Homework 1

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1 Problem 1

In order to achieve equilibrium all the momentums must be counteracted, and the net momentum has to be zero. For this to happen the following condition must be satisfied:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau yx}{\partial y} = 0$$
$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau yx}{\partial x} = 0$$

Since $\delta_x = \delta_y$ because it is a cube, and $\sigma_x = \sigma_y \rightarrow \frac{\partial \tau_{xy}}{\partial x} = \frac{\partial \tau_{yx}}{\partial y}$

2 Problem 2

a

$$\widetilde{S} = \begin{pmatrix} \frac{1}{E_L} & -\frac{\nu_{TL}}{E_T} & \frac{\nu_{TL}}{E_T} & 0 & 0 & 0 \\ -\frac{\nu_{LT}}{E_L} & \frac{1}{E_T} & \frac{\nu_T}{E_T} & 0 & 0 & 0 \\ \frac{\nu_{LT}}{E_L} & \frac{\nu_T}{E_T} & \frac{1}{E_T} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_T} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{LT}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{LT}} \end{pmatrix}$$

Since $G = \frac{E}{2(1+\nu)} \rightarrow G_{LT} = \frac{E_{xy}}{2(1+\nu_{xy})}$ and $G_T = \frac{E_{yz}}{2(1+\nu_{yz})}$

Problem 3

a

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{bmatrix}$$

$$\sigma = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix} = \begin{bmatrix} 100 & 10 & 20 \\ 10 & -100 & 0 \\ 20 & 0 & 50 \end{bmatrix}$$

$$\begin{bmatrix} 100-\sigma & 10 & 20 \\ 10 & -100-\sigma & 0 \\ 20 & 0 & 50-\sigma \end{bmatrix}$$

$\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3$

$I_1 = \sigma_{11} + \sigma_{22} + \sigma_{33} = 100 - 100 + 50 = 50$

$I_2 = \sigma_{11}\sigma_{22} + \sigma_{22}\sigma_{33} + \sigma_{33}\sigma_{11} - \sigma_{12}^2 - \sigma_{23}^2 - \sigma_{31}^2 = 100(-100) + (-100)50 + 50(100) - 10^2 - 0^2 - 20^2 = 10000 - 5000 + 5000 - 100 - 400 = -10500$

$I_3 = \sigma_{11}\sigma_{22}\sigma_{33} - \sigma_{11}\sigma_{23}^2 - \sigma_{22}\sigma_{31}^2 - \sigma_{33}\sigma_{12}^2 + 2\sigma_{12}\sigma_{23}\sigma_{31} = 100(-100)(50) - 100(0)^2 - (-100)(20)^2 - 50(10^2) + 2(10)(0)(20) = -465000$

$\phi = \frac{1}{3}cos^{-1}\left(\frac{2I_1^3 - 9I_1I_2 + 27I_3}{2(I_1^2 - 3I_2)^{\frac{3}{2}}}\right) = \frac{1}{3}cos^{-1}\left(\frac{2*50^3 - 9*50*(-10500) + 27*(-465000)}{2(50^2 - 3(-10500))^{\frac{3}{2}}}\right) = 0.739992414341282\text{rad}$

$\sigma_I = \frac{I_1}{3} + \frac{2}{3}(\sqrt{I_2^2 - 3I_2})cos\phi = 107.445$

$\sigma_{II} = \frac{I_1}{3} + \frac{2}{3}(\sqrt{I_1^2 - 3I_2})cos(\phi - \frac{2\pi}{3}) = 43.060227$

$\sigma_{III} = \frac{I_1}{3} + \frac{2}{3}(\sqrt{I_1^2 - 3I_2})cos(\phi - \frac{4\pi}{3}) = -100.505$

b

$\sigma_e = \sqrt{\frac{(\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2}{2}} = 184.4\text{MPa}$

Based on Von Mises criteria, the body fails.