY_S | 0.3 | 3 | 2005 |
| Rockfish | Vermilion Rockfish, North | *Sebastes miniatus* | VRML_N | 0.1 | 6 | 2005 |
| Rockfish | Vermilion Rockfish. South | *Sebastes miniatus* | VRML_S | 0.1 | 6 | 2005 |
| Rockfish | Widow Rockfish | *Sebastes entomelas* | WDOW | 0.125 | 12 | 2009 |
| Rockfish | Yelloweye Rockfish | *Sebastes ruberrimus* | YEYE | 0.047 | 15 | 2009 |

## 2. Methods

We implement Depletion-Based SRA using a delay-difference model of the form

$$B_t = B_{t-1} + P(B_{t-a}) - C_{t-1} \quad (1)$$

where $C$ is catch, $B_t$ is biomass at time $t$, and $P$ is latent annual production based on the parental biomass $a$ (median age at entry to the reproductive biomass) years earlier. The latent production function $P$ can take a variety of forms, based on computational convenience and tractability, biological understanding and local conventions; any production model or stock-recruitment model could be used (e.g., Eq. (A.1)). Although it is not critical to the method we are proposing, this description of DB-SRA uses a novel production function that is partially derived from a standard stock-recruitment relationship.

Data-rich assessments of northeast Pacific groundfish stocks off the US west coast have conventionally assumed a Beverton–Holt stock-recruitment relationship (BHSRR), which has several inherent and undesirable limitations. Specifically, the BHSRR restricts peak latent productivity to $B_{mnpl} < 0.5$ ($B_{mnpl}$ is defined as $B_{msy}/K$, where $K$ is unfished biomass) and also requires the value of $B_{mnpl}$ to decline systematically as maximum surplus production increases, so that $B_{mnpl}$ and $F_{msy}$ are tightly linked. There is no objective reason to believe that $B_{mnpl}$ and $F_{msy}$ should be as strongly linked as is required by two-parameter SRRs such as the BHSRR. Also, there is no reason to believe that actual populations are restricted to $B_{mnpl} < 0.5$. For example, the multispecies models of Walters and Kitchell (2001) and MacCall (2002) demonstrate that intraguild predation can result in values of $B_{mnpl} > 0.5$. In the case of marine mammals, Taylor and DeMaster (1993) concluded that $0.5 < B_{mnpl} < 0.85$ for most pinnipeds and odontocetes.

Following the advice of Martell et al. (2008), we develop a model with "leading parameters" that are directly useful to management. Also, we adopt a production model approach for computational simplicity. The Pella–Tomlinson–Fletcher (PTF) production model, first developed by Pella and Tomlinson (1969) and later reparameterized by Fletcher (1978), allows flexible specification of peak latent productivity ($0 < B_{mnpl} < 1$) and MSY. However, Fletcher (1978) observed that the PTF model tends to predict excessive productivity at low biomasses in the case of highly skewed production curves (specifically, where $B_{mnpl} < e^{-1}$). In the present context, we note that high skewness is a condition that is often encountered under typical high values of BHSRR steepness ($h > 0.5$, where $h$ is Mace–Doonan steepness, the ratio of recruitment at $B = 0.2K$ to recruitment at $B = K$; e.g., Dorn, 2002 and Punt et al., 2008). McAllister et al. (2000) adjusted the PTF model by proposing a hybrid Schaefer-PTF model where the Schaefer model applies at biomasses below $B_{mnpl}$. However, the hybrid Schaefer-PTF model of McAllister et al. appears to overcompensate for this problem and at low abundances underestimates productivity relative to the BHSRR (Fig. 1). In Appendix A we develop an alternative hybrid Schaefer-PTF model that provides a latent production function that has properties similar to the BHSRR while allowing full flexibility in specifying $B_{mnpl}$. This function has the form of a PTF production model for abundances above a join-point ($B_{join}$) and has the form of a Schaefer model for abundances below $B_{join}$, where the value of $B_{join}$ is chosen to produce a good approximation to the BHSRR model.

The analysis begins with a set of four Monte Carlo-drawn input parameters similar to those used in calculating Depletion-Corrected Average Catch (MacCall, 2009). These parameters are the natural mortality rate ($M$), the ratio of MSY fishing rate to $M$ ($F_{msy}/M$), the relative biomass at maximum latent productivity ($B_{mnpl} = B_{msy}/K$), and the relative depletion level ($B_T/K$) in a specific recent year $T$ which does not have to be the final year in the time series. As shown in Fig. 2, the product of $M$ and $F_{msy}/M$ gives $F_{msy}$. In converting from continuous to annual accounting, we replace the

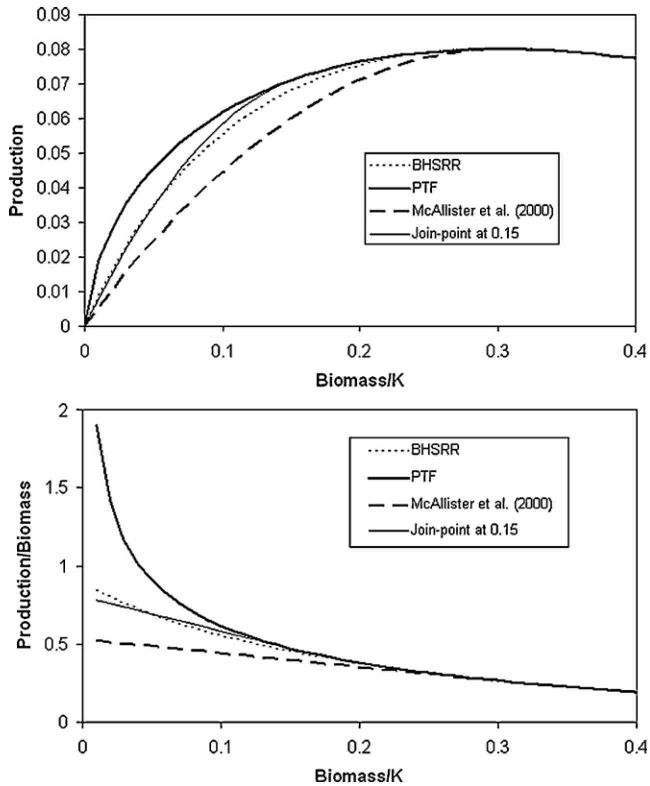


**Fig. 1.** Comparison of latent production curves (upper) and production-to-biomass ratios (lower) for alternative production functions. Peak productivity occurs at $0.3K$ (BHSRR $h = 0.58$), and the hybrid production model places the Schaefer join-point at $0.15K$ (details in Appendix A).

fishing mortality rate ($F$) with the annual exploitation rate, $U$, so that the value of MSY is given by $\text{MSY} = U_{msy} \cdot B_{mnpl} \cdot K$, where

$$U_{msy} = \left[ \frac{F_{msy}}{M + F_{msy}} \right] \cdot [1 - \exp(-(M + F_{msy}))] \tag{2}$$

This leaves only one unknown parameter to be estimated in the production function, the unfished biomass, $K$. Given the time series of historical catches, Eq. (1) is applied sequentially from time $t = 0$ to time $t = T$, and the value of $K$ is determined by a numerical solution that produces the recent relative depletion level. The abundance trajectory in years after time $T$ (including possible projection of future years) can also be modeled by sequential use of Eq. (1). Because the four input parameters (further described in Section

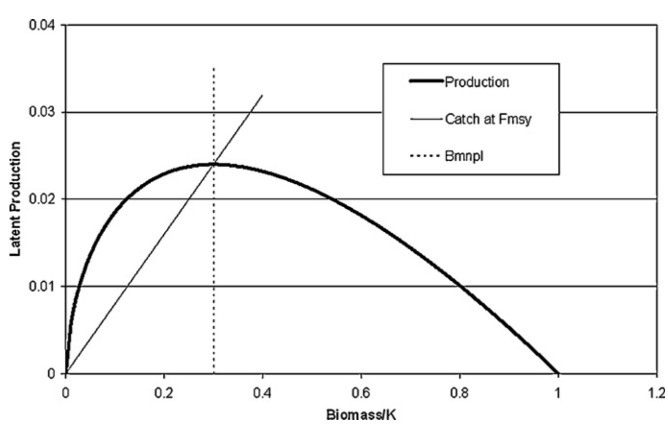


**Fig. 2.** Diagram showing how values of $B_{mnpl}$, $M$ and $F_{msy}/M$ specify the production function. The diagonal line is the catch at $F_{msy}$, and the slope of this line is the product of $M$ and $F_{msy}/M$.

3) are not known precisely, a Monte Carlo approach is used to draw many sets of plausible alternative from respective prior probability distributions.

As Walters et al. (2006) observe, not all combinations of the input parameters will permit the model to match the assumed relative stock depletion in the target year. Trajectories that predict negative biomass in any year (implicit stock extinction) are removed from the set of plausible runs. From the successful trajectories that are retained, we obtain updated distributions of $K$, $B_{msy}$, $F_{msy}$, MSY and $C_{Fmsy}$. The updated distributions could be considered posterior distributions resulting from a Bayesian framework in which the likelihood function has a value of 0 if negative estimated biomasses are present and has a value of 1 otherwise. The detailed algorithm for DB-SRA is presented in Appendix B.

Since reference points from DB-SRA ($C_{Fmsy}$, MSY, and $K$) may depend on the relative depletion level, $E\{B_T/K\}$, we test sensitivity to this by examining model performance for each of nine depletion levels (0.1, 0.2, ..., 0.9) applied to each of the 31 data-rich stock assessments in Table 1 and evaluate sensitivity of model outputs to this input distribution. For each reference point we calculate the difference between the median and the point estimate from the stock assessment and divide the absolute value by the assessment result ("relative absolute error", RAE). Using the overfishing limit as an example,

$$RAE_{OFL} = \frac{\left| median(\text{OFL}) - \text{OFL}_{assessment} \right|}{\text{OFL}_{assessment}} \tag{3}$$

Performance evaluations based on RAE require fewer simulations (5000) per run, because estimates of median values converge rapidly relative to the tails of the distributions. All other results that we present are based on 10,000 simulated trajectories.

## 3. Application and results

We tested the performance of DB-SRA by comparing outputs with corresponding estimates from recent data-rich assessments of 31 species/stocks of groundfish managed by the Pacific Fishery Management Council (PFMC) on the west coast of the United States. We examine one case (cowcod) in greater detail. Species-specific model inputs ($M$ and $a$) are taken from the corresponding data-rich assessments (Table 1). Many assessments reported length-based maturity schedules, in which case we used the length-at-age relationship from the assessment to approximate the age at 50% maturity. Productivity parameters ($B_{mnpl}$, $F_{msy}/M$) are given distributions typical of west coast groundfish.

We use 10,000 Monte Carlo simulations to propagate imprecision in parameters governing stock productivity and status, producing distributions of management quantities that reflect some (but certainly not all) of the major sources of uncertainty. The input data and parameter distributions used for Monte-Carlo exploration of DB-SRA are as follows:

*Catches*: Early historical catches were reconstructed from various sources and are documented in the individual assessments. Although historical catch estimates are imprecise, this analysis did not explore this aspect of uncertainty. Catches were also treated as known without error in the corresponding data-rich assessments.

*Natural mortality rate (M)*: Prior distributions for natural mortality rate were defined by setting the expectation of a lognormal distribution equal to the point estimate of total mortality rate used in the data-rich assessment and assuming a log-scale standard deviation (SD) of 0.4. This estimate of uncertainty is based on reanalysis of Hoenig's (1982, 1983) data relating total mortality rates to maximum age. MacCall (in press) found that the log-scale standard deviation of residuals from Hoenig's study was approximately 0.5. Hoenig (Pers. Comm. cited by MacCall, in press) notes that the mortality estimates in his studies were based on catch curve analysis and therefore include a nontrivial amount of estimation error. For this analysis we assume SD = 0.4, following Hoenig's suggestion that true mortality estimates would exhibit less variability about the mean, shrinking the prediction interval. Appropriate bias corrections were applied to transform the expected value of the lognormal distribution to the expected value of the log-scale normal distribution (Crow and Shimizu, 1988).

$F_{msy}/M$: Walters and Martell (2004) suggested that the ratio of $F_{msy}$ to $M$ for demersal species in the northeastern Pacific is about 0.8. We assume that $F_{msy}/M$ is lognormally distributed with an expectation of 0.8 and log-scale standard deviation of 0.2 for all species. Our estimate of uncertainty for $F_{msy}/M$ follows that of MacCall (2009). MacCall's estimate is based on an analysis of $F_{msy}/M$ for 16 west-coast groundfish species (MacCall, 2007). Profile likelihood analysis of these data supports a 95% interval for the log-scale standard deviation of (0.18, 0.35). MacCall's (2009) estimate falls within this interval and is consistent with the optimal value (0.24) to single-digit precision. Since a ratio of lognormal distributions is itself lognormally distributed (Crow and Shimizu, 1988), our assumptions also define $F_{msy}$ as a lognormally distributed random variable.

$B_{mnpl}$: The PFMC has defined proxy values for the biomass that generates maximum sustainable yield, $B_{msy}$, relative to unfished biomass ($K$). We refer to the ratio $B_{msy}/K$ as $B_{mnpl}$. The PFMC defines target biomass as 40% of unfished biomass (0.4$K$) for all groundfish species other than flatfish, for which maximum production is assumed to occur at 0.25$K$, reflecting the typically high productivity of flatfish species. Since $B_{mnpl}$ is constrained between 0 and 1, we assume that this parameter follows a bounded beta distribution. Lower and upper bounds of 0.05 and 0.95 were used to exclude simulations with extremely skewed yield curves. We set the expectation of $B_{mnpl}$ equal to 0.4 or 0.25 for non-flatfish and flatfish species, respectively, based on the proxies adopted by the PFMC (PFMC, 2010). MacCall (2009) treated $B_{mnpl}$ as a fixed value, but we recognize uncertainty in this parameter, using a standard deviation of 0.05 on the untransformed scale. The 95% intervals based on this assumption are approximately (0.31, 0.49) for non-flatfish and (0.17, 0.34) for flatfish species.

*Relative depletion level* ($B_T/K$): The relative depletion level is the biomass at time $T$ relative to the unfished level, $K$. We set $T$ equal to the assessment year in our comparison to data-rich stock assessments. To characterize uncertainty in $B_T/K$, we use a bounded beta distribution with $E\{B_T/K\} = 0.4$, $SD\{B_T/K\} = 0.1$ and upper and lower bounds of 0.99 and 0.01, respectively. Scientific advice related to catch-based management of data-poor stocks has emphasized the need for explicit prior information on this quantity (Restrepo et al., 1998). Although little information is currently available to inform stock status for data-poor groundfish stocks on the US west-coast, reference points from DB-SRA appears to be robust to a wide range of assumed depletion levels (see performance evaluation results).

For performance evaluation, we compiled estimated management reference points and other quantities from the 31 assessments (Table 2). We rescaled all DB-SRA results by the corresponding assessment values, so that perfect agreement has unit value. Comparison of DB-SRA outputs with corresponding assessment quantities posed a number of difficulties, often because the simple delay-difference model (Eq. (1)) could not reproduce quantities that were described in data-rich assessments. Unfished biomass ($K$) was often reported in units of "spawning output" (parental fecundity) rather than biomass or used a summary minimum age other than the age at maturity used in DB-SRA. We derived comparable values where feasible. Maximum Sustainable Yield and MSY fishing rate ($F_{msy}$) can be expressed in alternative ways. We did not use freely estimated values of these two quantities from the assessments due to their imprecision; they also were seldom reported. Because the PFMC uses an SPR (spawning potential ratio) proxy basis for nominal $F_{msy}$ we compared DB-SRA results to proxy-based estimates of MSY and $C_{Fmsy}$ (Table 2). Under PFMC management policy, the value of $C_{Fmsy}$ is used as an overfishing threshold (a.k.a., the overfishing level, OFL; NMFS, 2009). Unfortunately, assessments conducted for the PFMC rarely document the model-estimated OFL in the assessment year (they report the Council-adopted OFL instead) because of a two-year lag between assessments and implementation of resulting management. Orig-

**Table 2**
Estimates of maximum sustainable yield (MSY) and catch resulting from fishing the current stock at the MSY fishing rate ($C_{Fmsy}$ or OFL) from data-rich stock assessments, based on PFMC proxies (see text for details). Units of unfished biomass ($K$) vary among assessments, so reported values were adjusted to approximate the combined biomass [mt] of male and female spawners. Data-rich estimates of current biomass relative to unfished ($B_T/K$) and the Beverton–Holt (B–H) steepness parameter are provided for comparison.

| Species code | MSY [mt] | OFL [mt] | $K$ [mt] | $B_T/K$ | B–H steepness |
|---|---|---|---|---|---|
| ARTH | 5833 | 16,741 | 80,313 | 0.79 | 0.90 |
| BLCK_N | 408 | 535 | 11,390 | 0.53 | 0.60 |
| BLCK_S | 1035 | 1636 | 29,100 | 0.70 | 0.60 |
| BLGL | 223 | 285 | 19,006 | 0.52 | 0.65 |
| BLUR | 275 | 227 | 13,223 | 0.30 | 0.58 |
| CBZN_CA_N | 119 | 137 | 1835 | 0.45 | 0.70 |
| CBZN_CA_S | 26 | 33 | 470 | 0.60 | 0.70 |
| CBZN_OR | 49 | 60 | 803 | 0.52 | 0.70 |
| CLPR | 2099 | 2700 | 33,390 | 0.71 | 0.57 |
| DOVR | 17,651 | 37,400 | 598,108 | 0.63 | 0.80 |
| EGLS | 3877 | 15,221 | 72,024 | 0.95 | 0.80 |
| GPHR | 126 | 404 | 1995 | 0.97 | 0.65 |
| LCOD_N | 1710 | 2846 | 33,075 | 0.62 | 0.80 |
| LCOD_S | 1492 | 2705 | 25,331 | 0.74 | 0.80 |
| LSKT | 787 | 1157 | 14,068 | 0.66 | 0.40 |
| LSPN | 3687 | 2838 | 210,314 | 0.71 | 0.75 |
| SABL | 4871 | 10,593 | 489,594 | 0.39 | 0.48 |
| SNOS | 1244 | 2125 | 87,584 | 0.66 | 0.58 |
| SSPN | 2009 | 2474 | 230,500 | 0.66 | 0.70 |
| STRY_N | 818 | 1234 | 9648 | 0.44 | 0.80 |
| STRY_S | 396 | 792 | 4668 | 0.62 | 0.80 |
| VRML_N | 131 | 245 | 5627 | 0.65 | 0.83 |
| VRML_S | 171 | 359 | 9677 | 0.59 | 0.83 |
| BCAC | 1258 | 831 | 44,070 | 0.28 | 0.57 |
| CNRY | 954 | 587 | 51,986 | 0.24 | 0.51 |
| CWCD | 54 | 7 | 4366 | 0.05 | 0.60 |
| DBRK | 575 | 483 | 32,783 | 0.27 | 0.60 |
| POP | 1124 | 811 | 75,560 | 0.29 | 0.51 |
| PTRL | 2286 | 3254 | 50,668 | 0.12 | 0.95 |
| WDOW | 3031 | 3500 | 220,930 | 0.39 | 0.41 |
| YEYE | 49 | 48 | 8492 | 0.20 | 0.42 |

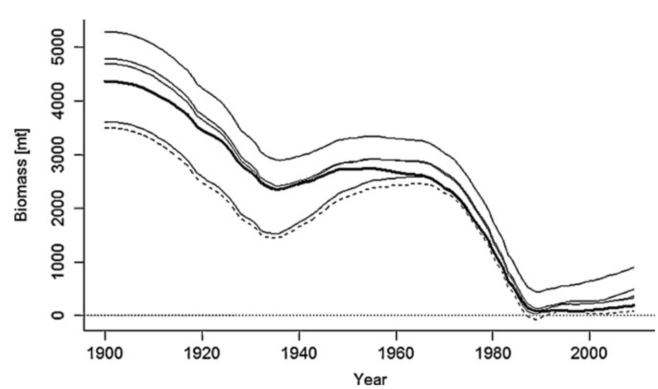


**Fig. 3.** Examples of abundance trajectories from DB-SRA for cowcod with $E\{B_T/K\} = 0.1$ (thin solid lines). Thick solid line is abundance history from the age-structured stock assessment, in which estimated $B_T/K = 0.05$. Dashed line is a DB-SRA trajectory that was rejected (negative biomass estimates).

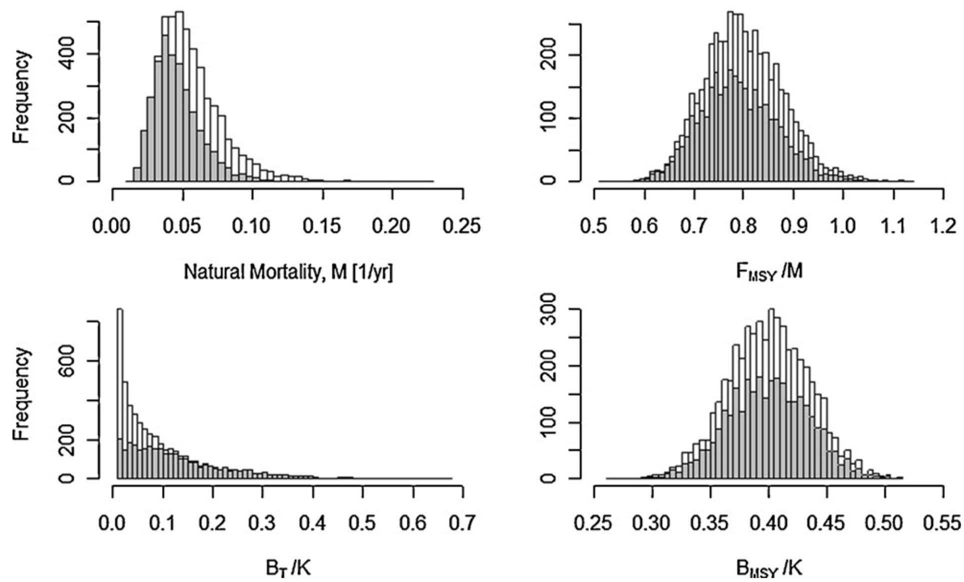


**Fig. 4.** Frequency distributions of parameter values, with accepted (grey) and rejected (white) regions (expected $B_T/K = 0.1$). The sum of the two bars corresponds to the underlying prior distribution, and the grey area reflects the proportion of retained runs in a region.

inal model estimates of OFL ($C_{Fmsy}$) were recovered where possible.

For lack of a consistent alternative, we treat the data-rich point estimates as fixed "true" values of the reference points despite their acknowledged imprecision. In some cases the data-rich assessments provide nominal asymptotic standard errors for some of the estimated quantities, but these underestimate the true variability because all of the assessments use fixed values of important and admittedly imprecise parameters such as $M$ and $h$, which results in severe underestimation of standard errors. In contrast, DB-SRA recognizes imprecision in $M$ and $h$. Some of the 31 data-rich assessments utilize the method of maximum likelihood with various prior probability distributions on input parameters, so the data-rich results used for comparison are formally "maximum posterior density" (MPD) estimates. Ideally, our DB-SRA results would be compared with MCMC estimates of posterior probability distributions from those data-rich models. However, few successful MCMC explorations have been conducted, and very few (e.g. Hamel, 2009) included uncertainty in all major parameters (i.e., including $M$ and $h$).

### 3.1. Example application to cowcod

We first describe DB-SRA results in detail for a single case, that of cowcod, which was determined to be "overfished" in 1999 and is presently being managed for stock rebuilding (Dick et al., 2009). Example DB-SRA abundance trajectories are shown in Fig. 3, and nearly all of the accepted cases were congruent with the data-rich stock assessment results, demonstrating the strong influence of the catch history. An example trajectory from one case that was rejected ($M = 0.053$ and $F_{msy}/M = 0.968$ is also shown. As shown in Fig. 3, the assessment for cowcod suggests that the stock experienced very low abundances in the late 1980s and 1990s. DB-SRA parameter draws based on the assumption of low stock status

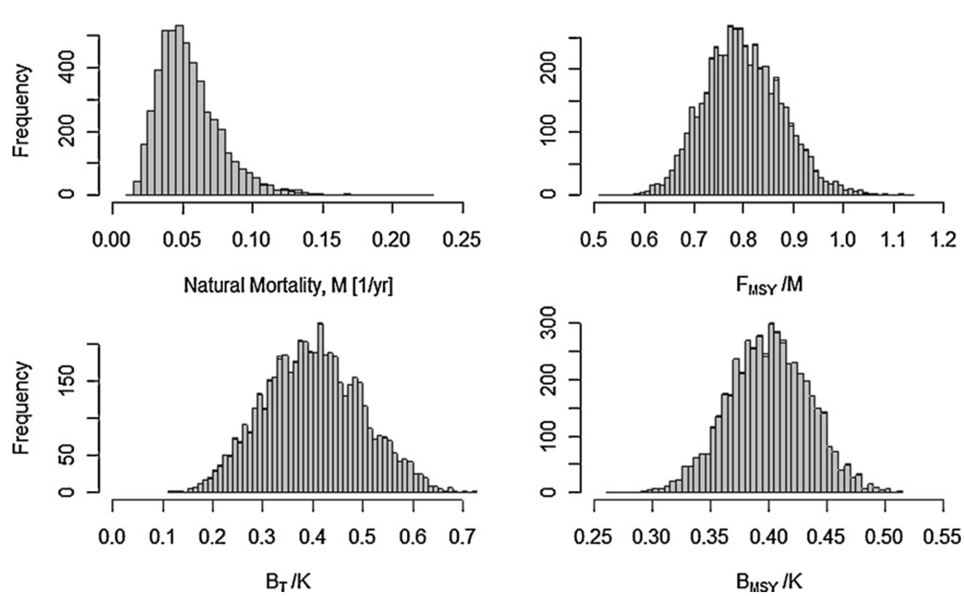


**Fig. 5.** Frequency distributions of parameter values, with accepted (grey) and rejected (white) regions (expected $B_T/K = 0.4$). The sum of the two bars corresponds to the underlying prior distribution, and the grey area reflects the proportion of retained runs in a region.

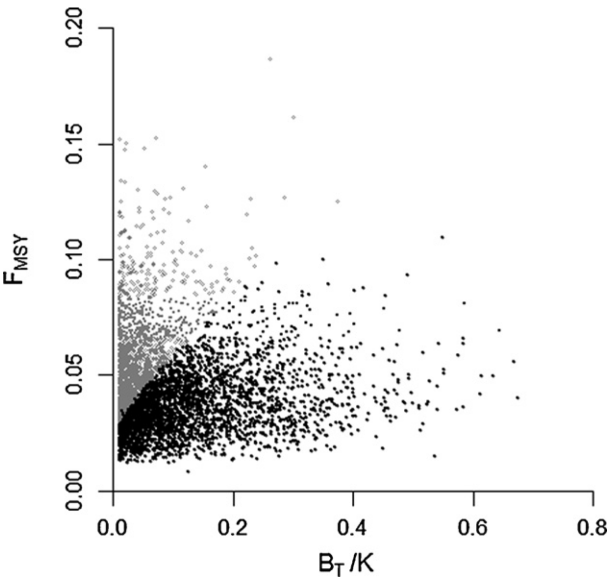


**Fig. 6.** $F_{msy}$ versus $B_T/K$, with rejection region in grey ($E\{B_T/K\} = 0.1$).

have a high rate of rejection, a property of stochastic SRA that was described by Walters et al. (2006). Drawn parameter values (both accepted and rejected) for $E\{B_T/K\} = 0.10$ and $E\{B_T/K\} = 0.4$ are shown in Figs. 4 and 5. A high rate of rejection occurs for low terminal abundances ($B_T/K$) and for high values of $M$ and $F_{msy}/M$ (Fig. 4). Because $F_{msy}$ is the product of the latter two values, Fig. 6 shows that, in effect, it is high values of $F_{msy}$ that tend to be rejected, especially at low values of $B_T/K$. Useful posterior information is gained for three of the four distributions; but the pattern of rejections provides little information on the value of $B_{mnpl}$ (Fig. 4).

### *3.2. Comparison with 31 data-rich assessments*

Assessments have been conducted for 31 stocks of west coast groundfish, some of which represent different geographic regions of the same species (Table 1). Although the quality and quantity of data varies substantially among these cases, these assessments are referred to nominally as being "data-rich" in that available data were sufficient to support a conventional likelihood-based stock assessment acceptable for the purposes of the Pacific Fishery Management Council. In each case, the MPD estimates from the full stock assessment are taken as benchmarks to which performance of DB-SRA can be compared. In order to express results on a comparable scale, each DB-SRA output distribution (MSY, $C_{Fmsy}$ and $K$) is expressed relative to the corresponding MPD value given by the assessment. We acknowledge that this rough standardization fails to account for the additional uncertainty in the original "data-rich" assessments themselves, which is undoubtedly substantial.

The DB-SRA distributions in Figs. 7–10 are based on the assumption that current stock biomass is 40% of the unfished biomass ($E\{B_T/K\} = 0.4$). Although the input distribution of relative stock depletion reflects considerable uncertainty (Fig. 5, lower left panel), it is interesting to note that estimates of MSY, $C_{Fmsy}$, and $K$ based on low values of $E\{B_T/K\}$ tend to minimize the absolute relative error between the DB-SRA median and the assessment MPD (Fig. 11).

## 4. Discussion

Stock reduction analysis is unusual among stock assessment methods in that it is well suited to analysis of so-called "one-way trips," i.e., cases with nearly monotonic declines in abundance. By initiating the model time series with an unfished resource, it is possible to have one parameter ($K$) serve two purposes, both as a key parameter of the production function and as the initial condition, thus reducing the problem to estimation of that single parameter. A drawback (also shared with data-rich approaches) is that DB-SRA requires knowledge of the entire history of catches, and estimated catches for early years tend to be poorly documented, requiring best-guess approximations.

Perhaps the most surprising feature of DB-SRA is that useful information can be gained from a catch history despite little knowledge of current abundance. A fundamental property of Stock Reduction Analysis in all of its forms is that the progressive reduction in abundance provides a basis for separating ongoing sustainable production from a non-sustainable, once-only harvest associated with reducing the population size. If the analysis is given a value of $B_T$ that is near $K$, there appears to have been no net impact from harvesting, and it produces implausibly high estimates of MSY (Fig. 11). For stocks that have recovered to $B_T$ that is near $K$, DB-SRA

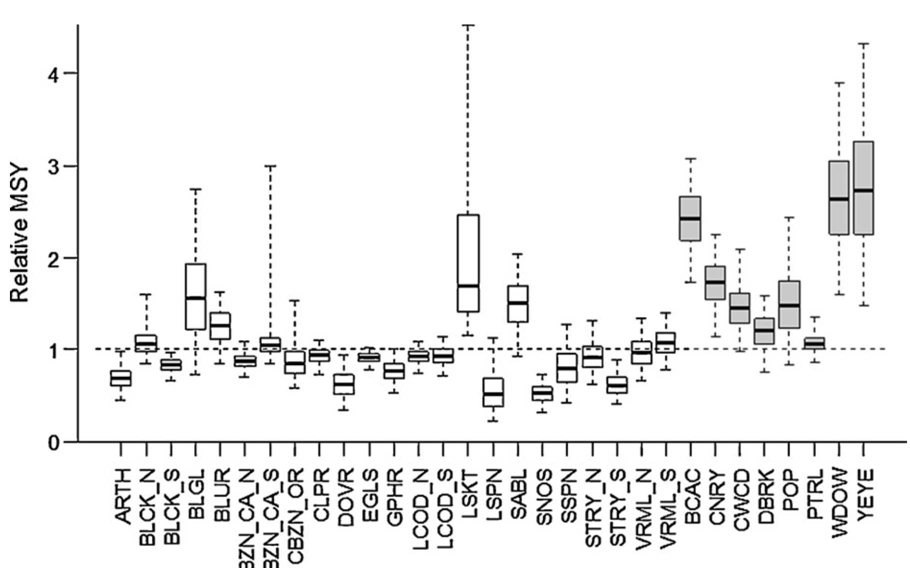


**Fig. 7.** Distributions of *MSY* from DB-SRA ($E\{B_T/K\} = 0.4$) scaled relative to *MSY* point estimates from age-structured stock assessments (SPR proxy values). Relative *MSY* = 1 (dashed line) represents agreement. Boxplots with grey shading indicate species with assessed status below the minimum stock size threshold.

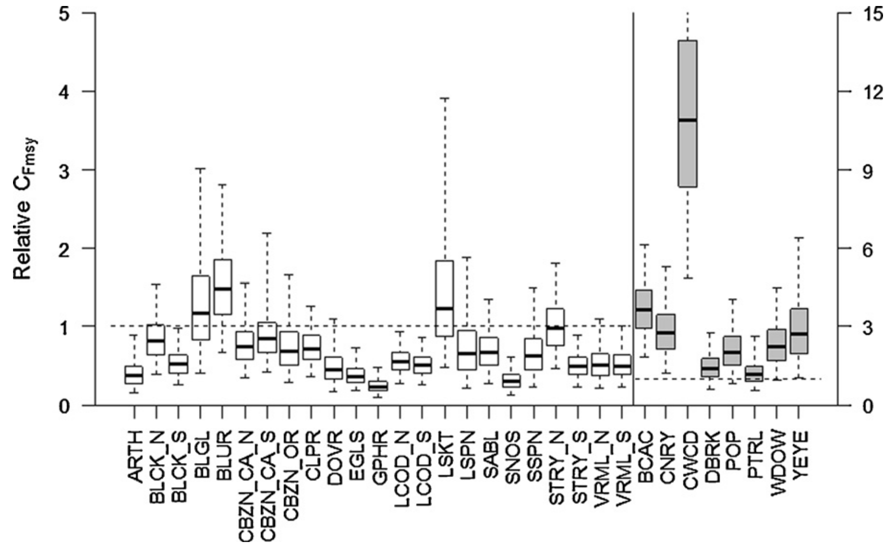


**Fig. 8.** Distributions of $C_{Fmsy}$ from DB-SRA ($E\{B_T/K\} = 0.4$) relative to $C_{Fmsy}$ point estimates from age-structured stock assessments (SPR proxy values). Relative $C_{Fmsy} = 1$ (dashed line) represents agreement. Vertical axis is rescaled for overfished species (grey boxplots).

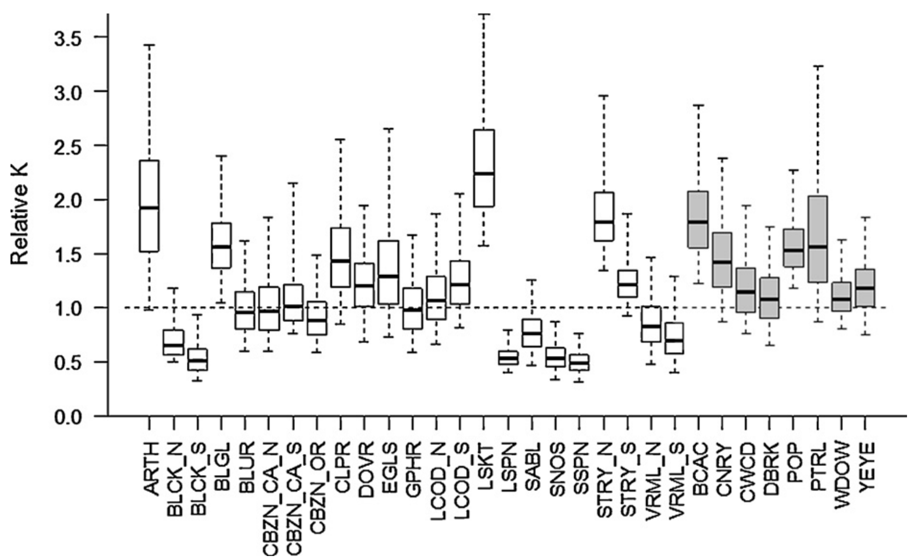


**Fig. 9.** Distributions of unfished spawning biomass ($K$) from DB-SRA, scaled relative to estimates from most recent stock assessments. All models assumed that $E\{B_T/K\} = 0.4$. Overfished species (below their minimum stock size thresholds) are shaded in grey.

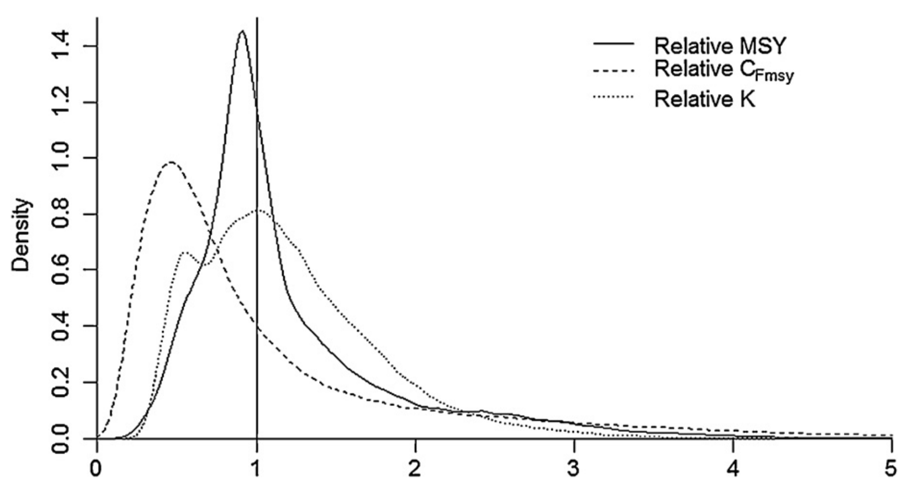


**Fig. 10.** Probability densities of DB-SRA reference points. Values are expressed as an equally weighted composite of retained runs, expressed relative to the MPD estimates from 31 corresponding data-rich assessments.

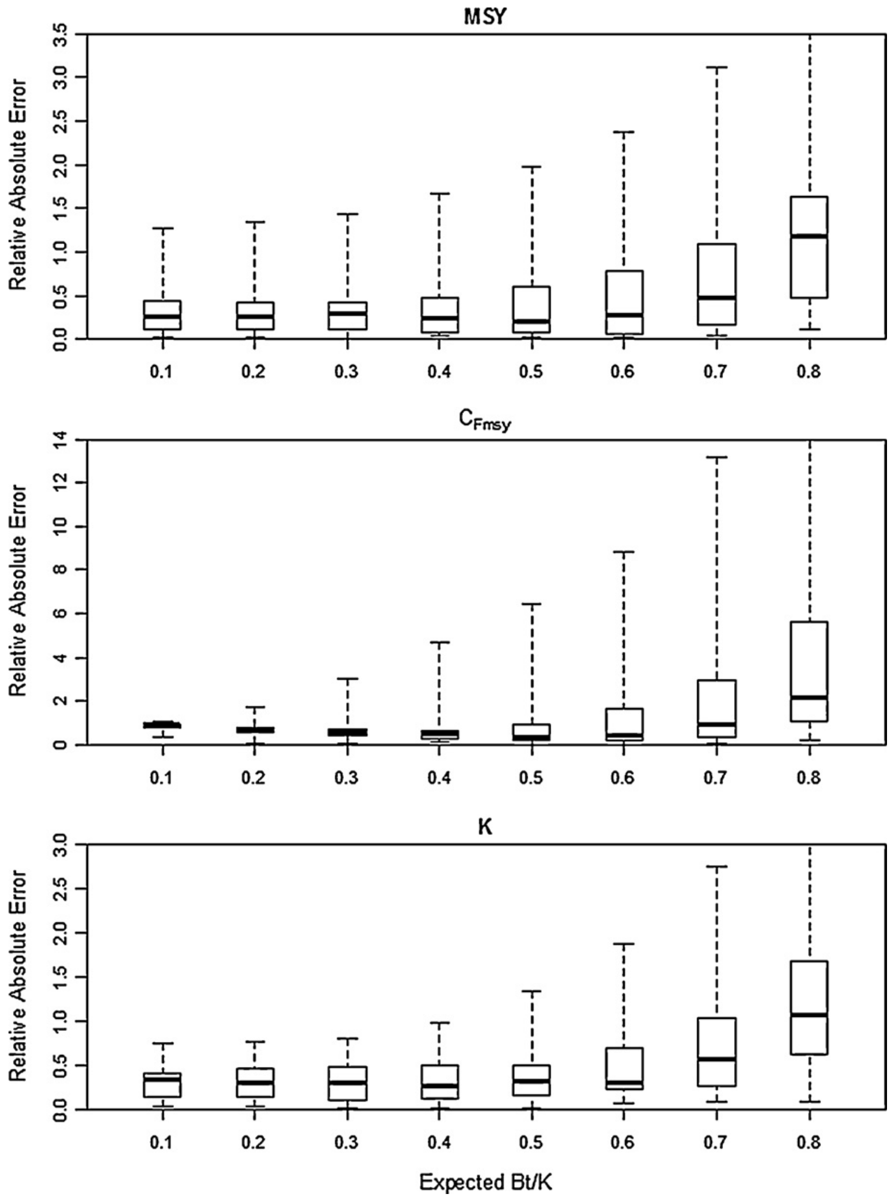


**Fig. 11.** Distributions of relative absolute error for *MSY*, $C_{Fmsy}$, and *K* among 31 stock assessments, for different values of relative stock depletion, $E\{B_T/K\}$ based on 5000 simulated trajectories. Results for runs with ending biomass at 90% of unfished biomass are not displayed for purposes of scale, due to extreme discrepancies with assessment results.

surprisingly gives better estimates if the analysis is falsely given a much lower value of $B_T$. As was described for stochastic SRA by Walters et al. (2006), assessment of stocks that have experienced extensive historical depletion gains precision from a high rate of rejected parameter draws (Fig. 4 and Table 3).

Management decisions (here, we discuss only estimates of $C_{Fmsy}$ = OFL = $U_{msy}B_T$) can utilize two different aspects of DB-SRA: the assumed relative depletion, $B_T/K$, and the appropriate percentile of the probability distribution of estimated $C_{Fmsy}$. In order to maintain transparency and avoid hidden embedding of precautionary aspects of the analysis (Restrepo et al., 1998), these specifications of DB-SRA would ideally be pre-established by an explicit management policy, for example setting assumed relative depletion at a target abundance (e.g., $B_T = 0.4K$) or more precautionarily at a threshold abundance (e.g., $B_T = 0.2K$). Similarly, the value of $C_{Fmsy}$ could be taken as the median output value, or as a lower percentile of the distribution (cf. Shertzer et al., 2008). For west-coast groundfish species, low values of assumed relative depletion appear to be most consistent with traditional assessment results (Table 2). The accuracy of $C_{Fmsy}$ estimates for these species is relatively stable for lower assumed values of $B_T/K$ (Fig. 11).

Estimation of sustainable yield is of primary importance to policy makers, and DB-SRA can reduce uncertainty about this quantity when only catch and life-history data are available. Our results suggest that uncertainty about other important reference points (MSY and *K*) is also greatly reduced through application of DB-SRA. For most stocks we evaluated, median estimates of MSY and *K* from DB-SRA tend to be between one-half and double the assessment value (Figs. 7 and 9). Assuming similar performance for unassessed stocks, this is a considerable improvement in our understanding of stock biomass and potential productivity.

DB-SRA estimates of MSY are most consistent with data-rich assessment results (Fig. 10). If the status of a data-poor stock is assumed to be above $B_{mnpl}$, then median values of $C_{Fmsy}$ will exceed MSY, by definition. This amount of catch (in numbers or weight) is unsustainable in the long term, also by definition. If abundance and

**Table 3**
Percentage of simulations that were not consistent with the catch history (i.e. negative biomass), by stock assessment and expected depletion assumption. Percentages were zero for all stocks when status was assumed to be 60% or greater of unfished biomass, with the exception of longnose skate, for which a small number (<3%) of simulations had negative biomass.

| Species code | $E\{B_T/K\}$ | | | | |
|---|---|---|---|---|---|
| | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
| ARTH | 1% | 0% | 0% | 0% | 0% |
| BLCK_N | 19% | 2% | 1% | 1% | 0% |
| BLCK_S | 11% | 0% | 0% | 0% | 0% |
| BLGL | 26% | 3% | 0% | 0% | 0% |
| BLUR | 11% | 0% | 0% | 0% | 0% |
| CBZN_CA_N | 1% | 1% | 1% | 0% | 0% |
| CBZN_CA_S | 12% | 1% | 1% | 0% | 0% |
| CBZN_OR | 29% | 0% | 0% | 0% | 0% |
| CLPR | 0% | 0% | 0% | 0% | 0% |
| DOVR | 0% | 0% | 0% | 0% | 0% |
| EGLS | 0% | 0% | 0% | 0% | 0% |
| GPHR | 32% | 1% | 0% | 0% | 0% |
| LCOD_N | 0% | 0% | 0% | 0% | 0% |
| LCOD_S | 0% | 0% | 0% | 0% | 0% |
| LSKT | 30% | 16% | 10% | 6% | 4% |
| LSPN | 2% | 0% | 0% | 0% | 0% |
| SABL | 3% | 0% | 0% | 0% | 0% |
| SNOS | 0% | 0% | 0% | 0% | 0% |
| SSPN | 1% | 0% | 0% | 0% | 0% |
| STRY_N | 0% | 0% | 0% | 0% | 0% |
| STRY_S | 0% | 0% | 0% | 0% | 0% |
| VRML_N | 25% | 0% | 0% | 0% | 0% |
| VRML_S | 15% | 0% | 0% | 0% | 0% |
| BCAC | 0% | 0% | 0% | 0% | 0% |
| CNRY | 11% | 0% | 0% | 0% | 0% |
| CWCD | 37% | 9% | 2% | 1% | 0% |
| DBRK | 0% | 0% | 0% | 0% | 0% |
| POP | 0% | 0% | 0% | 0% | 0% |
| PTRL | 4% | 0% | 0% | 0% | 0% |
| WDOW | 52% | 11% | 2% | 1% | 0% |
| YEYE | 52% | 10% | 1% | 0% | 0% |

$F_{msy}$ were known precisely (which is not true for data-poor stocks) fishing at the $F_{msy}$ harvest rate would, in theory, cause the stock to equilibrate at $B_{mnpl}$. However, given uncertainty in historical catch, unfished abundance, and $F_{msy}$, as well as our inability to update estimated current depletion for the data-poor stock, it may be reasonable to adopt a precautionary policy under which OFL is defined as the smaller of $C_{Fmsy}$ or MSY. Long-lived, low-productivity, and data-poor stocks such as those in the US west-coast groundfish fisheries may benefit from this type of harvest control rule, in that it discourages overcapitaliztion due to short-term (unsustainable) yields, and it is directly tied to the reference point with highest apparent precision (MSY). Even for data-rich cases, a catch control rule that is capped near MSY may be desirable (e.g., Froese et al., 2010). Short-lived, data-poor stocks that exhibit long-term cycles in productivity would not benefit from such a rule.

There are several opportunities for further modification of DB-SRA. As the model stands, uncertainty in historical catches is not addressed adequately, but the solution is not straightforward. There is a temporal structure to the covariance of catch estimates (especially for reconstructed catches), so that it is unlikely that Monte Carlo draws of catches from independent annual probability distributions would be an acceptable representation of inter-annual variability. Rather, the draws would ideally recognize the covariance matrix of catches, and that matrix has yet to be constructed. Similarly, the analysis could utilize a history of fishing efforts derived from comparable data-rich stock assessments, again according to a defined covariance matrix. For some guilds of species that share similar recruitment patterns, time-varying recruitment could be added to the production function by resampling recruitment deviations from a set of comparable data-rich assessments. Of course, the model could include more traditional goodness-of-fit to abundance indexes, in which case it quickly approaches Stochastic SRA as described by Walters et al. (2006).

## Acknowledgements

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## Appendix A. Derivation of a hybrid Schaefer-PTF model emulating latent production from a Beverton–Holt SRR-driven model

First, we establish the generic shape of a BHSRR-driven production function. Mangel et al. (2009) developed the following simplified model, which we reinterpret as an annual difference model rather than the original differential equation model:

$$P = \frac{\alpha B}{1+\beta B} - MB \tag{A.1}$$

where $P$ is surplus production, $B$ is biomass, $\alpha$ and $\beta$ are model parameters, and $M$ is the natural mortality rate of spawning biomass on an annual basis. Mangel et al. show that unfished biomass ($K$) is

$$K = \frac{1}{\beta}\left(\frac{\alpha}{M} - 1\right) \tag{A.2}$$

and if we want to scale the model to a value of $K=1$, this has the solution

$$\beta = \frac{\alpha}{M} - 1 \tag{A.3}$$

We also want to specify biomass producing MSY as a fractional value $B_{mnpl} = B_{msy}/K$, which in the special case of unit unfished biomass ($K=1$) is given by

$$B_{mnpl} = \frac{\sqrt{(\alpha/M)} - 1}{(\alpha/M) - 1} \tag{A.4}$$

so that a value of $\alpha/M$ can be obtained for a given value of $B_{mnpl}$. An arbitrary value of $M$ (such as 0.2) can be used, which establishes the value of $\alpha$. The value of $\beta$ is obtained from the solution of Eq. (A.3), so we have now specified the generic shape of the BHSRR-driven latent production function with a specified value of $B_{mnpl}$.

We now replace this BHSRR with a nearly exact equivalent based on the Pella–Tomlinson–Fletcher (PTF) generalized production model (Fletcher, 1978). Fletcher's reparameterization of the Pella–Tomlinson model (Pella and Tomlinson, 1969), again cast as an annual difference model, gives latent annual production as

$$P = gm\left(\frac{B_{t-a}}{K}\right) - gm\left(\frac{B_{t-a}}{K}\right)^n \tag{A.5}$$

where exponent $n$ ($n>0$) determines the skewness and $g = n^{n/(n-1)}/(n-1)$ (which is positive if $n>1$, and is negative for $0<n<1$). Parameter $m$ is the MSY, and as before, $K$ is the unfished biomass. Note that $B_{mnpl}$ is a function only of $n$: $B_{mnpl} = n^{1/(1-n)}$ for $n \neq 1$, and $B_{mnpl} = e^{-1}$ if $n=1$.

As noted by Fletcher (1978), a major drawback of the PTF model is that modeled productivity near the origin can be unrealistically high, especially when $n<1$ (i.e., $B_{mnpl} < e^{-1}$). To address this problem, McAllister et al. (2000) proposed a hybrid Schaefer-PTF model in which a PTF model is used at values $B > B_{msy}$, and that a Schaefer model ($n=2$) be used for $0<B<B_{msy}$, with a "join point" ($B_{join}$) at $B_{msy}$. From a comparison of production-to-biomass ratios from the

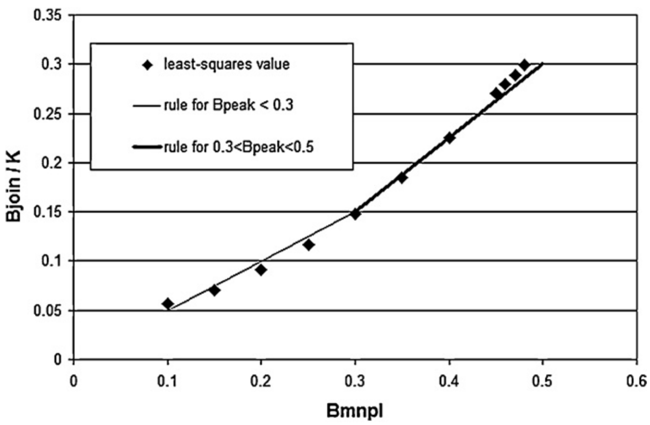


**Fig. A.1.** Least squares values of hybrid Schaefer-PTF join-points to approximate a Beverton–Holt SRR, and linear approximations to optimal join-points.

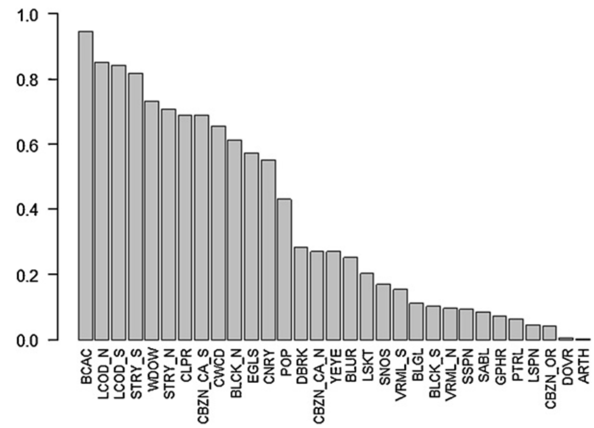


**Fig. A.2.** Proportion of simulated biomass trajectories that fell below the biomass "$B_{join}$".

PTF model and the hybrid model of McAllister et al. relative to that from a BHSRR (Fig. 1), it appears that their hybrid model has too low a productivity at low biomass.

We propose a closer approximation to a BHSRR-driven model by a modification of the hybrid model of McAllister et al., specifically by choosing a join-point below $B_{msy}$ $(0 < B_{join} < B_{msy})$ based on goodness of fit to the BHSRR production-to-biomass ratio at low abundances. The Schaefer model has a linear production-to-biomass ratio, so the slope of the Schaefer model for $B < B_{join}$ is equal to the slope of the PTF production-to-biomass ratio ($c$) evaluated at the join point, which for the PTF model is

$$c = ((1-n)gmB_{join}^{n-2}K^{-n} \tag{A.6}$$

For $B < B_{join}$, the corresponding Schaefer model is

$$P(B_{t-a} < B_{join}) = B_{t-a}\left(\frac{P(B_{join})}{B_{join}} + c(B_{t-a} - B_{join})\right) \tag{A.7}$$

The optimal values of $B_{join}$ were determined by means of a least-squares solution of the difference between the $P/B$ values of the BHSRR model and those of the Schaefer model evaluated at ten evenly spaced locations on the interval [0.05, $B_{join}/K$]; these values of $B_{join}/K$ are shown in Fig. A.1. Rather than re-estimating the optimal value of $B_{join}$ for each Monte Carlo draw, we use the following set of linear rules:

if $B_{mnpl} < 0.3$, $B_{join}/K = 0.5B_{mnpl}$;
if $0.3 < B_{mnpl} < 0.5$, $B_{join}/K = 0.75B_{mnpl} - 0.075$;
if $B_{mnpl} > 0.5$, use PTF model for all $B$.

Note that as $B_{mnpl}$ approaches 0.5, the PTF model approaches being a Schaefer model, and the hybrid model becomes insensitive to the location of $B_{join}$. However, the set of species cases we used in this study frequently visited the region $B_t < B_{join}$ (Fig. A.2), indicating the importance of this model feature in the present application.

## Appendix B. Stepwise description of DB-SRA algorithm

Iterate the following steps 10,000 times.
Draw parameter values from their assumed distributions:

(1) Draw a natural mortality rate ($M$).
(2) Draw a ratio of MSY fishing rate to $M$ ($F_{msy}/M$).
(3) Draw a value of $B_{mnpl}$.
(4) Draw a relative abundance level ($B_T/K$) in a specific recent year $T$ which does not have to be the final year in the time series.

Specify the production function:

(5) Based on the value of $B_{mnpl}$, calculate the value of the PTF skewness parameter, $n$, by numerical solution of $B_{mnpl} = n^{1/(1-n)}$.
(6) Calculate $g = n^{n/(n-1)}/(n-1)$ which is positive if $n > 1$, and is negative for $n < 1$.
(7) Obtain an approximate initial estimate of $K$ (an improved initial approximation should be possible after experience is gained from the first few iterations).
(8) Calculate MSY as $m = k \cdot B_{mnpl} \cdot U_{msy}$, where $U_{msy} = (F_{msy}/(F_{msy}+M))(1-e^{-(F_{msy}+M)})$.
(9) If $B_{mnpl} < 0.5$, calculate the join-point, $B_{join}$ according to the rule in Appendix A.

Iteratively solve for unfished abundance, $K$:

(10) Use the delay-difference equation (Eq. (1)) to estimate the time series of abundances from $B_0 = K$ to $B_T$.
   a. Note that because of the delay term, the latent production in the first $a$ years of the time series is zero because the parental biomass was at the unfished level.
(11) Iteratively adjust the trial value of $K$ until the value of $B_T$ satisfies the given value of $B_T/K$.
(12) If successful, add the parameter set and derived management quantities to the collection of successful trials, else go to step 13.
   a. $B_{msy} = K(B_{msy}/K)$
   b. $F_{msy} = M(F_{msy}/M)$
   c. $\text{MSY} = B_{msy} \cdot U_{msy}$
   d. $C_{Fmsy} = U_{msy} \cdot B_t$
   e. Project abundances to the present or future times $t > T$, if desired, based on observed or assumed catches (this is necessary to estimate current sustainable yields).
(13) If the drawn set of parameters is rejected ($B_t \leq 0$ for any $t$), add the parameter set and derived management quantities to the collection of unsuccessful trials.

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