Homework 1

Mauro Patimo

January 28, 2024

1 Problem 1

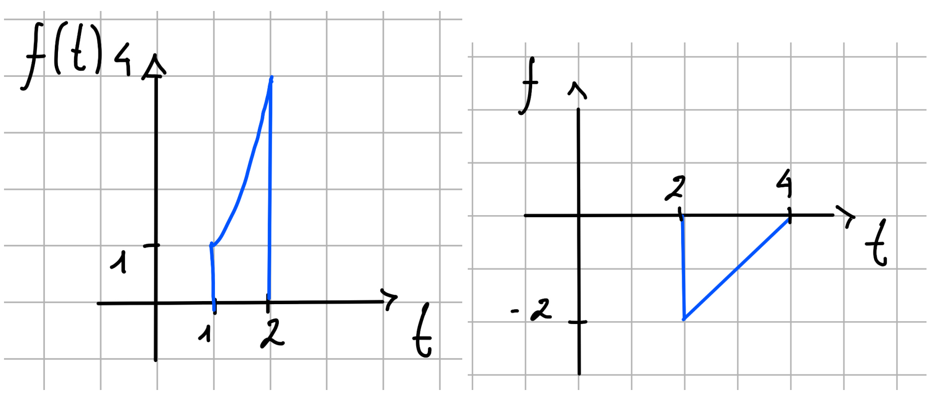
$f_3 = f_1(t+1) + f(t-1)$

$f_4 = f(t - \frac{1}{2}) + f_1(t + \frac{1}{2})$

$f_5 = 1.5f(\frac{t}{2} - 1)$

2 Problem 2

2a



(a) Problem 2a (b) Problem 2b

Figure 1: Problem 2

3 Problem 3

3.1 a

$\int_{-\infty}^{\infty} \delta(\tau) f(t-\tau) d\tau$

$t - \tau = \alpha \rightarrow \tau = t - \alpha$

$-\int_{-\infty}^{\infty} \delta(t-\alpha) f(\alpha) d\alpha$

Since $\delta(-\alpha) = \delta(\alpha)$

$-\int_{-\infty}^{\infty} \delta(-\alpha - (-t)) f(-\alpha)$

$-\alpha = \sigma$

$-\int_{-\infty}^{\infty} \delta(\sigma - (-t)t) f(\sigma) d\sigma$

$f(\sigma) = f(-\alpha) = f(t-\tau)$

3.2 b

$\int_{-\infty}^{\infty} \delta(t+3) e^{-t} dt$

e^3

3.3 c

$\int_{-\infty}^{\infty} f(\tau)\delta(t-\tau) d\tau$

$\int_{-\infty}^{\infty} f(t)\delta(\tau - t) d\tau \ f(t)$

3.4 d

$\int_{-\infty}^{\infty} f(2-t)\delta(3-t)$

$3 - t = \alpha$

$\int_{-\infty}^{\infty} f(\alpha+1)\delta(\alpha)$

$f(-1)$

4 Problem 4

a

$\frac{sin(0)}{0^2+2}\delta(t) = 0$

b

$\frac{1}{-3j+2}\delta(t+3) = \frac{1}{-3j+2}\delta(w+3)$

c

$\frac{sin(0k)}{0}\delta(\omega)$
$\frac{kcos(0k)}{1}\delta(\omega) = k\delta(\omega)$

5 Problem 5

$P = \lim_{t\to\infty} \frac{1}{T} \int_{-\infty}^{\infty} (Ccos(\omega t + \theta))^2 dt$

$P = \frac{1}{2\pi} \int_{0}^{2\pi} (Ccos(\omega t + \theta))^2 dt$

$P = \frac{1}{2\pi} C^2 \pi$

$P = \frac{C^2}{2}$

6 Problem 6

a

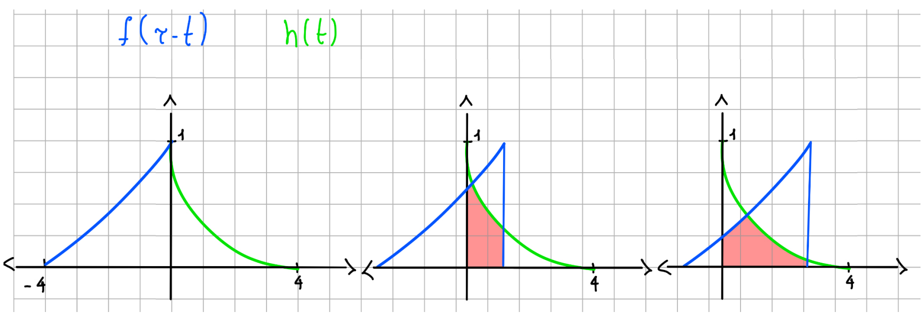


Figure 2: Problem 6a

b

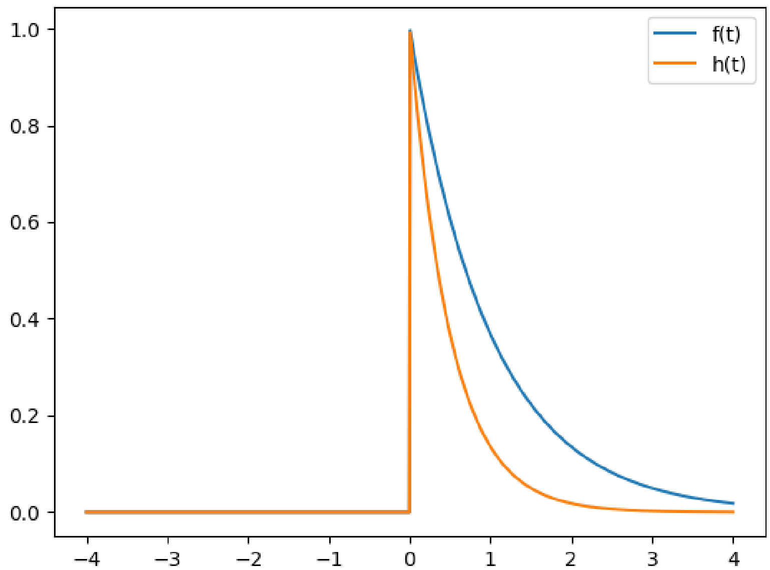


Figure 3: Pre-convolution

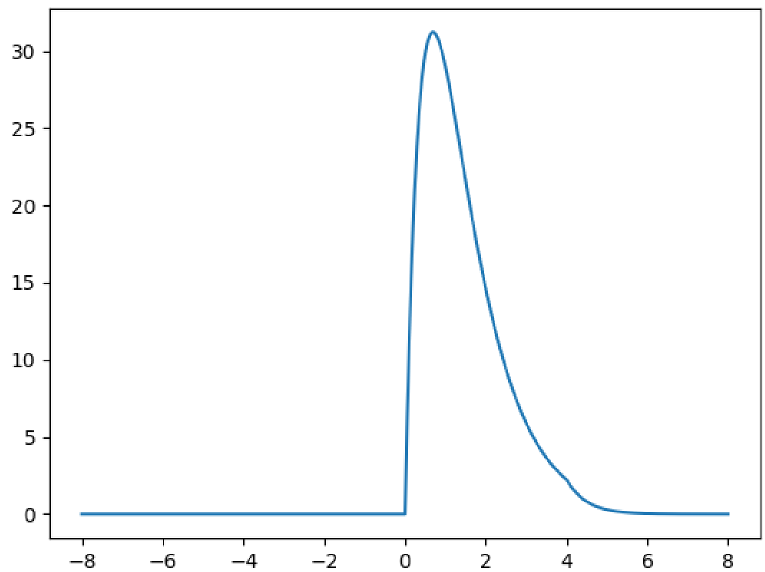


Figure 4: Convolution

597

Problem 1

a

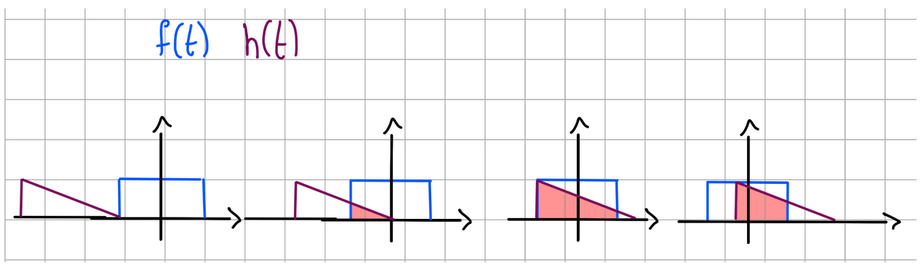


Figure 5: Problem 1a

b

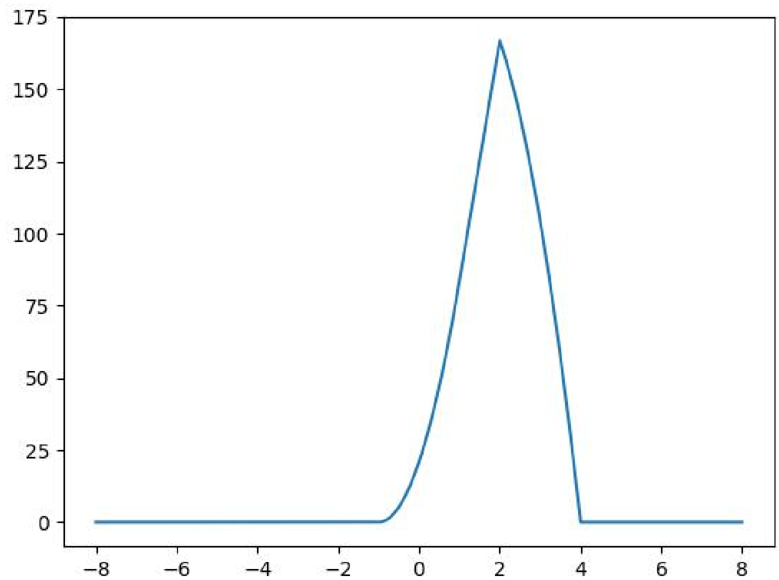


Figure 6: Problem 1b

The length is 5, as expected by summing the lengths of the two functions.

Problem 2

b

$P = \frac{1}{T}\lim_{T\to\infty}\int_{-\infty}^{\infty}(f_1(t)+f_2(t))^2dt =$
$= \frac{1}{2\pi}\Big(\int_0^{2\pi}(C_1cos(\omega_1 t+\theta_1))^2 + \int_0^{2\pi}(C_1cos(\omega_2 t+\theta_2))^2\Big) + \frac{1}{\pi}\int_0^{2\pi}C_1C_2cos(\omega_1 t+$
$\theta_1)cos(\omega_2 t+\theta_2)dt$
$= \frac{C_1^2}{2} + \frac{C_2^2}{2}\frac{1}{\pi}\int_0^{2\pi}C_1C_2cos(\omega_1 t+\theta_1)cos(\omega_2 t+\theta_2)dt$
In case $\omega_1 = n\omega_2$ where "n" is not irrational and non-zero the function is periodic.

c

As explained in part b, if $\omega_1 = \omega_2$ the function is periodic.