The deflection is 0 at y=0 and y=L. Also, d^2w/dy^2=0 at those points as well. So we will use sin(y *pi/L). The deflection is 0 at x=0 and x=nL. lso, d^2w/dx^2=0 at those points as well. But d^2w/dx^2=1/p* E*t^3/(12(1-v^2))z

In [501]:

```
import numpy as np
import sympy as spy
import matplotlib.pyplot as plt

x, y, z, L, n = spy.symbols('x y z L n')
E, t, p, v = spy.symbols('E t p v')
c_1, c_2, c_m = spy.symbols('c_1 c_2 c_m')
m = spy.symbols('m')

x_0, x_max = 0, n*L
y_0, y_max = 0, L
# w(0)=0, and d^2w/dx^2 at 0=0
w = c_1*spy.sin(spy.pi*y/L)*spy.sin(spy.pi*x/(n*L)) + c_2*spy.sin(spy.pi*y/L)*spy.sin(2*
spy.pi*x/(n*L))
w_x = spy.diff(w,x)
w_y = spy.diff(w,y)
w_x_x = spy.diff(spy.diff(w,x),x)
w_y_y = spy.diff(spy.diff(w,y),y)
w_x_y = spy.diff(spy.diff(w,x),y)
```

Since only N_x is applied and all sides are simply supported $w = \sum_{n=1}^{\infty} c_n\, sin\frac{y\pi}{L} sin\frac{nx\pi}{L}$

We only want 2 modes so $w = c_1\, sin\frac{y\pi}{L} sin\frac{x\pi}{L} + c_2\, sin\frac{y\pi}{L} sin\frac{2x\pi}{L}$

And therefore $U_{strain} = \frac{n\pi^4}{8}\frac{Et^3}{12(1-\nu^2)}\sum_{m=1}^{2} c_m^2\,(\frac{m}{n^2}+1)^2$

$U = \frac{\pi^4}{8n}\frac{Et^3}{12(1-\nu^2)}(c_1^2(1+n^2) + c_2(2+n^2))$

Since $n = 1, 2$ $\quad U_{strain} = \frac{\pi^4}{8L^2}\frac{Et^3}{12(1-\nu^2)}(4c_1^2 + 25c_2^2)$

For $U_{spring} = \frac{1}{2}k\int_0^L w^2\,dx$

In [502]:

```
U_strain = spy.pi**4*E*t**3/(8*L**2*12*(1-v**2)) * (4*c_1**2+25*c_2**2)
```

In [503]:

```
k = 1/p * (E*t**3)/(12*(1-v**2))
U_spring = 1/2 * k * (spy.integrate(w**2, (x, 0, L)))
U_spring = U_spring.subs(spy.sin(spy.pi*n), 0)
U_spring = U_spring.subs(spy.cos(spy.pi*n), 1)
U_spring.simplify()
```

Out[503]:

$$0.00347222222222222ELt^3 \cdot \left(3c_1^2 n\sin\left(\frac{2\pi}{n}\right) - 6\pi c_1^2 - 12c_1c_2 n\sin\left(\frac{\pi}{n}\right) + 4c_1c_2 n\sin\left(\frac{3\pi}{n}\right) + \frac{3c_2^2 n\sin\left(\frac{4\pi}{n}\right)}{2} - 6\right.$$

$$\frac{\qquad\qquad}{\pi p\left(v^2-1\right)}$$

$U_{spring} = \frac{ELt^3 sin\frac{n\pi}{L}\sum_{m=1}^{2} c_m^2}{48p(\nu^2-1)}$

But the total number of springs depends on n. So \ $U_{spring} = \frac{(n-1)ELt^3 sin\frac{n\pi}{L}\sum_{m=1}^{2} c_m^2}{48p(\nu^2-1)}$

So total U

$$= \frac{n\pi^4}{8}\frac{Et^3}{12(1-\nu^2)}\sum_{m=1}^{2} c_m^2\left(\frac{m}{n^2}+1\right)^2 + \frac{(n-1)ELt^3 sin\frac{n\pi}{L}\sum_{m=1}^{2} c_m^2}{48p(\nu^2-1)}$$

In [504]:

```
W = 1/2 * spy.integrate(spy.integrate(N_x * w_x**2, (x, 0, L)), (y, 0, n*L))
W = W.subs(spy.sin(spy.pi*n), 0)
W = W.subs(spy.cos(spy.pi*n), 1)
W.simplify()
```

Out[504]:

$$\frac{0.0416666666666667\pi N_x\left(\frac{3c_1^2 n\sin\left(\frac{2\pi}{n}\right)}{2}+3\pi c_1^2+12c_1c_2n\sin\left(\frac{\pi}{n}\right)+4c_1c_2n\sin\left(\frac{3\pi}{n}\right)+3c_2^2n\sin\left(\frac{4\pi}{n}\right)+12\pi c_2^2\right.}{n}$$

$W = \frac{n\pi^2 N_x}{8}\sum_{m=1}^{2}(c_m^2 m^2)$

$= \frac{n\pi^2 N_x}{8}(c_1^2 + 4c_2^2)$

$$\Pi = \frac{n\pi^4}{8}\frac{Et^3}{12(1-\nu^2)}\sum_{m=1}^{2} c_m^2\left(\frac{m}{n^2}+1\right)^2 + \frac{(n-1)ELt^3 sin\frac{n\pi}{L}\sum_{m=1}^{2} c_m^2}{48p(\nu^2-1)} - \frac{n\pi^2 N_x}{8}\sum_{m=1}^{2}(c_m^2 m^2)$$

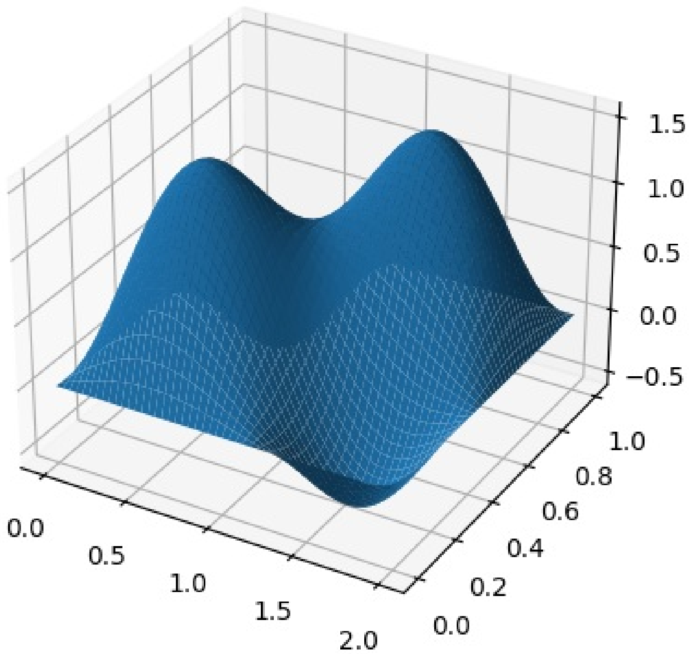
$$\frac{\partial\Pi}{\partial c_m} = \frac{n\pi^4}{4}\frac{Et^3}{12(1-\nu^2)}\sum_{m=1}^{2} c_m\left(\frac{m}{n^2}+1\right)^2 + \frac{(n-1)ELt^3}{24(\nu^2-1)}sin\frac{n\pi}{L}\sum_{m=1}^{2} c_m - \frac{n\pi^2 N_x}{4}\sum_{m=1}^{2} c_m m^2$$

N_x =

In [506]:

```
import numpy as np
```

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

L = 1
n = 2
# Example with all Cs equal to 1
def f(x, y):
    return np.sin(np.pi*y/L)*np.sin(np.pi*x/(n*L)) + np.sin(2*np.pi*y/L)*np.sin(2*np.pi
*x/(n*L))

# Generate the x and y values
x = np.linspace(0, n*L, 100*n)
y = np.linspace(0, L, 100)
X, Y = np.meshgrid(x, y)

# Calculate the corresponding z values
Z = f(X, Y)

# Create a 3D plot
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
ax.plot_surface(X, Y, Z)

ax.set_label('x')
ax.set_label('y')
# Show the plot
plt.show()
```



In [507]:

```
L, n = spy.symbols('L n')
E, t = spy.symbols('E t')
c_m = spy.symbols('c_m')
a, b = spy.symbols('a b')
v, m = spy.symbols('v m')
x, y = spy.symbols('x y')
a = n*L
b = L
k = 1/p * (E*t**3)/(12*(1-v**2))

w = c_m * spy.sin(spy.pi*y/b) * spy.sin(m*spy.pi*x/a)
w
```

Out[507]:

$$c_m \sin\left(\frac{\pi y}{L}\right) \sin\left(\frac{\pi m x}{L n}\right)$$

In [508]:

```
U_strain = a*b*spy.pi**4/8 * E*t**3/(12*(1-v**2)) * c_m**2 * (m**2/(a**2) + 1**2/(b**2)
)**2
U_spring = 1/2 *spy.integrate(k*w**2, (y, 0, L))
U_spring = U_spring.subs(spy.sin(spy.pi*m), 0)
W = b*spy.pi**2/(8*a) * N_x * c_m**2 * m**2
display(U_spring)
display(U_strain)
display(W)
```

$$0.5\left(\begin{cases} \frac{ELc_m^2 t^3 \sin^2\left(\frac{\pi m x}{Ln}\right)}{2p(12-12v^2)} & \text{for } L > -\infty \wedge L < \infty \wedge L \neq 0 \\ 0 & \text{otherwise} \end{cases}\right)$$

$$\frac{\pi^4 EL^2 c_m^2 n t^3 \left(\frac{m^2}{L^2 n^2} + \frac{1}{L^2}\right)^2}{8 \cdot (12 - 12v^2)}$$

$$\frac{\pi^2 N_x c_m^2 m^2}{8n}$$

In [509]:

```
PI = U_strain + U_spring - W
PI_c = spy.diff(PI, c_m)
numerator = PI_c.as_numer_denom()[0]
numerator = numerator/(L**6*c_m*n**2)
numerator = numerator.simplify()
numerator = numerator.subs(x, n-1)
numerator
```

Out[509]:

$$\begin{cases} \frac{2.0EL^3n^3t^3\sin^2\left(\frac{\pi m(n-1)}{Ln}\right) + \pi^2 p\left(\pi^2 Et^3\left(m^2+n^2\right)^2 + 12L^2 N_x m^2 n^2\left(v^2-1\right)\right)}{p} & \text{for } (L > -\infty \vee L > 0) \wedge (L > -\infty \vee L < \\ \pi^2\left(\pi^2 Et^3\left(m^2+n^2\right)^2 + 12L^2 N_x m^2 n^2\left(v^2-1\right)\right) & \text{othe} \end{cases}$$

$$-2EL^3n^3t^3(n-1)sin^2(\frac{\pi m x}{Ln}) - \pi^2 p(\pi^2 Et^3(m^2+n^2)^2$$
$$+ 12L^2 N_x m^2 n^2(\nu^2-1))$$