Homework 0

Exercise 1

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ 0 indipendent rows, so rank is 0.

Exercise 4

$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \quad B = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix}$

$AB = \begin{bmatrix} A_{11}B_{11} + A_{12}B_{21} + A_{13}B_{31} & A_{11}B_{12} + A_{12}B_{22} + A_{13}B_{32} & A_{11}B_{13} + A_{12}B_{23} + A_{13}B_{33} \\ A_{21}B_{11} + A_{22}B_{21} + A_{23}B_{31} & A_{21}B_{12} + A_{22}B_{22} + A_{23}B_{32} & A_{21}B_{13} + A_{22}B_{23} + A_{23}B_{33} \\ A_{31}B_{11} + A_{32}B_{21} + A_{33}B_{31} & A_{31}B_{12} + A_{32}B_{22} + A_{33}B_{32} & A_{31}B_{13} + A_{32}B_{23} + A_{33}B_{33} \end{bmatrix}$

$trAB = A_{11}B_{11} + A_{12}B_{21} + A_{13}B_{31} + A_{21}B_{12} + A_{22}B_{22} + A_{23}B_{32} + A_{31}B_{13} +$

$A_{32}B_{23} + A_{33}B_{33}$

$\frac{\partial trAB}{\partial A} = \begin{bmatrix} B_{11} & B_{21} & B_{31} \\ B_{12} & B_{22} & B_{32} \\ B_{13} & B_{23} & B_{33} \end{bmatrix} = B^T$

Exercise 13

$p = p_0 + \dot{p}t + \frac{1}{2}\ddot{p}t^2$

a

$$x = \begin{bmatrix} p_0 + \dot{p}t + \frac{1}{2}\ddot{p}t^2 \\ \dot{p} + \ddot{p}t \\ \ddot{p} \end{bmatrix} \quad A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} x$$

b

$$e^{At} = \sum_{j=0}^{\infty} \frac{At^j}{j!} = I + At + \frac{At^2}{2!} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} t + \frac{1}{2} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} t^2$$

$$= \begin{bmatrix} 1 & t & \frac{1}{2}t^2 \\ 0 & 1 & t \\ 0 & 0 & 1 \end{bmatrix}$$

Exercise 19

a

$x_1(t) = 0 \quad x_2(t) = 0$

$0 = \frac{Ku - gx_2}{x_3} - \frac{GM}{(R+x_1)^2}$

$0 = \frac{Ku}{x_3} - \frac{GM}{R^2} \rightarrow x_3 = \frac{KuR^2}{GM}$

Since $\dot{x}_3 = -u \quad u(t) = e^{-\frac{GM}{kR^2}t}$

b

$x_3 = \frac{kR^2}{GM} e^{-\frac{GM}{kR^2} t}$

c

$A = \left[\frac{\partial f}{\partial x}\right] = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2GM}{(R+x_1)^3} & -\frac{g}{x_2} & -\frac{Ku - gx_2}{x_3^2} \\ 0 & 0 & 0 \end{bmatrix}$

$B = \left[\frac{\partial f}{\partial u}\right] = \begin{bmatrix} 0 \\ \frac{K}{x_3} \\ -1 \end{bmatrix}$

$\dot{x} = Ax + Bu$

```
import numpy as np; from scipy.integrate import
    solve_ivp; import matplotlib.pyplot as plt


k, g, G, M, R, delta_u_list, t_span, y0 = 1000, 50,
    6.673e-11, 5.98e24, 6.37e6, [10, 100, 300], [0,5],
    [0,0]
u, u_int = lambda t: np.exp(-G*M/(k*(R**2))*t), lambda
     t, delta_u: (G*M)/(k*(R**2))*u(t)*10000+np.abs(np.
    cos(t))*delta_u
non_linear_system = lambda t, y, delta_u: [y[1], -((k
    * u(t) - g * y[1]) / u_int(t, delta_u) - G * M / (R
     + y[0]) ** 2)]
```

```
linear_system = lambda t, y, delta_u: [y[1], -2*G*M*y
    [0]/(R**3)-g*y[1]/(u_int(t, delta_u))+(k*10000*u(t)
    /u_int(t, delta_u))-k*10000*u(t)/u_int(t, delta_u)]
solution = lambda delta_u, function: solve_ivp(
    function, t_span, y0, args=(delta_u,), t_eval=np.
    linspace(0, 5, 1000))


delta_u = 10
print(solution(delta_u, linear_system))
[plt.plot(np.linspace(0, 5, len(solution(delta_u,
    linear_system).y[0])), solution(delta_u,
    linear_system).y[0], '--', label=r'$\Delta␣u={}$'.
    format(delta_u)) for delta_u in delta_u_list]
[plt.plot(np.linspace(0, 5, len(solution(delta_u,
    non_linear_system).y[0])), solution(delta_u,
    non_linear_system).y[0], label=r'$\Delta␣u={}$'.
    format(delta_u)) for delta_u in delta_u_list]
plt.xlabel('Time'), plt.ylabel('Altitude'), plt.xlim
    (0, 5), plt.legend(), plt.savefig('Nonlinear␣System
    '), plt.show()
```

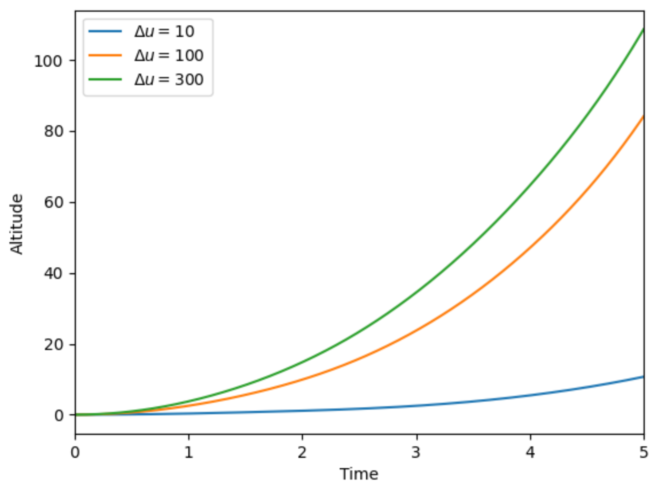

Figure 1: Nonlinear System