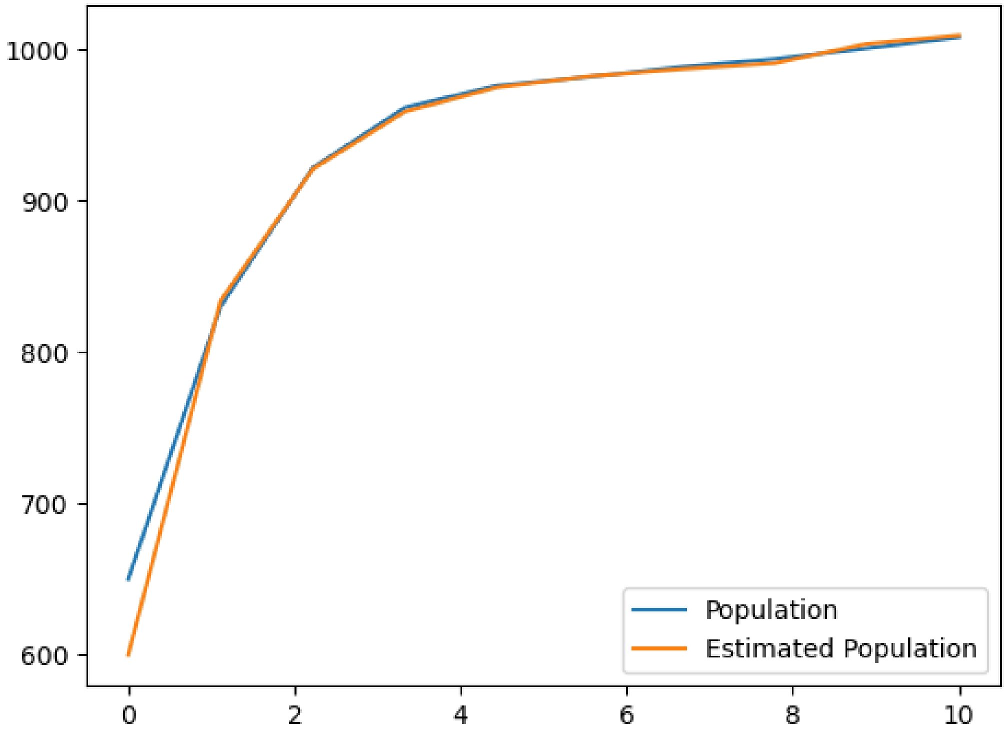
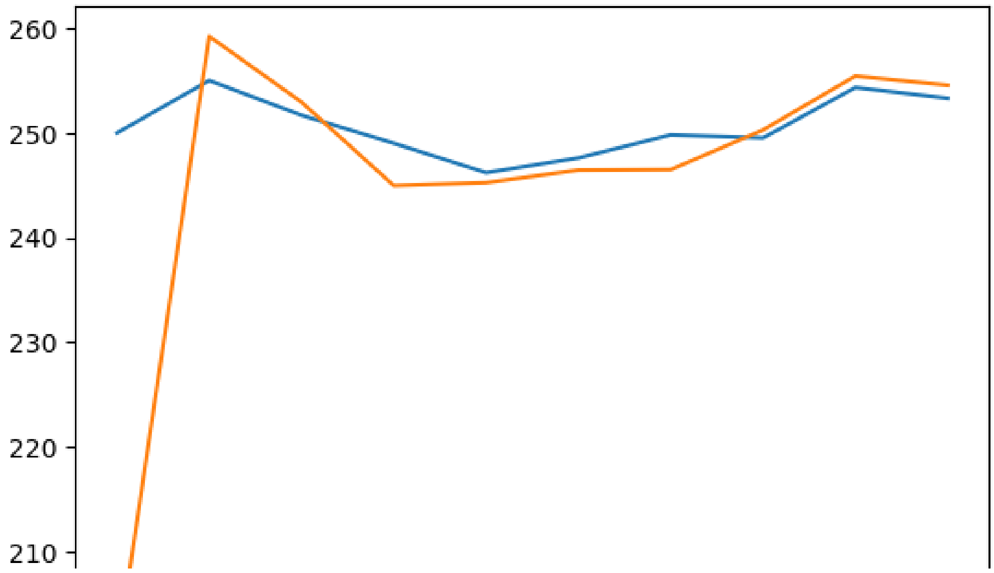
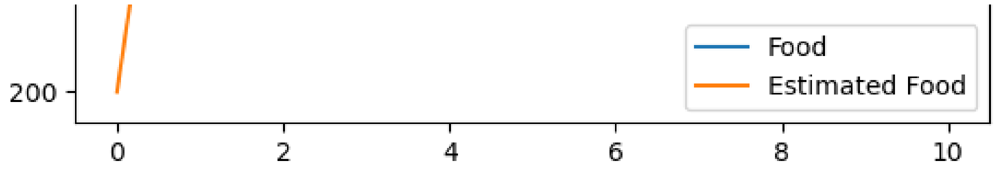
In [18]:

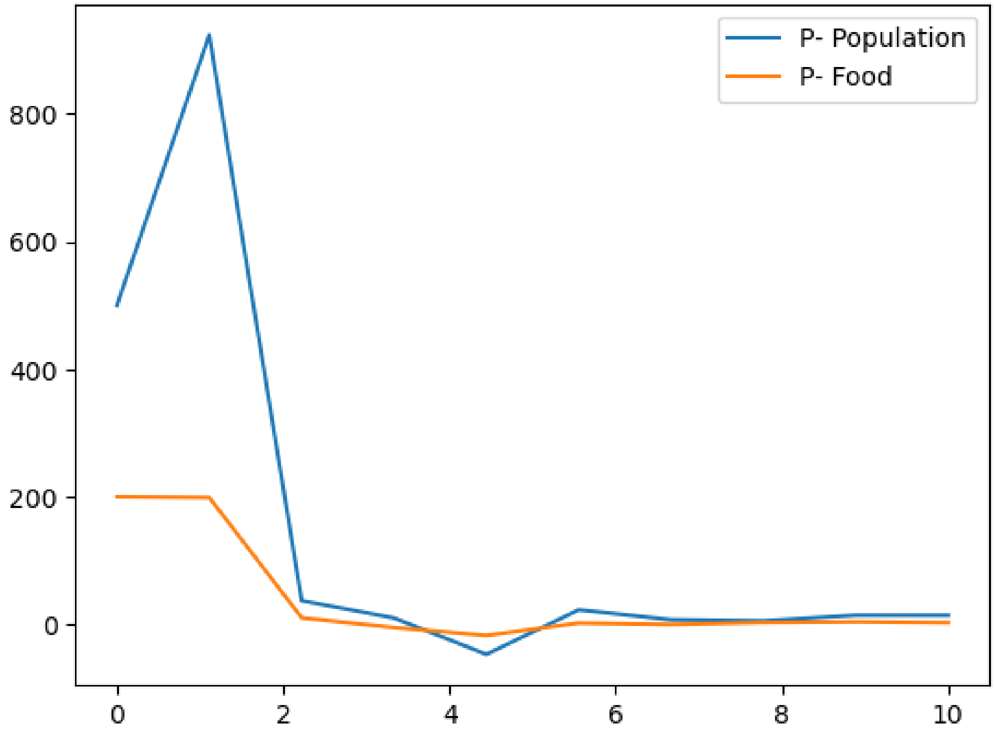
```
import numpy as np
import matplotlib.pyplot as plt
A = np.array([[1]])
B = np.array([[3]])
A @ B

time_steps = 10

F = np.array([[0.5, 2],
              [0, 1]])

x = np.zeros((2, 1, time_steps))
x[:, :, 0] = np.array([[650],
                       [250]])

x_minus = np.zeros((2, 1, time_steps))
x_minus[:, :, 0] = np.array([[600],
                             [200]])

x_plus = np.zeros((2, 1, time_steps))
x_plus[:, :, 0] = np.array([[600],
                            [200]])

P_minus = np.zeros((2, 2, time_steps))
P_minus[:, :, 0] = np.array([[500, 0],
                             [0, 200]])

P_plus = np.zeros((2, 2, time_steps))
P_plus[:, :, 0] = np.array([[500, 0],
                            [0, 200]])

y = np.zeros((2, 1, time_steps))
y[:, :, 0] = np.array([[600],
                       [200]])
H = np.array([[1, 0],
              [0, 1]])

K = np.zeros((2, 2, time_steps))
v = np.random.normal(0, np.sqrt(10))
K[:, :, 0] = P_minus[:, :, 0]@np.transpose(H)@np.linalg.inv(H@P_minus[:, :, 0]@np.transp
ose(H)+np.array([[v], [v]]))

Q, R = 10, 10
def kalman_filter(x, y, x_minus, x_plus, P_minus, P_plus, F, H, Q, R, K, time_steps):
    for i in range(1, time_steps):
        v = np.random.normal(0, np.sqrt(Q))
        x[:, :, i] = F @ x[:, :, i-1] + np.array([[v], [v]])
        y[:, :, i] = H @ x[:, :, i] + np.array([[v], [v]])
        w = np.random.normal(0, np.sqrt(R))
        x_minus[:, :, i] = F @ x_plus[:, :, i-1] + np.array([[w], [w]])
        P_minus[:, :, i] = F @ P_plus[:, :, i-1] @ np.transpose(F) + np.array([[w], [w]]
)
        K[:, :, i] = P_minus[:,:, i]@np.transpose(H)@np.linalg.inv(H@P_minus[:,:, i]@np.
transpose(H)+np.array([[10, 0], [0, 10]]))
        x_plus[:, :, i] = x_minus[:, :, i] + K[:, :, i]@(y[:, :, i]-H@x_minus[:, :, i])
        P_plus[:, :, i] = (np.eye(2)-K[:, :, i]@H)@P_minus[:, :, i]
    return x, y, x_minus, x_plus, P_minus, P_plus, K

x, y, x_minus, x_plus, P_minus, P_plus, K = kalman_filter(x, y, x_minus, x_plus, P_minus
, P_plus, F, H, Q, R, K, time_steps)

plt.plot(np.linspace(0, 10, time_steps), x[0, 0, :], label='Population')
plt.plot(np.linspace(0, 10, time_steps), x_plus[0, 0, :], label='Estimated Population')
plt.legend()
plt.show()

plt.plot(np.linspace(0, 10, time_steps), x[1, 0, :], label='Food')
```

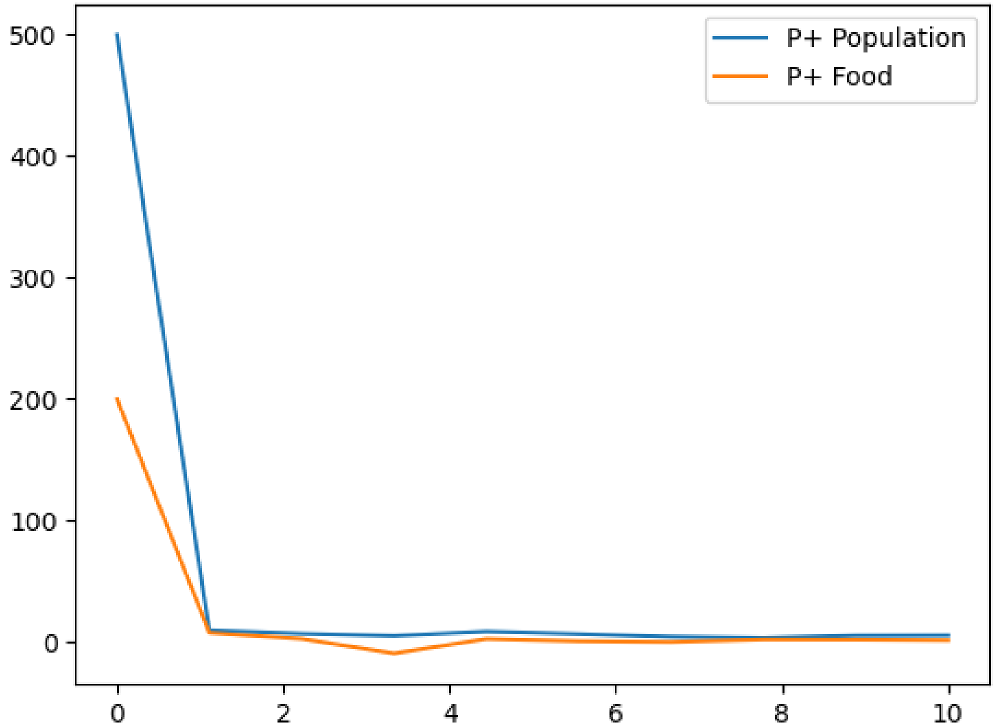
```python
plt.plot(np.linspace(0, 10, time_steps), x_plus[1, 0, :], label='Estimated Food')
plt.legend()
plt.show()

plt.plot(np.linspace(0, 10, time_steps), P_minus[0, 0, :], label='P- Population')
plt.plot(np.linspace(0, 10, time_steps), P_minus[1, 1, :], label='P- Food')
plt.legend()
plt.show()

plt.plot(np.linspace(0, 10, time_steps), P_plus[0, 0, :], label='P+ Population')
plt.plot(np.linspace(0, 10, time_steps), P_plus[1, 1, :], label='P+ Food')
plt.legend()
plt.show()

plt.plot(np.linspace(0, 10, time_steps), K[0, 0, :], label='K Population')
plt.plot(np.linspace(0, 10, time_steps), K[1, 1, :], label='K Food')
plt.legend()
plt.show()

# find standard deviation of the error and plot it
error_pop = x[0, 0, :]-x_plus[0, 0, :]
error_food = x[1, 0, :]-x_plus[1, 0, :]
plt.plot(np.linspace(0, 10, time_steps), error_pop, label='Population Error')
plt.plot(np.linspace(0, 10, time_steps), P_plus[0, 0, :], label='P Population')
print(np.std(error_pop), P_plus[0, 0, -1])
plt.legend()
plt.show()
```

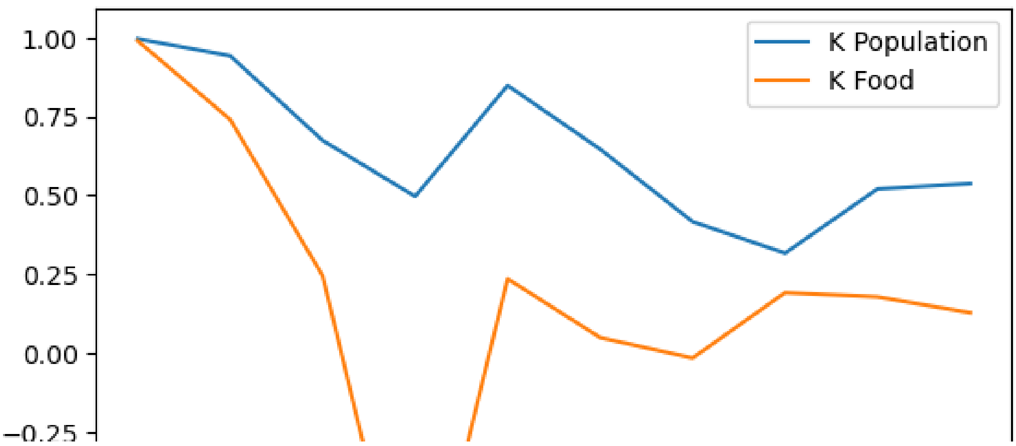


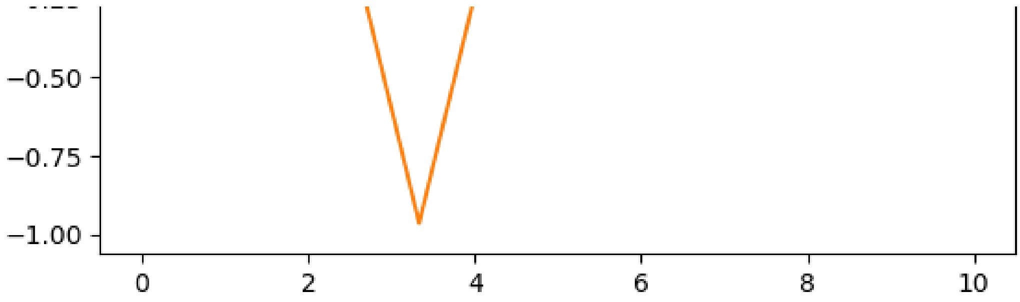



Food
Estimated Food

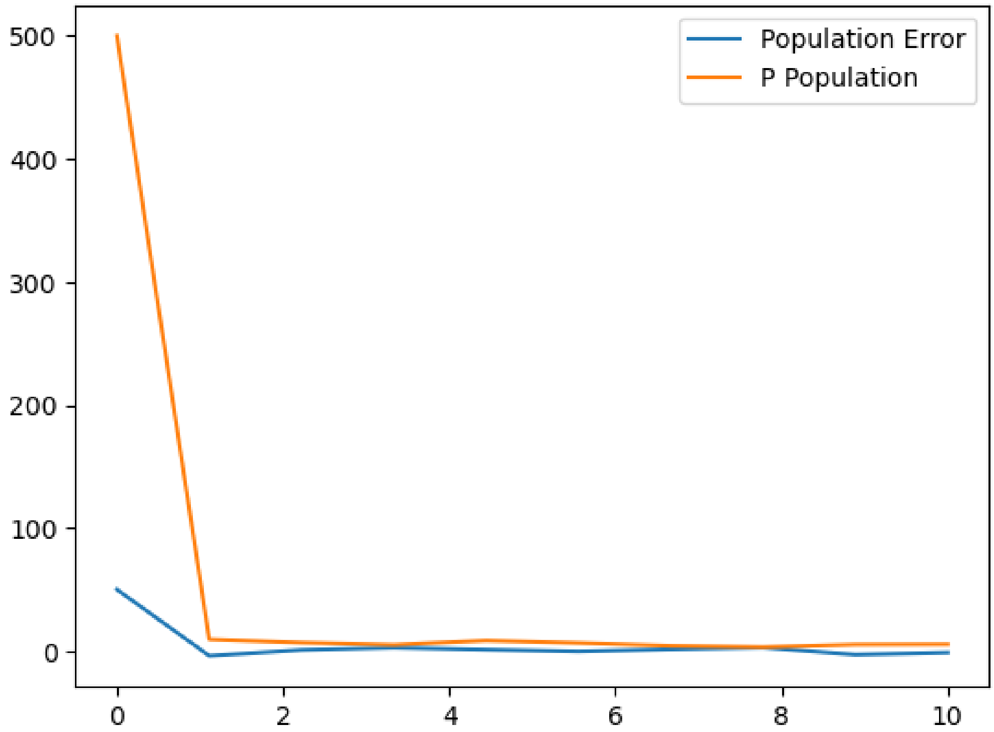

P- Population
P- Food


P+ Population
P+ Food


K Population
K Food



15.133538207641662 5.372345796721074



P^+ is a smaller value because it's a derived by both the estimates and the measurement, so the error will be smaller. However, if ran forever, the std of the population will approach P^+

In [19]:

```
import numpy as np
import matplotlib.pyplot as plt
A = np.array([[1]])
B = np.array([[3]])
A @ B

time_steps = 1000

F = np.array([[0.5, 2],
              [0, 1]])

x = np.zeros((2, 1, time_steps))
x[:, :, 0] = np.array([[650],
                        [250]])

x_minus = np.zeros((2, 1, time_steps))
x_minus[:, :, 0] = np.array([[600],
                              [200]])

x_plus = np.zeros((2, 1, time_steps))
x_plus[:, :, 0] = np.array([[600],
                             [200]])

P_minus = np.zeros((2, 2, time_steps))
P_minus[:, :, 0] = np.array([[500, 0],
                              [0, 200]])
```

```python
P_plus = np.zeros((2, 2, time_steps))
P_plus[:, :, 0] = np.array([[500, 0],
                            [0, 200]])

y = np.zeros((2, 1, time_steps))
y[:, :, 0] = np.array([[600],
                       [200]])
H = np.array([[1, 0],
              [0, 1]])

K = np.zeros((2, 2, time_steps))
v = np.random.normal(0, np.sqrt(10))
K[:, :, 0] = P_minus[:, :, 0]@np.transpose(H)@np.linalg.inv(H@P_minus[:, :, 0]@np.transp
ose(H)+np.array([[v], [v]]))

Q, R = 10, 10
def kalman_filter(x, y, x_minus, x_plus, P_minus, P_plus, F, H, Q, R, K, time_steps):
    for i in range(1, time_steps):
        v = np.random.normal(0, np.sqrt(Q))
        x[:, :, i] = F @ x[:, :, i-1] + np.array([[v], [v]])
        y[:, :, i] = H @ x[:, :, i] + np.array([[v], [v]])
        w = np.random.normal(0, np.sqrt(R))
        x_minus[:, :, i] = F @ x_plus[:, :, i-1] + np.array([[w], [w]])
        P_minus[:, :, i] = F @ P_plus[:, :, i-1] @ np.transpose(F) + np.array([[w], [w]]
)
        K[:, :, i] = P_minus[:,:, i]@np.transpose(H)@np.linalg.inv(H@P_minus[:,:, i]@np.
transpose(H)+np.array([[10, 0], [0, 10]]))
        x_plus[:, :, i] = x_minus[:, :, i] + K[:, :, i]@(y[:, :, i]-H@x_minus[:, :, i])
        P_plus[:, :, i] = (np.eye(2)-K[:, :, i]@H)@P_minus[:, :, i]
    return x, y, x_minus, x_plus, P_minus, P_plus, K

x, y, x_minus, x_plus, P_minus, P_plus, K = kalman_filter(x, y, x_minus, x_plus, P_minus
, P_plus, F, H, Q, R, K, time_steps)

# find standard deviation of the error and plot it
error_pop = x[0, 0, :]-x_plus[0, 0, :]
error_food = x[1, 0, :]-x_plus[1, 0, :]
print(np.std(error_pop[:9]), P_plus[0, 0, 9])
print(np.std(error_pop), P_plus[0, 0, -1])
```

```
15.846089395150642 10.506957488385458
100.43860127491796 7.273164258077111
```

Due to some calculations rounding by the software, the standard deviation goes crazy since at certain points the error is very high, but it should decrease.