## Linear models

#### Example dataset: forest trees

► Go to https://tinyurl.com/treesdata

```
trees <- read.csv("data-raw/trees.csv")
head(trees)</pre>
```

```
      site
      dbh
      height
      sex
      dead

      1
      4
      29.68
      36.1
      male
      0

      2
      5
      33.29
      42.3
      male
      0

      3
      2
      28.03
      41.9
      female
      0

      4
      5
      39.86
      46.5
      female
      0

      5
      1
      47.94
      43.9
      female
      0

      6
      1
      10.82
      26.2
      male
      0
```

#### Example dataset: forest trees

- ► Go to https://tinyurl.com/treesdata
- Download zip file and uncompress (within your project folder!)

```
trees <- read.csv("data-raw/trees.csv")
head(trees)</pre>
```

```
      site
      dbh
      height
      sex
      dead

      1
      4
      29.68
      36.1
      male
      0

      2
      5
      33.29
      42.3
      male
      0

      3
      2
      28.03
      41.9
      female
      0

      4
      5
      39.86
      46.5
      female
      0

      5
      1
      47.94
      43.9
      female
      0

      6
      1
      10.82
      26.2
      male
      0
```

#### Questions

▶ What is the relationship between DBH and height?

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- ▶ Do taller trees have bigger trunks?

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- ▶ Do taller trees have bigger trunks?
- ► Can we predict height from DBH? How well?

Always plot your data first!

## Always plot your data first!

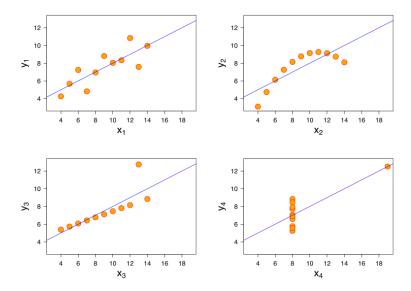
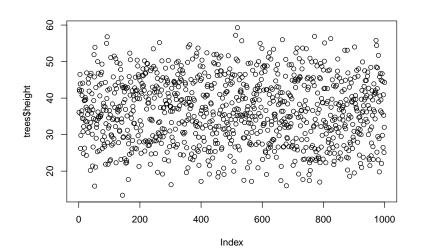


Figure 1

## Exploratory Data Analysis (EDA)

Outliers

```
plot(trees$height)
```



#### Outliers impact on regression

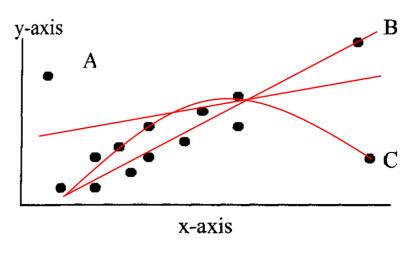
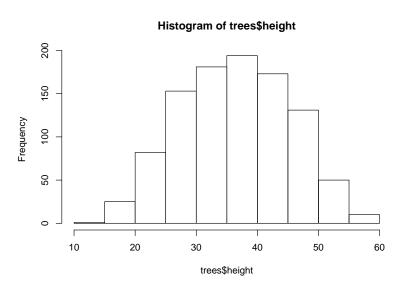


Figure 2

See http://rpsychologist.com/d3/correlation/

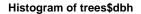
## Histogram of response variable

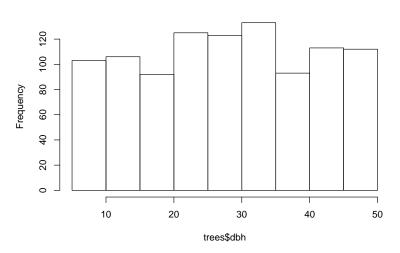
hist(trees\$height)



## Histogram of predictor variable

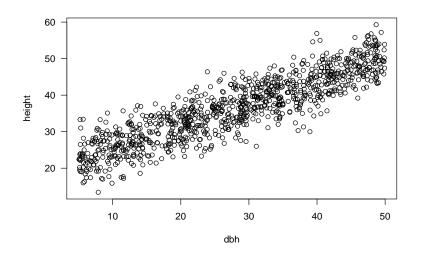
hist(trees\$dbh)



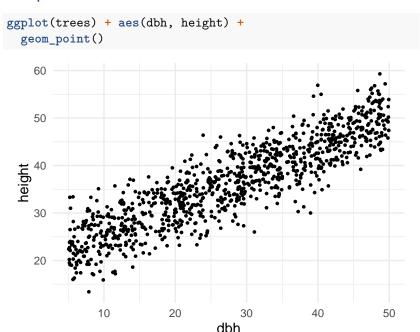


#### Scatterplot

```
plot(height ~ dbh, data = trees, las = 1)
```



#### Scatterplot





#### Now fit model

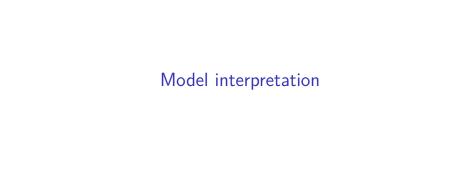
Hint: 1m

#### Now fit model

Hint: 1m

which corresponds to

$$Height_{i} = a + b \cdot DBH_{i} + \varepsilon_{i}$$
$$\varepsilon_{i} \sim N(0, \sigma^{2})$$



#### What does this mean?

```
Call:
lm(formula = height ~ dbh, data = trees)
Residuals:
    Min
            10 Median
                            30
                                   Max
-13.3270 -2.8978 0.1057 2.7924 12.9511
Coefficients:
          Estimate Std. Error t value Pr(>|t|)
(Intercept) 19.33920  0.31064  62.26  <2e-16 ***
dbh 0.61570 0.01013 60.79 <2e-16 ***
Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

Residual standard error: 4.093 on 998 degrees of freedom Multiple R-squared: 0.7874, Adjusted R-squared: 0.7871 F-statistic: 3695 on 1 and 998 DF, p-value: < 2.2e-16

#### Avoid dichotomania of statistical significance



Never conclude there is 'no difference' or 'no association' just because p > 0.05 or CI includes zero'

#### Avoid dichotomania of statistical significance



# significance

- Never conclude there is 'no difference' or 'no association' just because p > 0.05 or CI includes zero'
- Estimate and communicate effect sizes and their uncertainty

#### Avoid dichotomania of statistical significance



# It's time to talk about ditching statistical significance

- Never conclude there is 'no difference' or 'no association' just because p > 0.05 or CI includes zero'
- Estimate and communicate effect sizes and their uncertainty
- https://doi.org/10.1038/d41586-019-00857-9

#### Communicating results

We found a significant positive relationship between DBH and Height (p < 0.05) (b = 0.61, SE = 0.01).

## Presenting model results

kable(xtable::xtable(m1), digits = 2)

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	19.34	0.31	62.26	0
dbh	0.62	0.01	60.79	0

## Presenting model results

```
texreg::texreg(m1, single.row = TRUE)
```

	Model 1	
(Intercept)	19.34 (0.31)***	
dbh	0.62 (0.01)***	
R <sup>2</sup>	0.79	
Adj. R <sup>2</sup>	0.79	
Num. obs.	1000	
RMSE	4.09	
*** p < 0.001, ** p < 0.01, *p < 0.05		

Table 2: Statistical models

## Retrieving model coefficients

#### coef(m1)

(Intercept) dbh 19.3391968 0.6157036

#### Tidy up model coefficients with broom

0.787 0.787 4.09

library(broom)

<dbl>

```
tidy(m1)
# A tibble: 2 x 5
 term estimate std.error statistic p.value
 <chr>
        <dbl> <dbl>
                             <dbl>
                                   <dbl>
1 (Intercept) 19.3 0.311 62.3
2 dbh 0.616 0.0101 60.8
glance(m1)
# A tibble: 1 x 11
 r.squared adj.r.squared sigma statistic p.value df logLik AIC BIC
```

# ... with 2 more variables: deviance <dbl>. df.residual <int>

<dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> < 3695. 0

2 -2827, 5660, 5675,

#### Confidence intervals

#### confint(m1)

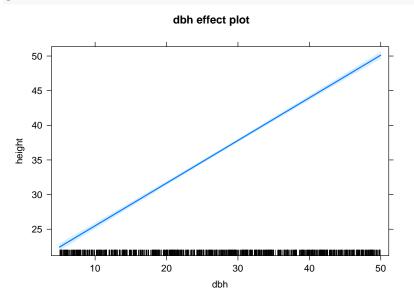
#### Using effects package

```
library(effects)
summary(allEffects(m1))
model: height ~ dbh
dbh effect
dbh
      5
              20 30
                               40
                                        50
22.41771 31.65327 37.81030 43.96734 50.12438
Lower 95 Percent Confidence Limits
dbh
              20
                    30
                             40
                                        50
21.89682 31.35487 37.55287 43.61733 49.61669
Upper 95 Percent Confidence Limits
dbh
              20
                       30
                               40
                                        50
22.93861 31.95167 38.06774 44.31735 50.63207
```



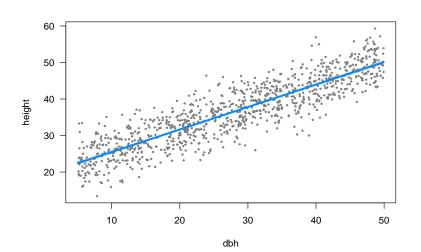
#### Plot effects





## Plot model (visreg)

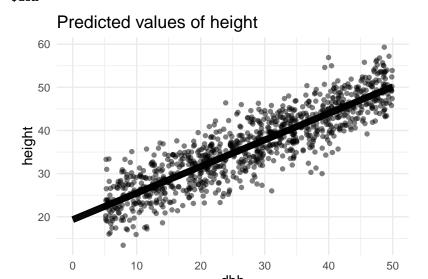
```
library(visreg)
visreg(m1)
```

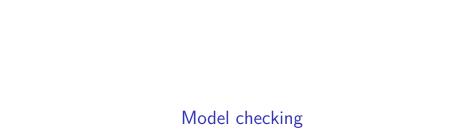


### Plot model (sjPlot - ggplot2)

```
sjPlot::plot_model(m1, type = "eff", show.data = TRUE, line.size
```

\$dbh





## Linear model assumptions

► Linearity (transformations, GAM...)

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- ► Linearity (transformations, GAM...)
- ► Residuals:

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- ► Residuals:
  - Independent

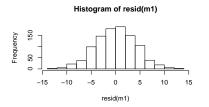
- ► Linearity (transformations, GAM...)
- ► Residuals:
  - Independent
  - Equal variance

- ► Linearity (transformations, GAM...)
- ► Residuals:
  - Independent
  - ► Equal variance
  - Normal

- ► Linearity (transformations, GAM...)
- ► Residuals:
  - Independent
  - Equal variance
  - Normal
- ► No measurement error in predictors

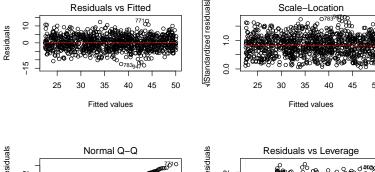
#### Are residuals normal?

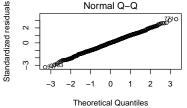
#### hist(resid(m1))

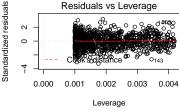


SD of residuals = 4.09 coincides with estimate of sigma.

#### Model checking: residuals



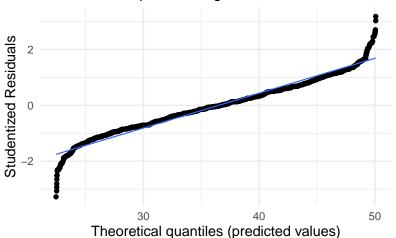




### Model checking (sjPlot)

```
plot_model(m1, type = "diag")[[1]]
```

#### Non-normality of residuals and outliers Dots should be plotted along the line



# Model checking (ggResidpanel)

0.000 0.001 0.002 0.003 0.004

Leverage

ggResidpanel::resid\_panel(m1, plots = "all") Residual Plot Histogram Q-Q Plot Sample Quantiles 0.100 10 Residuals 0.075 0.050 0.025 5 0.025 0.000 30 -10 -10 Residuals Predicted Values Theoretical Quantiles / Standardized Residuals **Boxplot** COOK's D Plot Location-Scale Plot Residuals COOK's D 0.010 5 0 -10 0.000 500 750 1000 30 Observation Predicted Values Standardized Residuals Leverage Plot Response vs Predicted Index Plot 60 50 height 3 Residuals

30

40

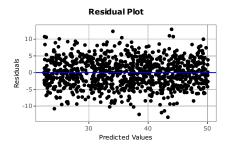
Predicted Values

50

Observation Number

# Interactive model checking (ggResidpanel)

ggResidpanel::resid\_interact(m1)



# Using model for prediction

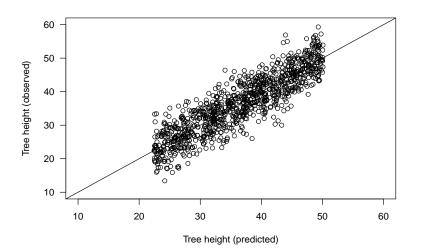
## How good is the model in predicting tree height?

fitted gives predictions for each observation

```
trees$height.pred <- fitted(m1)
head(trees)</pre>
```

	site	dbh	height	sex	dead	height.pred
1	4	29.68	36.1	male	0	37.61328
2	5	33.29	42.3	male	0	39.83597
3	2	28.03	41.9	${\tt female}$	0	36.59737
4	5	39.86	46.5	${\tt female}$	0	43.88114
5	1	47.94	43.9	${\tt female}$	0	48.85603
6	1	10.82	26.2	male	0	26.00111

## Calibration plot: Observed vs Predicted values



# Using fitted model for prediction

```
Q: Expected tree height if DBH = 39 \text{ cm}?
new.dbh <- data.frame(dbh = c(39))</pre>
predict(m1, new.dbh, se.fit = TRUE)
$fit
43.35164
$se.fit
[1] 0.1715514
$df
[1] 998
$residual.scale
[1] 4.092629
```

# Using fitted model for prediction

► Visualise data

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- ▶ Understand fitted model (summary, allEffects...)

- Visualise data
- ▶ Understand fitted model (summary, allEffects...)
- ▶ Visualise model (plot(allEffects), visreg, plot\_model...)

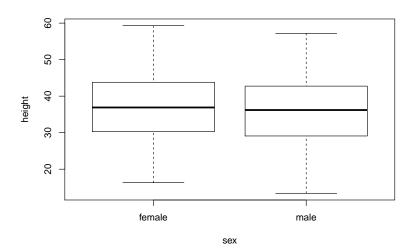
- Visualise data
- ▶ Understand fitted model (summary, allEffects...)
- ▶ Visualise model (plot(allEffects), visreg, plot\_model...)
- ► Check model (plot, resid\_panel, calibration plot...)

- Visualise data
- ▶ Understand fitted model (summary, allEffects...)
- Visualise model (plot(allEffects), visreg, plot\_model...)
- Check model (plot, resid\_panel, calibration plot...)
- Predict (fitted, predict)

# Categorical predictors (factors)

#### Q: Does tree height vary with sex?

```
plot(height ~ sex, data = trees)
```



#### Model height ∼ sex

```
m2 <- lm(height ~ sex, data = trees)
Call:
lm(formula = height ~ sex, data = trees)
Residuals:
          1Q Median 3Q
    Min
                                     Max
-22.6881 -6.7881 -0.0097 6.7261 22.3687
Coefficients:
           Estimate Std. Error t value Pr(>|t|)
(Intercept) 36.9312 0.3981 92.778 <2e-16 ***
sexmale -0.8432 0.5607 -1.504 0.133
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 8.865 on 998 degrees of freedom
```

Multiple R-squared: 0.002261, Adjusted R-squared: 0.001261 F-statistic: 2.261 on 1 and 998 DF, p-value: 0.133

## Linear model with categorical predictors

corresponds to

$$Height_i = a + b_{male} + \varepsilon_i$$

$$\varepsilon_i \sim N(0, \sigma^2)$$

#### Model height ∼ sex

```
m2 <- lm(height ~ sex, data = trees)
Call:
lm(formula = height ~ sex, data = trees)
Residuals:
          1Q Median 3Q
    Min
                                     Max
-22.6881 -6.7881 -0.0097 6.7261 22.3687
Coefficients:
           Estimate Std. Error t value Pr(>|t|)
(Intercept) 36.9312 0.3981 92.778 <2e-16 ***
sexmale -0.8432 0.5607 -1.504 0.133
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 8.865 on 998 degrees of freedom
```

Multiple R-squared: 0.002261, Adjusted R-squared: 0.001261 F-statistic: 2.261 on 1 and 998 DF, p-value: 0.133

# Presenting model results

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	36.93	0.40	92.78	0.00
sexmale	-0.84	0.56	-1.50	0.13

# Effects: Height $\sim$ sex

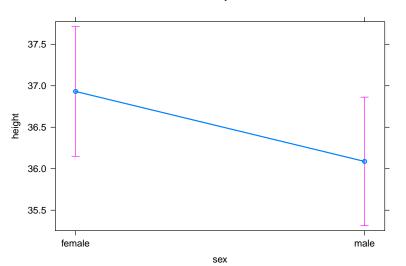
Compare Cls

```
summary(allEffects(m2))
model: height ~ sex
sex effect
sex
 female male
36.93125 36.08810
Lower 95 Percent Confidence Limits
sex
 female male
36.15012 35.31319
Upper 95 Percent Confidence Limits
sex
 female male
37,71238 36,86300
```

#### Plot

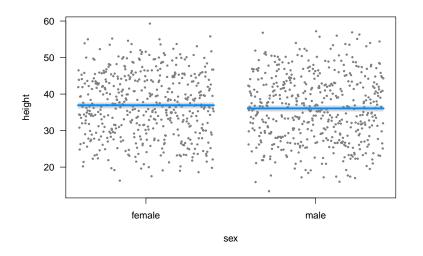
#### plot(allEffects(m2))

#### sex effect plot



# Plot (visreg)

#### visreg(m2)



# Plot model (sjPlot)

```
plot_model(m2, type = "eff")
```

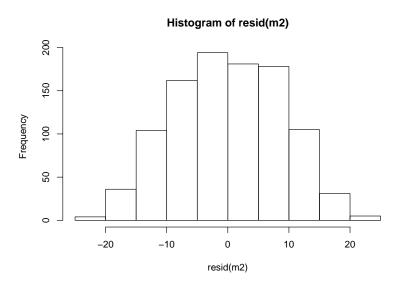
\$sex

#### Predicted values of height

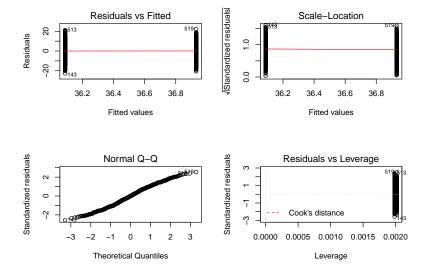


# Model checking: residuals

hist(resid(m2))

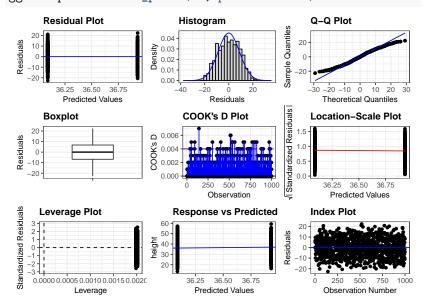


#### Model checking: residuals



# Model checking (ggResidpanel)

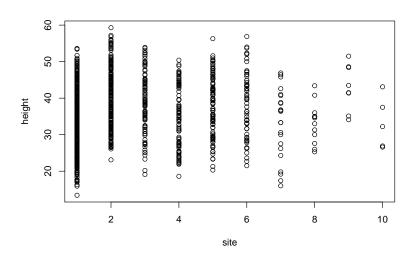
ggResidpanel::resid\_panel(m2, plots = "all")



Q: Does height differ among field sites?

#### Plot data first

```
plot(height ~ site, data = trees)
```



#### Linear model with categorical predictors

$$y_i = a + b_{site2} + c_{site3} + d_{site4} + e_{site5} + ... + \varepsilon_i$$
  
 $\varepsilon_i \sim N(0, \sigma^2)$ 

## Model Height ~ site

All right here?

```
m3 <- lm(height ~ site, data = trees)
```

#### Call:

```
lm(formula = height ~ site, data = trees)
```

#### Residuals:

```
Min 1Q Median 3Q Max -22.4498 -6.7049 0.0709 6.7537 23.0640
```

#### Coefficients:

--- Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 8.842 on 998 degrees of freedom Multiple R-squared: 0.007429, Adjusted R-squared: 0.006435 site is a factor!

trees\$site <- as.factor(trees\$site)</pre>

### Model Height ~ site

```
Call:
lm(formula = height ~ site, data = trees)
Residuals:
    Min
             1Q Median
                            30
                                   Max
-20.4416 -6.9004 0.0379 6.3051
                               19.7584
Coefficients:
          Estimate Std. Error t value Pr(>|t|)
(Intercept) 33.8416
                     0.4266 79.329 < 2e-16 ***
site2
           6.3411 0.7126 8.899 < 2e-16 ***
site3
           4.9991 0.9828 5.086 4.36e-07 ***
site4
           0.5329 0.9872 0.540 0.58949
site5
           4.3723 0.9425 4.639 3.97e-06 ***
site6 4.7601 1.1709 4.065 5.18e-05 ***
site7 -0.7416 1.8506 -0.401 0.68871
site8 -0.6832 2.4753 -0.276 0.78258
site9
          9.1709 3.0165 3.040 0.00243 **
site10
         -0.5816 3.8013 -0.153 0.87843
---
Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
Residual standard error: 8.446 on 990 degrees of freedom
Multiple R-squared: 0.1016, Adjusted R-squared: 0.09344
F-statistic: 12.44 on 9 and 990 DF, p-value: < 2.2e-16
```

# Presenting model results

kable(xtable::xtable(m3), digits = 2)

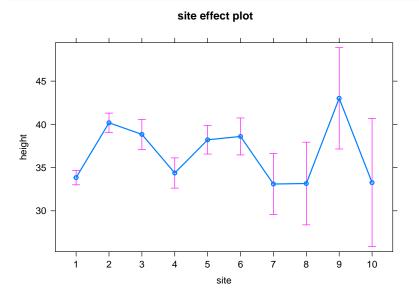
	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	33.84	0.43	79.33	0.00
site2	6.34	0.71	8.90	0.00
site3	5.00	0.98	5.09	0.00
site4	0.53	0.99	0.54	0.59
site5	4.37	0.94	4.64	0.00
site6	4.76	1.17	4.07	0.00
site7	-0.74	1.85	-0.40	0.69
site8	-0.68	2.48	-0.28	0.78
site9	9.17	3.02	3.04	0.00
site10	-0.58	3.80	-0.15	0.88
			· ·	

### Estimated tree heights for each site

```
summarv(allEffects(m3))
model: height ~ site
 site effect
site
33.84158 40.18265 38.84066 34.37444 38.21386 38.60167 33.10000 33.15833
               10
43.01250 33.26000
Lower 95 Percent Confidence Limits
site
33.00444 39.06264 37.10317 32.62733 36.56463 36.46190 29.56629 28.37367
               10
37.15251 25.84764
Upper 95 Percent Confidence Limits
site
34.67872 41.30265 40.57814 36.12156 39.86309 40.74143 36.63371 37.94299
               10
48.87249 40.67236
```

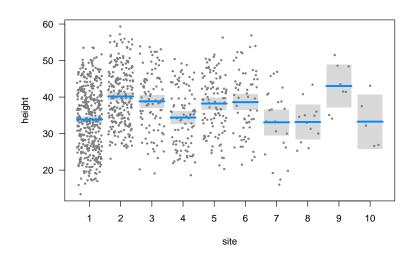
# Plot

#### plot(allEffects(m3))



# Plot (visreg)

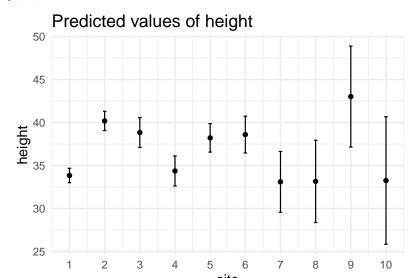
#### visreg(m3)



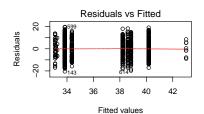
# Plot model (sjPlot)

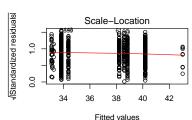
```
plot_model(m3, type = "eff")
```

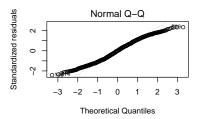
\$site

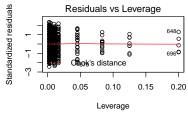


# Model checking: residuals











# Predicting tree height based on dbh and site

```
lm(height ~ site + dbh, data = trees)
```

corresponds to

$$y_i = a + b_{site2} + c_{site3} + d_{site4} + e_{site5} + ... + k \cdot DBH_i + \varepsilon_i$$
  
 $\varepsilon_i \sim N(0, \sigma^2)$ 

### Predicting tree height based on dbh and site

```
Call:
lm(formula = height ~ site + dbh, data = trees)
Residuals:
    Min
             10
                Median
                             30
                                    Max
-10.1130 -1.9885 0.0582 2.0314 11.3320
Coefficients:
           Estimate Std. Error t value Pr(>|t|)
(Intercept) 16.699037
                     0.260565 64.088 < 2e-16 ***
site2
          6.504303 0.256730 25.335 < 2e-16 ***
site3
         4.357457 0.354181 12.303 < 2e-16 ***
         1.934650 0.356102 5.433 6.98e-08 ***
site4
site5 3.637432 0.339688 10.708 < 2e-16 ***
site6 4.204511
                     0.421906 9.966 < 2e-16 ***
site7 -0.176193
                     0.666772 -0.264 0.7916
site8
       -5.312648
                     0.893603 -5.945 3.82e-09 ***
site9
         5.437049 1.087766 4.998 6.84e-07 ***
site10 2.263338 1.369986 1.652 0.0988.
           0.617075
                     0.007574 81.473 < 2e-16 ***
dbh
___
Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
Residual standard error: 3.043 on 989 degrees of freedom
Multiple R-squared: 0.8835. Adjusted R-squared: 0.8823
F-statistic: 750 on 10 and 989 DF, p-value: < 2.2e-16
```

# Presenting model results

kable(xtable::xtable(m4), digits = 2)

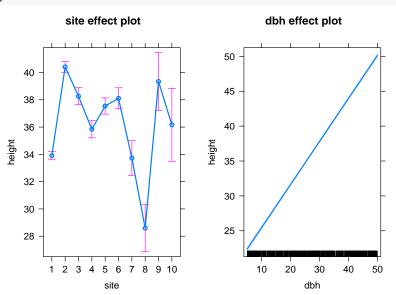
	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	16.70	0.26	64.09	0.00
site2	6.50	0.26	25.34	0.00
site3	4.36	0.35	12.30	0.00
site4	1.93	0.36	5.43	0.00
site5	3.64	0.34	10.71	0.00
site6	4.20	0.42	9.97	0.00
site7	-0.18	0.67	-0.26	0.79
site8	-5.31	0.89	-5.95	0.00
site9	5.44	1.09	5.00	0.00
site10	2.26	1.37	1.65	0.10
dbh	0.62	0.01	81.47	0.00

### Estimated tree heights for each site

```
summary(allEffects(m4))
 model: height ~ site + dbh
 site effect
site
33.90437 40.40868 38.26183 35.83902 37.54181 38.10889 33.72818 28.59173
39.34142 36.16771
Lower 95 Percent Confidence Limits
site
33,60276,40,00512,37,63569,35,20858,36,94739,37,33787,32,45495,26,86438
              10
37,22831 33,49623
Upper 95 Percent Confidence Limits
site
34.20599 40.81223 38.88798 36.46947 38.13622 38.87990 35.00141 30.31907
      9 10
41.45454 38.83919
dbh effect
dbh
              20
                       30
22.38634 31.64246 37.81321 43.98396 50.15471
```

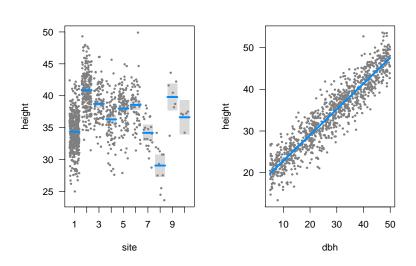
### **Plot**

#### plot(allEffects(m4))



# Plot (visreg)

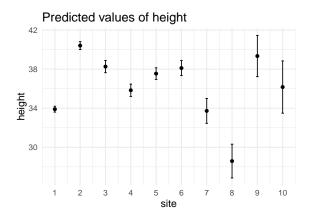
#### visreg(m4)



# Plot model (sjPlot)

```
plot_model(m4, type = "eff")
```

#### \$site

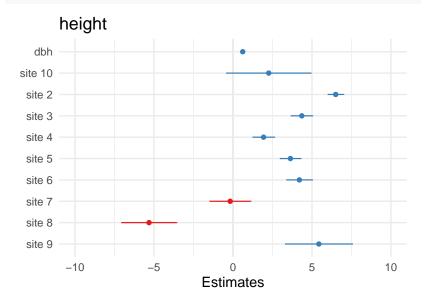


\$dbh

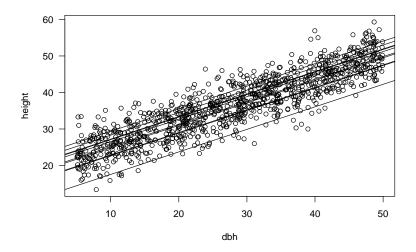
Predicted values of height

# Plot model (sjPlot)

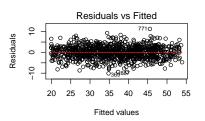
```
plot_model(m4, type = "est")
```

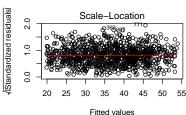


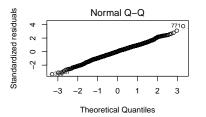
# We have fitted model w/ many intercepts and single slope

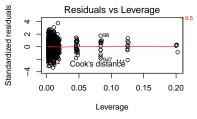


# Model checking: residuals



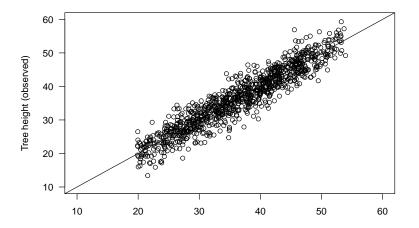






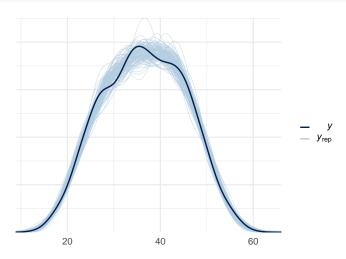
## How good is this model? Calibration plot

```
trees$height.pred <- fitted(m4)
plot(trees$height.pred, trees$height, xlab = "Tree height (predi
abline(a = 0, b = 1)</pre>
```



### Model checking with simulated data

```
library(bayesplot)
sims <- simulate(m4, nsim = 100)
ppc_dens_overlay(trees$height, yrep = t(as.matrix(sims)))</pre>
```



Q: Does allometric relationship between DBH

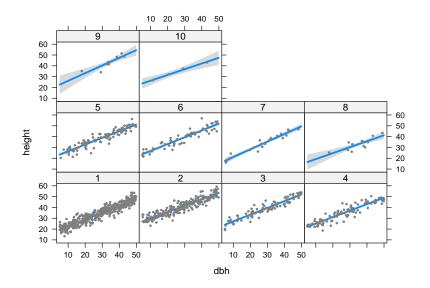
and Height vary among sites?

#### Model with interactions

```
Call.
lm(formula = height ~ site * dbh. data = trees)
Residuals:
    Min
             1Q Median
                                     Max
-10.1017 -1.9839 0.0645
                          2.0486 11.1789
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 16.359437  0.360054  45.436  < 2e-16 ***
site2
           7.684781 0.609657 12.605 < 2e-16 ***
site3
           4.518568 0.867008 5.212 2.28e-07 ***
site4
           site5
          3.917607 0.870983 4.498 7.68e-06 ***
site6
          4.155161
                    1.009379 4.117 4.17e-05 ***
site7
          -2.306799
                    1.551303 -1.487 0.137334
          -2.616095
site8
                    4.090671 -0.640 0.522630
site9
          2.621560 5.073794 0.517 0.605492
          4.662340 2.991072 1.559 0.119378
site10
dbh
           0.629299 0.011722 53.685 < 2e-16 ***
site2:dbh =0.042784 0.020033 =2.136 0.032950 *
site3:dbh
          -0.006031 0.027640 -0.218 0.827312
site4.dbh
          -0.031633 0.028225 -1.121 0.262677
site5:dbh
          -0.010173 0.027887 -0.365 0.715334
site6:dbh
          0.001337 0.032109 0.042 0.966797
site7:dbh
          0.079728 0.052056 1.532 0.125951
site8:dbh -0.079027 0.113386 -0.697 0.485984
site9.dbh
           0.081035 0.146649
                             0.553.0.580679
site10:dbh -0.101107 0.114520 -0.883 0.377522
---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 3.041 on 980 degrees of freedom
Multiple R-squared: 0.8847. Adjusted R-squared: 0.8825
F-statistic: 395.7 on 19 and 980 DF, p-value: < 2.2e-16
```

# Does slope vary among forests?

```
visreg(m5, xvar = "dbh", by = "site")
```



paperplanes: How does flight distance differ with age, gender or paper type?

- paperplanes: How does flight distance differ with age, gender or paper type?
- mammal sleep: Are sleep patterns related to diet?

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- paperplanes: How does flight distance differ with age, gender or paper type?
- mammal sleep: Are sleep patterns related to diet?
- ▶ iris: Predict petal length ~ petal width and species
- racing pigeons: is speed related to sex?