A Bayesian hierarchical formulation of the De Lury stock assessment model for abundance estimation of Falkland Islands' squid (*Loligo gahi*)

Murdoch K. McAllister, Simeon L. Hill, David J. Agnew, Geoffrey P. Kirkwood, and John R. Beddington

Abstract: In stock assessments of short-lived species, De Lury depletion models are commonly applied in which commercial catches and changing catch rates are used to estimate resource abundance. These methods are applied within fishing seasons to decide when to close the fishery and can be reliable if the data show a distinct decline in response to the catch removals. However, this is not always the case, particularly when sampling error variation masks trends in abundance. This paper presents a Bayesian hierarchical formulation of the De Lury model in which data from previous years are combined hierarchically in the same stock assessment model to improve parameter estimation for future stock assessments. The improved precision in parameter estimates is demonstrated using data for the Falkland Islands' *Loligo gahi* squid fishery.

Résumé : Les modèles d'épuisement de De Lury sont couramment appliqués aux prises commerciales et aux taux de capture changeants pour estimer l'abondance de la ressource dans l'évaluation des stocks des espèces à vie brève. Ces méthodes sont utilisées au cours de la saison de pêche pour décider quand mettre fin à la pêche et elles sont fiables lorsque les données montrent un net déclin en réaction aux retraits dus à la pêche. Cependant, cela n'est pas toujours le cas, surtout lorsque la variation de l'erreur d'échantillonnage masque les tendances dans l'abondance. Nous présentons une formulation hiérarchique bayésienne du modèle de De Lury dans laquelle les données des années antérieures sont combinées de façon hiérarchique dans un même modèle d'évaluation du stock afin d'améliorer l'estimation des paramètres pour les évaluations futures. Des données provenant de la pêche commerciale du calmar *Loligo gahi* des îles Falklands nous servent à démontrer l'amélioration obtenue dans l'estimation des paramètres.

[Traduit par la Rédaction]

Introduction

Leslie–De Lury depletion models (hereafter referred to as De Lury models) for estimating population size rely on the fact that removing individuals from a population often has a noticeable effect on some observable index of abundance. In a fisheries context, such methods can be used when commercial catch rates show a distinct decline in response to catch removals (see overview in Hilborn and Walters 1992). For example, these methods have been successfully used in the assessment of squid stocks around the Falkland Islands (also referred to hereafter as the Falklands) (Rosenberg et al. 1990; Basson et al. 1996; Agnew et al. 1998). In this case, catch per unit effort (CPUE) is assumed to be directly proportional to abundance. The scaling factor, known as catchability, represents the proportion of the population that will be caught per unit effort. A maximum likelihood method is used to fit a model to the declining period in the CPUE data and estimate parameters representing initial abundance and catchability (Rosenberg et al. 1990). With these parameters, the data can also be used to estimate abundance up to the last recorded catch removal and to predict the abundance in future weeks. Prediction of the total removals that would leave the stock at or above an agreed biological reference point can facilitate in-season management of the fishery.

De Lury models are useful provided that (*i*) the catch rate data show a distinct decline in response to catch removals, (*ii*) there is relatively little emigration and immigration during the depletion, (*iii*) a linear relationship holds between catch rate and abundance, and (*iv*) the natural mortality rate and catchability remain constant over the period in the data. However, these conditions seldom hold all at once, which can produce biased results.

For years in which the first condition does not hold, for example, because sampling error in CPUE data masks declines in abundance, and the other three conditions hold, ad hoc methods have been proposed in which the value for the catchability coefficient (q) is fixed at some previously esti-

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M.K. McAllister,[1] D.J. Agnew, G.P. Kirkwood, and J.R. Beddington. Department of Environment Science and Technology, Imperial College, RSM Building, Prince Consort Road, London, SW7 2BP UK.
S.L. Hill. British Antarctic Survey, High Cross, Madingley Road, Cambridge, CB3 0ET, UK.

[1]Corresponding author (e-mail: m.mcallister@imperial.ac.uk).

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mated value (Agnew et al. 1998). Whereas this sensibly makes use of historical data to confine parameter estimates, it does not formally account for the interannual variability in values for the parameter and the effect of this on estimation uncertainty in the present year.

In contrast, methods since the early 1990s have been developed to probabilistically share information among populations and improve the estimation properties of stock assessments (e.g., Collie and Walters 1991; McAllister et al. 1994; Rivot and Prévost 2002). Among these, Bayesian hierarchical modeling (BHM) methods have been devised to model the cross-population variability in parameters that underlie different sets of historical data and to develop posterior predictive probability density functions (pdfs) of the parameters in unsampled populations (Gelman et al. 1995; Adkison and Su 2001; Harley and Myers 2001). Posterior predictive pdfs obtained from BHM can serve as prior pdfs of the parameters in analyses of data sets for newly sampled populations to help improve precision in parameter estimation (Michielsens 2003). The resulting posterior pdfs of model parameters can then help to reduce uncertainty when evaluating the potential consequences of alternative fishery management actions that can be taken.

The squid species *Illex argentinus* and *Loligo gahi* are the main commercial stocks exploited in the Falkland Islands' conservation zones and have been major contributors to the economy of the Islands since the Falkland Islands government began the active management of fishery resources in 1987. Both squid are short lived, with life spans of about 1 year. They are both migratory species and the fisheries operate in adult feeding areas where the squid populations are resident for several months. There are two fishing seasons for *L. gahi* each year, exploiting two main pulses of recruitment resulting from spawning events in spring and autumn. De Lury estimation methods are applied to the fisheries data to provide in-season management advice (Rosenberg et al. 1990; Agnew et al. 1998). Because they are annual species, all historical data are excluded in each new assessment. Whereas informative depletions in catch rates occur in most years, there are others where the De Lury estimations fail to provide sensible results because the data are not well behaved (Agnew et al. 1998).

In this paper, we develop a Bayesian hierarchical formulation of the De Lury depletion model, which incorporates historical CPUE series to improve estimation of the catchability coefficient and preseason abundance. The application of the approach is illustrated using data for the Falkland Islands' *L. gahi* squid fishery.

Bayesian methodology

There are eight steps in the Bayesian hierarchical estimation of De Lury model parameters. These are summarized below and explained in detail afterwards. It is necessary to use some technical terms in the summary. These are defined in the detailed explanation. (1) Specify the De Lury model of interest and identify the key parameters to be estimated. (2) Construct a likelihood function of the catch rate data. (3) Construct a hierarchical probability model for the De Lury model parameters that incorporates historical data. (4) Identify the hyper-parameters and marginal posterior predictive distributions of interest (i.e., marginal posterior probability distributions for parameters (e.g., catchability) that could be used as priors for a new analysis and that summarize all that is known about the parameter). (5) Apply a method to integrate the joint posterior pdf of the hierarchical De Lury model parameters to produce marginal posterior pdfs of the parameters of interest. (6) Simulation test the estimation procedure using fixed known parameter values to simulate data series and then evaluate the precision and bias in hierarchical model parameters. Consider readjusting hierarchical model parameterization if poor estimation performance is detected. (7) Compute and evaluate convergence diagnostics for the Bayesian integration method used. (8) Evaluate the characteristics of the marginal and joint posterior probability distributions obtained from the hierarchical modeling of the De Lury model, e.g., the interannual variation in key model parameters and the precision in the predictive distribution. Each of these steps is outlined in further detail below.

(1) Specifying the De Lury model of interest and identifying the parameters to be estimated

We apply the De Lury model formulation in Rosenberg et al. (1990) that is applied in the stock assessment of Falkland Islands' squid stocks. The predicted CPUE in a given week w for year y is obtained by

$$(1)\quad \widehat{\mathrm{CPUE}}_{y,w} = q_y N_{0,y} \exp[-(w+0.5)M] - q_y \sum_{k=0}^{w-1} C_{y,k} \exp[-(w-k)M]$$

where q_y is the catchability coefficient, $N_{0,y}$ is the abundance at the beginning of the season in week 0, M is the weekly rate of natural mortality, and $C_{y,k}$ is the catch in numbers taken in week k. The estimable parameters are $N_{0,y}$ and q_y. M is typically not known with certainty; however, for the sake of simplifying the illustration, M will be fixed at 0.06, as is typically done for the Falklands' *L. gahi* (Agnew et al. 1998). This need not be the case and an informative prior pdf of M could be applied in future applications to take into account uncertainty in this parameter.

(2) Constructing a likelihood function of the catch rate data

It is assumed that the deviations between predicted and observed catch rates are lognormally distributed (Rosenberg et al. 1990). The likelihood function of the CPUE data in year y is as follows:

$$(2)\quad L(\underline{\mathrm{CPUE}}_y | q_y, N_{0,y}, \tau_L) = \prod_{w=1}^{nwk(y)} \frac{1}{\mathrm{CPUE}_{y,w}\sqrt{2\pi\tau_L^{-1}}} \exp\{-0.5\tau_L[\ln(\mathrm{CPUE}_{y,w}) - \ln(\widehat{\mathrm{CPUE}}_{y,w})]^2\}$$

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where $\underline{\text{CPUE}}_y$ represents a vector containing several weeks of CPUE data for year y, τ_L is the precision ($1/\sigma^2$ and σ is the standard deviation (SD)) in the natural logarithm of the deviations between observed and predicted CPUE, $nwk(y)$ is the number of weeks in year y, and $\text{CPUE}_{y,w}$ is the observed catch rate in week w. τ_L is also an estimated parameter and is assumed to be constant across years. In the example below, we had tried to model τ_L using a hierarchical structure in which a value for $\tau_{L,y}$ for each year was estimated as well as the mean and variance in $\tau_{L,y}$ across years. However, because of the relatively limited time series (e.g., <20 weeks) and hence insufficient information in the data about $\tau_{L,y}$ for several of the years (Table 1), this hierarchical model was overparameterized. For example, the posterior correlations among the parameters q_y, $N_{0,y}$, and $\tau_{L,y}$ became very high and Monte Carlo integration became intractable. In contrast, the high posterior correlations vanished and it became feasible to reliably estimate the parameters q_y and $N_{0,y}$ when a single value for τ_L was estimated across all years.

(3) Constructing a hierarchical probability model for the De Lury model parameters from historical data

Here we develop a Bayesian hierarchical probability model for q_y and $N_{0,y}$ (Gelman et al. 1995; Rivot and Prévost 2002). This assumes that there is a population of q_y and $N_{0,y}$ across years and that a value in any one year comes from some particular parametric pdf. We illustrate the hierarchical approach using normal and lognormal density functions and keep the example simple for illustrative purposes. A lognormal probability density function is applied to model the variation in values for q_y over years with a median q_y of μ_q and precision in the natural logarithm of q_y of τ_q. $N_{0,y}$ are lognormally distributed over years with a median $N_{0,y}$ of μ_{N_0} and precision of τ_{N_0}. It is also assumed that q_y and $N_{0,y}$ are uncorrelated. Thus,

$$(3) \qquad \begin{aligned} q_y|\mu_q,\tau_q &\sim \log\text{Norm}(\mu_q,\tau_q) \\ N_{0,y}|\mu_{N_0},\tau_{N_0} &\sim \log\text{Norm}(\mu_{N_0},\tau_{N_0}) \end{aligned}$$

In BHM, the parameters describing cross-population distributions, such as the parameters μ_q, τ_q, μ_{N_0}, and τ_{N_0}, are typically called hyper-parameters (Gelman et al. 1995). In empirical Bayes' methods, which preceded BHM, hyper-parameters were estimated from data for related populations but then treated as fixed and known in subsequent estimation. In contrast, one of the goals of BHM is to estimate posterior pdfs of the hyper-parameters from historical data, and BHM requires that prior pdfs, i.e., hyper-priors, are specified for the hyper-parameters. Thus, we need to specify hyper-priors for μ_q, τ_q, μ_{N_0}, and τ_{N_0}. These hyper-priors are denoted by $P(\mu_q)$, $P(\tau_q)$, $P(\mu_{N_0})$, and $P(\tau_{N_0})$, respectively. Another estimated parameter is τ_L in the lognormal likelihood function of the data (eq. 2), and a prior pdf was assigned to τ_L, denoted as $P(\tau_L)$.

The set of parameters to be estimated in hierarchical analysis is then $\underline{q}_y$, $\underline{N}_{0,y}$, μ_q, τ_q, μ_{N_0}, τ_{N_0}, and τ_L, where the underlined parameters indicate the vector over the years of historical data. The joint posterior density for this vector of parameters, i.e., the hierarchical probability model for the De Lury model parameters, is given by

$$(4) \qquad P(\underline{q}_y,\underline{N}_{0,y},\mu_q,\tau_q,\mu_{N_0},\tau_{N_0},\tau_L|\underline{\text{CPUE}}) = P(\mu_q)P(\tau_q)P(\mu_{N_0})P(\tau_{N_0})P(\tau_L)\prod_{y=1}^{nyr}\left(P(q_y|\mu_q,\tau_q)P(N_{0,y}|\mu_{N_0},\tau_{N_0})\prod_{w=1}^{nwk(y)}P(\text{CPUE}_{y,w}|q_y,N_{0,y},\tau_L)\right)$$

where nyr is the number of years of historical data, and $P(\text{CPUE}_{y,w}|q_y,N_{0,y},\tau_L)$ is the likelihood function of the CPUE data in year y and week w. In practice, it is common to assign relatively uninformative priors to the hyper-parameters (Gelman et al. 1995). The prior and hyper-prior pdfs applied in the hierarchical model (eq. 4) were assumed to be normally distributed with a very large prior coefficient of variation (CV) (CV = (prior SD)/(prior mean) = 100) (Table 2). The posterior pdfs where thus determined almost entirely by the data. The prior and posterior CVs provide standardized measures of spread in the pdfs and enable comparisons of uncertainty between quantities with different units or different means. The prior mean for the initial abundances was chosen to be a few orders of magnitude as large as the observed weekly catches, and the prior mean value for q_y was chosen to be a few orders of magnitude less than the observed CPUE divided by the mean observed weekly catch.

(4) Identify the hyper-parameters and posterior predictive distributions of interest

As noted above, for assessments in new years, the use of an informative prior pdf of q_y and $N_{0,y}$ based on the analysis of historical data could help to improve precision in these values particularly in the first few weeks of the fishing season. For the purpose of constructing a prior pdf of these parameters for use in future assessments, this work focuses on the development of a hierarchical probability model for $N_{0,y}$ and q_y, so only the estimates for quantities associated with these parameters will be evaluated. For example, the posterior CV in the range of historical estimates for these parameters will be computed, since this will show how well historical estimates can help to inform us about the potential range of future estimates. Posterior distributions for μ_q, τ_q, μ_{N_0}, and τ_{N_0} will also inform us about the central tendency and spread of historical estimates for q_y and $N_{0,y}$.

One output of BHM is a predictive marginal posterior (PMP) pdf of each key parameter. The PMP pdf summarizes what can be inferred about the value of a parameter for some unsampled population given the estimated mean and variance in the parameter across sampled populations and the uncertainty in the mean and variance. PMP pdfs can serve as prior pdfs of key parameters in sequential Bayesian data analysis (McAllister and Kirkwood 1999; Michielsens 2003). PMP pdfs were obtained for q_y and $N_{0,y}$ in the De Lury stock assessment model. The PMP pdf for q_y was obtained first by integrating the joint pdf with respect to all parameters except μ_q and τ_q:

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Table 1. The total catch (in tonnes (t)) and catch per unit effort (CPUE) (in tonnes per hour towed, for medium-sized Spanish- and Falkland-registered trawlers) data for Falkland Islands' *Loligo gahi* from 1987 until 2000.

Year	Week	CPUE ($t \cdot h^{-1}$)	Catch (t)	Mass (g)[a]	Year	Week	CPUE ($t \cdot h^{-1}$)	Catch (t)	Mass (g)[a]
1987	11	5.43	8755	43	1993	15	2.76	3636	53
1987	12	4.52	7581	44	1993	16	2.16	2750	55
1987	13	4.20	4476	43	1993	17	1.60	2546	56
1987	14	1.91	3116	46	1993	18	1.46	1664	58
1987	15	1.48	3365	47	1994	7	2.30	3001	32
1987	16	1.57	3261	49	1994	8	2.34	2829	32
1987	17	0.73	1291	57	1994	9	1.42	2291	32
1987	18	0.56	956	58	1994	10	1.36	2010	32
1988	6	2.90	3914	37	1994	11	1.19	1536	32
1988	7	2.25	4965	38	1994	12	0.99	1417	32
1988	8	1.63	4382	40	1994	13	1.49	2038	32
1988	9	1.68	4686	41	1995	11	14.98	4243	53
1988	10	1.65	3872	42	1995	12	9.72	4885	54
1988	11	1.45	3771	43	1995	13	6.91	3579	54
1988	12	1.29	3454	44	1995	14	6.06	4663	55
1988	13	0.69	1548	43	1995	15	5.19	4281	56
1988	14	1.07	2034	46	1995	16	4.80	3197	57
1988	15	0.72	1503	47	1995	17	4.47	3620	57
1988	16	0.68	1684	49	1995	18	3.59	2983	58
1988	17	0.65	1358	57	1995	19	3.23	2796	59
1989	10	10.60	6540	42	1996	7	1.59	2041	43
1989	11	8.54	7992	43	1996	8	2.16	2362	46
1989	12	4.46	7856	44	1996	9	1.49	1860	48
1989	13	4.55	7412	43	1996	10	0.34	453	50
1989	14	4.24	7170	46	1996	11	0.32	378	53
1989	15	4.21	7020	47	1996	12	0.48	690	55
1989	16	3.15	6743	49	1997	10	1.40	1306	43
1989	17	2.91	5277	57	1997	11	1.33	1416	44
1989	18	2.31	4773	55	1997	12	1.10	1160	44
1989	19	1.95	3076	57	1997	13	0.87	837	44
1989	20	1.44	4010	59	1997	14	0.73	662	45
1990	7	7.75	7472	38	1997	15	0.76	756	45
1990	8	7.31	6708	40	1998	9	3.25	3271	43
1990	9	5.00	5173	41	1998	10	2.73	2566	44
1990	10	3.01	4738	42	1998	11	2.31	2812	46
1990	11	5.85	6914	43	1998	12	2.94	2987	47
1990	12	4.04	4774	44	1998	13	3.02	2411	48
1990	13	3.55	3113	43	1998	14	1.20	1202	50
1990	14	2.20	3818	46	1999	6	2.01	2569	39
1990	15	2.50	4515	47	1999	7	1.20	1576	40
1991	6	6.47	3686	37	1999	8	1.09	1518	41
1991	7	6.35	3142	38	1999	9	1.15	1561	42
1991	8	3.42	2577	40	1999	10	1.30	1618	43
1991	9	4.18	2808	41	1999	11	1.01	1214	44
1991	10	2.73	2608	42	1999	12	0.79	1062	45
1991	11	2.88	2243	43	1999	13	0.96	1091	45
1991	12	1.93	1522	44	1999	14	0.76	1186	46
1992	8	5.14	4055	30	1999	15	0.55	892	47
1992	9	2.75	3166	31	1999	16	0.54	934	48
1992	10	4.89	4583	31	2000	5	1.76	1883	27
1992	11	3.05	3734	32	2000	6	1.86	2953	29
1992	12	1.44	1264	33	2000	7	1.70	3102	30
1992	13	1.58	1693	33	2000	9	1.25	1921	34
1992	14	2.20	2170	34	2000	11	1.80	2465	37
1992	15	1.97	1894	34	2000	12	1.62	1909	38
1992	16	2.15	1778	35	2000	13	1.39	1299	40
1993	14	2.54	2799	51	2000	14	1.41	1658	42

[a]Smoothed mean mass of squid. Mean mass data for 1987–1991 are illustrative.

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$$(5)\qquad P(\mu_q,\tau_q|\underline{\mathrm{CPUE}}) = \int\int\int\int\int P(\mu_q)P(\tau_q)P(\mu_{N_0})P(\tau_{N_0})P(\sigma_L)\prod_{y=1}^{nyr}\left[P(q_y|\mu_q,\tau_q)P(N_{0,y}|\mu_{N_0},\tau_{N_0})\prod_{w=1}^{nwk(y)}P(\mathrm{CPUE}_{y,w}|q_y,N_{0,y},\tau_y)\right]\mathrm{d}\underline{N}_{0,y}\,\mathrm{d}\underline{q}_y\,\mathrm{d}\tau_L\,\mathrm{d}\mu_{N_0}\,\mathrm{d}\tau_{N_0}$$

The marginal predictive posterior pdf of q_y is then given by integrating the following joint pdf:

$$(6)\qquad P(q_y|\underline{\mathrm{CPUE}}) = \int\int P(q|\mu_q,\tau_q)P(\mu_q,\tau_q|\underline{\mathrm{CPUE}})\,\mathrm{d}\mu_q\,\mathrm{d}\tau_q$$

The marginal predictive posterior pdf of N_0 is obtained similarly.

$$(7)\qquad P(N_{0,y}|\underline{\mathrm{CPUE}}) = \int\int P(N_0|\mu_{n_0},\tau_{N_0})P(\mu_{N_0},\tau_{N_0}|\underline{\mathrm{CPUE}})\,\mathrm{d}\mu_{N_0}\,\mathrm{d}\tau_{N_0}$$

The conditional density functions $P(q_y|\mu_q,\tau_q)$ and $P(N_{0,y}|\mu_{N_0}\tau_{N_0})$, were taken to be lognormal. The prespecified values for the median and precision in these density functions, where required, are given in Table 2.

Additionally, the sensitivity of the central tendency and spread of these historical estimates to relatively low sample size (in terms of number of historical data sets) is also of interest and was also evaluated. This was done by repeating the hierarchical analysis, each time eliminating one of the historical data sets from the hierarchical model. The sensitivity of estimates of the properties for the meta-population to low sample size was evaluated by comparing the degree of similarity between the resulting posterior predictive distributions for N_0 and q.

(5) Integrating the hierarchical probability model to produce marginal posterior pdfs

The integrations required in eqs. 5–7 were approximated using importance sampling (Rubin 1988; McAllister et al. 1994; McAllister and Ianelli 1997). The importance functions used were multivariate log-*t* pdfs based on the posterior mode, which was obtained from nonlinear minimization and the estimate of the posterior covariance matrix at the mode using the Hessian matrix. Further details on the minimizer used, how the Hessian matrix was calculated, and the steps applied in the importance sampling procedure applied can be found in McAllister and Ianelli (1997).

The rate at which the pdf from importance sampling approaches the estimated posterior pdf and the stability of the results obtained from importance sampling are determined by the closeness of approximation of the importance function to the posterior pdf (Geweke 1989). Inefficiency and instability can occur if the importance function is considerably more diffuse than the posterior pdf and a large proportion of the sampling is from regions of parameter space with negligible posterior density. Instability will also result if the tails of the importance function taper off faster than the tails of the posterior pdf. Instability will be manifested by occasional large changes in the computed posterior expected values even after hundreds of thousands of draws from the importance function. If this occurs, sampling should be stopped, the run discarded, and the importance function adjusted (Geweke 1989; McAllister and Ianelli 1997).

Convergence of the approximated pdf to a stable result was evaluated using diagnostics developed in previous works (Geweke 1989; McAllister and Ianelli 1997). One diagnostic is the percentage of the maximum importance weight from the set of draws relative to the sum of the importance weights, where the importance weight, $W(\theta_k)$, for draw k is the product of the prior pdf and likelihood, $P(\theta_k)L(\mathrm{data}|\theta_k)$, divided by the density of the importance function, $h(\theta_k)$, all evaluated at the parameter values θ_k in draw k (i.e., $W(\theta_k) = P(\theta_k)L(\mathrm{data}|\theta_k)/h(\theta_k)$). If draws were to be obtained from the posterior pdf as the importance function, this percentage should be equal to $100\%/m$, where m is the number of importance samples. This is because $W(\theta_k)$ would be constant across draws if the importance function was the posterior pdf. Thus, the percent maximum weight should drop to a low value within several hundred thousand draws from a well-chosen importance function. In this application, importance sampling was stopped after this percentage dropped below 3% and this occurred after several hundred thousand draws. Also, the CV in the importance weights (CV(W)) was computed and compared with the CV in the product of the prior and likelihood function (CV($P{\cdot}L$)). If draws of parameter values were to be taken from the posterior pdf, the CV(W) should be 0. If the importance function is to give reliable results, then CV(W) can be expected to be not much larger than the CV($P{\cdot}L$). We have found from numerical experiments that when CV(W) is approximately five or more times larger than the CV($P{\cdot}L$), the marginal posterior estimates can be strongly biased or numerically unstable, and an importance function that is either more similar in central tendency and spread to the posterior pdf or has less sharp tails is required.

Hierarchical modeling of catch rate data from the Falklands' *L. gahi* trawl fishery

Implementation of BHM

The hierarchical model described above was applied using a subset of the Falkland Islands' *L. gahi* squid fishery weekly catch and CPUE data (Table 1). These data relate to February to April when the fishery exploits the first of the two annual and unrelated recruitment pulses (the first cohort). Weeks listed in Table 1 are sequential 7-day periods, with week 1 beginning 1 January, week 2 beginning 8 January, and so on. These are the time units commonly used in assessments of this stock. Data for years 1987–2000 were used. Total removals by week and CPUE in tonnes per hour fished by week were compiled from data provided by the industry as part of the license requirements. The trawl vessels in this fishery are assigned to a number of fleets on the basis of flag nation and vessel size, and a separate catchability coefficient is calculated for each fleet in routine assessments. For simplicity, the current study considers a single-fleet implementation, so CPUE data were used only for the most common

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Table 2. Specifications of the priors and hyper-priors used in the De Lury stock assessment of Falkland Islands' fisheries for *Loligo gahi*.

Hyper-parameters	Symbol	Value assigned
Prior median for μ_q	$\mu(\mu_q)$	0.00005
Prior CV for μ_q	$CV(\mu_q)$	100
Prior median for τ_q	$\mu(\tau_q)$	5
Prior CV for τ_q	$CV(\tau_q)$	100
Prior median for μ_{N_0}	$\mu(\mu_{N_0})$	5000 million
Prior CV for μ_{N_0}	$CV(\mu_{N_0})$	100
Prior median for τ_{N_0}	$\mu(\tau_{N_0})$	5
Prior CV for τ_{N_0}	$CV(\tau_{N_0})$	100
Prior median for τ_L	$\mu(\tau_L)$	25
Prior CV for τ_L	$CV(\tau_L)$	100

Note: CV, coefficient of variation; q_y, catchability of squid in year y; μ_q, median q_y; τ_q, precision in q_y; $N_{0,y}$, the initial abundance of squid in year y; μ_{N_0}, median $N_{0,y}$; τ_{N_0}, precision in $N_{0,y}$; τ_L, the precision of the lognormal likelihood function; and τ, precision ($1/\sigma^2$, where σ is standard deviation).

fleet (Spanish- and Falklands-registered vessels with a gross registered tonnage of 1000–2000 t). The mean mass per animal at week was calculated from data collected by shipboard observers. A smoothed growth function was obtained by fitting a linear model to these data. Total catch and CPUE data were expressed in numbers of squid by dividing by the smoothed mass at week (Table 1). Estimations were implemented only on the set of weeks in which a decline in catch rates was observed in a given fishing season (Table 1).

The hierarchical modeling procedure was first tested using simulated data to evaluate whether it could be effectively implemented and was reasonably unbiased. Because of the enormous volume of results produced, only the key findings will be reported. Reasonably unbiased results were obtained using importance sampling after reparameterizing the variances in the normal and lognormal density functions as precision τ, where $\tau = 1/\sigma^2$ and σ is the standard deviation. For example, when a diffuse lognormal or normal prior was placed on σ_q, and the true simulated value for σ_q was set at 0.2 or more, the posterior modal estimate for σ_q obtained with simulated catch and catch rate data was often 0. This is because $1/\sigma$ is part of the normal and lognormal density functions, and tiny values for σ when data are relatively uninformative can maximize the density function. With the prior placed instead on $\tau_q = 1/\sigma_q^2$ (Table 2), the joint posterior modal estimate for σ_q was in all trials larger than 0 and approximately equal to the true simulated value. To facilitate the numerical integration of eqs. 5–7, the normal and lognormal density functions were reparameterized to use τ_q instead of σ_q, and priors were placed on $\tau_q = 1/\sigma_q^2$ rather than on σ_q (Table 2). The same was also done in the normal and lognormal density functions for $N_{0,y}$. This same reparameterization of σ in the normal and lognormal density functions has been used in other Bayesian estimation problems (e.g., Meyer and Millar 1999).

The Bayesian hierarchical model for q_y and $N_{0,y}$ was fitted to the *L. gahi* fishery data from 1987 to 2000. Diffuse hyper-priors with normal pdfs were assigned to τ_L, μ_q, τ_q, μ_{N_0}, and τ_{N_0} (Table 2). Importance sampling was applied to produce a marginal predictive posterior distributions for q_y and $N_{0,y}$ based on the data from 1987 to 2000 (eqs. 5–7). The analysis was then repeated, each time leaving out one of the years from 1987 to 2000 to evaluate the sensitivity of the results to the relatively low number of years included in the analysis.

To illustrate the features of BHM, a Bayesian nonhierarchical estimation was conducted for comparison. In the nonhierarchical model, the parameters $N_{0,y}$ and q_y were independently estimated for each individual year, i.e., the parameters for the cross-year median value and variance for these parameters were not included in the nonhierarchical model. The same De Lury model and likelihood function (eqs. 1 and 2) were applied. The estimation for all years, however, was conducted in the same statistical model assuming that the precision parameter in the likelihood function was the same across years.

In Bayesian estimation, the most commonly used value for central tendency is the posterior mean value, and we have reported these in the Results section. However, to facilitate comparisons of results with those from the Bayesian nonhierarchical estimators, we report posterior modes when making these comparisons.

Results

BHM produces marginal posterior pdfs of the key parameters of each individual population included, taking into account cross-population variance in these parameters. Marginal posteriors are shown for the parameters q_y and $N_{0,y}$ for the years 1987–2000 (Fig. 1). For comparison, the diffuse prior pdfs applied for each of these quantities are also plotted. The central tendencies of the marginal posterior pdfs of $N_{0,y}$ vary considerably from year to year and are far more precise than the flat prior density. The posterior mean values for $N_{0,y}$ range from 0.3 to 4.4 billion animals (Table 3). The posterior CVs range from 0.02 to 0.36, and many of the distributions have extremely low overlap with others. This indicates that some of the data are highly informative. However, not all of the data are informative. The large range of posterior CVs for the marginal distributions for $N_{0,y}$ indicates that the information value in the data varies considerably across years. The central tendencies for q_y vary considerably less (i.e., from 0.3×10^{-4} to 1.8×10^{-4}) (Table 3). The posterior CV in the estimates of q_y varies less, from 0.12 to 0.42.

Hierarchical modeling also quantifies how a parameter varies across different members of a population. In this case, the population is the set of years in which De Lury assessments have been applied. It is assumed that a parametric distribution can be used to describe how the values of the parameter vary across members of the population. In this analysis, it is assumed that the population of true values for $N_{0,y}$ across years is normally distributed. The same is assumed about the distribution for q_y. The median and precision of these lognormal distributions were estimated for $N_{0,y}$ and q_y (i.e., μ_q, τ_q, μ_{N_0}, and τ_{N_0}, respectively). The posterior mean estimates for μ_q and μ_{N_0} were 9.3×10^{-5} and 2.1×10^9 squid, respectively (Fig. 2, Table 4). μ_q and μ_{N_0} are precisely estimated with posterior CVs of 0.16 and 0.15, re-

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Fig. 1. Marginal posterior probability density functions (pdfs) for (*a*) the initial abundance of squid in year *y* ($N_{0,y}$) and (*b*) catchability coefficient of squid in year *y* (q_y) from the Bayesian hierarchical De Lury model for years 1987–2000. The diffuse priors are shown as bold broken lines. The predictive posterior pdfs are shown are the bold lines.

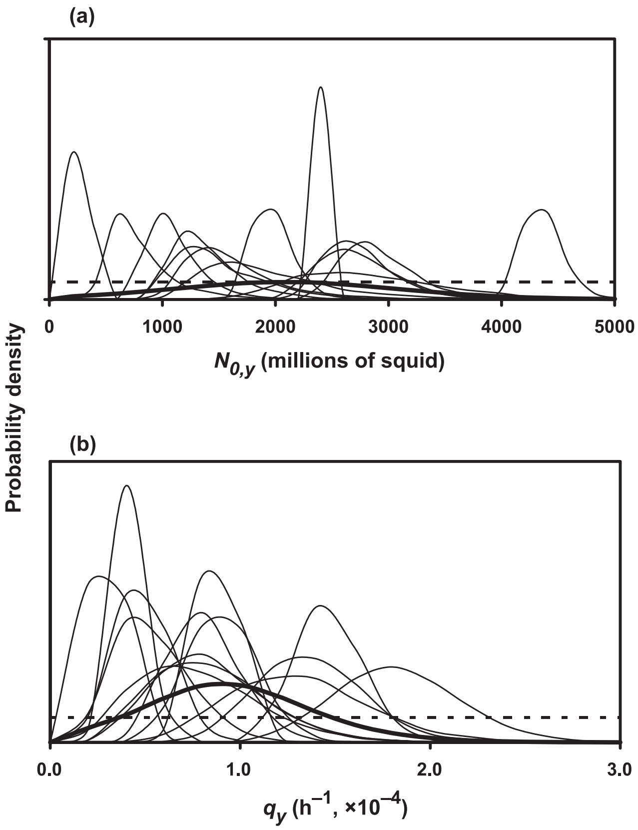

Table 3. Posterior mean estimates and posterior coefficients of variation (CVs) of hierarchical model quantities for years 1987–2000.

Year	$N_{0,y}$	CV	q_y	CV
1987	2398	0.02	1.50×10^{-4}	0.12
1988	1974	0.09	4.10×10^{-5}	0.17
1989	4360	0.04	8.80×10^{-5}	0.13
1990	2918	0.12	9.10×10^{-5}	0.20
1991	1385	0.23	1.30×10^{-4}	0.28
1992	2747	0.14	8.10×10^{-5}	0.23
1993	1502	0.28	8.40×10^{-5}	0.36
1994	1932	0.30	5.40×10^{-5}	0.37
1995	2743	0.12	1.40×10^{-4}	0.21
1996	293	0.06	1.80×10^{-4}	0.18
1997	796	0.36	7.80×10^{-5}	0.42
1998	1627	0.27	8.50×10^{-5}	0.35
1999	1100	0.22	4.90×10^{-5}	0.29
2000	2844	0.24	3.10×10^{-5}	0.31

Note: Abundance is in millions of animals. $N_{0,y}$ is the initial abundance of squid in year *y*, and q_y is the catchability of squid in year *y*.

spectively. The estimates of variance in values across years are also of interest because they indicate the spread of the distribution of historical estimates and the potential distribution of future values. The posterior mean values for the CVs in N_0 and q_y are intermediate and are 0.49 and 0.47, respectively. Whereas the prior CVs for τ for N_0 and q_y and τ_L were very large, i.e., 100, the posterior CVs for the corresponding quantities CV($N_{0,y}$), CV(q_y), and σ_L were much smaller, i.e., 0.22, 0.24, and 0.08, respectively (Table 4). This indicates that the data, not the priors, largely determined the posterior pdfs for these quantities.

The Bayesian nonhierarchical De Lury model provided plausible parameter estimates for all years except for 1998 and 2000. In these years, the catch rate series did not show a distinct decline and were quite variable over the time series (Table 1). The posterior modal estimates for $N_{0,y}$ (analogous to maximum likelihood estimates currently applied in the stock assessment) were implausibly large (millions of times larger than other values), and the estimates for *q* were millions of times less than the estimates in the other years. The individual posterior modal estimates of q_y and $N_{0,y}$ for the years that worked under nonhierarchical modeling were more variable than the values obtained from hierarchical modeling, but overall the two groups of estimates were largely similar (Table 5). For q_y, the most extreme estimates were pulled toward the central tendency in the hierarchical modeling. However, this was not apparent for $N_{0,y}$ because for all years where both were estimated, the estimates of $N_{0,y}$ were similar between the hierarchical and nonhierarchical methods. The cross-year CVs (excluding 1998 and 2000) in the posterior modal estimates for $N_{0,y}$ were 0.59 and 0.61 for the hierarchical and nonhierarchical methods, respectively (Table 5). The corresponding CVs in the posterior modal estimates for q_y were 0.45 and 0.61 for the hierarchical and nonhierarchical methods, respectively. The estimated posterior CVs for each annual value were similar between the two estimation approaches and similarly variable between years.

The hierarchical estimates of $N_{0,y}$ for years that did not work under nonhierarchical modeling (1998 and 2000) were within the range of estimates for other years (Table 5). However, the hierarchical estimates for *q* were slightly lower than the lowest nonhierarchical estimates obtained. This is not surprising given the very low decline in the CPUE data. The estimates of the posterior CVs for these parameters in these years were intermediate relative to the estimates in other years (Table 5). Therefore, the use of hierarchical information (e.g., among year variances in q_y and $N_{0,y}$) obtained by hierarchical modeling combined with the available catch rate and catch data for 1998 and 2000 to produce plausible estimates of q_y and $N_{0,y}$.

One of the key assumptions of hierarchical modeling is that the modeled populations are exchangeable (Gelman et al. 1995). This is reasonable if there are no reasons to suspect a priori that some populations are different from others. In this application, the main exchangeability assumption is that there is no temporal autocorrelation in the parameter values. Given experience in other fisheries, the existence of autocorrelation in recruitment and catchability could be reasonable assumptions. If $N_{0,y}$ was found to be related to some environmental variable (Agnew et al. 2000; Chang 2001), then this variable could be included as an explanatory variable, and the hierarchical application could become even more powerful at prediction (Myers et al. 2001). However,

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Fig. 2. Marginal posterior probability density functions (pdfs) for the median in the lognormal distribution of (*a*) the initial abundance of squid in year *y* ($N_{0,y}$) and (*b*) catchability coefficient of squid in year *y* (q_y) across the years 1987–2000. The broken line is the prior pdf and the solid line is the posterior pdf.

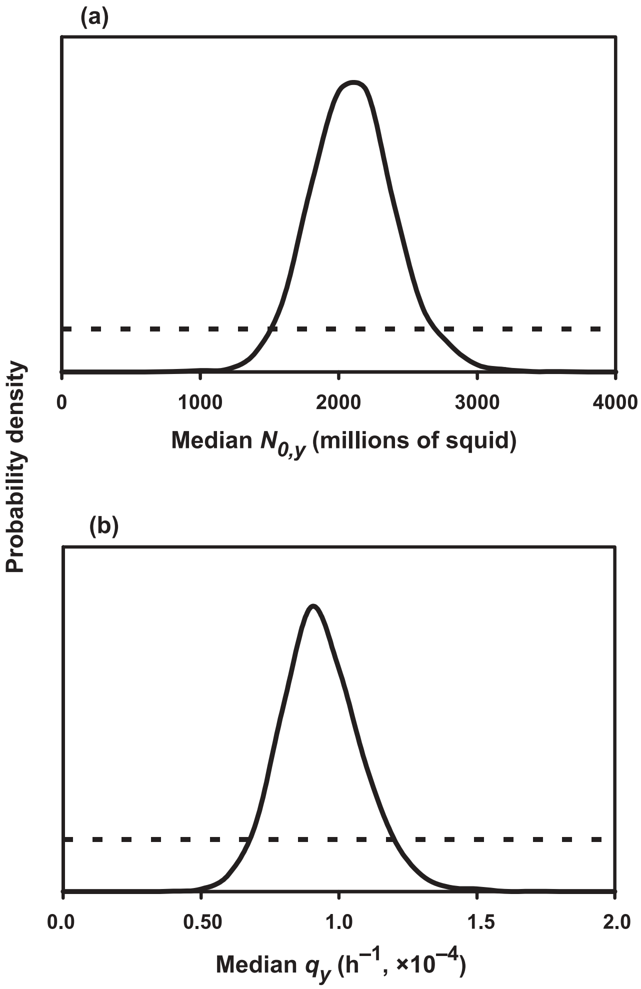

Table 4. Posterior mean estimates and posterior coefficients of variation (CVs) of the hierarchical De Lury model quantities.

Parameter	Mean	CV
Median q_y	9.40×10^{-5}	0.16
Median $N_{0,y}$	2100	0.15
CV(q_y)	0.47	0.24
CV($N_{0,y}$)	0.49	0.22
Predictive q_y	9.70×10^{-5}	0.45
Predictive $N_{0,y}$	2177	0.45
σ_L	0.22	0.08

Note: The medians q_y and $N_{0,y}$ refer to the median values for q_y and $N_{0,y}$, respectively, in the lognormal density function describing the distribution of q_y and $N_{0,y}$ across years. The CV(q_y) and CV($N_{0,y}$) refer to the CVs for q_y and $N_{0,y}$, respectively, in the lognormal density function describing the distribution of q_y and $N_{0,y}$ across years. Abundance is in millions of animals.

the hierarchical model would become more complex if the assumption of autocorrelation or an environmental covariate was adopted in the model structure. Thus, for the purpose of illustration, autocorrelation and environmental covariates were not modeled. However, autocorrelations were negligible and nonsignificant in the annual hierarchical posterior estimates of $N_{0,y}$ (e.g., for lag = 1 year, $\rho = 0.13$, *p* value = 0.60) and q_y (for lag = 1 year, $\rho = 0.06$, *p* value = 0.80) (Table 5).

It is also of interest to evaluate whether the estimates for μ_q, and μ_{N_0} are correlated. If they were, this would suggest that this correlation should be incorporated in the development of a prior pdf of q_y and $N_{0,y}$ for assessments in future years. The joint posterior distribution for μ_q and μ_{N_0} shows no appreciable correlation (corr = –0.09) (Fig. 3). This indicates that it is reasonable to assume no posterior correlation between μ_q and μ_{N_0}.

As mentioned above, the precision of parameter estimates in future assessments could be increased if historical data were used to develop informative priors, and hierarchical probability modeling does so by developing posterior predictive distributions for key parameters. Here we developed posterior predictive distributions for q_y and $N_{0,y}$, assuming that values for these are independent. Importance sampling was applied to do the required integrations in eqs. 5–7. The posterior mean of this distribution was similar to the posterior mean estimate of the μ_{N_0} at 1900 million individuals (Fig. 4). The posterior predictive distribution for $N_{0,y}$ was much wider than the posterior distribution for μ_{N_0} with a posterior CV of 0.45 (Table 4). The posterior predictive mean value for q_y was also similar to the posterior mean estimate of μ_q at 9.7×10^{-5}. The CV for the posterior predictive distribution for q_y was 0.45 (Fig. 4).

The posterior predictive distributions for key parameters can be sensitive when the number of populations for which data exist are relatively few. In this case, 14 different years of data are available. The sensitivity of the posterior predictive distributions for q_y and $N_{0,y}$ to few years being included was evaluated by redoing the analysis, each time leaving out a single year from the analysis. The resulting set of 14 predictive distributions for q_y and $N_{0,y}$ were very similar (Fig. 5). The years making the most difference were not the same for q_y and $N_{0,y}$, i.e., 1996 and 1989, respectively. The 1989 catch rate data had one of the lowest gradients, so when these data were removed, the predictive distribution for $N_{0,y}$ shifted to the left. The marginal posterior estimate of q_y for 1996 was the highest, and when 1996 was left out, the marginal predictive posterior for q_y shifted to the left.

Discussion

The development of a Bayesian hierarchical approach to the estimation of De Lury model parameters offers to provide more reliable estimates of population abundance than nonhierarchical methods. This is demonstrated, for instance, by the successful versus unsuccessful results from 1998 and 2000 with the hierarchical and nonhierarchical models, respectively. We have previously encountered this problem with the 1998 and 2000 populations, which resisted attempts to apply either simple or multifleet De Lury likelihood models (Rosenberg et al. 1990). In the past, we have developed a

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Table 5. Posterior modal estimates and posterior coefficients of variation (CVs) for catchability coefficient of squid in year y (q_y) and the initial abundance of squid in year y ($N_{0,y}$) from hierarchical and nonhierarchical De Lury models.

	Hierarchical		Nonhierarchical		Hierarchical		Nonhierarchical	
Year	$N_{0,y}$	CV	$N_{0,y}$	CV	q_y	CV	q_y	CV
1987	2385	0.02	2371	0.01	0.00015	0.11	0.00016	0.11
1988	1932	0.08	1962	0.09	0.00004	0.17	0.00004	0.17
1989	4334	0.04	4417	0.04	0.00009	0.12	0.00008	0.13
1990	2829	0.11	2958	0.14	0.00009	0.20	0.00009	0.24
1991	1232	0.19	973	0.13	0.00014	0.27	0.00021	0.23
1992	2643	0.13	2873	0.20	0.00008	0.24	0.00007	0.32
1993	1290	0.24	1225	0.27	0.00009	0.36	0.00010	0.43
1994	1751	0.31	2128	0.49	0.00005	0.42	0.00004	0.62
1995	2621	0.11	2434	0.10	0.00014	0.20	0.00017	0.21
1996	286	0.06	268	0.04	0.00019	0.17	0.00025	0.15
1997	644	0.28	623	0.33	0.00009	0.41	0.00009	0.49
1998	1427	0.24	NA	NA	0.00009	0.37	NA	NA
1999	1006	0.18	1035	0.19	0.00005	0.28	0.00005	0.28
2000	2781	0.24	NA	NA	0.00003	0.31	NA	NA
CV_{est}	0.59	0.66	0.61	0.83	0.45	0.49	0.61	0.56
Mean	1913	0.14	1939	0.17	0.00010	0.30	0.00011	0.28

Note: The posterior CVs were obtained from the Hessian matrix evaluated at the mode of the posterior. NA, not obtainable; CV, the posterior CV in the estimate; CV_{est}, the CV in the set of point estimates in the column above (standard deviation of estimates/mean of estimates). Abundance is in millions of animals.

Fig. 3. Contour plot of the joint posterior estimate of the median values for the initial abundance of squid in year y ($N_{0,y}$) and catchability coefficient of squid in year y (q_y). Darker shades indicate higher joint posterior probability density.

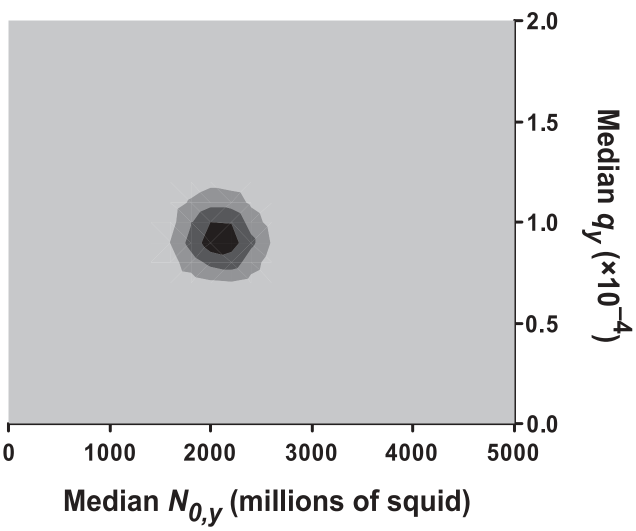
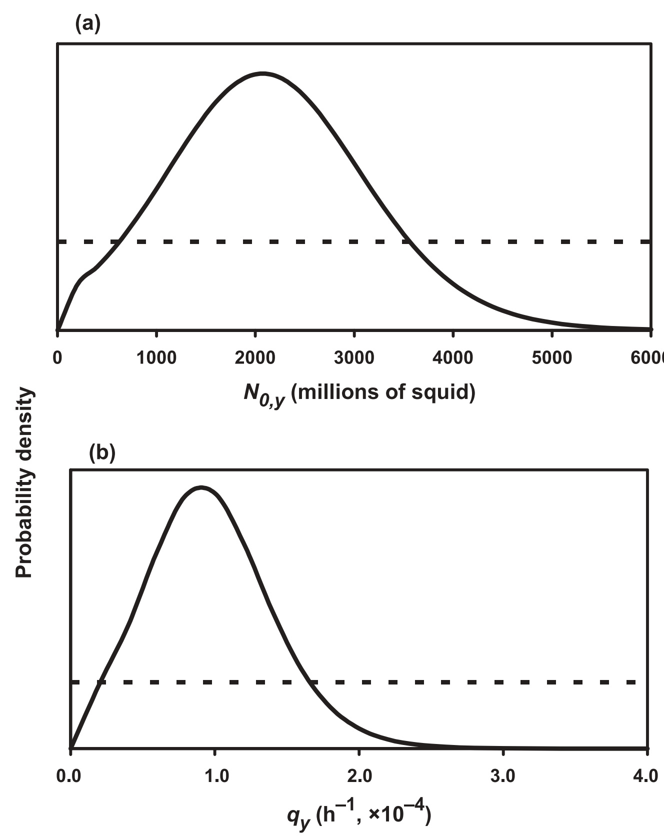


Fig. 4. Plot of the marginal posterior predictive probability density functions (pdfs) for (*a*) the initial abundance of squid in year y ($N_{0,y}$) and (*b*) catchability coefficient of squid in year y (q_y). The broken line is the prior pdf and the solid line is the posterior pdf.



pragmatic response to the problem of nonconvergence of a multifleet De Lury model (Agnew et al. 1998), which applies prior knowledge of the catchabilities of each fleet to CPUE and derives from this an estimate of stock size. This pragmatic solution is similar to the method used in the Bayesian hierarchical approach, in that the latter uses prior information about the properties of catchabilities across all years. The Bayesian hierarchical approach developed in the present paper is a formalized and much more robust realization of that philosophy and appears to perform well.

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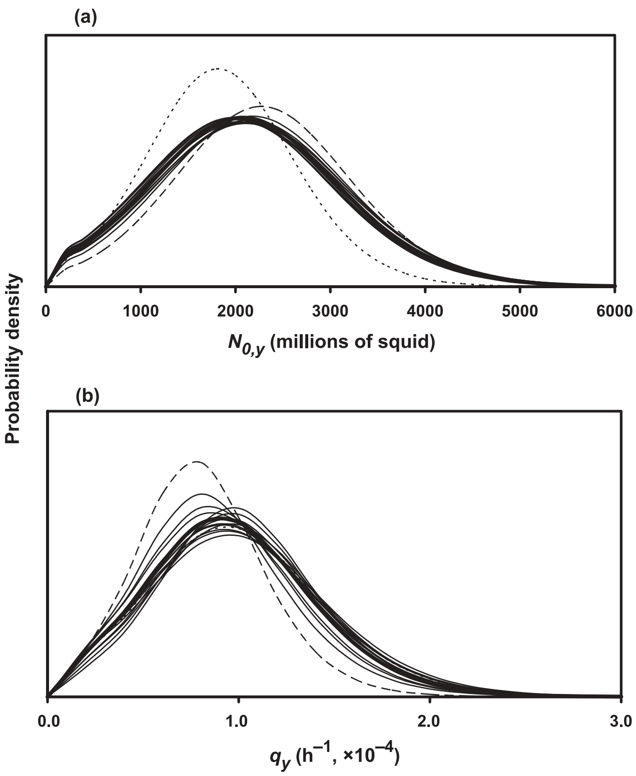
Fig. 5. Plot of the marginal posterior predictive probability density functions (pdfs) for (*a*) the initial abundance of squid in year *y* ($N_{0,y}$) and (*b*) catchability coefficient of squid in year *y* (q_y) with a missing year of data. For $N_{0,y}$, the most influential year is 1989. The dotted line illustrates the result when data for 1989 are left out. For q_y, the most influential year is 1996. The broken line illustrates the result when data for 1996 are left out. The bold line illustrates the result when all data are included.



The use of BHM (Gelman et al. 1995; Rivot and Prévost 2002) to construct prior distributions for q_y and $N_{0,y}$ from historical data is one of the main strengths of the approach. BHM is particularly useful when the De Lury model is applied for in-season management controls. For example, BHM is helpful near the beginning of the fishing season when only a few weeks of catch rate data are available and estimates from the conventional estimation method for the De Lury model can be highly imprecise. BHM is useful for years when the catch rate data are not well behaved. For example, they show a decline for only a very short part of the weekly time series, relatively little decline, or excessive decline.

There are at least two options for applying a BHM in stock assessment. A stock assessment could employ the hierarchical model of all previous years and the current year's data. If the hierarchical model were very quick and simple to operate, this would be a viable and straightforward option. However, because the Bayesian Monte Carlo integration in the analysis of historical data can be time consuming owing to slow convergence, an in-season De Lury stock assessment application that included the data for the current year and all historical years would be cumbersome. The full historical data analysis would have to be rerun with each new addition of weekly data. It would instead be more efficient to first analyze all of the historical data to produce a predictive posterior pdf of the De Lury stock assessment model parameters given the historical data. In Bayesian analysis, the posterior pdf contains all of the information in the data about the model parameters. This posterior pdf could then serve as a prior pdf for the current year's assessment. It would be much easier with each new week's data to rerun the stock assessment model that incorporated a prior pdf based on the hierarchical analysis of the historical data and was fitted only to the current year's data. Thereby, the historical data analysis is conducted only once at the beginning of the season rather than repeatedly with the reapplication of the De Lury model in each new week of the season. This type of estimation in which the posterior from one data analysis becomes the prior for a new data analysis is one form of sequential Bayesian analysis (Berger 1985; McAllister and Kirkwood 1999; Michielsens 2003).

Markov Chain Monte Carlo (MCMC) methods have recently been taken to be synonymous with Bayesian estimation (e.g., Meyer and Millar 1999; Harley and Myers 2001; Rivot and Prévost 2002). However, other reliable non-MCMC methods of Bayesian integration also exist (Rubin 1988; Walters and Ludwig 1994; Rivot et al. 2001), and the choice of method is a matter of personal preference. Thus, in this paper, importance sampling (Rubin 1987, 1988; McAllister and Ianelli 1997) was used rather than a MCMC method. Whichever integration method is applied, it is important to apply diagnostics to evaluate whether convergence has been achieved. If possible, it is even better to apply both approaches to check for consistency in results after diagnostics of each suggest convergence (McAllister and Ianelli 1997).

There are a few hazards when applying the suggested methodology to develop prior pdfs in future stock assessments. The first occurs if the range of hierarchical estimates from the historical years does not reflect that in future years. This is conceivable if the properties of the fishery changed systematically. For example, the efficiency of the fleet often increases systematically over time. Priors that assume constant catchability could then bias the parameter estimates. This could be overcome by conducting Bayesian De Lury assessments with both informative and uninformative priors and (or) by weighting the posterior predictive distributions towards the most recent years. The second occurs if the catch rate data do not follow a distinctive declining pattern over a series of several weeks into the fishing season. This may not necessarily imply that abundance is extremely high or catchability very low for that season. It could be that other processes are occurring. For example, this could reflect a lagged migration of some part of the population. In reality, all data sets subjected to De Lury assessment, either by traditional or Bayesian methods, must be selected carefully to ensure that only the declining phases of the fishery are included. The initial recruitment to the fishing grounds and spawning emigration at the end of the fishing season are excluded from the analysis. It might also be advisable to construct some alternative structural models that account for a limited set of alternative processes to those considered in the De Lury depletion model and fit these also to the data. Auxiliary data such as maturity and mean length data could be helpful for distinguishing among them. Model-checking methods such as those suggested in Chapter 6 of Gelman et al. (1995) could be applied to evaluate whether the model choice is appropriate.

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In this particular application, the catch rate data from only one of the fleets was used. Catch rate data from other fleets exist and are used in routine assessments of Falkand Islands' *L. gahi* and *I. argentinus*. Because of the information in these additional data, the next step in the hierarchical modeling would be to extend the model structure to model CPUE data from the different fleets to help to estimate posterior predictive distributions for q_y for each fleet and $N_{0,y}$.

The posterior predictive distributions obtained for q_y and $N_{0,y}$ appear to be sufficiently precise but not too precise for use as prior pdfs in future stock assessments (McAllister and Kirkwood 1998). For example, it has been found that informative prior pdfs of key parameters in stock assessment, e.g., constants of proportionality for relative abundance indices, should have prior CVs of no more than about 0.7 to be useful as informative priors (McAllister et al. 1994; McAllister and Ianelli 1997; McAllister and Kirchner 2002). However, prior CVs should be no less than about 0.5 to avoid potential bias in posterior estimates because of being "precisely wrong" in the prior distributions used (McAllister and Kirkwood 1998). With this in mind, it would also be of interest to evaluate how the prior estimates of $N_{0,y}$ and q_y are updated as new catch and CPUE data are gathered in each week. In particular, it would be of interest to identify whether priors for both $N_{0,y}$ and q_y or only one or the other of these parameters should be incorporated in new estimations, taking into account the potential effects of bias and precision of these alternative estimation approaches.

As more years go by, the number of years in the hierarchical probability model for De Lury model parameters could be increased to improve estimates of the hierarchical model's parameters, but weighted towards the most recent years. In sequential Bayesian estimation, it is common for priors to narrow as more data are incorporated (McAllister and Kirkwood 1999). In the present situation, it is unlikely that the posterior predictive distributions will narrow considerably. This is because one of the key parameters in determining the predictive distributions is the cross-year variance in q_y and $N_{0,y}$, and it is expected that the CVs of the resulting posterior predictive distributions for these parameters should not decrease progressively and should instead stabilize at values reflecting the actual long-run interannual variances in these parameters. However, as more years of data become available, the presence of autocorrelation should be tested for and incorporated if found to be substantial.

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References

Adkison, M.D., and Su, Z. 2001. A comparison of salmon escapement estimates using a hierarchical Bayesian approach versus separate maximum likelihood estimation of each year's return. Can. J. Fish. Aquat. Sci. **58**: 1663–1671.

Agnew, D.J., Baranowski, R., Beddington, J.R., des Clers, S., and Nolan, C.P. 1998. Approaches to assessing stocks of *Loligo gahi* around the Falkland Islands. Fish. Res. **35**: 155–169.

Agnew, D.J., Hill, S., and Beddington, J.R. 2000. Predicting the recruitment strength of an annual squid stock: *Loligo gahi* around the Falkland Islands. Can. J. Fish. Aquat. Sci. **57**: 2479–2487.

Basson, M., Beddington, J.R., Crombie, J.A., Holden, S.J., Purchase, L.V., and Tingley, G.A. 1996. Assessment and management techniques for migratory squid stocks: the *Illex argentinus* fishery in the Southwest Atlantic as an example. Fish. Res. **28**: 3–27.

Berger, J.O. 1985. Statistical decision theory and Bayesian analysis. 2nd ed. Springer-Verlag New York Inc., New York.

Chang, C. 2001. Environmental influences on the recruitment of *Illex argentinus* in the Southwest Atlantic. Ph.D. thesis, Imperial College of Science, Technology and Medicine, London.

Collie, J.S., and Walters, C.J. 1991. Adaptive management of spatially replicated groundfish populations. Can. J. Fish. Aquat. Sci. **48**: 1273–1284.

Gelman, A., Carlin, J., Stern, H., and Rubin, D. 1995. Bayesian data analysis. Chapman & Hall Ltd., London.

Geweke, J.F. 1989. Bayesian inference in econometric models using Monte Carlo integration. Econometrica, **57**: 1317–1340.

Harley, S.J., and Myers, R.A. 2001. Hierarchical Bayesian models of length-specific catchability of research trawl surveys. Can. J. Fish. Aquat. Sci. **58**: 1569–1584.

Hilborn, R., and Walters, C.J. 1992. Quantitative fisheries stock assessment: choice, dynamics and uncertainty. Chapman & Hall Inc., New York.

McAllister, M.K., and Ianelli, J.N. 1997. Bayesian stock assessment using catch-age data and the sampling – importance resampling algorithm. Can. J. Fish. Aquat. Sci. **54**: 284–300.

McAllister, M.K., and Kirchner, C.H. 2002. Accounting for structural uncertainty to facilitate precautionary fishery management: illustration with Namibian orange roughy. Bull. Mar. Sci. **70**(2): 499–540.

McAllister, M.K., and Kirkwood, G.P. 1998. Using Bayesian decision analysis to help achieve a precautionary approach for managing developing fisheries. Can. J. Fish. Aquat. Sci. **55**: 2642–2661.

McAllister, M.K., and Kirkwood, G.P. 1999. Applying multivariate conjugate priors in fishery-management system evaluation: how much quicker is it and how does it bias the ranking of management options? ICES J. Mar. Sci. **56**: 884–889.

McAllister, M.K., Pikitch, E.K., Punt, A.E., and Hilborn, R. 1994. A Bayesian approach to stock assessment and harvest decisions using the sampling/importance resampling algorithm. Can. J. Fish. Aquat. Sci. **51**: 2673–2687.

Meyer, R., and Millar, R.B. 1999. BUGS in Bayesian stock assessments. Can. J. Fish. Aquat. Sci. **56**: 1078–1086.

Michielsens, C.G.J. 2003. Bayesian decision theory for fisheries management of migratory species with multiple life histories. Imperial College, London.

Myers, R.A., MacKenzie, B.R., Bowen, K., and Barrowman, N.J. 2001. What is the carrying capacity of fish in the ocean? A meta-analysis of population dynamics of North Atlantic cod. Can. J. Fish. Aquat. Sci. **58**: 1464–1476.

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Rivot, E., and Prévost, E. 2002. Hierarchical Bayesian analysis of capture–mark–recapture data. Can. J. Fish. Aquat. Sci. **59**: 1768–1784.

Rivot, E., Prévost, E., and Parent, E. 2001. How robust are Bayesian posterior inferences based on a Ricker model with regards to measurement errors and prior assumptions about parameters? Can. J. Fish. Aquat. Sci. **58**: 2284–2297.

Rosenberg, A.A., Kirkwood, G.P., Crombie, J.A., and Beddington, J.R. 1990. The assessment of stocks of annual squid species. Fish. Res. **8**: 335–350.

Rubin, D.B. 1987. Comment on 'The calculation of posterior distributions by data augmentation'. J. Am. Stat. Assoc. **82**: 543–546.

Rubin, D.B. 1988. Using the SIR algorithm to simulate posterior distributions. *In* Bayesian Statistics 3: Proceedings of the Third Valencia International Meeting, 1–5 June 1987, Valencia, Spain. *Edited by* J.M. Bernardo, M.H. Degroot, D.V. Lindley, and A.M. Smith. Clarendon Press, Oxford. pp. 385–402.

Walters, C.J., and Ludwig, D. 1994. Calculation of Bayes' posterior probability distributions for key population parameters: a simplified approach. Can. J. Fish. Aquat. Sci. **51**: 713–722.

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