Wave Motion

* Any disturbance is called as wave and propagation of that disturbance is called wave motion.

Mechanical waves:

* These are the waves which require material medium for their propagation.
* Position, velocity, acceleration, ~~vary with time~~ of particles vary with time. Such variation is called disturbance. In an elastic medium, if disturbance is created at one point, then remaining medium particles also get disturbed after some time
* Propagation of this disturbance is called wave motion
* In pressure waves, pressure of an element of a medium varies with time. This is disturbance and is transmitted

Non mechanical waves:

* These are the waves, which donot require material medium for their propagation.

Ex: Electromagnetic waves. In this case, electric field $\vec{E}$, magnetic field $\vec{B}$ at a point vary with time. This is a disturbance. Propagation of these variations is electromagnetic wave motion

* In any wave motion, energy and momentum are transmitted.
* Mechanical wave velocity $v = \sqrt{\frac{E}{\rho}}$. Here E, ρ are elastic modulus and density of the medium. v is independent of frequency, wave length of medium but depends on nature of medium only

Ex: Infra sonics, Audible sounds, Ultra sonics travel with same speed in air

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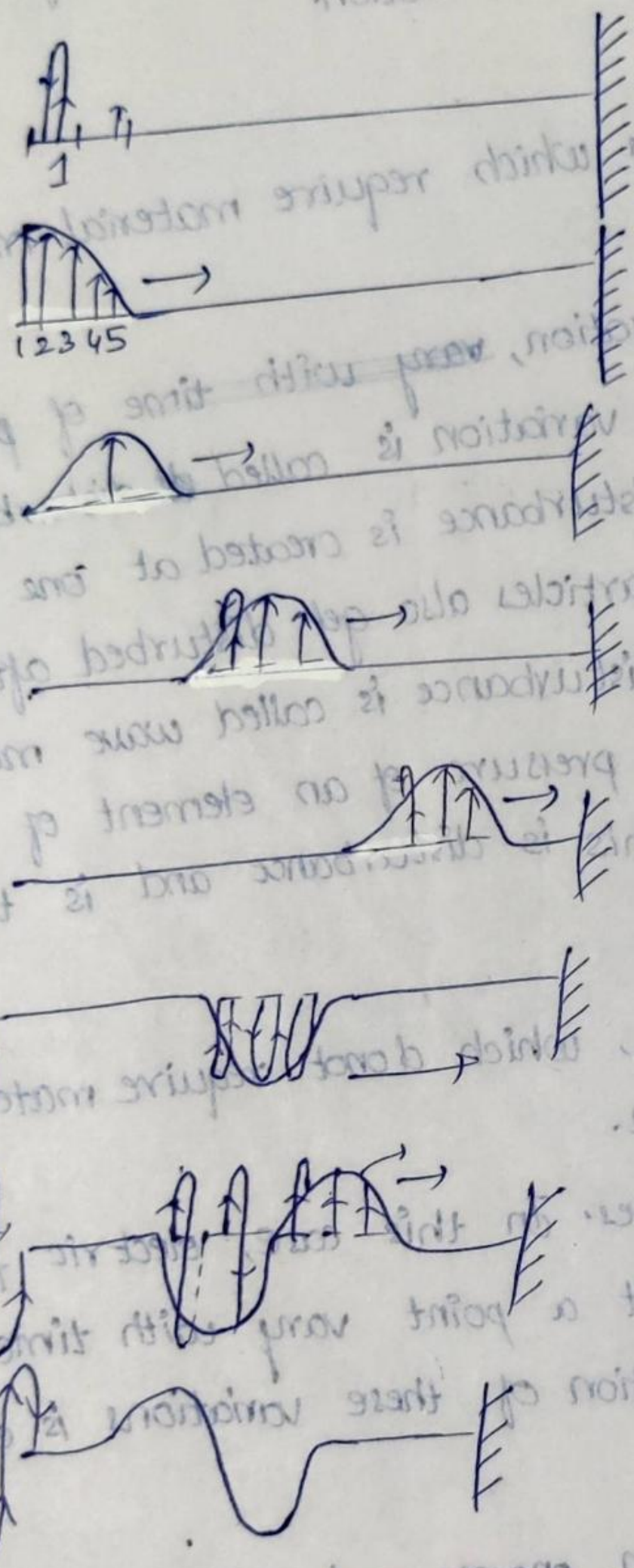
Transverse waves:

* These are the waves in which particles are oscillated ⊥ar to the direction of propagation of waves.

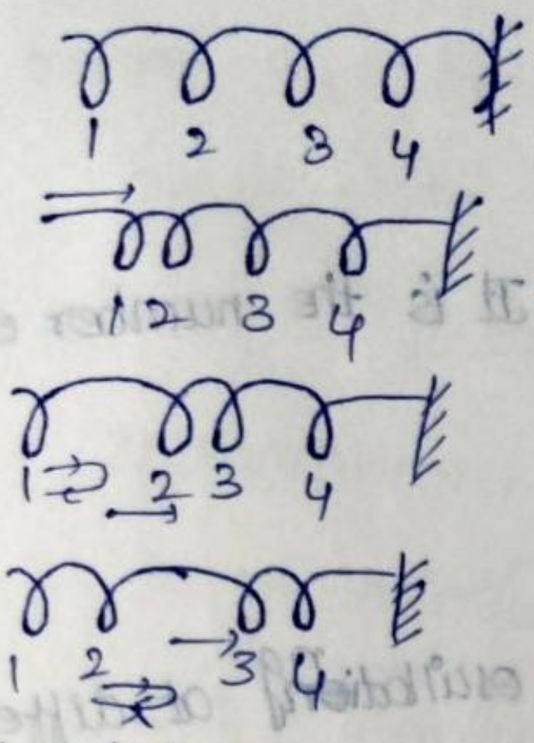


* If a stiff string is plucked, then, transverse vibrations are generated that point propagations of these vibrations are transverse waves
* All electromagnetic waves are transverse waves only

Longitudinal waves:

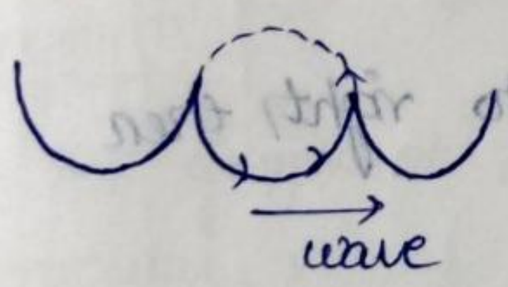
* These are the waves in which the vibrations of particles are parallel to the direction of propagation of wave

Ex: Compressional waves in spring
Sound waves in air



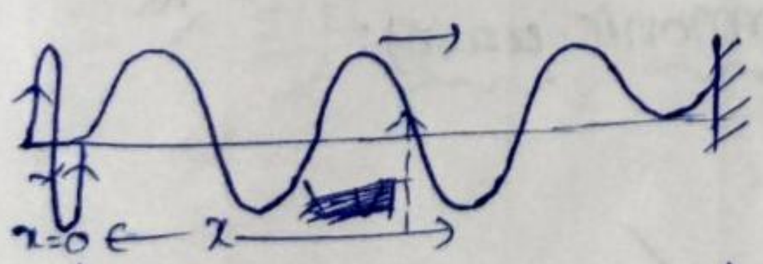
spring in relaxed state

* ~~Mechanical~~ Longitudinal waves are ~~longitudinal~~ mechanical waves only
* Transverse mechanical waves can travel through solids only but not through liquids and gases. This is because liquids & gases cannot sustain shear.
* Longitudinal waves can travel through solids, liquids & gases
* Waves on liquid surface will have transverse and longitudinal components. The particle moves along circle. These are called surface waves



Harmonic Waves:

* During wave motion, if the particles of medium execute SHM, then it is called harmonic wave. It can be generated by oscillating element 1 simple harmonically



* When there are transverse harmonic waves in a streched string, its snap shot at any instant will be in the form of sine curve
* Distance travelled by a wave in a time equal to time period

of oscillation T is called wave length (λ).

$$\lambda = vT$$

$$= \frac{v}{n}$$

$$\boxed{v = n\lambda}$$

* Here n = frequency of wave also. It is the number of waves crossing a point/sec.

Phase difference (ϕ):

* As the particles of medium start oscillating at different instants of time, phases of different particles will be different at an instant.
* If λ is the distance b/w any two particles, then phase difference b/w them is 2π
* If x is the distance b/w two particles, then phase difference b/w them $= \frac{2\pi x}{\lambda} = kx$. Here $k = \frac{2\pi}{\lambda}$ is called angular wave number (or) propagation constant

S.I units → rad/m

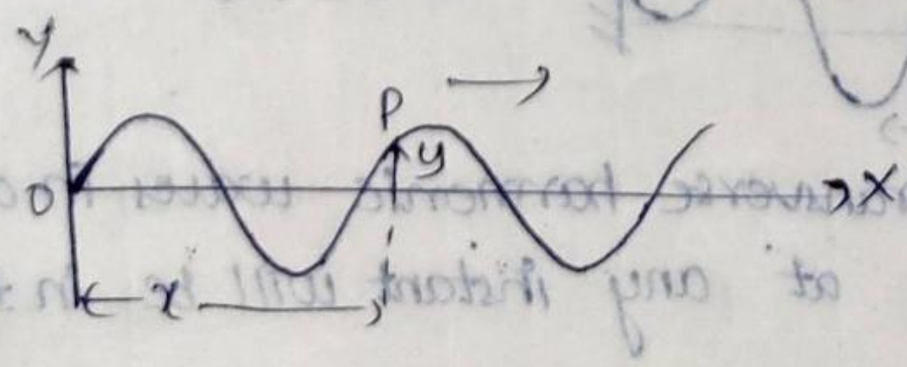
* If the wave travels from left to right, then

$$(\text{phase})_A - (\text{phase})_B = kx$$

Similarly if the wave is travelling from right to left, then

$$(\text{phase})_B - (\text{phase})_A = kx$$

Progressive wave Equation (Harmonic wave):



Let the wave is propagating along +ve X-axis

$(\text{phase})_O > (\text{phase})_P$

For particle at "O", $y = A\sin(\omega t + \phi_0)$

where ϕ_0 = initial phase of particle at $x = 0$

$0 \le \phi_0 \le 2\pi$

$(\text{phase})_P = (\omega t + \phi_0) - kx$

Instantaneous displacement of particle P is,

$$y = A\sin[\omega t + \phi_0 - kx]$$

$$\boxed{y = A\sin((\omega t - kx) + \phi_0)}$$

* If the wave travels along -ve x-axis,

$$(\text{phase})_P = (\omega t + kx) + \phi_0$$

$$y = A\sin[(\omega t + kx) + \phi_0]$$

On generalising, $\boxed{y = A\sin((\omega t \pm kx) + \phi_0)}$ → valid for both transverse & longitudinal waves travelling harmonic wave equation

* This equation represents the motion of all particles participating in wave motion.

wave-diagram:

* It is y-x graph at a particular time t (or) y-t graph at a particular x.

Slope of y-x graph $\frac{dy}{dx} = \pm Ak\cos[(\omega t + \phi_0) \pm kx] = S$

Slope of y-t graph $\frac{\partial y}{\partial t} = A\omega\cos[\omega t + \phi_0 \pm kx]$

$= \text{velocity of particle } (v_p)$

$\frac{v_p}{S} = \frac{\pm\omega}{k}$, $\frac{\omega}{k} = \frac{2\pi n}{\frac{2\pi}{\lambda}} = n\lambda$

$= V$ = wave velocity

$v_p = \pm VS$

* -ve sign is used when wave travels along +ve x-axis
* +ve sign is used when wave travels along -ve x-axis
* This relation is valid for wave of any shape

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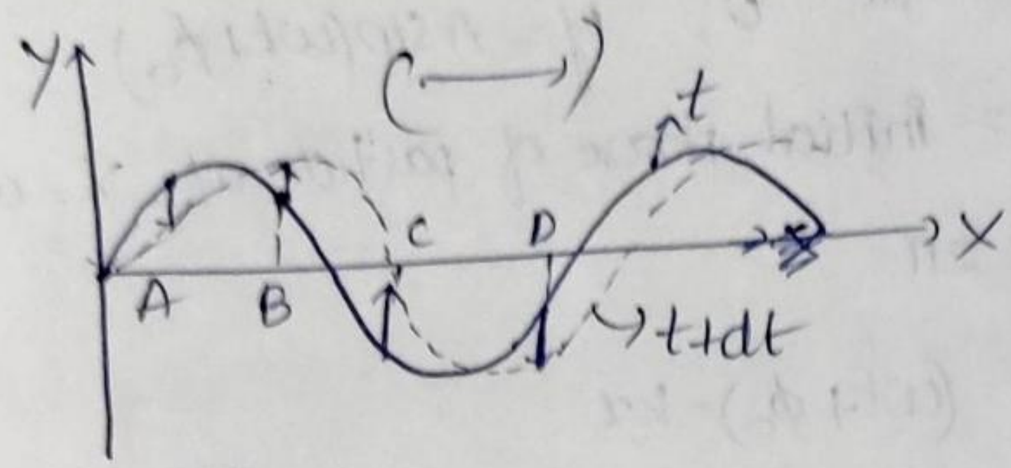
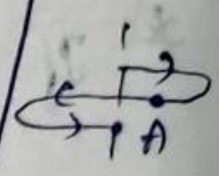
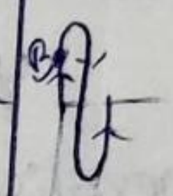
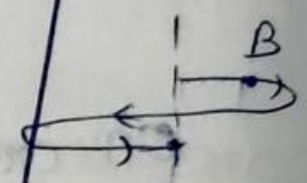
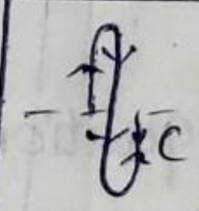
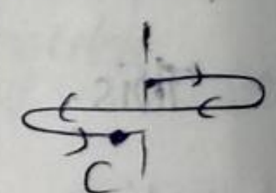
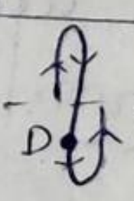
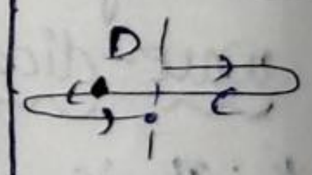
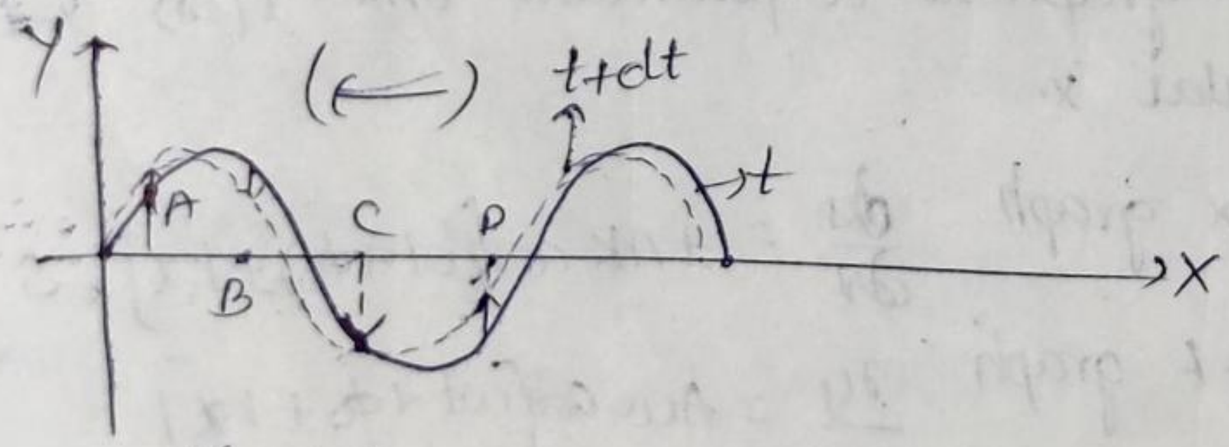
finding direction of v_p from wave diagram:

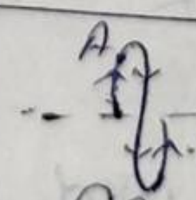
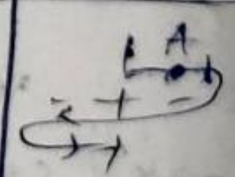
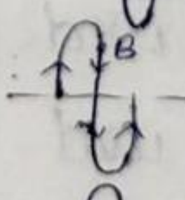
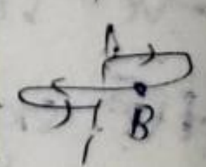
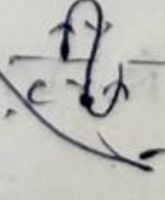
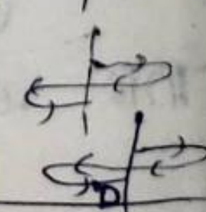
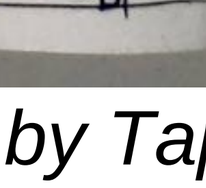


$v_p = -vs$

Particle	sign of Y	Sign of S	Sign of v_p	State of oscillation (transverse)	longitudinal
A	+	+	−		
B	+	−	+		
C	−	−	+		
D	−	+	−		



$v_p = +vs$

Particle	Sign of Y	S	v_p	State of oscillation (transverse)	longitudinal
A	+	+	+		
B	+	−	−		
C	−	−	−		
D	−	+	+		

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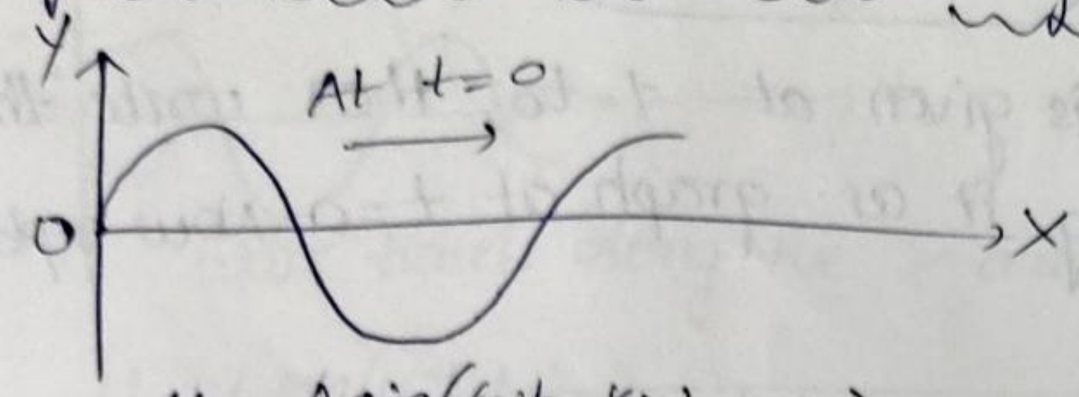
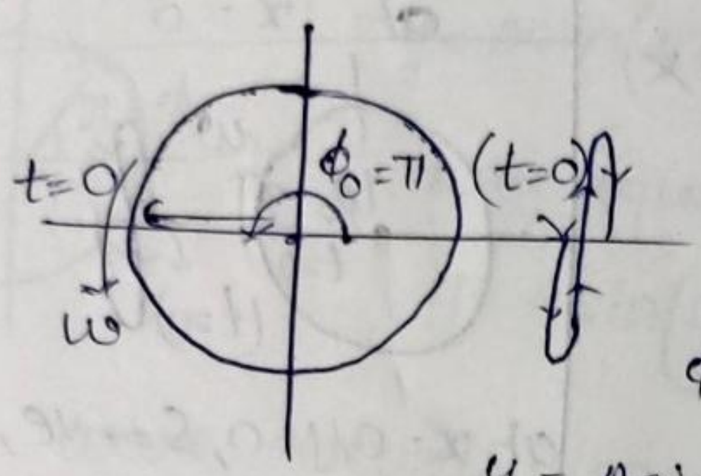
Writing wave equation from wave diagram:



$$y = A\sin((\omega t - kx) + \phi_0)$$

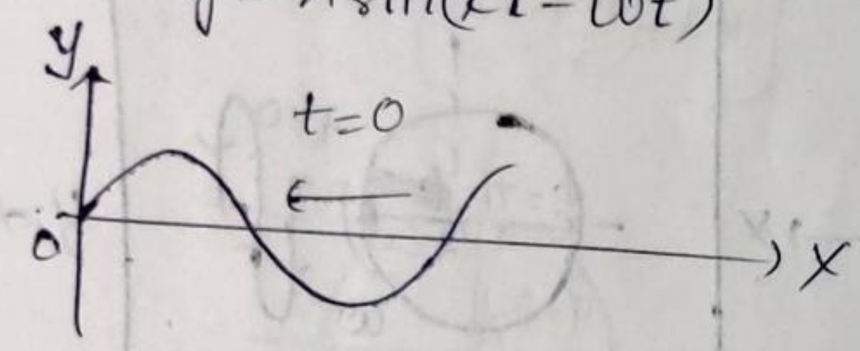
At $x=0$, $y=0$, "v_p" is -ve as "s" is +ve



$\phi_0 = \pi$

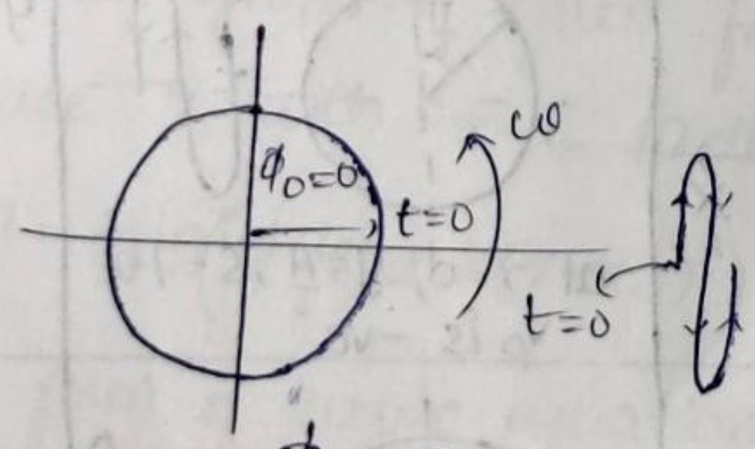
$$y = A\sin(\omega t - kx + \pi)$$

$$y = A\sin(kx - \omega t)$$



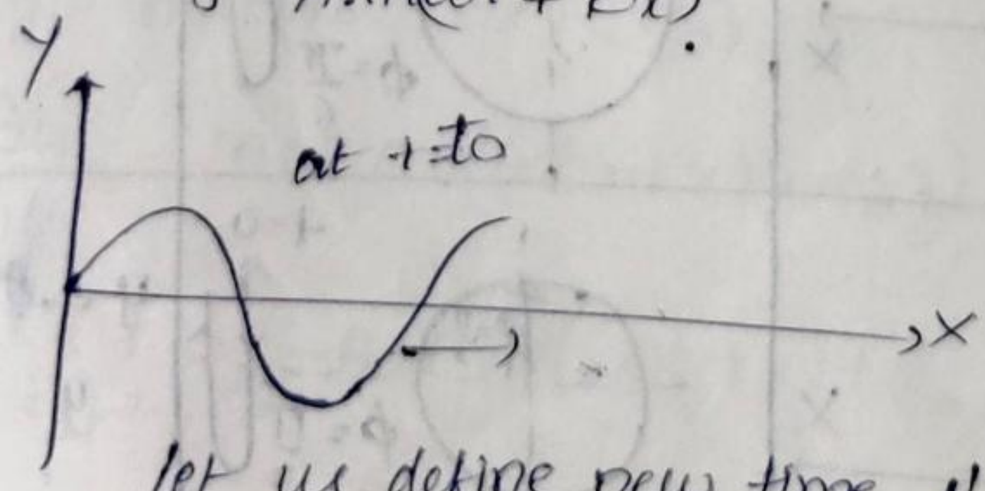
$$y = A\sin(\omega t + kx + \phi_0)$$

At $x=0$, $y=0$, v_p is +ve as s is +ve



$\phi_0 = 0$

$$y = A\sin(\omega t + kx)$$



let us define new time t' as $t'=0$ at time $t=t_0$

$$y = A\sin(kx - \omega t')$$

$$t = t_0 + t' \Rightarrow t' = t - t_0$$

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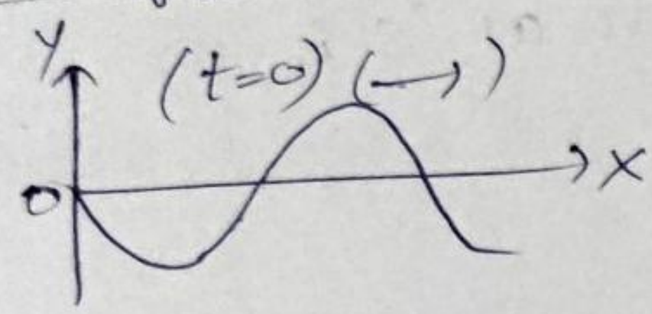
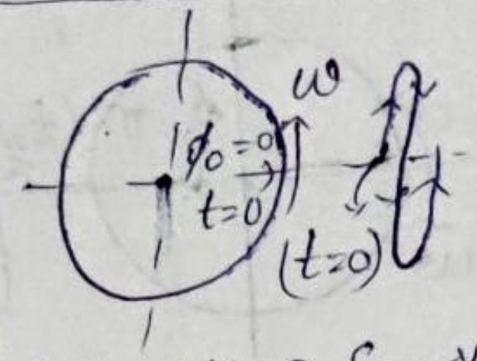
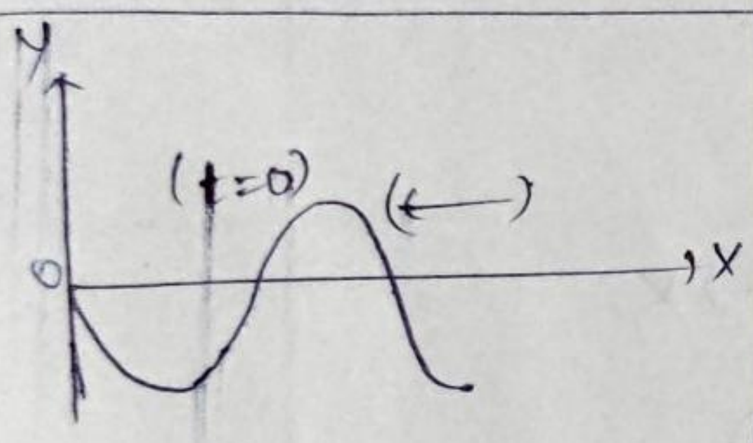
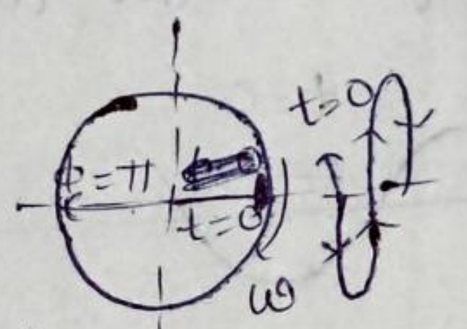
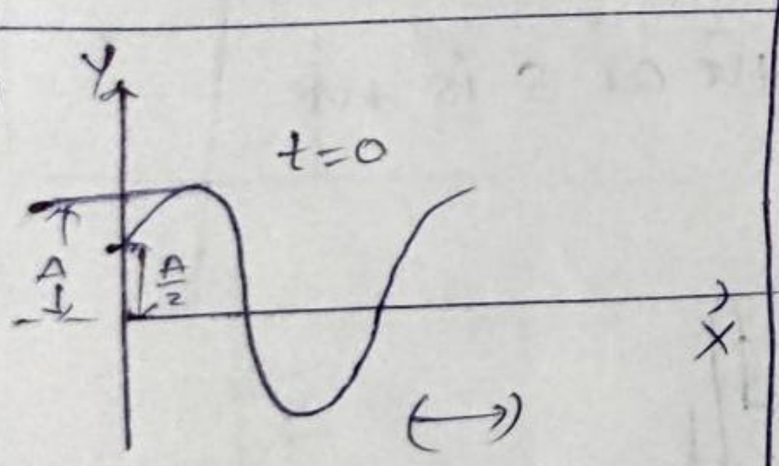
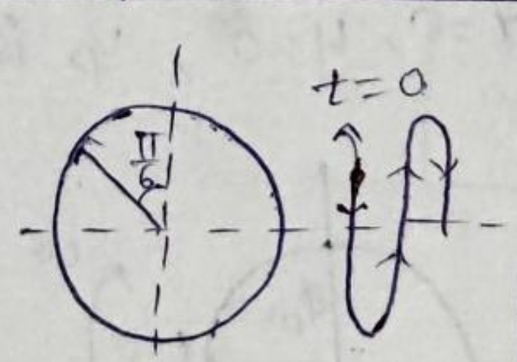
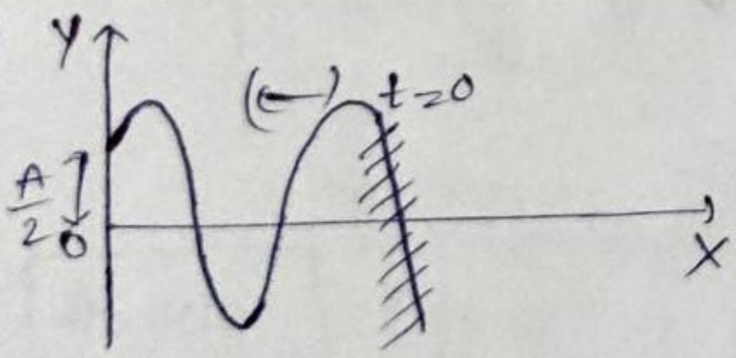
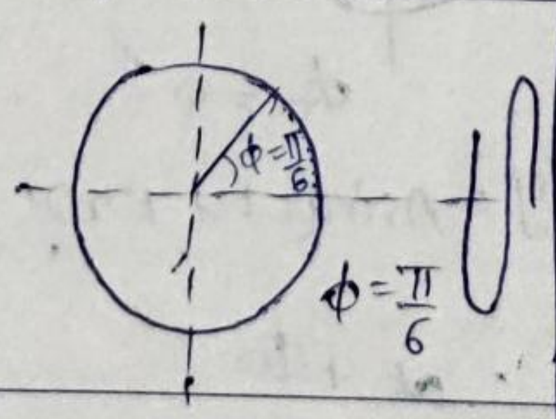
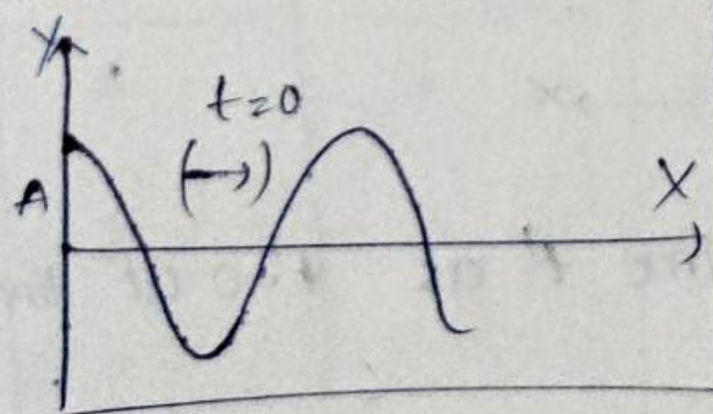
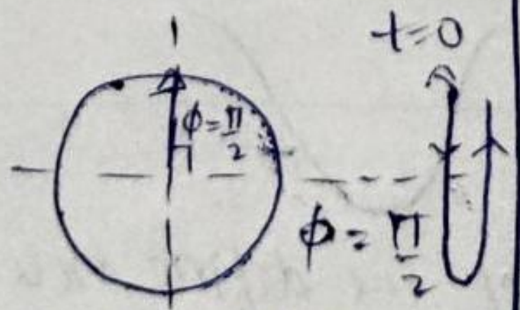
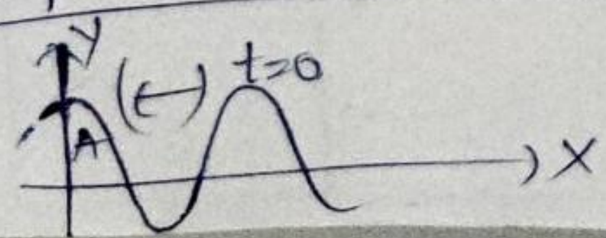
$$y = A\sin(kx - \omega(t - t_0))$$

* If y-x graph is given at $t = t_0$, then write the wave equation treating it as graph at $t = 0$. Now replace t with $t - t_0$

Wave diagram	ϕ_0, state of oscillation at $x = 0$	Wave equation
1) 	 at $x = 0, y = 0, S = -ve$, v_p is +ve	$y = A\sin(\omega t - kx)$
2) 	 at $x = 0$, $y = 0$, $S = -ve$ v_p is -ve	$y = A\sin(\omega t + kx)$
3) 	 at $x = 0$, $y = \frac{A}{2}$, $S = +ve$ v_p is -ve	$y = A\sin(\omega t - kx + \frac{\pi}{6})$
4) 		$y = A\sin(\omega t + kx + \frac{\pi}{6})$
5) 	 at $x = 0$, $y = +A$, $S = -ve$ v is +ve	$y = A\sin(\omega t - kx + \frac{\pi}{2})$ $y = A\cos(\omega t - kx)$
6) 	$\phi = \frac{\pi}{2}$	$y = A\sin(\omega t + kx + \frac{\pi}{2})$ $= A\cos(\omega t + kx)$

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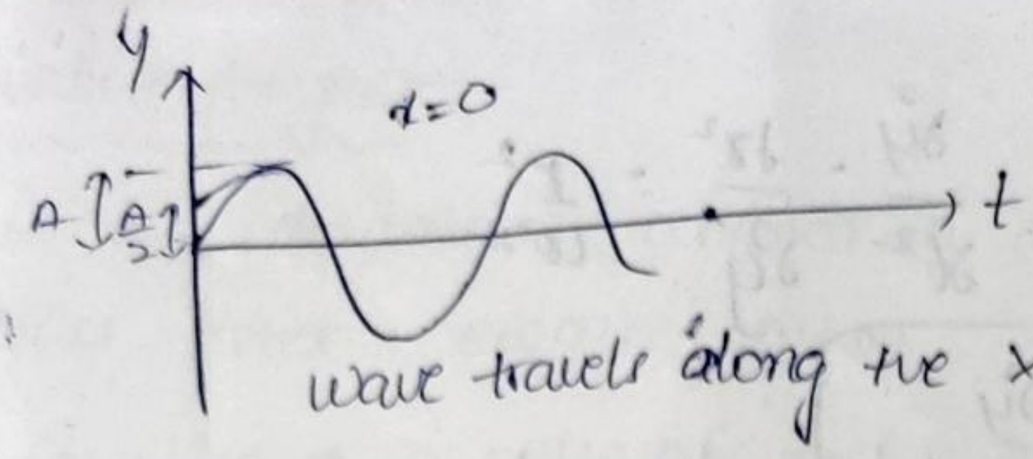
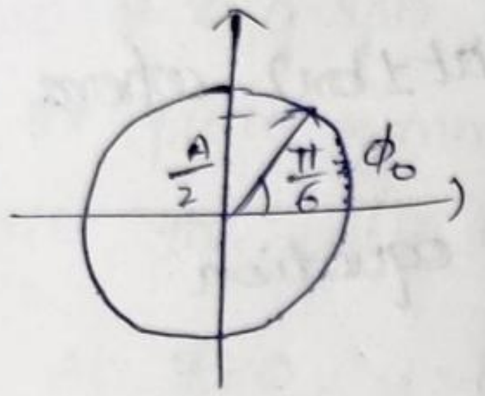


wave travels along the x-axis

$$y = A\sin(\omega t + \phi_0)$$

at $t=0$, $y = +\frac{A}{2}$, v_p is +ve



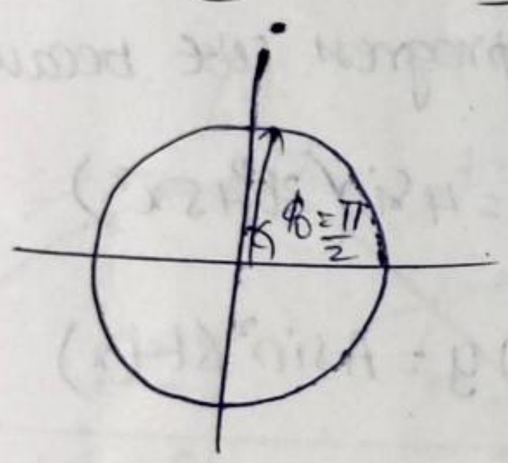
$$y = A\sin\left(\omega t + \frac{\pi}{6}\right)$$

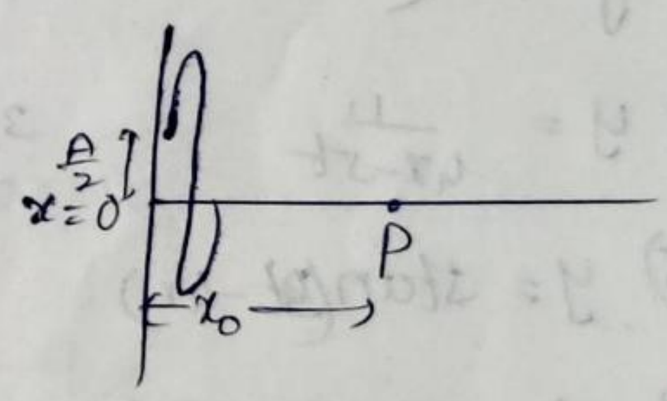
wave equation is

$$y = A\sin\left(\omega t + \frac{\pi}{6} - kx\right)$$

If the wave travels along ~~the~~ -ve x-axis, then

$$y = A\sin\left[\left(\omega t + \frac{\pi}{6}\right) + kx\right]$$





* If the same graph is given for the particle at $x = x_0$ then replace x with $x - x_0$ in the above equation

$$y = A\sin\left(\omega t + \frac{\pi}{6} + k(x - x_0)\right)$$

Differential form of wave equations

$$y = A\sin((\omega t - kx) + \phi_0)$$

$$\frac{\partial y}{\partial t} = A\omega\cos((\omega t - kx) + \phi_0))$$

$$\frac{\partial^2 y}{\partial t^2} = -A\omega^2\sin((\omega t - kx) + \phi_0) \quad \text{— (1)}$$

$$\frac{\partial y}{\partial x} = -Ak\cos((\omega t - kx) + \phi_0)$$

$$\frac{\partial^2 y}{\partial x^2} = -Ak^2\sin((\omega t - kx) + \phi_0) \quad \text{— (2)}$$

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$$\frac{①}{②} \Rightarrow \cancel{\frac{\partial x^2}{\partial t^2}} \quad \frac{\partial^2 y}{\partial t^2} \cdot \frac{\partial x^2}{\partial^2 y} = \frac{1}{\omega^2}$$

$$\boxed{\frac{\partial^2 y}{\partial t^2} = v^2 \cdot \frac{\partial^2 y}{\partial x^2}}$$

* An equation represents progressive wave equation if

(i) It is in the form of $y = f(\pm at \pm bx)$ where a, b are +ve constants

(ii) y satisfies above differential equation

(iii) y is continuous

(iv) y is differentiable

(v) y is finite

Ex 1) $y = \ln(3t + 4x)$ → Not progress w.e because it is not defined at $x = t = 0$

2) $y = \frac{1}{4x - 5t}$ 3) $y = 4\sin(3t^2 + 5x^2)$ 4) $y = 2xt$

5) $y = \tan(3t - 4x)$ 6) $y = A\sin^2(3t - 4x)$

1) Sol: Not P.W.E because as x, t increase ~~cont~~ $t \to \infty \Rightarrow y \to \infty$

2) Sol: Not P.W.E because at $x = 0$, $t = 0$ y = not defined

3) Sol: Not P.W.E as $y \neq f(\pm at \pm bx)$

4) Sol: Not P.W.E as $y \neq f(\pm at \pm bx)$

5) Sol: Not P.W.E as y = Not defined at $3t - 4x = \frac{\pi}{2}$

6) Sol: It is P.W.E as $y = f(\pm at \pm bx)$ and it satisfies all conditions

* If $y = f(\pm at \pm bx)$ represents a progressive wave equation, then wave velocity $v = \frac{a}{b}$ 18/11

Equation for pulse:

* Pulse is a disturbance confined in a finite region. The particles donot execute S.H.M.

Ex: Equation of a pulse at $t=0$ is given as $y = \frac{8}{5+4x^2}$. This pulse is travelling along -ve x-axis with a speed of 3m/s

(i) Draw y-x graph at $t=0$, $t=0.2$sec

(ii) Is the pulse symmetric?

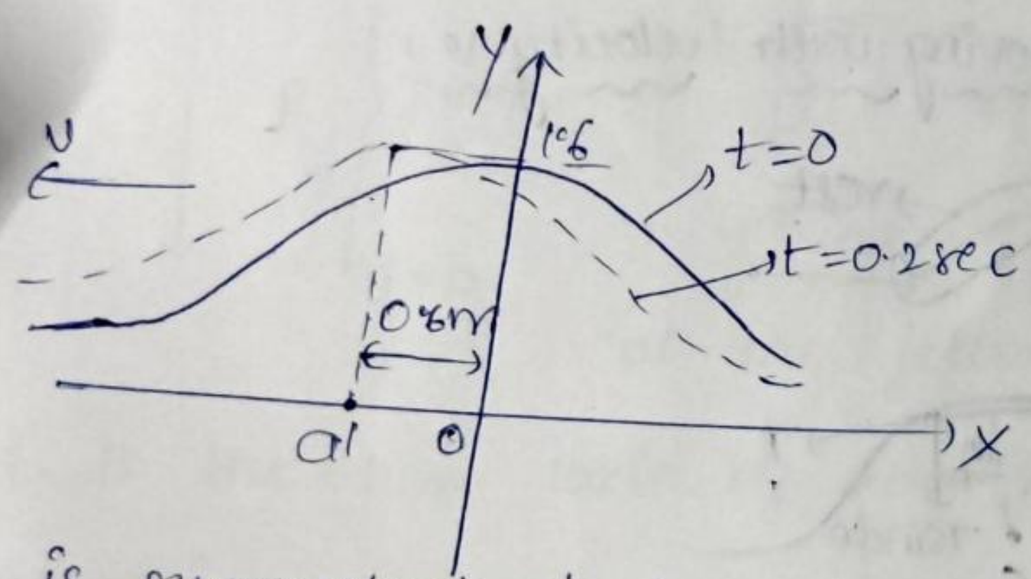
(iii) Write the pulse equation

Sol: (i) At $x=0$, $y=\frac{8}{5} = 1.6$ units $= y_{max}$

at $x=\infty$, $y=0$

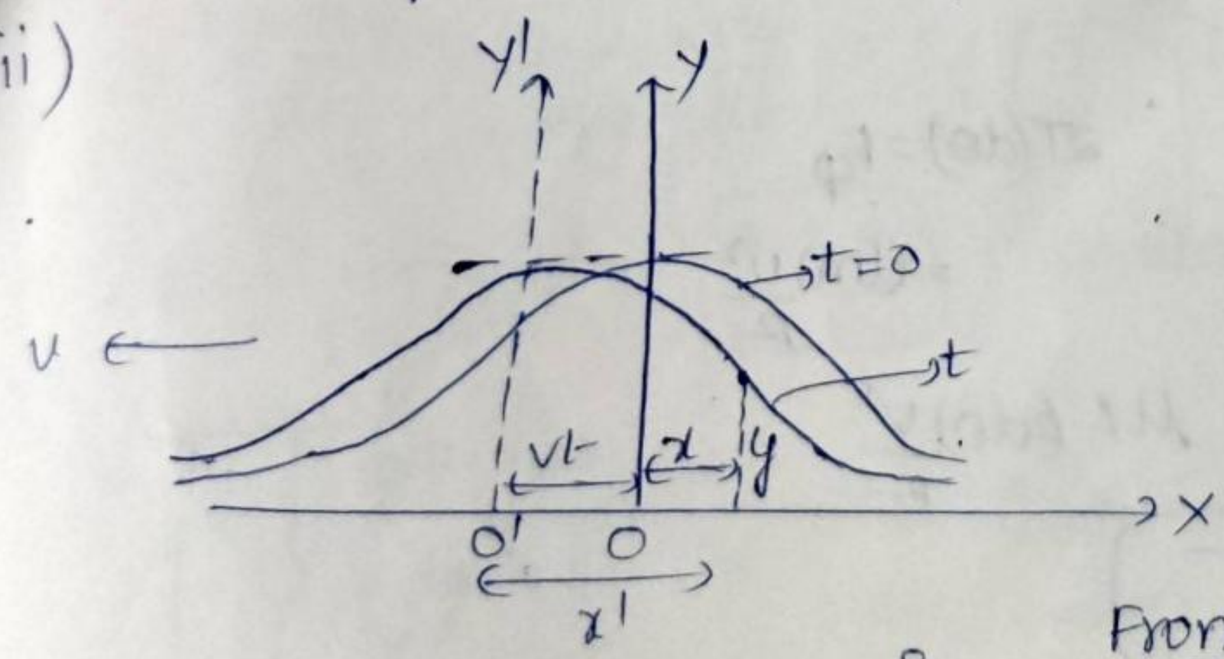
In $\Delta t = 0.2$sec, distance travelled by pulse $\Delta s = v(\Delta t)$

$= 3\times 0.2 = 0.6$m



(ii) Pulse is symmeteric about peak as $f(x) = f(-x)$ at $t=0$

(iii)



From the figure, $x+vt = x'$

At time t, $y = \frac{8}{5+4(x')^2}$ $\boxed{y = \frac{8}{5+4(x+vt)^2}}$

* It can be generalised that a wave equation can be obtained by replacing x with $x \pm vt$ in the equation of the wave given at $t=0$

* In the same way, replace t with $t \pm \frac{x}{v}$ in the equation

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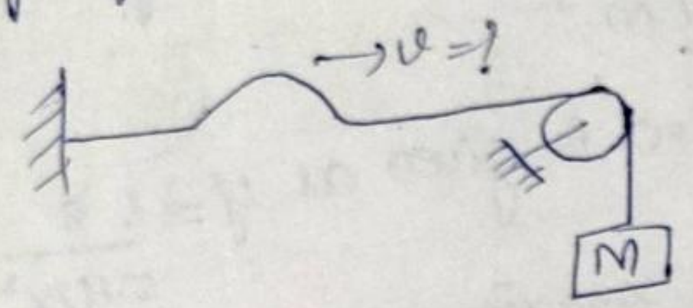
given at $x=0$

Velocity of transverse wave in a stretched string:



v = velocity of transverse pulse in the string w.r.t string

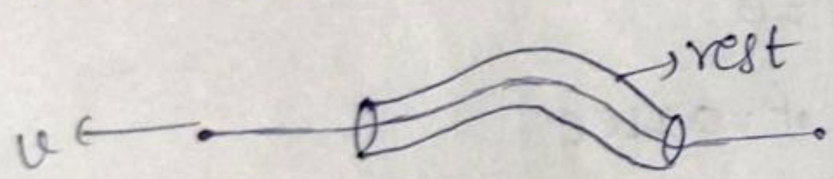
T = Tension in string

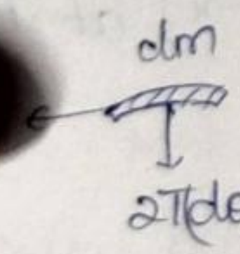
μ = Linear mass density of string

$= \frac{dm}{dl} = \frac{m}{l}$

G frame:

w.r.t a frame moving with velocity 'v':





$2T(d\theta) = F_{cp}$

$= (dm)\frac{v^2}{R}$

$2T(d\theta) = \mu R(d\theta)\frac{v^2}{R}$

$$\boxed{v = \sqrt{\frac{T}{\mu}}}$$

* This is valid for small amplitude pulses only. T is the tension in string in the absence of wave. Even in the presence of wave, Tension can be assumed to be same because the change is very small due to small amplitude pulses

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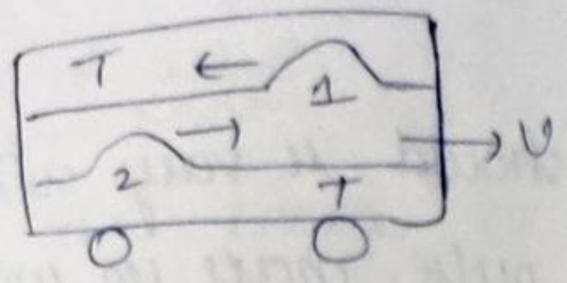
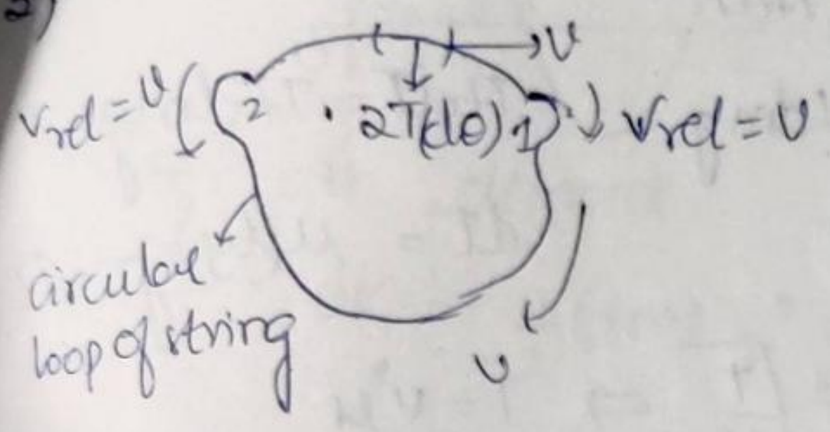
Ex(1)



w.r.t. ground, $v_1 = \left|v - \sqrt{\frac{T}{\mu}}\right|$

$v_2 = v + \sqrt{\frac{T}{\mu}}$

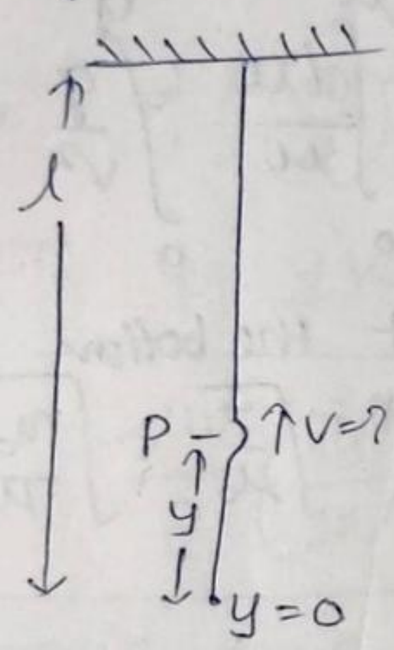
2)



$V_1 = V + V = 2V$

$V_2 = V - V = 0$

3) Vertical string :



At P:

$T = \mu y g$

$V = \sqrt{\frac{T}{\mu}} = \sqrt{gy}$

$y\uparrow$ as $v\uparrow$

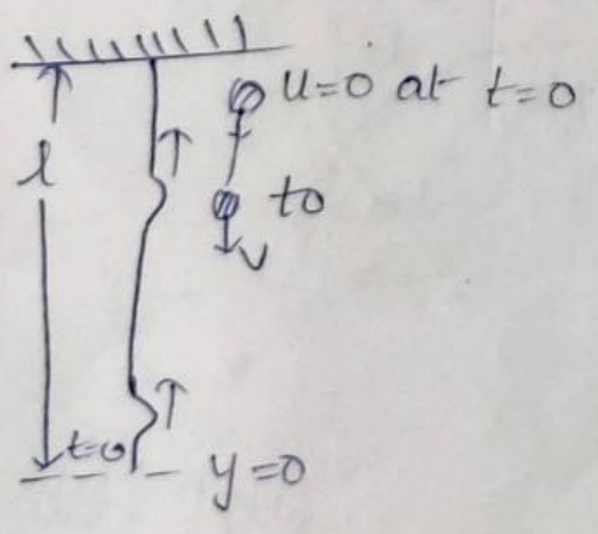
at the bottom, $y=0 \Rightarrow v=0$

Let t_0 is the time taken by pulse to move from top to bottom

$$a = \frac{dv}{dt} = \sqrt{g}\cdot\frac{1}{2\sqrt{y}}\cdot\frac{dy}{dt} = \frac{1}{2}\sqrt{\frac{g}{y}}\cdot v = \frac{1}{2}\sqrt{\frac{g}{y}}\cdot\sqrt{gy}$$

$$\boxed{a = \frac{g}{2}} = \text{constant} \Rightarrow l = \frac{1}{2}\frac{g}{2}(t_0^2) \Rightarrow \boxed{t_0 = \sqrt{\frac{4l}{g}}}$$

4)



what is the height "h" at which pulse crosses the stone?

Sol: $a_{pulse} = \frac{g}{2}$, $a_{stone} = -g$

~~a rel~~ $a_{rel} = \frac{3g}{2}$, $S_{rel} = l$

$$S_{rel} = \frac{1}{2}\left(\frac{3g}{2}\right)t_0^2 \Rightarrow t_0^2 = \frac{4l}{3g} \Rightarrow \boxed{h = \frac{1}{2}\left(\frac{g}{2}\right)t_0^2 = \frac{l}{3}}$$

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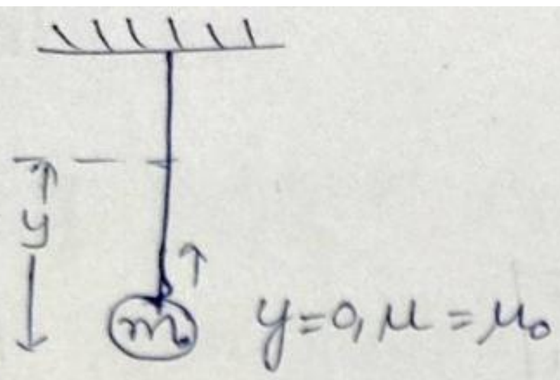
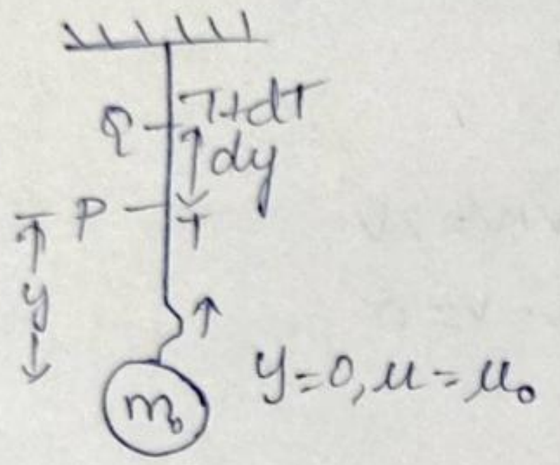
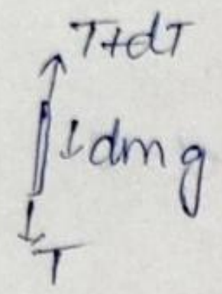
5)



How should μ vary with y so that pulse moves up with const velocity

Sol:





$T+dT = T+dmg$

$dT = \mu(dy)g \quad \text{--- (1)}$

$v = \sqrt{\frac{T}{\mu}} \Rightarrow T = v^2\mu$

$dT = v^2 d\mu$

$v^2 d\mu = \mu dy . g \Rightarrow \int_{\mu_0}^{\mu} \frac{d\mu}{\mu} = \int_0^y \frac{g}{v^2} . dy$

$\ln\left(\frac{\mu}{\mu_0}\right) = \frac{gy}{v^2}$

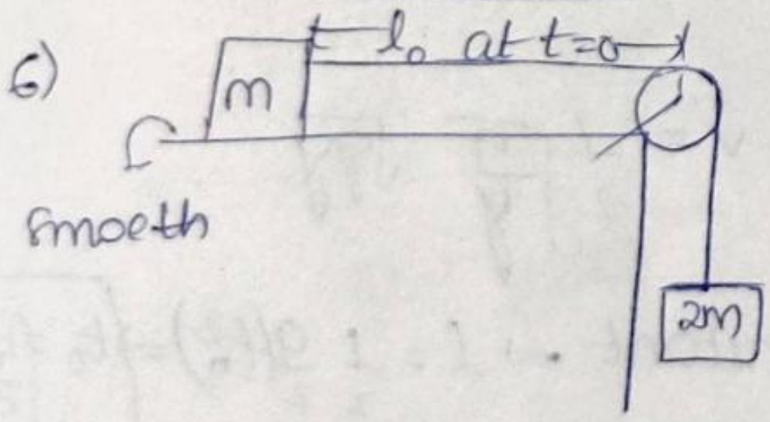
$= \frac{gy\mu_0}{m_0 g}$

at the bottom

$v = \sqrt{\frac{T}{\mu_0}} = \sqrt{\frac{m_0 g}{\mu_0}} \Rightarrow \boxed{v^2 = \frac{m_0 g}{\mu_0}}$

$$\boxed{\mu = \mu_0 e^{\frac{\mu_0 y}{m_0}}}$$

6)



If a pulse is generated in the string at m at $t=0$, what is the time at which pulse reaches the pulley. Blocks are also released at $t=0$

Sol:

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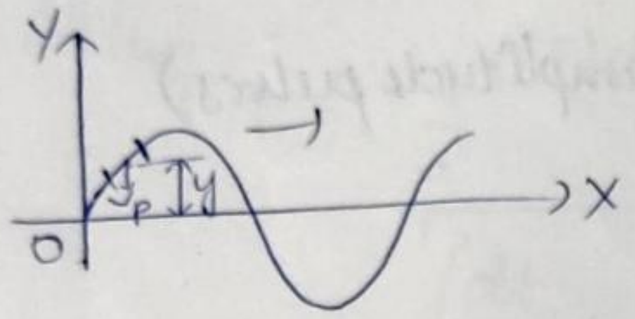
Energy of wave:

* It is the energy of medium in which wave is present
It is the sum of K.E and P.E



For an element,

$$dK = \frac{1}{2}(dm)\, v_p^2$$

$$= \frac{1}{2}(\mu dx)\left(\frac{\partial y}{\partial t}\right)^2$$

$$\frac{dK}{dx} = \frac{1}{2}\mu\left(\frac{\partial y}{\partial t}\right)^2$$

(or) $v_p = \pm v_s = v\left(\frac{\partial y}{\partial x}\right)$

$$\boxed{\frac{dK}{dx} = \frac{\mu v^2}{2}\left(\frac{\partial y}{\partial x}\right)^2}$$ → for any shape of wave

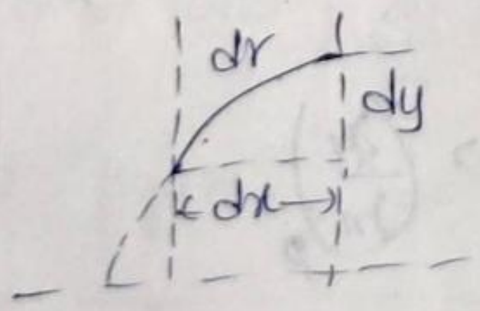
For harmonic wave, $y = A\sin(\omega t - kx + \phi_0)$

$$\frac{\partial y}{\partial x} = -Ak\cos(\omega t - kx + \phi_0)$$

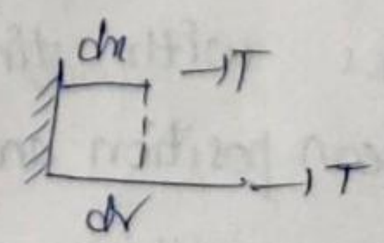
$$dK = \frac{1}{2}\mu v^2 dx\left[A^2k^2\cos^2(\omega t - kx + \phi_0)\right]$$

$$dK = \frac{1}{2}\mu A^2\omega^2 (dx)\left[\cos^2(\omega t - kx + \phi_0)\right]$$

Potential Energy:



when the length of the element is increased from dx to dr work done against T will be equal to its PE



$$dU = T(dl)$$

$$= T(dr - dx)$$

$$= v^2\mu\left(\sqrt{(dy)^2 + (dx)^2} - dx\right)$$

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$$dU = v^2\mu\, dx\left[\sqrt{1+\left(\frac{dy}{dx}\right)^2}-1\right]$$

$$= v^2\mu\, dx\left[1+\frac{1}{2}\left(\frac{dy}{dx}\right)^2-1\right]$$

$\left(\frac{dy}{dx} << 1\right.$ for small amplitude pulses)

$$dU = v^2\mu\, dx\left[\frac{1}{2}\left(\frac{dy}{dx}\right)^2\right]$$

$$\boxed{\frac{dU}{dx} = \frac{1}{2}v^2\mu\left(\frac{\partial y}{\partial x}\right)^2} = \frac{dK}{dx}$$

* For an element, $dK = dU$ at an instant

Total M.E of the element $dE = dU + dK$.

$$dE = v^2\mu\left(\frac{\partial y}{\partial x}\right)^2 \cdot dx$$

* For harmonic wave; $dE = v^2\mu\, dx \cdot A^2k^2\cos^2(\omega t - kx)$

$$dE = A^2\omega^2(s\rho)\, dx\cos^2(\omega t - kx)$$

s = Area of cross section of wire

ρ = Mass density of material

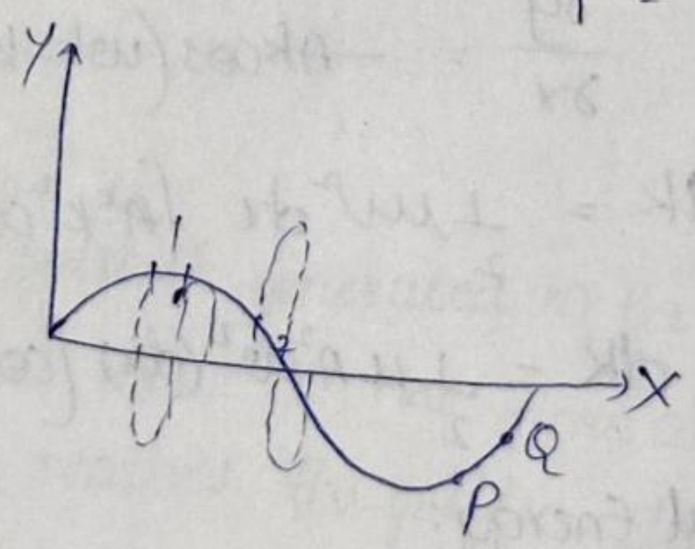
$$\mu = \frac{dm}{dx} = \frac{s(dx)\rho}{dx} = s\rho$$

For element 1,

$dK = 0$

$dU = 0$

$dE = 0$



For element 2,

dK, dU and dE are max

$$\left(\frac{dE}{dx}\right)_Q > \left(\frac{dE}{dx}\right)_P \quad \text{as} \quad \left(\frac{\partial y}{\partial x}\right)_Q > \left(\frac{\partial y}{\partial x}\right)_P$$

* During wave propagation, total energy of an element changes with time. It increases while moving from extreme to mean position and decreases while moving from mean to extreme position. But still it executes SHM

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Average energy:

* For a given element, mechanical energy is a function of time

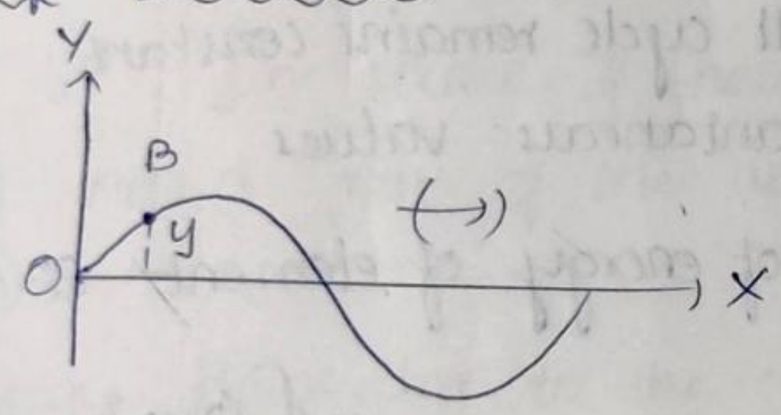
$$(dE)_{avg} = \frac{\int_0^T (dE)\,dt}{\int_0^T dt} = \frac{A^2\omega^2 s\rho\left(\frac{T}{2}\right)dx}{T}$$

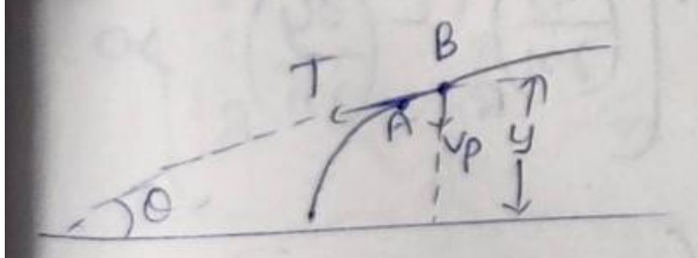
$$= \frac{1}{2}\rho A^2\omega^2 s(dx)$$

* This is independent of x and t
* Average mechanical energy confined in a length l of medium along wave propagation

$$(\Delta E)_{avg} = \int (dE)_{avg} = \frac{1}{2}\rho A^2\omega^2 s(l)$$

* Power of Transmittion:





Power delivered by left part of string on right part at B is

$$P = -(\sin\theta)v_p$$

As θ is very small; for small amplitude pulses

$$\sin\theta = \tan\theta = \frac{\partial y}{\partial x}$$

In figure, $v_p = -\frac{dy}{dt}$

$$\boxed{P = -T\frac{\partial y}{\partial x}\frac{\partial y}{\partial t}}$$ → general formulae for wave of any shape

For harmonic wave,

$$y = A\sin(\omega t - kx + \phi_0)$$

$$[-A\omega\cos(\omega t - kx + \phi_0)][+A\omega\cos(\omega t - kx + \phi_0)]$$

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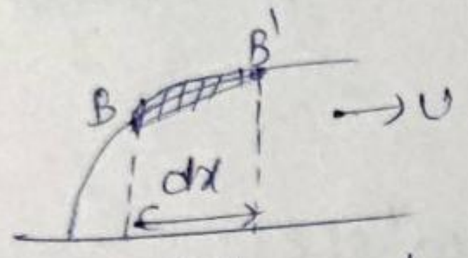
$$= + v^2 \mu A^2 k\omega \cos^2(\omega t - kx + \phi_0)$$

$$P = v s \rho A^2 \omega^2 \cos^2(\omega t - kx + \phi_0) \quad [\mu = \rho s,\ vk = \omega]$$

$$P_{avg} = \frac{\int_0^T P\,dt}{\int_0^T dt} = \rho A^2 \omega^2 s v \frac{\int_0^T \cos^2(\omega t - kx + \phi_0)}{T}$$

$$\boxed{P_{avg} = \frac{1}{2} \rho A^2 \omega^2 s v} \rightarrow \text{Independent of } x \text{ \& } t$$

* If we consider an element of string BB', avg energy delivered

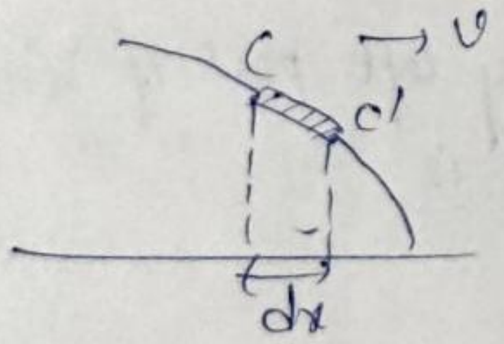


to it from left and avg energy delivered by it to the right will be same in one full cycle. That is why, avg energy of the element in one full cycle remains constant

* This is not true for instantaneous values

Rate of increase of energy of element (BB') $= \frac{dE}{dt}$

$$= TV\left[\left(\frac{\partial y}{\partial x}\right)_B^2 - \left(\frac{\partial y}{\partial x}\right)_{B'}^2\right] > 0$$



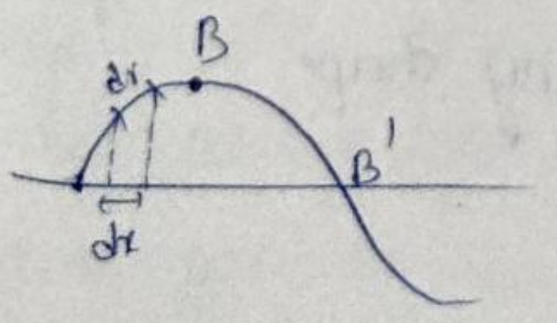
For element CC'

$$\frac{dE}{dt} = TV\left[\left(\frac{\partial y}{\partial x}\right)_C^2 - \left(\frac{\partial y}{\partial x}\right)_{C'}^2\right] < 0$$

i.e, $E_{CC'}$ decreases with time

* If the wave travels leftwards, the above conclusions are interchanged

Strain



$$\text{strain} = \frac{de}{dx} = \frac{dr - dx}{dx} = \frac{\sqrt{(dx)^2 + (dy)^2} - dx}{dx}$$

$$= \frac{dx\left[1 + \frac{1}{2}\left(\frac{dy}{dx}\right)^2\right]}{dx} - 1$$

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$$\text{strain} = \frac{1}{2}\left(\frac{\partial y}{\partial x}\right)^2$$

$(\text{strain})_B = 0$, $(\text{strain})_{B'}$ is max (for any direction of wave propagation)

Intensity of propagation:

* It is the power transmitted per unit area ⊥ar to the flow

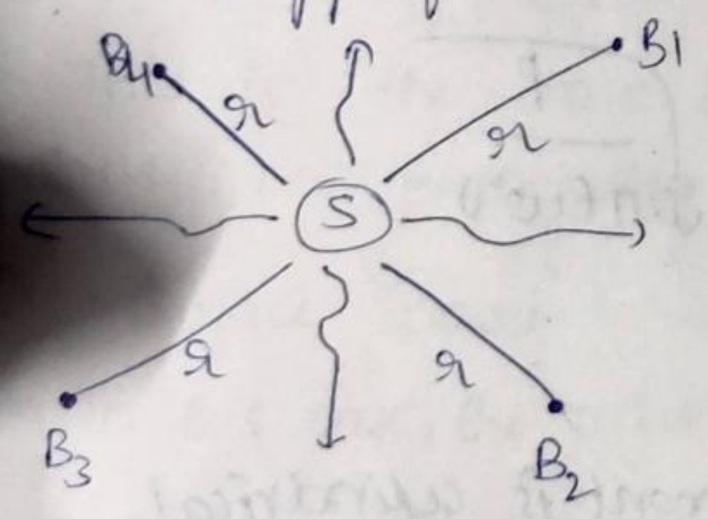
$$I = \frac{P}{S}$$

$$I_{avg} = \frac{P_{avg}}{S}$$

$$\boxed{I_{avg} = \frac{1}{2}\rho A^2 \omega^2 v}$$

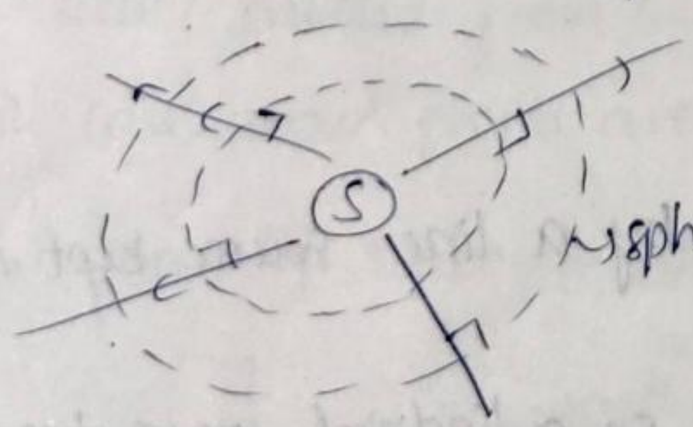
Wave front:

* It is the locus of all points which are in same phase.
* Wave fronts are spherical due to a point source and cylindrical due to a line source. If these sources are far from the observer, then a part of the wave front appears like a plane
* Wave energy flows ⊥ar to the wave front

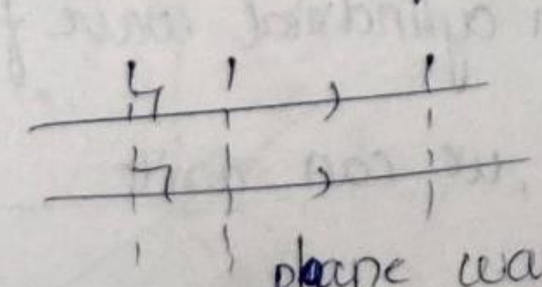


"S" is a point source of wave (sound)

B_1, B_2, B_3 are in same ~~plane~~ phase

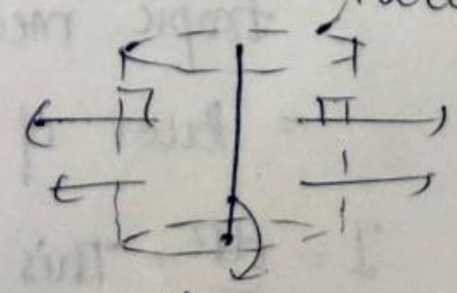


spherical wave front



plane wave fronts

cylindrical wave front



line source

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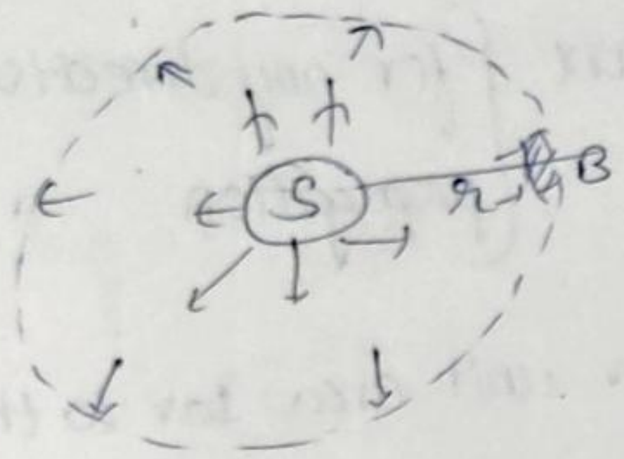
Amplitude Variation:



P = Power of emission by a point source kept in isotropic medium

= Power of incidence on spherical wave front

At B: $I = \frac{dP}{dA}$. As this is uniform, we can write

$$I_{avg} = \frac{P}{\text{Area}} = \frac{P}{4\pi r^2}$$

If the waves are harmonic

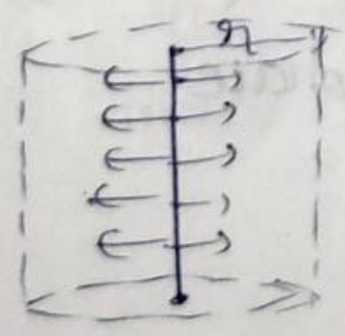
$$\frac{1}{2}\epsilon A^2\omega^2 v = \frac{P}{4\pi r^2}$$

$$A^2 = \frac{\left(\frac{P}{2\pi\epsilon\omega^2 v}\right)}{r^2}$$

$$A = \frac{A_0}{r} \quad \text{where} \quad A_0 = \sqrt{\frac{P}{2\pi\epsilon\omega^2 v}}$$

$$\boxed{y = \frac{A_0}{r}\sin(\omega t - kx + \phi_0)}$$

* In case of a line source, wave front is cylindrical



P = Power of emission by a line source kept in an isotropic medium

= Power of incident on cylindrical wave front

$I = \frac{dP}{dA}$. This is uniform, we can write

$$I_{avg} = \frac{P}{\text{Area}} = \frac{P}{2\pi r l}$$

If waves are harmonic,

$$\frac{1}{2}\rho A^2 \omega^2 v = \frac{P}{2\pi r l}$$

$$A^2 = \frac{\left(\frac{P}{\pi l \rho \omega^2 v}\right)}{r}$$

$$A = \frac{A_0}{\sqrt{r}} \quad \text{where} \quad A_0 = \sqrt{\frac{P}{\pi l \rho \omega^2 v}}$$

$$\boxed{y = \frac{A_0}{\sqrt{r}} \sin(\omega t - kx + \phi_0)}$$

Imp 12

* 1-Dimensional wave:

* It is the wave in which the disturbance is confined to a line

Ex: Transverse waves in a string.

* In this case, wave front is a point only

* Two-Dimensional wave:

* This is the wave in which disturbance is confined to a plane i.e, propagating in a plane

Ex: Surface waves.

* In this case, the wave front is a curve (or) a line

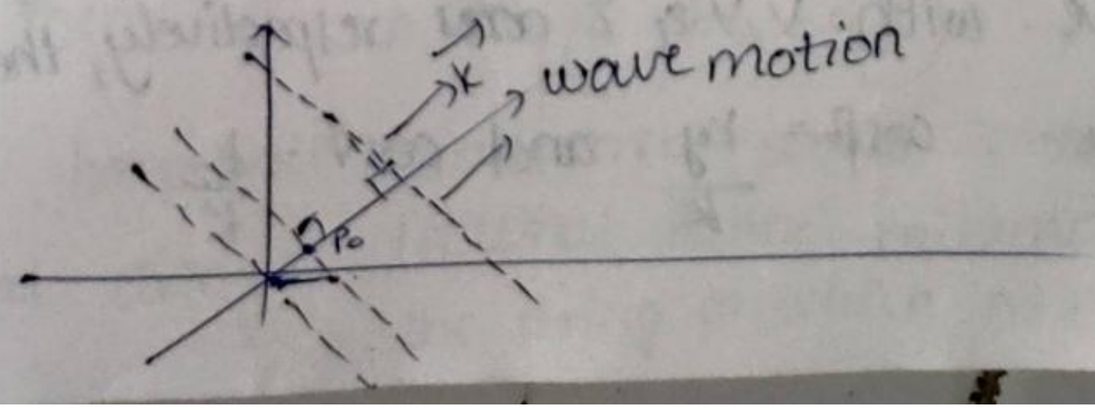
* 3-Dimensional waves:

* These are the waves in which, disturbance spreads in space

Ex: Sound waves emitted from a point source

* In this case, wave fronts are surfaces

Ex1) General form of wave equation (plane wave fronts)



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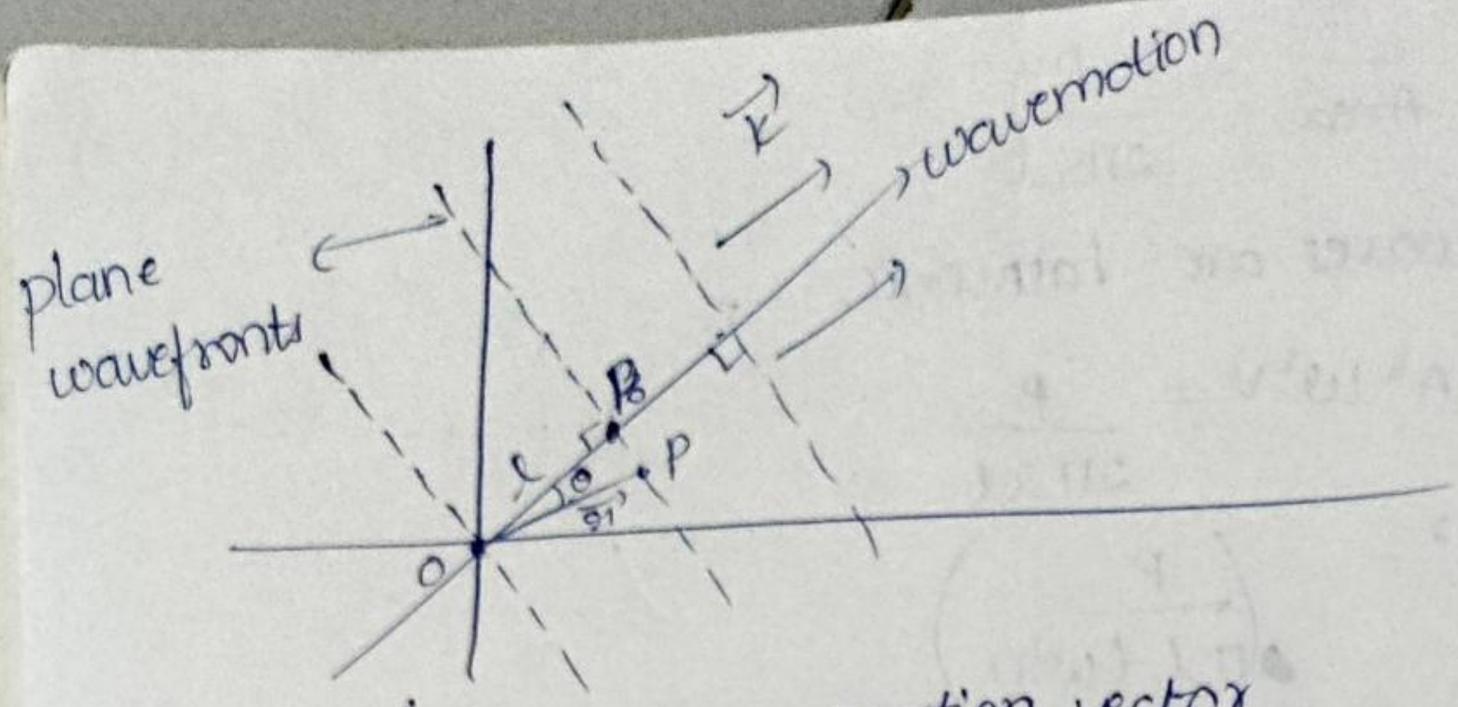

$\vec{k}$ is called propagation vector

$$\vec{k} = k\hat{k}$$

$$k = \frac{2\pi}{\lambda}$$

$\hat{k}$ is a unit vector along direction of motion of wave

For P_0, $y = A\sin[\omega t - kl + \phi_0]$

P and P_0 lie on same wave front

$y_P = y_{P_0}$ at any instant

$= A\sin[\omega t - kl + \phi_0]$

From figure, $\cos\theta = \frac{l}{r} \Rightarrow l = r\cos\theta$

$$y_{P_0} = A\sin[\omega t - kr\cos\theta + \phi_0]$$

$$= A\sin[\omega t - \vec{k}\cdot\vec{r} + \phi_0]$$

= instantaneous displacement of a particle at $\vec{r}$

$\vec{k} = k_x\hat{i} + k_y\hat{j} + k_z\hat{k}$, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$y_P = A\sin[\omega t - (k_x x + k_y y + k_z z) + \phi_0]$$

$k = \sqrt{k_x^2 + k_y^2 + k_z^2}$, $k_x = \frac{\omega}{v_x}$, $k_y = \frac{\omega}{v_y}$, $k_z = \frac{\omega}{v_z}$

$$v^2 \neq v_x^2 + v_y^2 + v_z^2$$

* If α, β, γ are the angles made by the direction of propagation of wave with x, y & z axes respectively, then

$\cos\alpha = \frac{k_x}{k}$ $\cos\beta = \frac{k_y}{k}$ and $\cos\gamma = \frac{k_z}{k}$.

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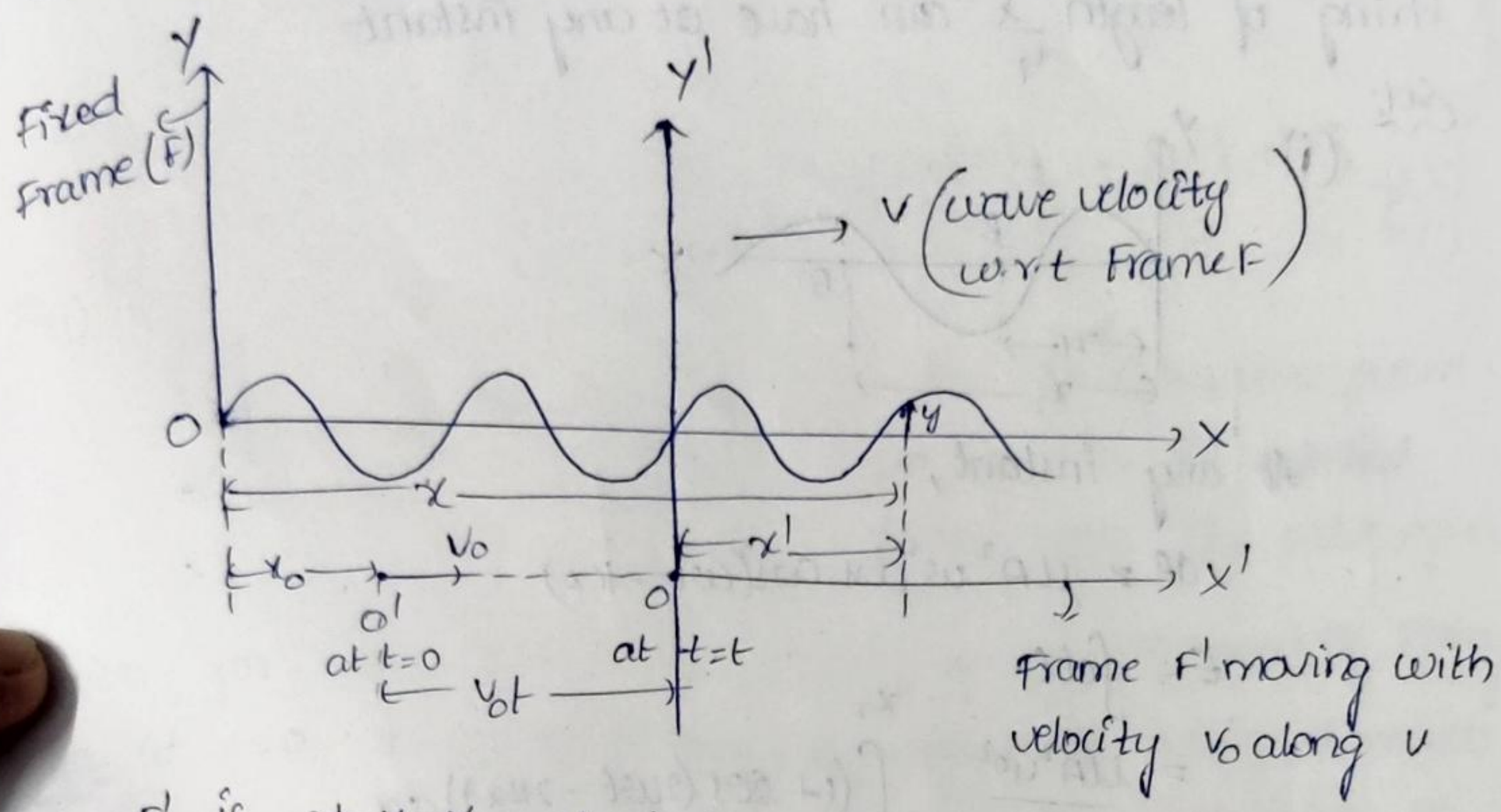
Wave equation from a moving frame of reference:



F' is at $x = x_0$ at $t = 0$

$$y = A\sin(\omega t - kx + \phi_0)$$

From figure $x_0 + v_0 t + x' = x$

$$y = A\sin(\omega t - k(x_0 + v_0 t + x') + \phi_0)$$

$$= A\sin\Big[\underbrace{(\omega - kv_0)}_{\omega'} t - kx' + \underbrace{(\phi_0 - kx_0)}_{\phi_0'}\Big] \qquad (\omega = vk)$$

$$y = A\sin\Big[\underbrace{k(v - v_0)}_{v'} t - kx' + (\phi_0 - kx_0)\Big]$$

* If the frame moves along -ve X-axis, then replace v_0 with $-v_0$ in the above formula

* If the frame moves along +ve X-axis with constant acceleration a_0, initial velocity v_0, then

$$x = x_0 + v_0 t + \frac{1}{2} a_0 t^2 + x'$$

then $y = A\sin\Big[\Big(\omega - kv_0 - \frac{a_0 t}{2}\Big)t - kx' + (\phi_0 - kx_0)\Big]$

Ex1) A harmonic transverse wave is travelling along a streched string (i) What is the minimum length along the string in which M.E remains constant

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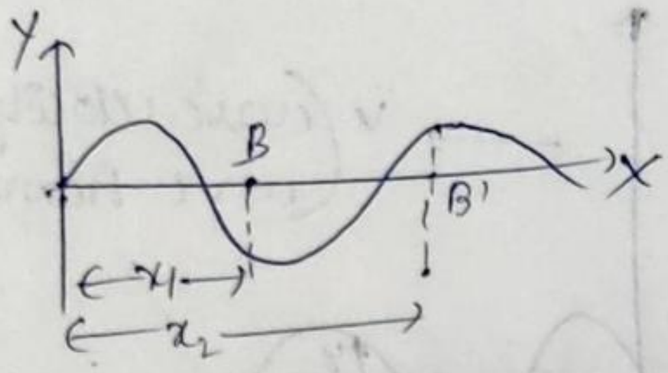
with time (ii) What is the maximum M.E that a string of length $\frac{\lambda}{4}$ can have at any instant

Sol:

(i)



At any instant,

$$dE = \mu A^2 \omega^2 dx \cos^2(\omega t - kx)$$

$$E = \int dE$$

$$= \frac{\mu A^2 \omega^2}{2} \int_{x_1}^{x_2} (1 + \cos(2\omega t - 2kx))\, dx$$

$$= \frac{\mu A^2 \omega^2}{2} \left[(x_2 - x_1) + \frac{1}{2k} \left(\sin(2\omega t - 2kx_2) - \sin(2\omega t - 2kx_1) \right) \right]$$

This is independent of time if

$$\sin(2\omega t - 2kx_2) = \sin(2\omega t - 2kx_1)$$

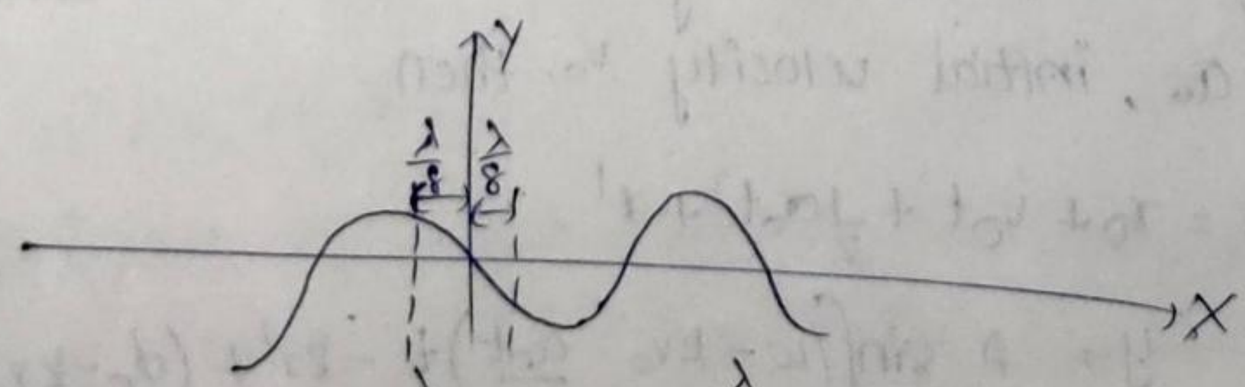
$$2\omega t - 2kx_2 + 2n\pi = 2\omega t - 2kx_1$$

$$2\left(\frac{2\pi}{\lambda}\right)(x_2 - x_1) = 2n\pi$$

$$\boxed{(x_2 - x_1)_{min} = \frac{\lambda}{2}}$$

(ii) M.E is maximum if the elements are at mean position

If the $\frac{\lambda}{4}$ segment is taken around mean position, then its M.E will be maximum



$$E = \int_{-\frac{\lambda}{8}}^{\frac{\lambda}{8}} dE = \int_{-\frac{\lambda}{8}}^{\frac{\lambda}{8}} \frac{\mu A^2 \omega^2}{2} (1 + \cos(2\omega t - 2kx))\, dx$$

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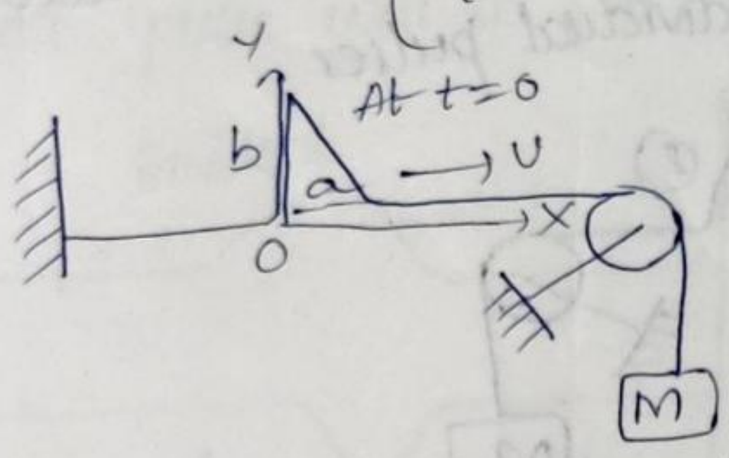
$$= \frac{\mu A^2\omega^2}{2}\left(\frac{\lambda}{4} + \frac{1}{2k}\left[\sin\left(2\omega t - \frac{\pi}{2}\right) - \sin\left(2\omega t + \frac{\pi}{2}\right)\right]\right)$$

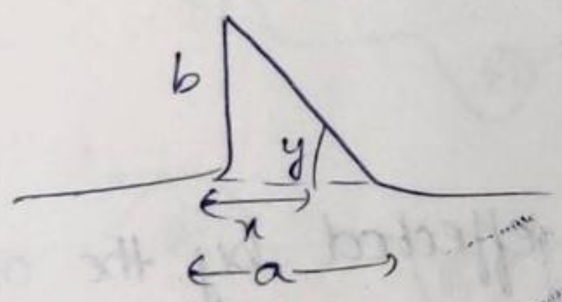
$$= \frac{\mu A^2\omega^2}{2}\left(\frac{\lambda}{4} + \frac{1}{2k}(+1+1)\right) = \frac{\mu A^2\omega^2}{2}\left(\frac{\lambda}{4} + \frac{\lambda}{2\pi}\right)$$

Ex1)



for the transverse pulse developed in the stretched string, write the pulse equation for $x>0$, $t>0$. Find velocity of an element of string at $t=0$, $x=\frac{a}{2}$. Find total momentum of string elements in the pulse at $t=0$. Find KE of string elements in the pulse at $t=0$

Sol: (a) At $t=0$,



$$\frac{y}{a-x} = \frac{b}{a}$$

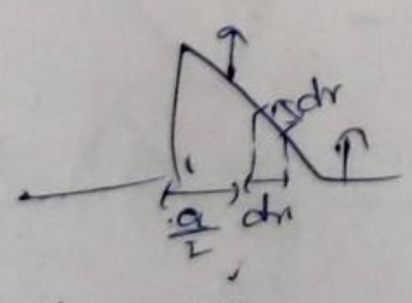
$$\frac{a-x}{a} = \frac{y}{b}$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

For pulse equation, replace x with $x-vt$

$$\boxed{\frac{x-vt}{a} + \frac{y}{b} = 1}$$

(b)



$$v_p = -vs = -v\left(\frac{-b}{a}\right) = \frac{vb}{a}$$

(c) $dp = (dm)v_p = \mu(dl)\frac{vb}{a}$

$$P = \int dp = \frac{\mu v b}{a}\int dl = \frac{\mu v b}{a}\sqrt{a^2+b^2}$$

d) $dK = \frac{1}{2}(dm)v_p^2$

$$= \frac{1}{2}(\mu dl)\frac{v^2b^2}{a} \Rightarrow K.E = \frac{1}{2}\frac{\mu v^2 b^2}{a^2}\sqrt{a^2+b^2}$$

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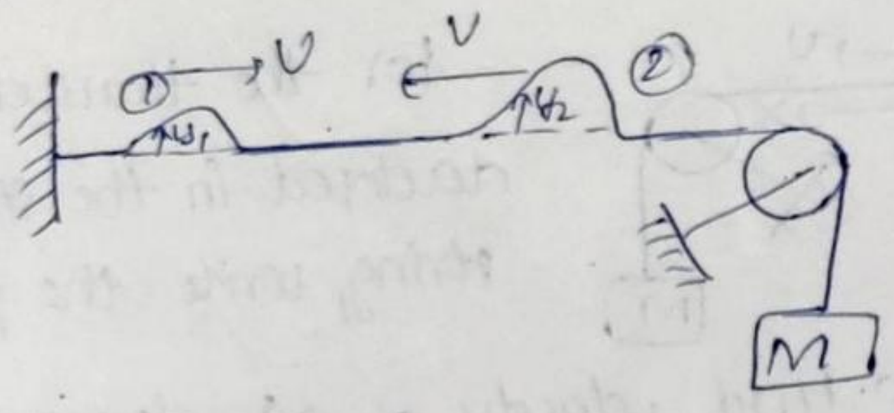
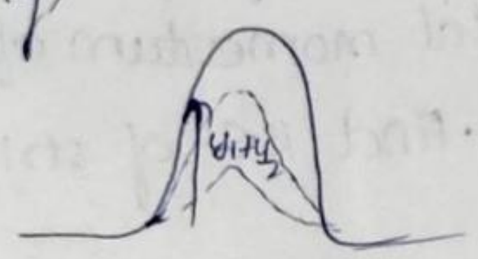
Principle of Superposition:

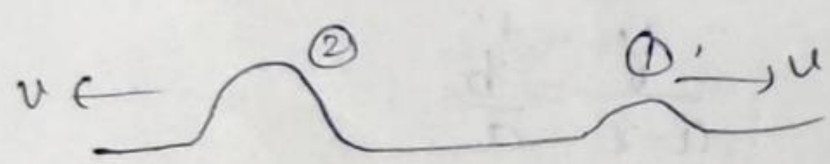
* If two pulses pass through a point, then displacement of the particle at the point will be equal to the vector sum of displacements due to individual pulses



while crossing,

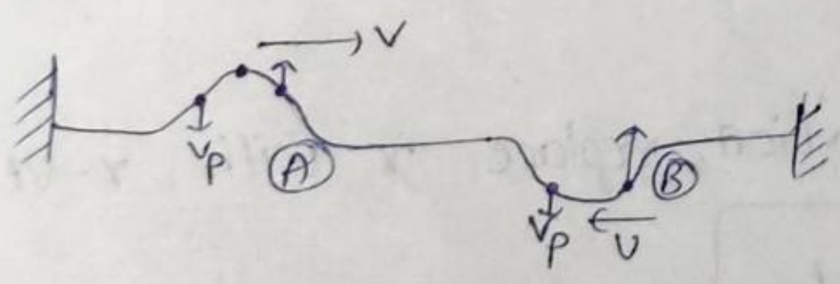


After crossing,



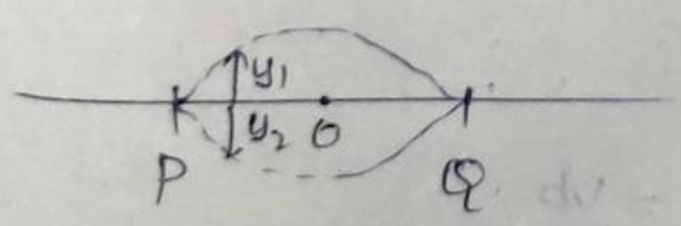
* Due to crossing, no pulse is effected by the other. Each pulse is travelled as if the other is absent

Ex1)



pulses A and B are identical in shape

while crossing:

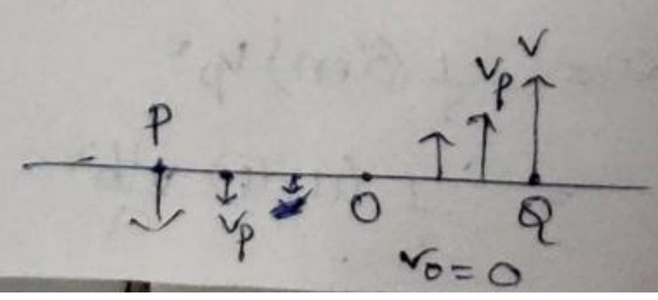


$y_2 = -y_1 \Rightarrow y = y_1 + y_2 = 0$ to every element

string is straight

$PE = 0$ as $\frac{\partial y}{\partial x} = 0$ everywhere

$T.E = K.E$ purely



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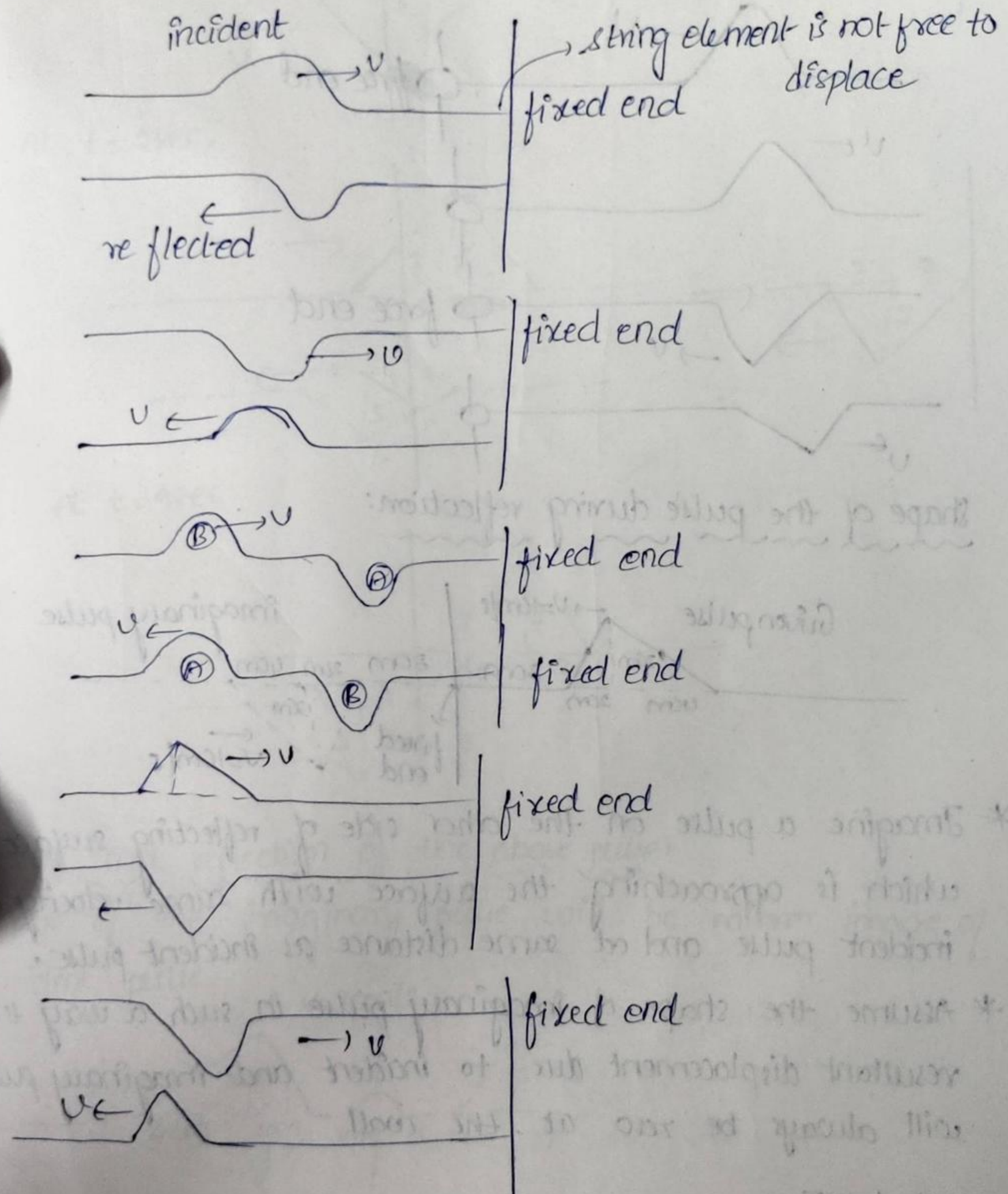
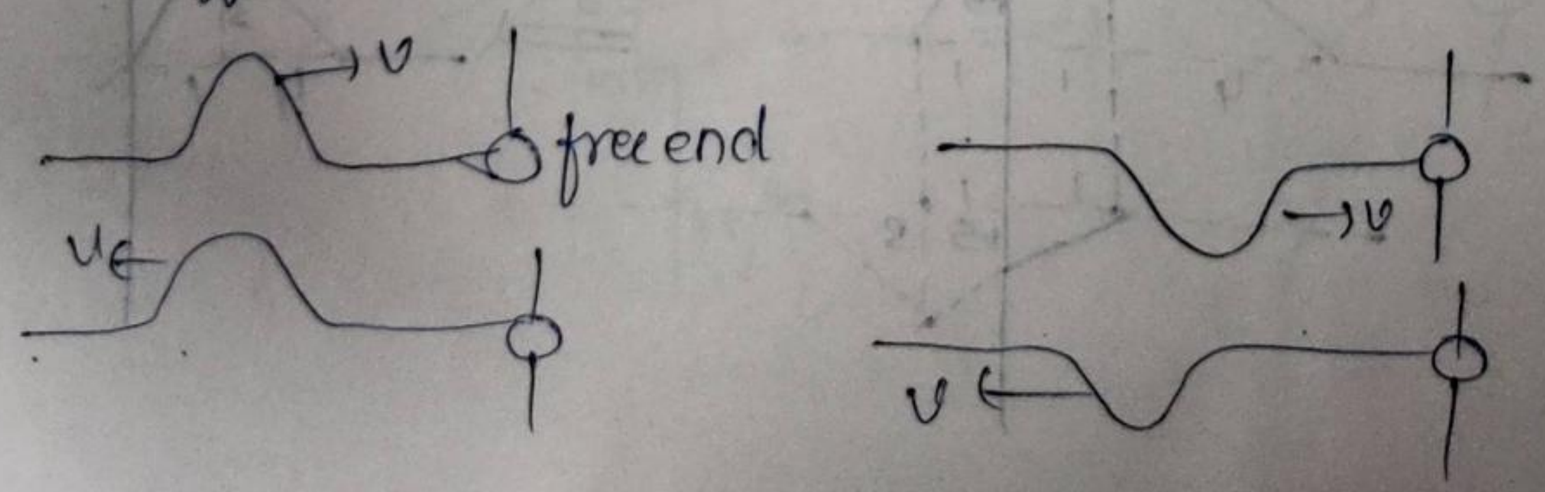
Reflection of the pulses:

* When a pulse reflects at a fixed end, then the pulse will be inverted on reflection. Phase difference b/w incident and reflected pulse will be π



* In case of free end reflection, pulses are not inverted and phase difference will be zero



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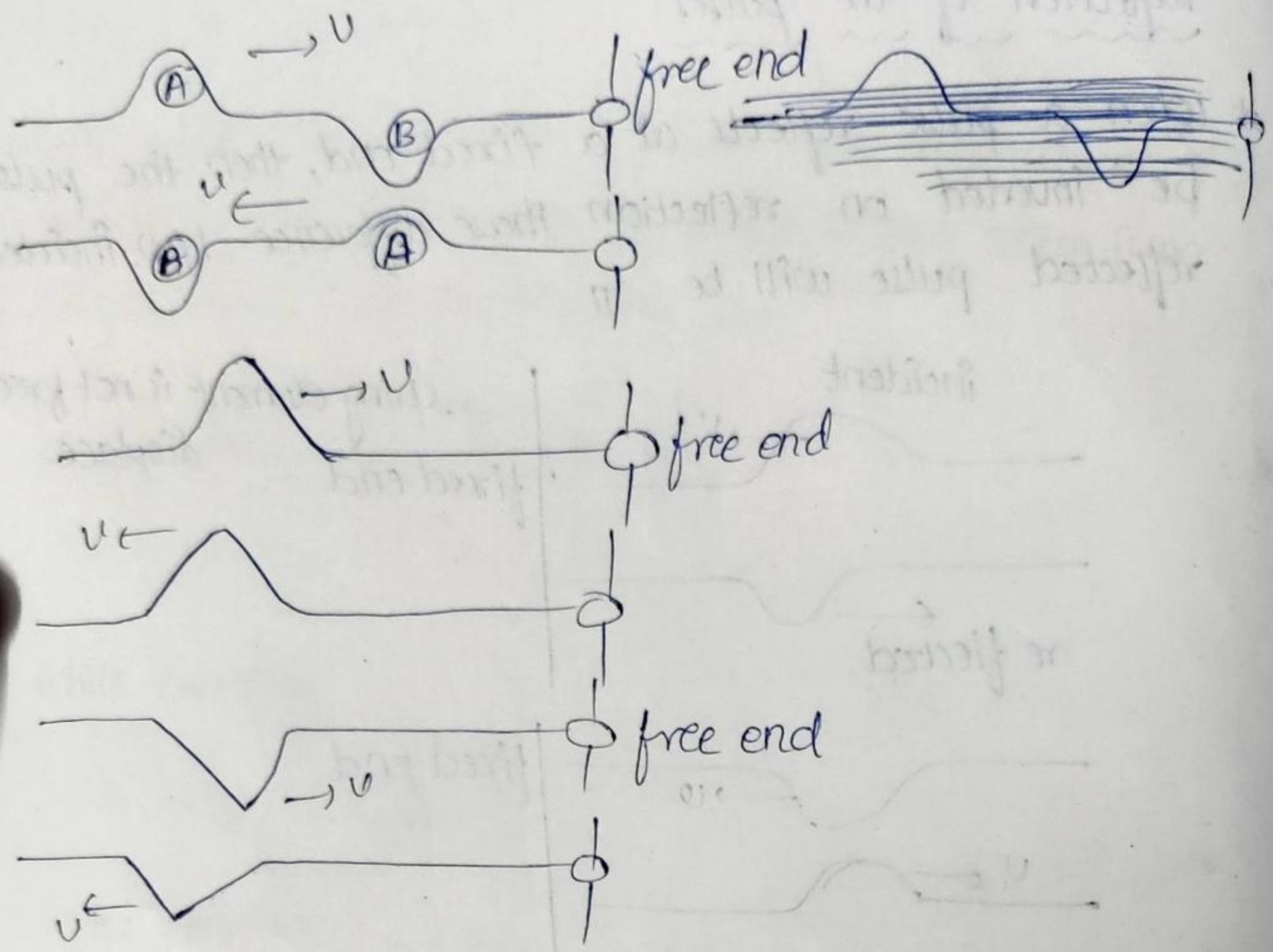
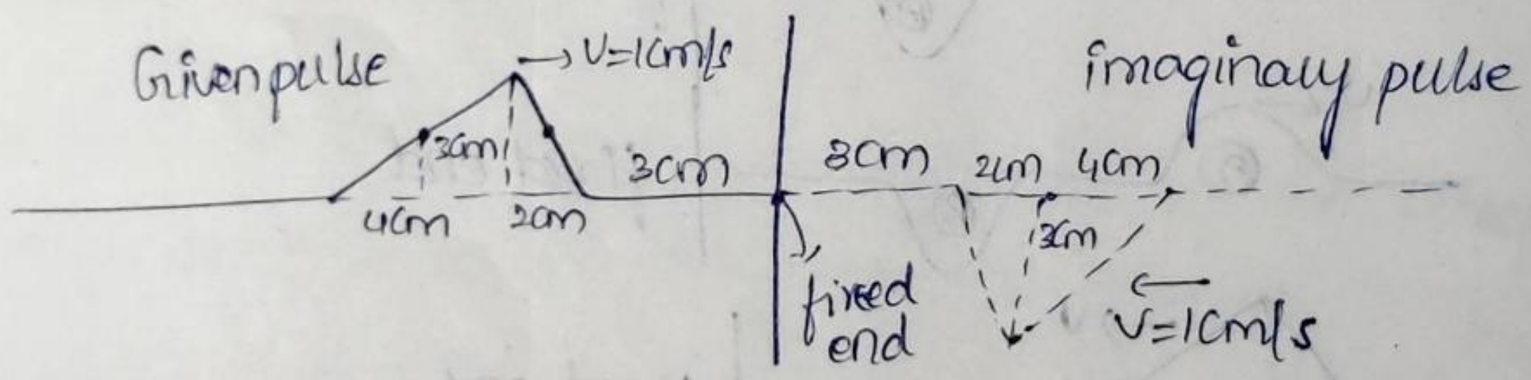
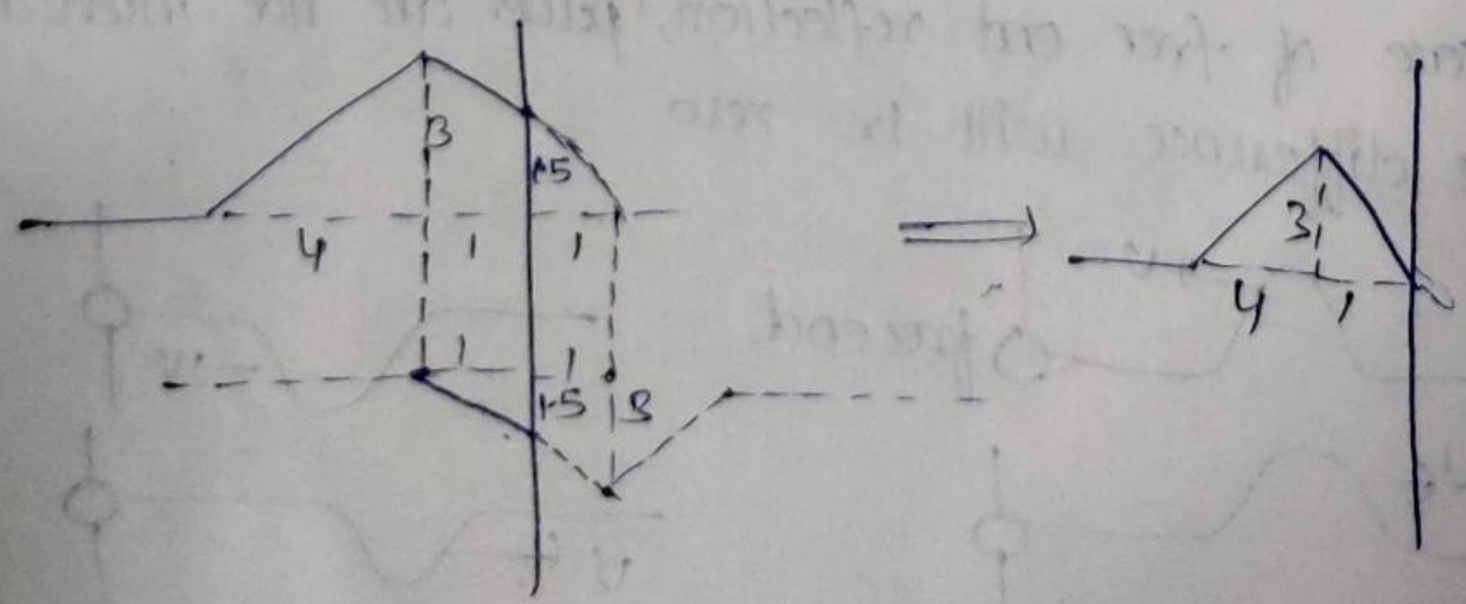


Shape of the pulse during reflection:



* Imagine a pulse on the other side of reflecting surface which is approaching the surface with same velocity as incident pulse and at same distance as incident pulse.
* Assume the shape of imaginary pulse in such a way that resultant displacement due to incident and imaginary pulses will always be zero at the wall

At t=4sec,



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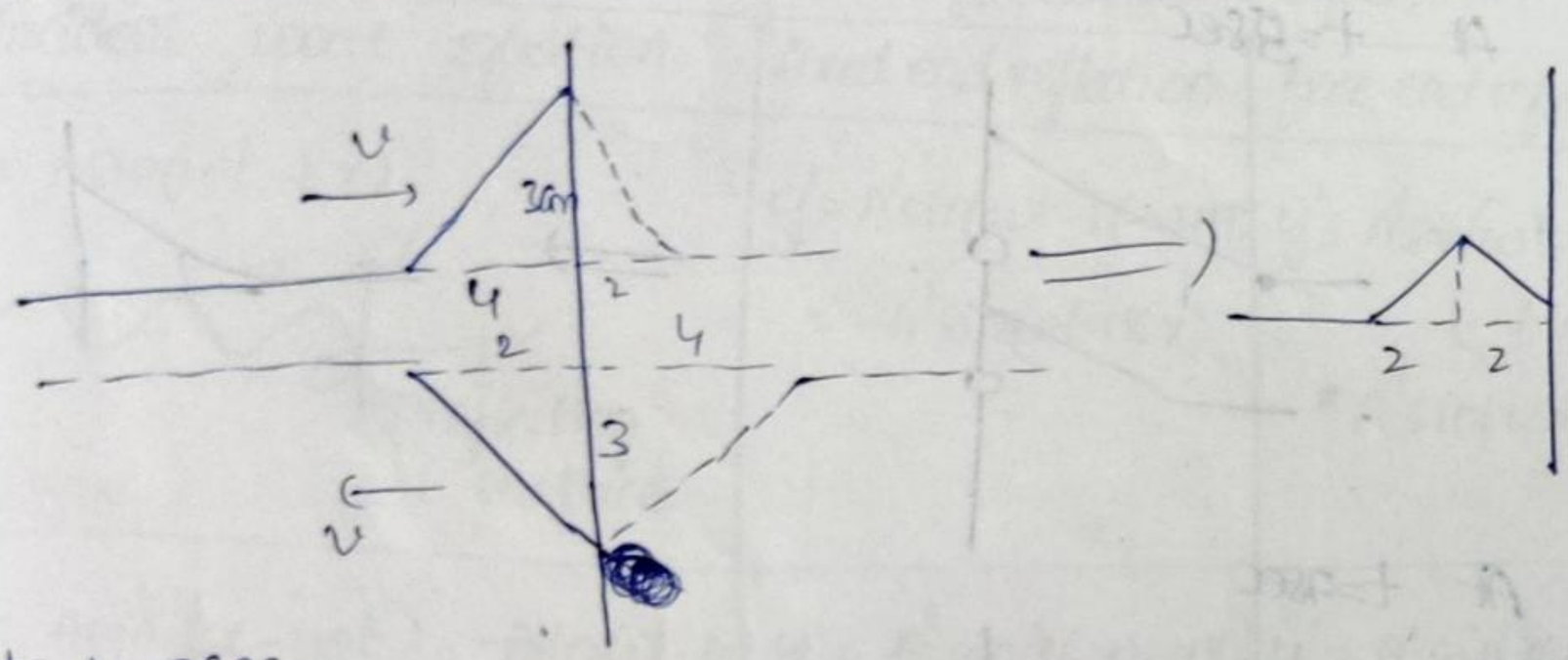
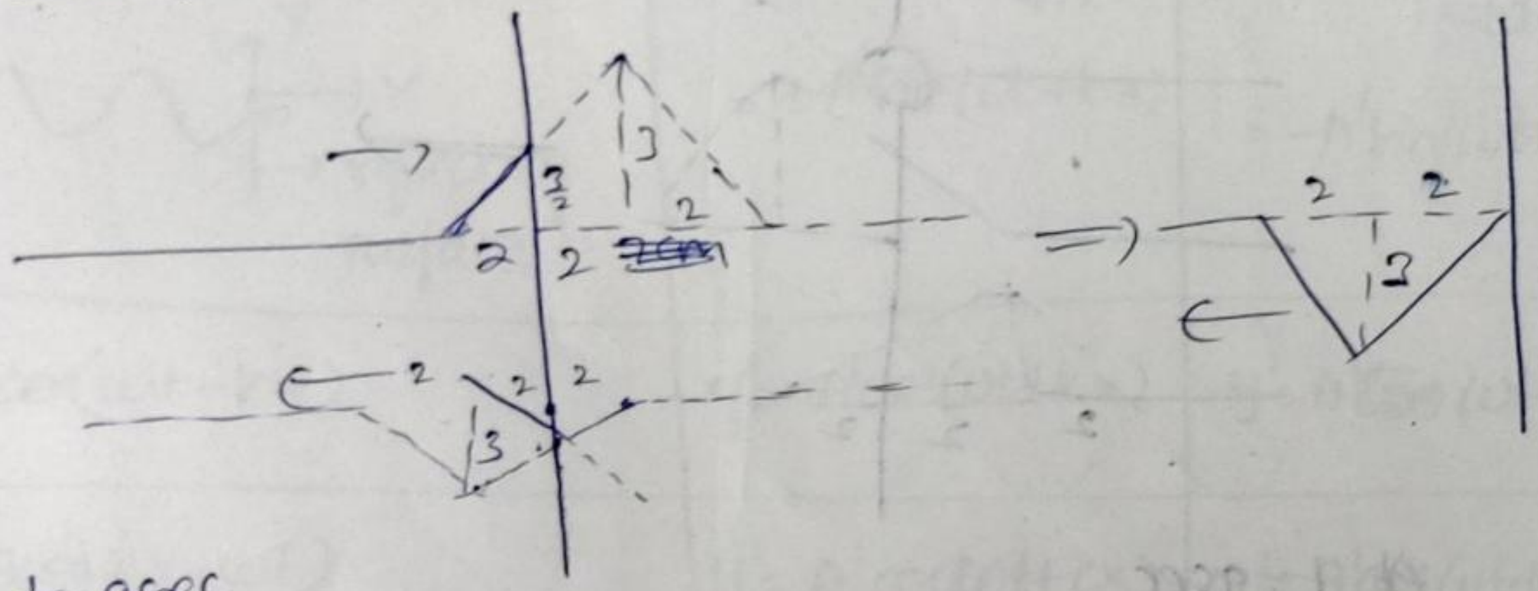
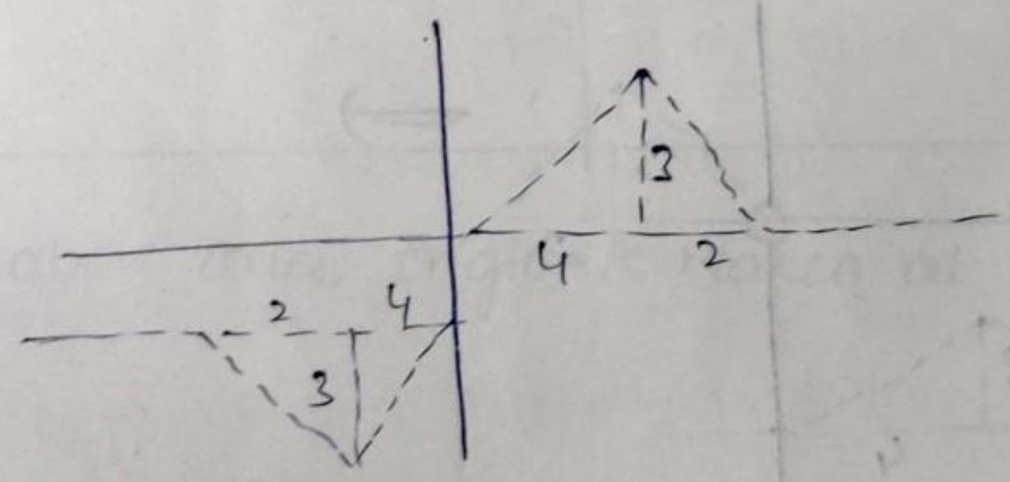
At t=5sec,



At t=7sec,

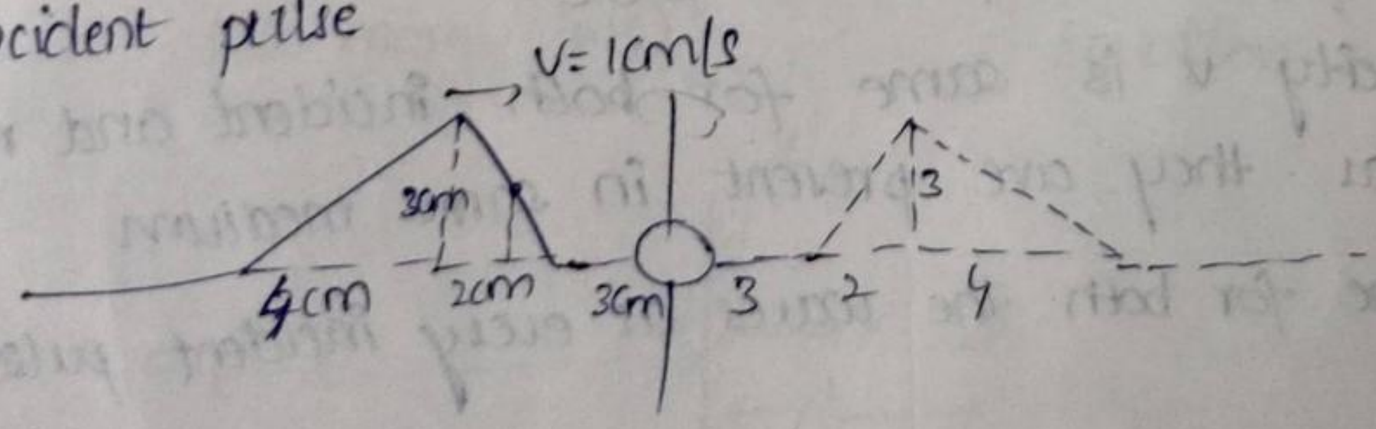


At t=9sec

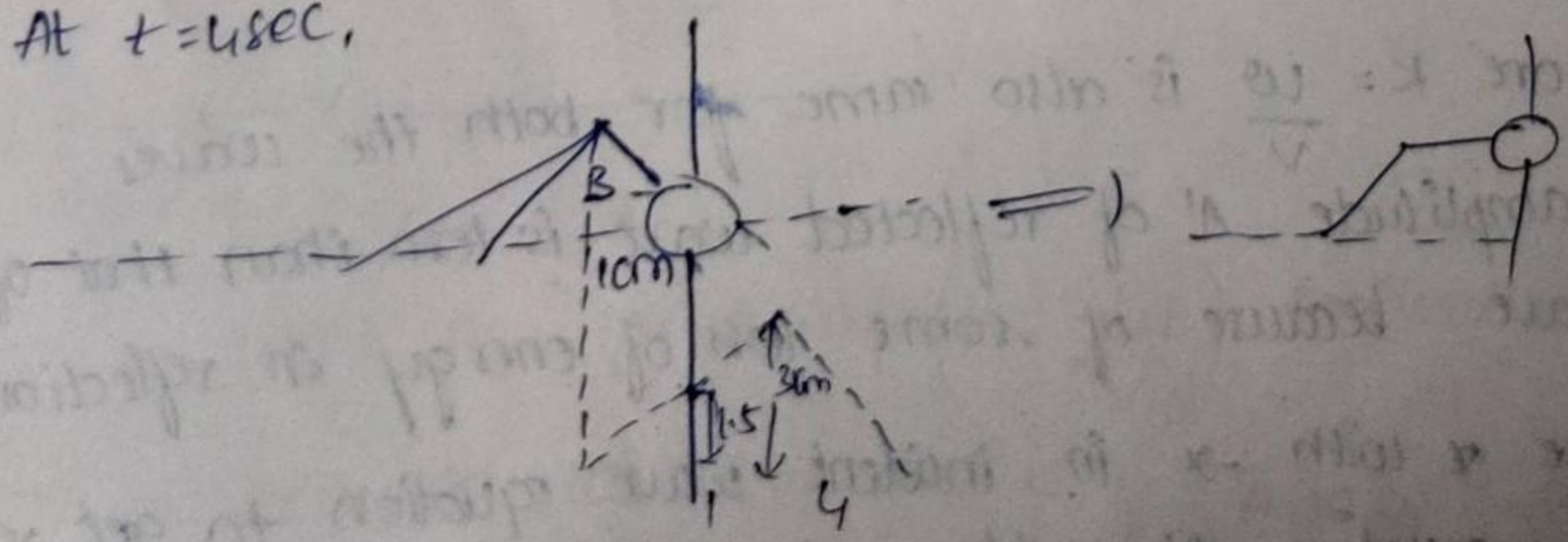


Free end reflection of the above pulse

* Shape of the imaginary pulse will be mirror image of incident pulse



At t=4sec,



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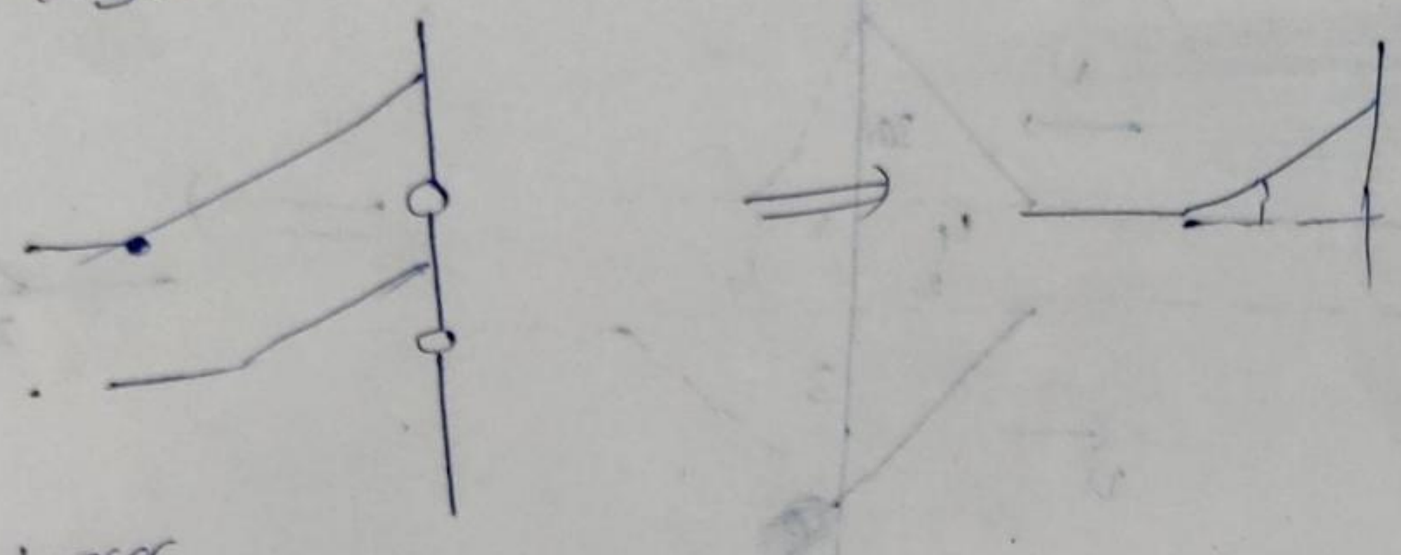
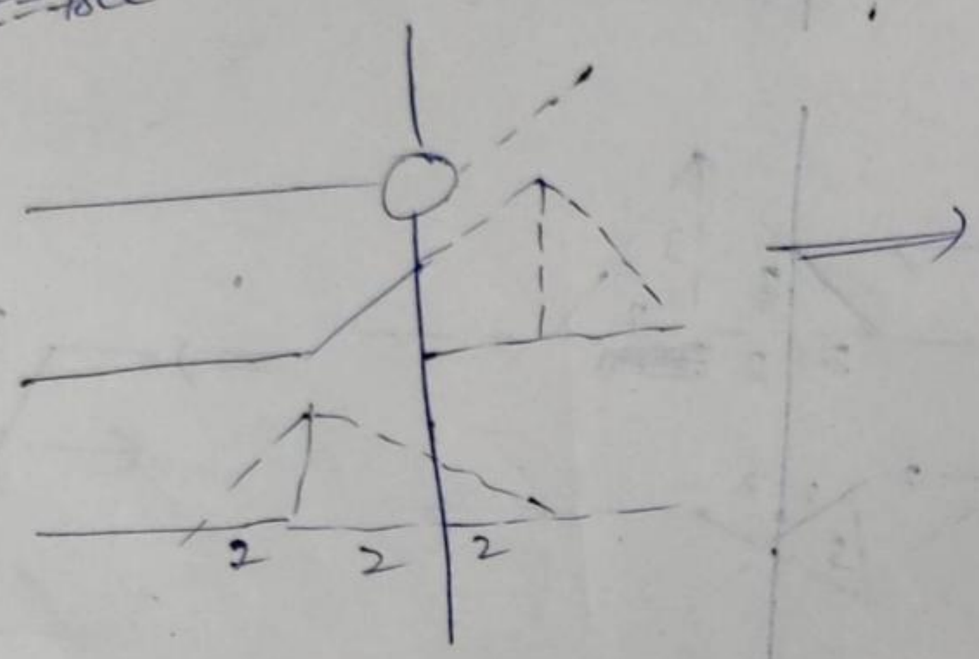
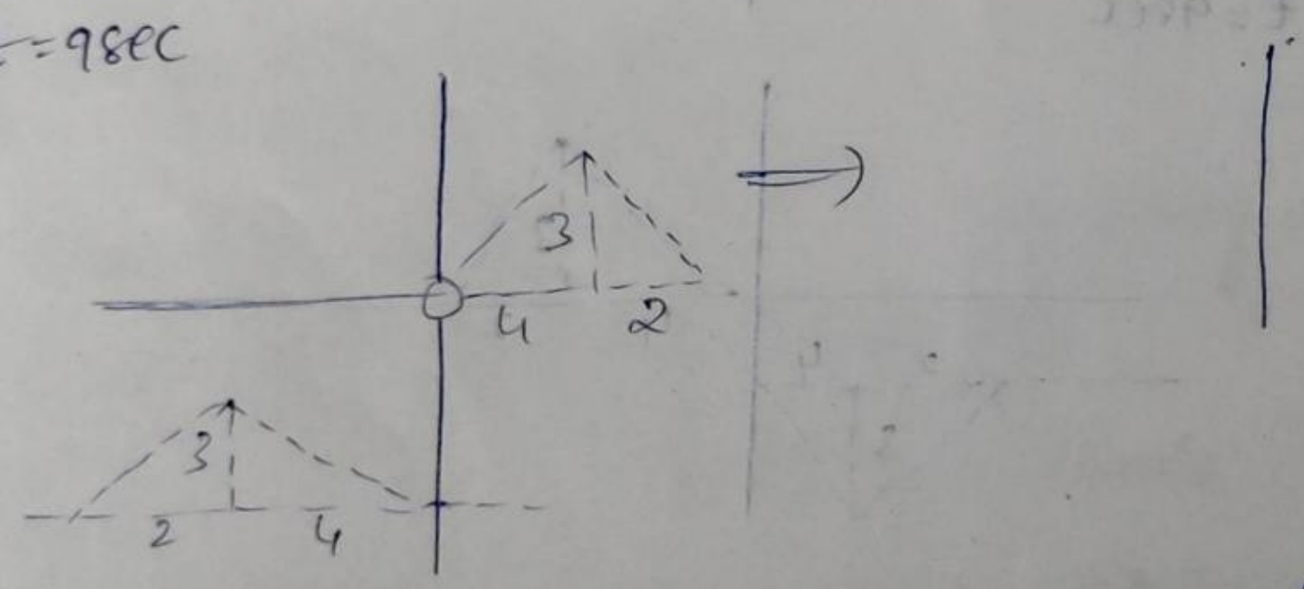
At $t = 5sec$



At $t = 7sec$



At $t = 9sec$



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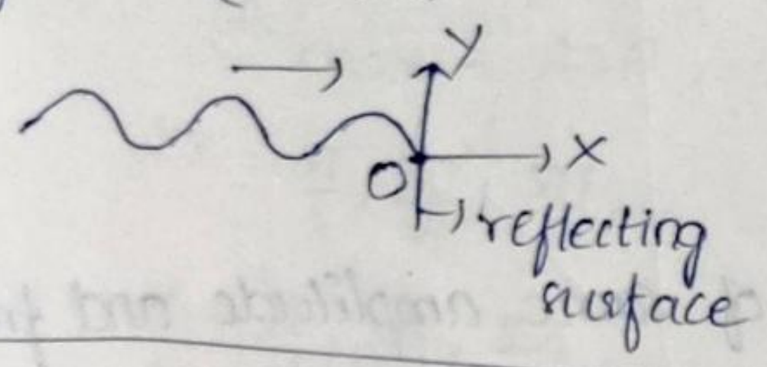
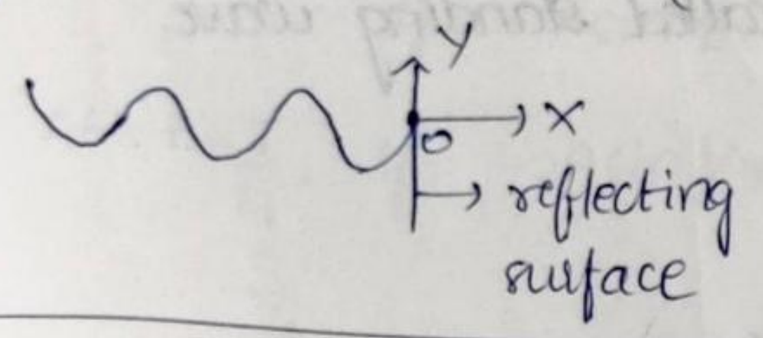
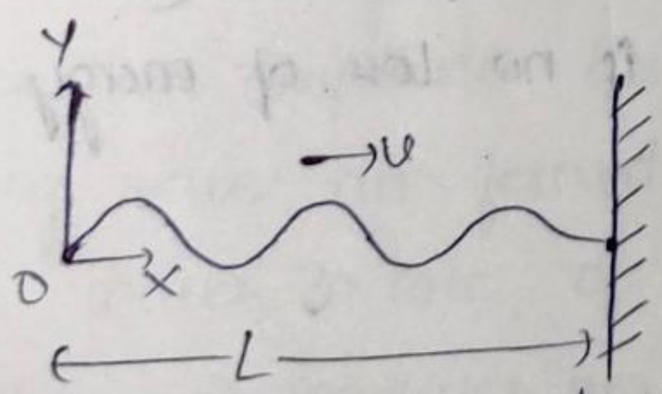
Equation of reflected harmonic wave:

* Wave velocity v is same for both incident and reflected wave as they are present in same medium
* ω is same for both the waves as every incident pulse is reflected
* Therefore $k = \frac{\omega}{v}$ is also same for both the waves
* But amplitude A' of reflected wave is less than that of incident wave because of some loss of energy in reflection
* Replace x with $-x$ in incident wave equation to get reflected wave equation. Also add a phase π if the reflection takes place at a fixed end and no phase addition for free end reflection

Incident wave equation	Reflected wave equation: fixed end reflection	Reflected wave equation: free end reflection
1) $y = A\sin(\omega t - kx)$ 	$y' = A'\sin(\omega t - k(-x) + \pi)$ $= -A'\sin(\omega t + kx)$	$y' = A'\sin(\omega t - k(-x) + 0)$ $= A'\sin(\omega t + kx)$
2) $y = A\sin(kx - \omega t) = -A\sin(\omega t - kx)$ 	$y' = A'\sin(k(-x) - \omega t) + \pi$ $= +A'\sin(\omega t + kx)$	$y' = A'\sin(k(-x) - \omega t + 0)$ $= -A'\sin(\omega t + kx)$
3) $y = A\cos(\omega t - kx)$	$y' = -A'\cos(\omega t + kx)$	$y' = A'\cos(\omega t + kx)$
4) $y = A\cos(kx - \omega t)$	$y' = A'\cos(\omega t + kx)$	$y' = A'\cos(\omega t + kx)$

* In all the above cases, origin is taken at reflecting surface

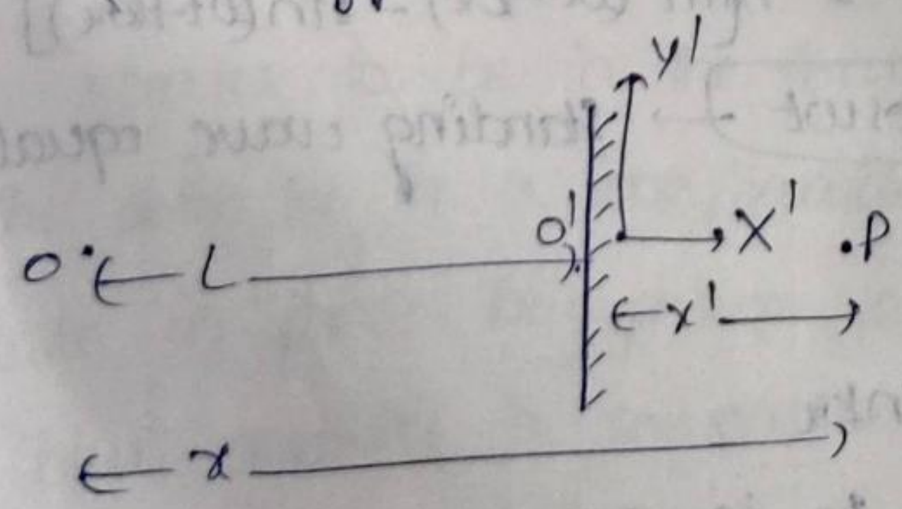
Ex)



(No energy loss) fixed end

If the incident wave equation is $y = A'\sin\omega\left(t - \frac{x}{v}\right)$ write the reflected wave equation.

Sol:



$L + x' = x$

w.r.t O', $y = A\sin\omega\left(t - \frac{x'+L}{v}\right)$ incident wave equation

Reflected wave equation is $y = A\sin\left[\omega t - \frac{(x'+L)}{v} + \pi\right]$

$= -A\sin\left(\omega\left(t + \frac{x'-L}{v}\right)\right)$

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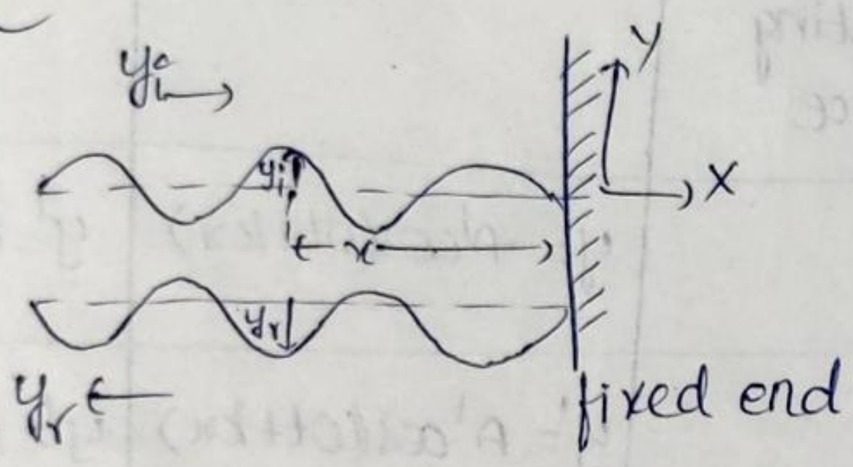
w.r.t given origin O, $y = -A\sin\omega\left(t + \frac{(x-l)-L}{v}\right)$

$$\boxed{y = -A\sin\left(t + \frac{x-2L}{v}\right)}$$

Standing waves:

* When two progressive waves of same amplitude and frequency travelling in opposite directions are super imposed, then the wave pattern formed is called standing wave

* Equation:



let incident wave equation $y_i = A\sin(\omega t - kx)$. If the reflection takes place at fixed end

reflected wave equation is $y_r = -A\sin(\omega t + kx)$

(Assumed that there is no loss of energy due to reflection)

Due to super imposition,

$$y = y_i + y_r$$
$$= A[\sin(\omega t - kx) - \sin(\omega t + kx)]$$

$\boxed{y = -2A\sin kx \cos\omega t}$ → standing wave equation

$y = -A_r\cos\omega t$

where $A_r = 2A\sin kx$

All particles will be in SHM with angular frequency ω but with different amplitudes

$|A_r|$ = resultant amplitude

Nodes: points of $(A_r)_{min}$

$(A_r)_{min} = 0 \Rightarrow \sin kx = 0 \Rightarrow kx = 0, \pi, 2\pi, \ldots$

2π

$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots$

Antinodes: Points of $(A)_{max}$

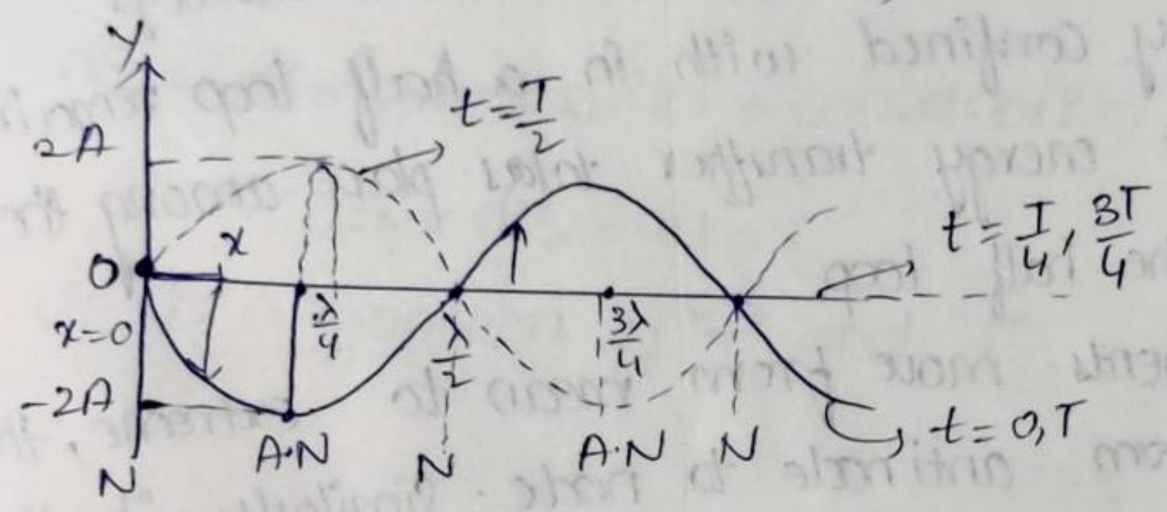
$(A)_{max} = \pm 2A, \quad \sin kx = \pm 1$

$kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$

At $t=0$, $y = -A\sin kx$

At $t = \frac{T}{4}$, $y = -2A\sin kx \cos\left(\frac{2\pi}{T}\cdot\frac{T}{4}\right) = 0$



* Particles at the nodes are permanently at rest and at antinodes ~~are~~, they oscillate with amplitude 2A. Remaining particles have amplitudes b/w 0 and 2A
* Standing waves are formed with both transverse and longitudinal waves. In case of transverse waves, y-x graph and locus of particles at an instant appear to be similar
* Because of peristance of vision, the string b/w consecutive nodes appeares to be in the form of loops
* All the particles in a loop (particles b/w consecutive nodes) oscillate in phase. Their phasors always coincide. Phase difference b/w particles in consecutive loops is π
* To an element at antinode, P.E is always zero as $\frac{\partial y}{\partial x} = 0$ always. Hence Mechanical energy, $dE = dK$ only. dE is maximum at mean position and zero at extreme position
* To an element at node, K.E (dK) is always zero as $V_p = 0$. Hence mechanical energy $dE = dU$ only. dE is max

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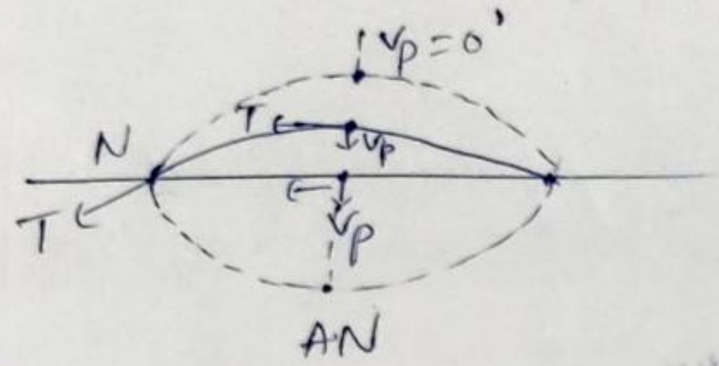
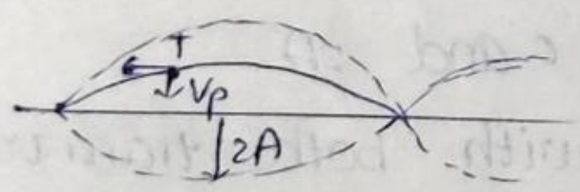
mum when the elements are at their extreme positions and zero when the elements are at their mean positions. Power transmitted through nodes and antinodes is zero at every instant



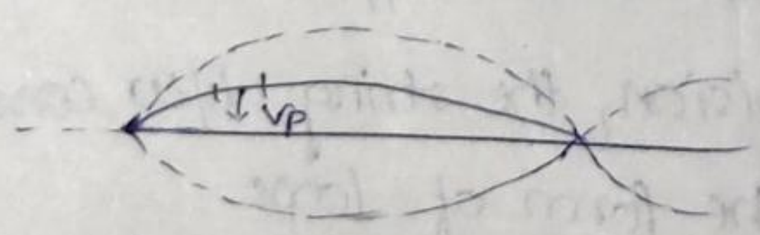
At antinodes, $P = Tv_p\cos 90^\circ = 0$ always

At nodes, $P = \vec{T}\cdot\vec{v} = 0$ as $\vec{v} = \vec{0}$ always

* Total energy confined with in a half loop remains constant always. But energy transfer takes place among the elements with in the half loop.
* If the elements move from mean to extreme, then energy transfers from antinode to node. Similarly, if the elements move from extreme to mean, then energy transfers from node to antinode



Energy of standing wave



let $y = -2A\sin kx\cos\omega t$

$$v_p = \frac{\partial y}{\partial t} = 2A\omega\sin kx \sin\omega t$$

$$S = \frac{\partial y}{\partial x} = -2Ak\cos kx\cos\omega t$$

KE of an element, $dK = \frac{1}{2}(dm)v_p^2$

$$= \frac{1}{2}\mu dx\, 4A^2(\omega^2\sin^2 kx\sin^2\omega t)$$

$$= (2A^2\omega^2\mu dx)\sin^2 kx\sin^2\omega t \quad \text{--- (1)}$$

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$$dK = dK_0 \sin^2 kx \sin^2 \omega t \quad - (1)$$

P.E of an element is $dU = \frac{1}{2}\mu v^2 dx \left(\frac{\partial y}{\partial x}\right)^2$

$$dU = \frac{1}{2}\frac{\omega^2}{k^2}\mu(dx)\left[4A^2k^2\cos^2 kx \cos^2\omega t\right]$$

$$= 2A^2\omega^2 \mu\, dx \cos^2 kx \cos^2 \omega t$$

$$dU = (dU_0)\cos^2 kx \cos^2 \omega t \quad - (2)$$

$dU_0 = dK_0$ but dU is not always equal to dK

Total mechanical energy is

$$dE = dK + dU$$

$$dE = dE_0\left[\sin^2\omega t \sin^2 kx + \cos^2\omega t \cos^2 kx\right]$$

here $dE_0 = dU_0 = dK_0$

dE is a function of "x" and "t"

Problem:

Ex) Find the values of x at which dE is independent of t?

Sol:

$$\sin^2\omega t \sin^2 kx + \cos^2\omega t \cos^2 kx = \text{constant}$$

so $\sin^2 kx = \cos^2 kx \Rightarrow 2\cos 2kx = 0$

$$2kx = (2n+1)\frac{\pi}{2} \Rightarrow \boxed{x = \frac{(2n+1)\pi}{4k} = \frac{(2n+1)\lambda}{8}}$$

at $x = \frac{\lambda}{8}, \frac{3\lambda}{8}, \frac{5\lambda}{8}, \ldots$ energy of element is independent of t

Similarly if dE has to be independent of x,

$$\sin^2\omega t = \cos^2\omega t \Rightarrow 2\cos 2\omega t = 0$$

$$2\omega t = (2n+1)\frac{\pi}{2} \Rightarrow \boxed{t = \frac{(2n+1)\pi}{4\omega} = \frac{(2n+1)T}{8}}$$

at $t = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}, \ldots$ energy of element is independent of x

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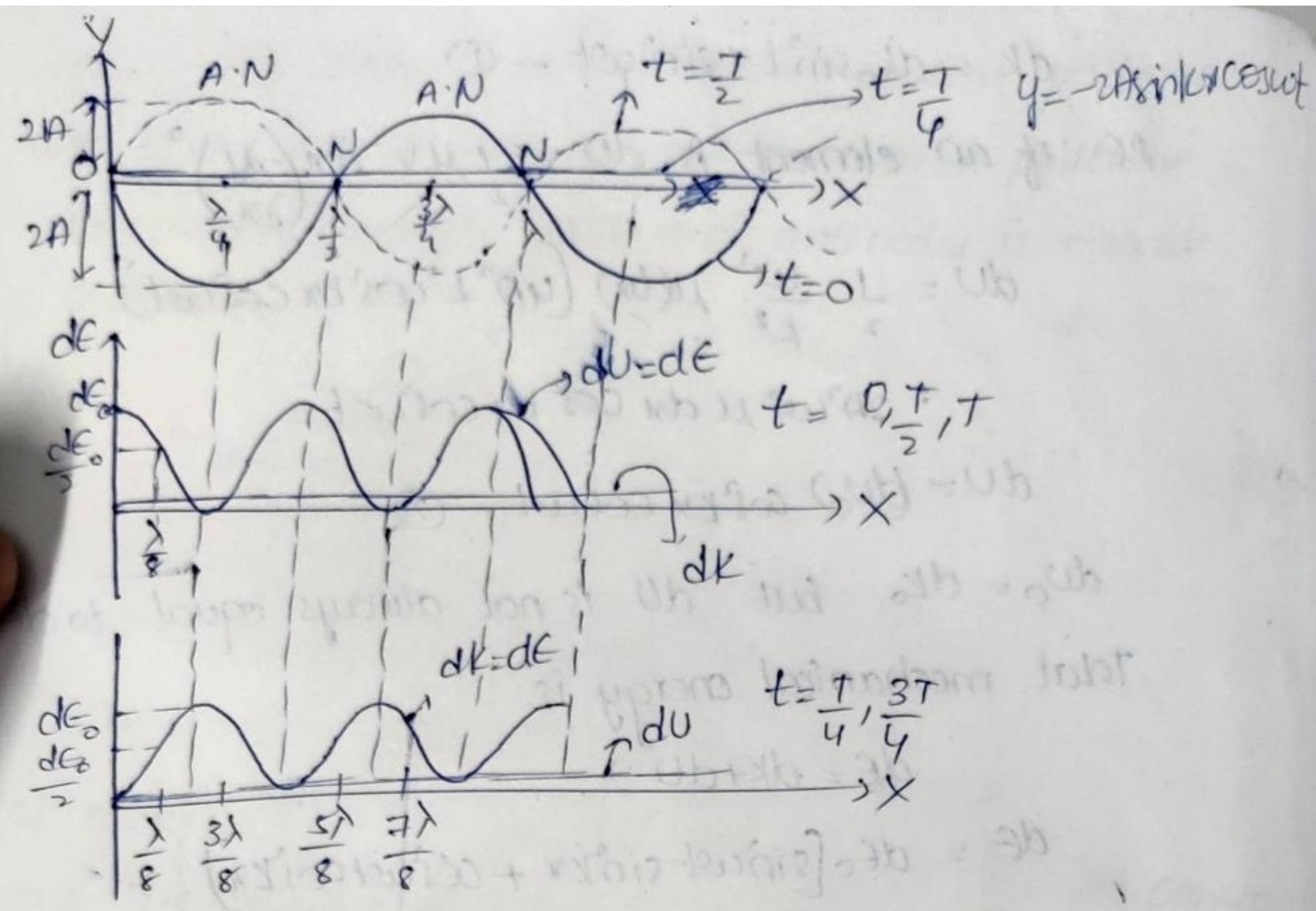
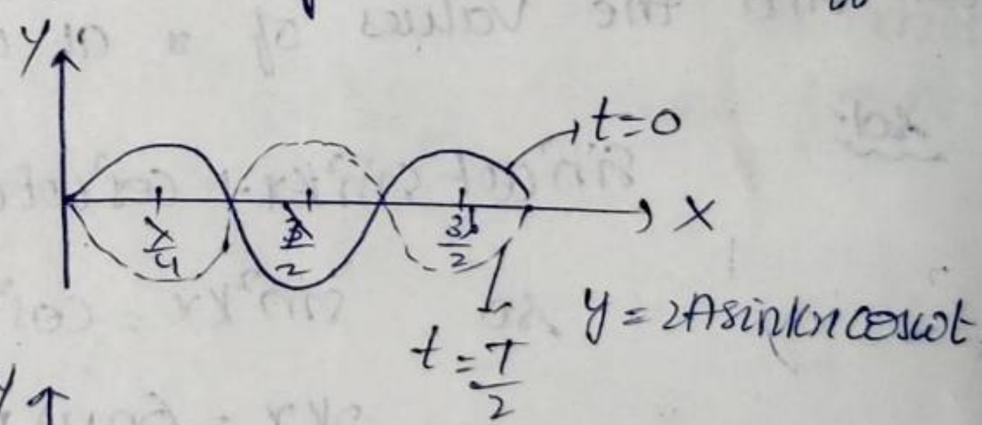


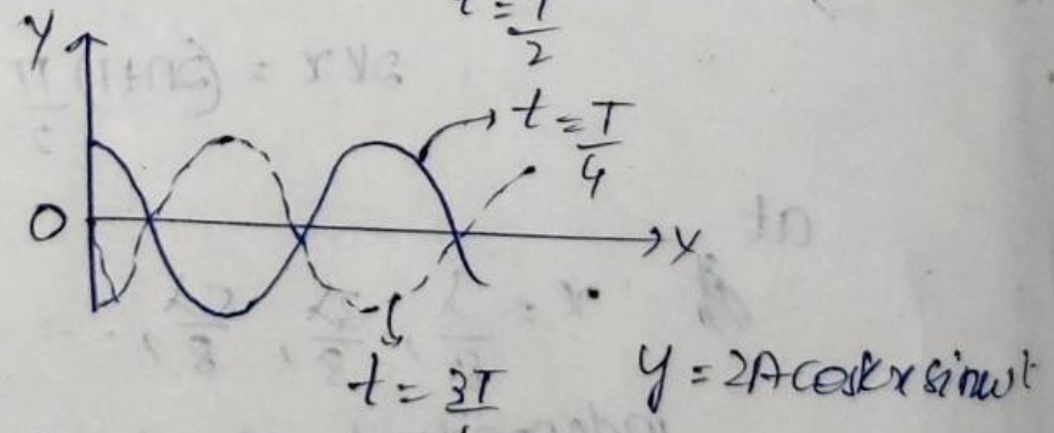
Different forms of standing wave equation:

* Depending upon incident wave equation and nature of reflecting surface, equation of standing wave will be different

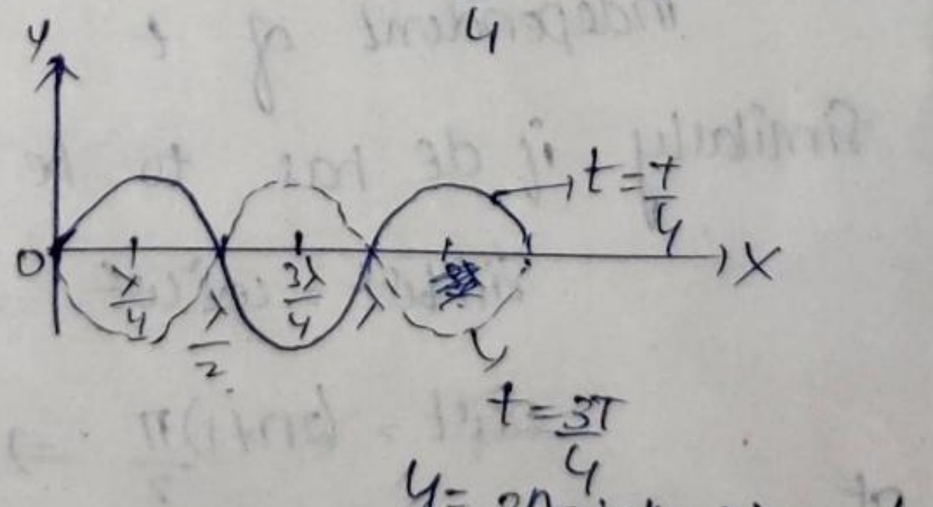
$y = \pm 2A \sin kx \cos \omega t$



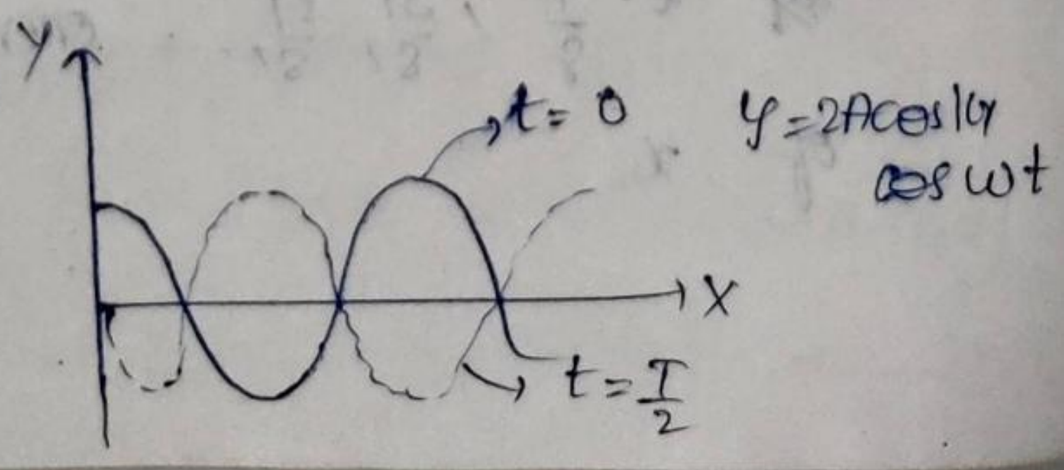
$y = \pm 2A \cos kx \sin \omega t$



$y = \pm 2A \sin kx \sin \omega t$



$y = 2A \cos kx \cos \omega t$



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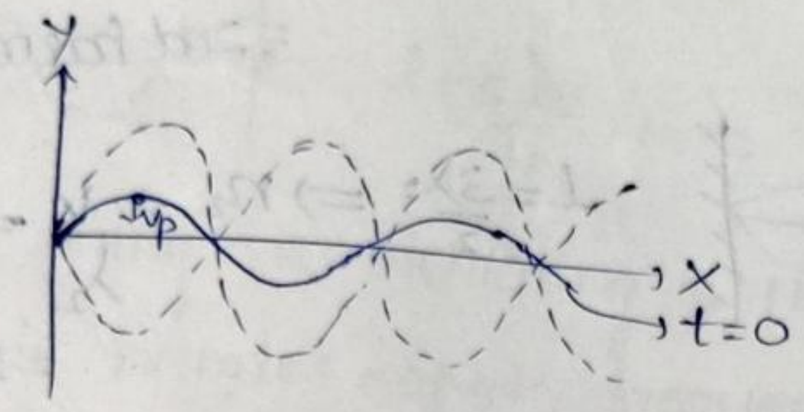
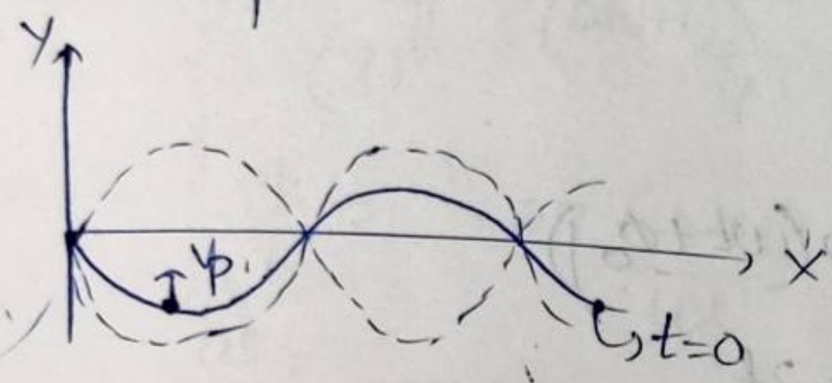
* If there is node at $x=0$, write $2A\sin kx$ as amplitude function. Similarly if there is antinode at $x=0$, write $2A\cos kx$ as amplitude function

Generally,



$y = 2A\sin kx \sin(\omega t+\phi_0)$ with $\frac{\pi}{2} < \phi_0 < \pi$

If v_p is upward, $0 < \phi_0 < \frac{\pi}{2}$



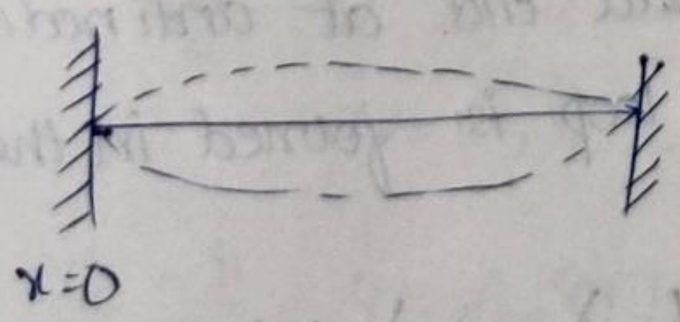
$\frac{3\pi}{2} < \phi_0 < 2\pi$

If v_p is downward

$\pi < \phi_0 < \frac{3\pi}{2}$

Transverse standing waves in a string fixed at both ends.

* Read H.C Verma pg- 312, 313, 314, 315
* As the string is fixed at both ends, always nodes are formed at ends. In fundamental mode, string vibrates in one loop. Corresponding frequency is called fundamental frequency
* All allowed frequencies which are higher than fundamental frequency are called overtones.
* The frequencies which are integral multiples of fundamental frequency are called harmonics.



$l = \frac{\lambda}{2} \Rightarrow \lambda = 2l$

$n = \frac{v}{\lambda} = \frac{v}{2l} = \frac{1}{2l}\sqrt{\frac{T}{\pi r^2 \rho}}$

= fundamental frequency

= 1st harmonic

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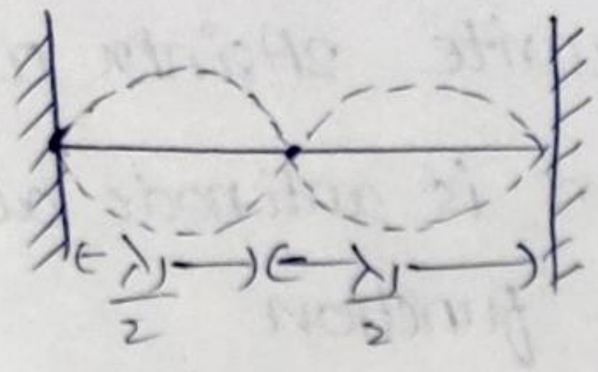
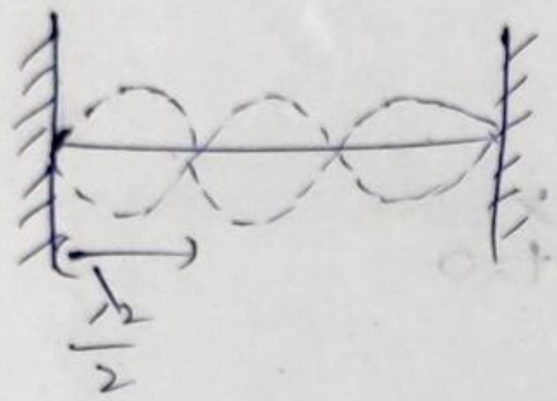


$l = \lambda_1, \quad n_1 = \frac{v}{\lambda_1} = \frac{v}{l} = 2n$

= 1st overtone

= 2nd harmonic



$l = \frac{3\lambda_2}{2} \Rightarrow n_2 = \frac{v}{\lambda_2} = 3n$

= 2nd overtone

= 3rd harmonic

* If the string oscillates in N th harmonic, loops = N
nodes = N+1 , Antinodes = N

* Standing wave equation,

$$y = A_r \sin kx \sin(\omega t + \phi_0)$$

$$\frac{N\lambda}{2} = l \Rightarrow \lambda = \frac{2l}{N}$$

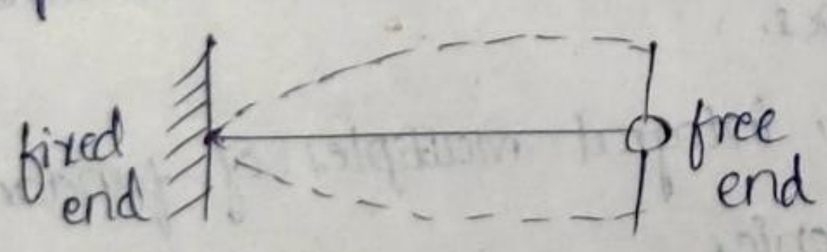
$$k = \frac{2\pi}{\lambda} = \frac{2\pi N}{2l} = \frac{\pi N}{l}$$

$$\omega = 2\pi(Nn)$$

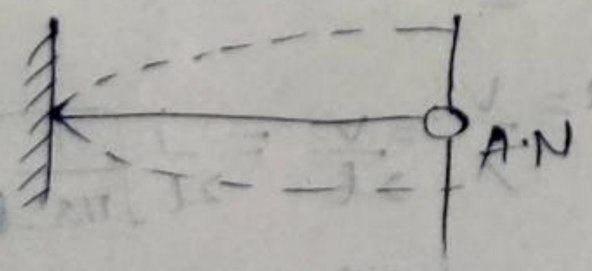
$$\boxed{y = A_r \sin \frac{\pi N}{l} x \sin(2\pi N n)t + \phi_0)}$$

* ϕ_0 is decided by state of the string at $t=0$

String fixed at one ~~end~~ end and free at other end:



* Node is always formed at fixed end at antinode at free end. In fundamental mode, half loop is formed in the string



$l = \frac{\lambda}{4} \Rightarrow \lambda = 4l, \; n = \frac{v}{\lambda} = \frac{v}{4l} = \frac{1}{4l}\sqrt{\frac{T}{\mu}}$

= fundamental frequency

= 1st harmonic

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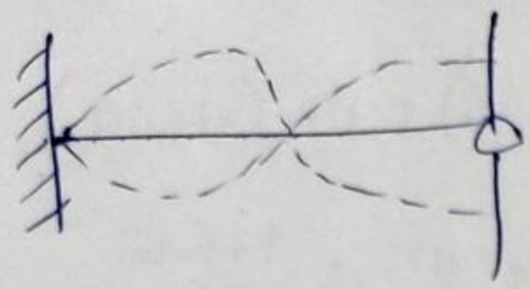
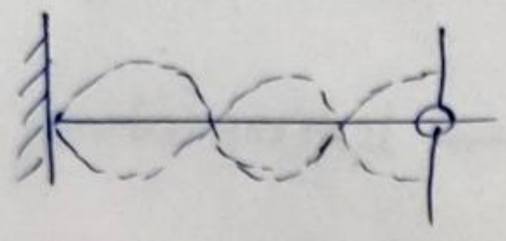


$l = \frac{3\lambda_1}{4} \Rightarrow \lambda_1 = \frac{4l}{3}$, $n_1 = 3n$

= 1st overtone

= 3rd harmonic



$l = \frac{5\lambda_2}{4}$, $\lambda_2 = \frac{4l}{5}$, $n_2 = 5n$ = 2nd overtone

= 5th harmonic

* If the string is oscillating in Nth overtone, (2N+1) harmonic number = 2N+1, frequency of oscillation = (2N+1)n
* No. of nodes = N+1
* No of antinodes = N+1
* length of string $(l) = (2N+1)\frac{\lambda}{4} \Rightarrow \lambda = \frac{4l}{2N+1}$

$$k = \frac{2\pi(2N+1)}{4l} = \frac{\pi(2N+1)}{2l}$$

$$\omega = 2\pi(2N+1)n$$

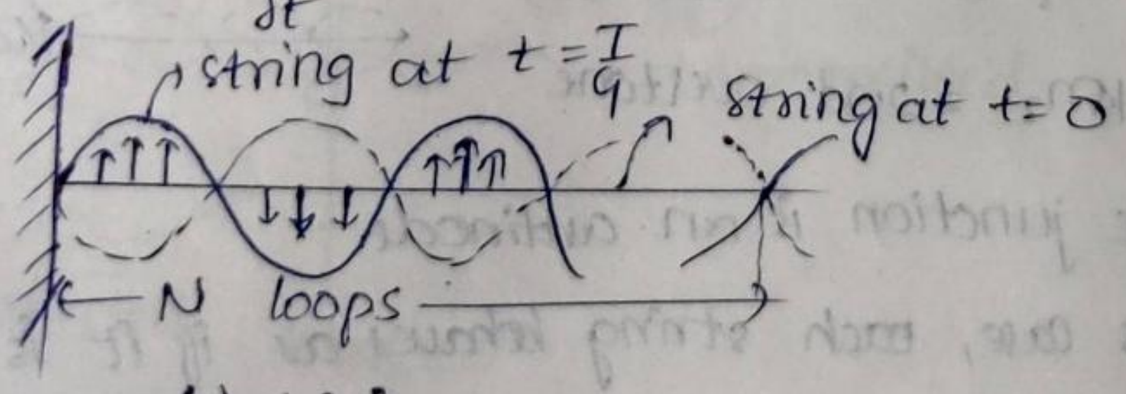
$$\boxed{y = A_r \sin\left(\frac{\pi(2N+1)}{2l}\right)x \, \sin\left((2\pi(2N+1)n)t + \phi_0\right)}$$

* If $\phi_0 = 0$, $y = A_r \sin kx \sin\omega t$

At $t=0$, $y=0$ for all particles

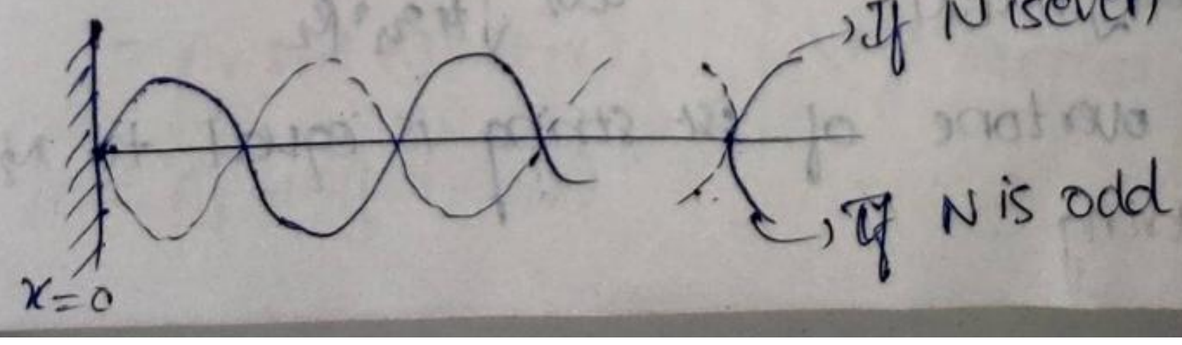
$$\frac{\partial y}{\partial t} = A_r \omega \sin kx \cos\omega t$$

At $t=0$, $\frac{\partial y}{\partial t} = A_r \omega \sin kx \rightarrow$ +ve $\quad 0 < x < \frac{\lambda}{2}$



$N = 1, 2, 3, \ldots$

* If $\phi_0 = \frac{\pi}{2}$, $y = A_r \sin kx \cos\omega t$



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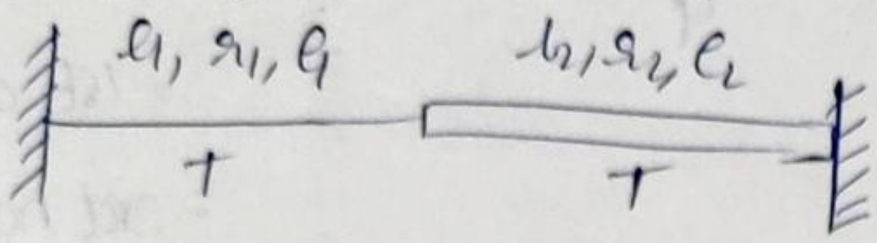
Composite strings:



Case(i): If the junction is a node:

* In this case, each string behaves as if it is fixed at both ends

* Fundamental frequencies of 1st and 2nd strings are $n_1 = \frac{1}{2l_1}\sqrt{\frac{T}{\pi r_1^2 \rho_1}}$

$$n_2 = \frac{1}{2l_2}\sqrt{\frac{T}{\pi r_2^2 \rho_2}}.$$

* n_1 and n_2 may not be equal but different harmonics may become equal. Let N_1th harmonic of 1st string is equal to N_2th harmonic of 2nd string

$$N_1 n_1 = N_2 n_2$$

$$N_1 \frac{1}{2l_1}\sqrt{\frac{T}{\pi r_1^2 \rho_1}} = N_2 \frac{1}{2l_2}\sqrt{\frac{T}{\pi r_2^2 \rho_2}}$$

* Let this ratio is obtained as $\frac{3}{5}$. then $\frac{N_1}{N_2} = \frac{3}{5} = \frac{6}{10}$ and soon

* In fundamental mode, $N_1 = 3$, $N_2 = 5$

$f = 3n_1 = 5n_2$ = fundamental frequency of composite string

$f_1 = 6n_1 = 10n_2$ = 1st overtone

$f_2 = 9n_1 = 15n_2$ = 2nd overtone

Case(ii): If the junction is an antinode

Now in this case, each string behaves as if it is fixed at one end and free at other end

$$n_1 = \frac{1}{4l_1}\sqrt{\frac{T}{\pi r_1^2 \rho_1}} \qquad n_2 = \frac{1}{4l_2}\sqrt{\frac{T}{\pi r_2^2 \rho_2}}$$

* Let N_1th overtone of 1st string is equal to N_2th overtone of 2nd string

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$$(2N_1+1)\, n_1 = (2N_2+1)\, n_2$$

$$\frac{2N_1+1}{2N_2+1} = \frac{n_2}{n_1} = \frac{l_1}{l_2} \cdot \frac{r_1}{r_2}\sqrt{\frac{\rho_1}{\rho_2}} = \frac{3}{5} \text{ (let)}$$

* In fundamental mode, $2N_1+1 = 3 \Rightarrow N_1 = 1$

$2N_2+1 = 5 \Rightarrow N_2 = 2$

* fundamental frequency $f = 3n_1 = 5n_2$

1st over tone $f_1 = 9n_1 = 15n_2$

2nd over tone $f_2 = 15n_1 = 25n_2$

Denser and Rarer media:

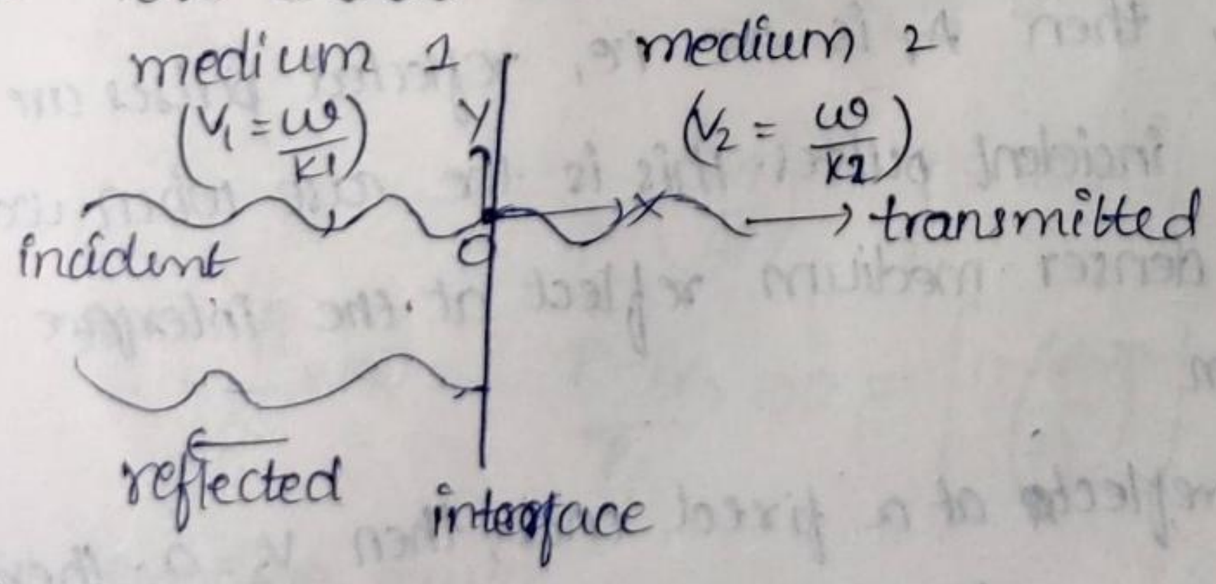
* When two media are compared, the medium in which wave velocity is more is called rarer medium and the other is denser

Ex: 1) Sound velocity is more in water than in air. Hence water is accortically rarer and air is denser

2) Speed of light is more in air than in water. Hence air is optically rarer and water is optically denser.

Amplitudes of reflected and transmitted waves:

*



let $y_i = A_i \sin(\omega t - k_1 x)$

$y_r = A_r \sin(\omega t - k_1(-x))$

$= A_r \sin(\omega t + k_1 x)$

$y_t = A_t \sin(\omega t - k_2 x)$

(assumed that pulses are not inverted)

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In medium 1, $y_1 = y_i + y_r$

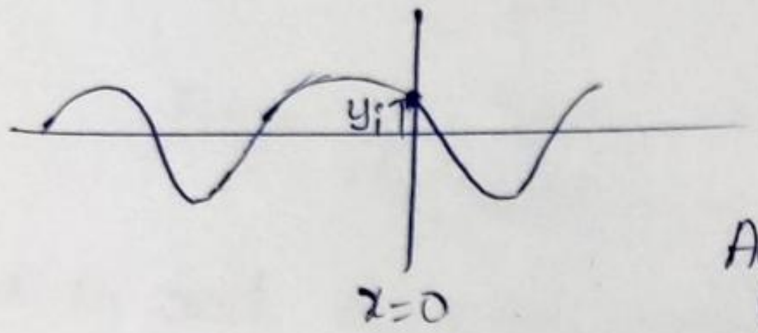
$= A_i \sin(\omega t - k_1 x) + A_r \sin(\omega t + k_1 x)$

In medium 2, $y_2 = y_t$

$= A_t \sin(\omega t - k_2 x)$

At the interface $(x=0)$ $y_1 = y_2$

$A_i \sin\omega t + A_r \sin\omega t = A_t \sin\omega t$



$A_i + A_r = A_t$ — ①

Again at $x=0$, $\left(\frac{\partial y}{\partial x}\right)_1 = \left(\frac{\partial y}{\partial x}\right)_2$

$-A_i k_1 \cos\omega t + A_r k_1 \cos\omega t = -A_t k_2 \cos\omega t$

$-A_i \frac{\omega}{v_1} + A_r \frac{\omega}{v_1} = -A_t \frac{\omega}{v_2}$ — ②

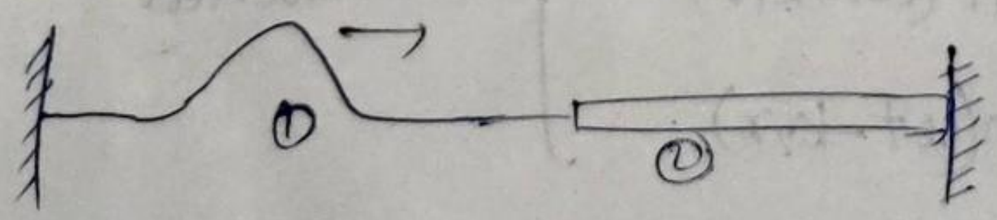
$A_r = \left(\frac{v_2 - v_1}{v_2 + v_1}\right) A_i$, $A_t = \left(\frac{2v_1}{v_2 + v_1}\right) A_i$

Case(i): If $v_2 < v_1$, then A_r is -ve i.e, reflected pulses are inverted w.r.t incident pulses. This happens when a wave travelling in rarer medium reflects at the interface with denser medium.

Case(ii): If $v_2 > v_1$, then A_r is +ve i.e, reflected pulses are not inverted w.r.t incident pulses. This is the case when waves travelling in denser medium reflect at the interface with rarer medium

Case(iii): If the waves reflect at a fixed end, then $v_2 = 0$. then A_r is -ve so the pulses are inverted

* Let a composite string is made of a thin and a thick string with same material



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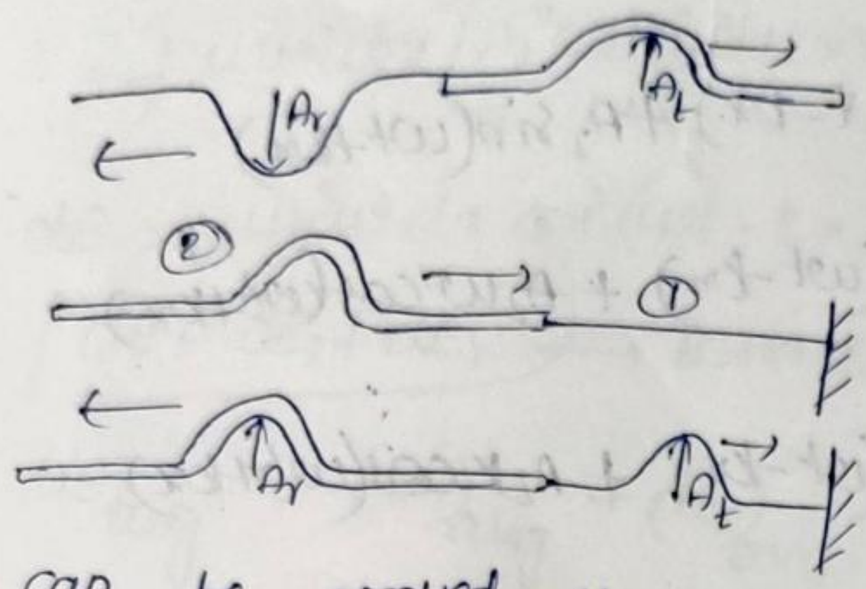
$$v_1 = \sqrt{\frac{T}{\pi r_1^2 \rho}}, \quad v_2 = \sqrt{\frac{T}{\pi r_2^2 \rho}}$$

$$r_1 < r_2 \Rightarrow v_1 > v_2$$

1 → rarer, 2 → denser



* It can be proved that $(P_{avg})_{incident} = (P_{avg})_{reflected} + (P_{avg})_{transmitted}$

* % power reflected $= \frac{P_r}{P_i} \times 100 = \frac{\frac{1}{2}\mu A_r^2 \omega^2 v_1}{\frac{1}{2}\mu_1 A_i^2 \omega^2 v_1} \times 100$ (for harmonic waves)

$$= \left(\frac{A_r}{A_i}\right)^2 \times 100$$

but % power transmitted $= \left(\frac{A_t}{A_i}\right)^2 \times 100$

$$= \frac{\frac{1}{2}\mu_2 A_t^2 \omega^2 v_2}{\frac{1}{2}\mu_1 A_i^2 \omega^2 v_1} \times 100$$

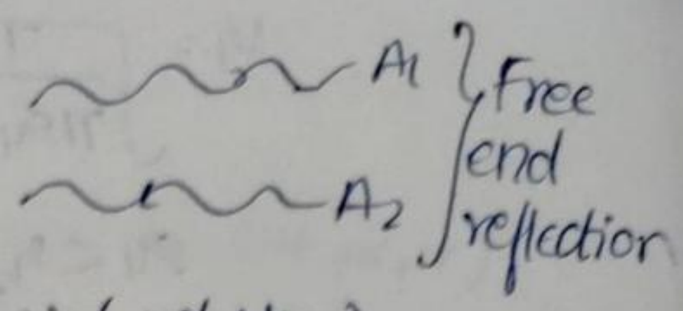
$$= \left(\frac{A_t}{A_i}\right)^2 \frac{v_1}{v_2} \times 100 \quad \left[\mu = \frac{T}{v^2}\right]$$

(or) $\frac{P_t}{P_i} \times 100 = \frac{P_i - P_r}{P_i} \times 100 = \left[1 - \left(\frac{A_r}{A_i}\right)^2\right] \times 100$

Partial Standing waves:

* If the amplitudes of two progressive waves whose super imposition form a standing wave are not equal then there will be some power transmittion through any cross section. This is called Partial standing wave

let incident wave equation is



$$y_1 = A_1 \sin(\omega t - kx)$$

reflected wave equation is $y_2 = A_2 \sin(\omega t + kx)$

$$y = y_1 + y_2$$
$$= A_1 \sin(\omega t - kx) + A_2 \sin(\omega t + kx)$$

$$\frac{\partial y}{\partial t} = A_1 \omega \cos(\omega t - kx) + A_2 \omega \cos(\omega t + kx)$$

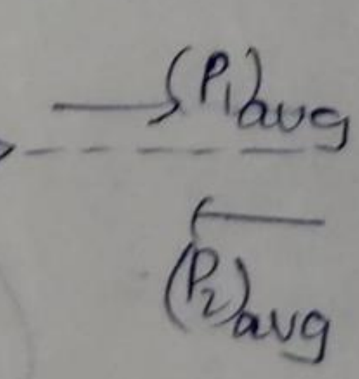
$$\frac{\partial y}{\partial x} = -A_1 k \cos(\omega t - kx) + A_2 k \cos(\omega t + kx)$$

Power transmitted $P_1 = \underbrace{-T}_{\mu v^2}\left(\frac{\partial y}{\partial t}\right)\left(\frac{\partial y}{\partial x}\right)$

$$P = -v^2\mu\left[-A_1^2 \omega k \cos^2(\omega t - kx) + A_2^2 \omega k \cos^2(\omega t + kx)\right]$$
$$= v^2\mu\, \omega \cdot \frac{\omega}{v}\left[A_1^2 \cos^2(\omega t - kx) - A_2^2 \cos^2(\omega t + kx)\right]$$

$$P_{avg} = \frac{\int_0^T P\,dt}{\int_0^T dt} = \mu\omega^2 v\left[A_1^2 \cdot \frac{1}{2} - A_2^2 \cdot \frac{1}{2}\right]$$
$$= \frac{1}{2}\mu\omega^2 v\left[A_1^2 - A_2^2\right]$$
$$= \frac{1}{2}\mu A_1^2 \omega^2 v_1 - \frac{1}{2}\mu A_2^2 \omega^2 v_2$$

$$P_{avg} = (P_1)_{avg} - (P_2)_{avg}$$



If $A_1 = A_2$, then $P_{avg} = 0$

Energy of a partial standing wave:

* $dK = \frac{1}{2}(dm)\left(\frac{\partial y}{\partial t}\right)^2$

$$= \frac{1}{2}\mu\, dx\left[A_1 \omega \cos(\omega t - kx) + A_2 \omega \cos(\omega t + kx)\right]^2$$
$$= \frac{1}{2}\mu\omega^2 dx\left[A_1 \cos(\omega t - kx) + A_2 \cos(\omega t + kx)\right]^2$$

$$dU = \frac{v^2}{2}\mu\, dx\left(\frac{\partial y}{\partial x}\right)^2$$

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$$\frac{\omega^2}{2k^2}\mu\, dx\left[-A_1k\cos(\omega t-kx)+A_2k\cos(\omega t+kx)\right]^2$$

$$=\frac{1}{2}\mu\omega^2 dx\left[A_2\cos(\omega t+kx)-A_1\cos(\omega t-kx)\right]^2$$

$$dE = dK + dU$$

$$=\frac{1}{2}\mu\omega^2 dx\left[A_2^2\cos^2(\omega t+kx)+A_1^2\cos^2(\omega t-kx)\right]2$$

$$dE = \mu A_1^2\omega^2 dx\cos^2(\omega t-kx)+\mu A_2^2\omega^2 dx\cos^2(\omega t+kx)$$

$\boxed{dE = dE_1 + dE_2}$ → instantaneous values

$$(dE)_{avg} = (dE_1)_{avg} + (dE_2)_{avg}$$

$$=\frac{1}{2}\mu\omega^2 dx\,(A_1^2+A_2^2)$$

Ex: Energy confined in one loop is

$$E_{avg} = \frac{1}{2}\mu\omega^2(A_1^2+A_2^2)\frac{\lambda}{2}$$

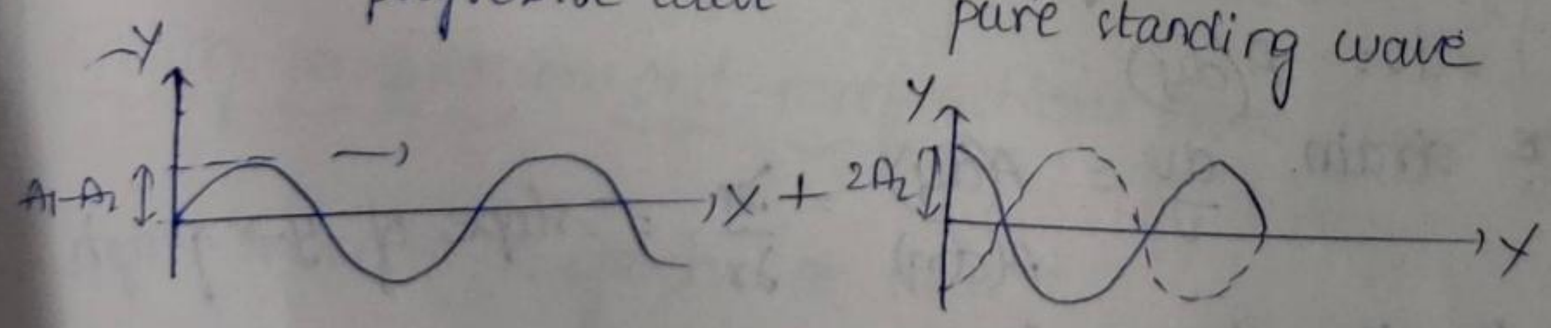
Amplitudes at nodes and antinodes

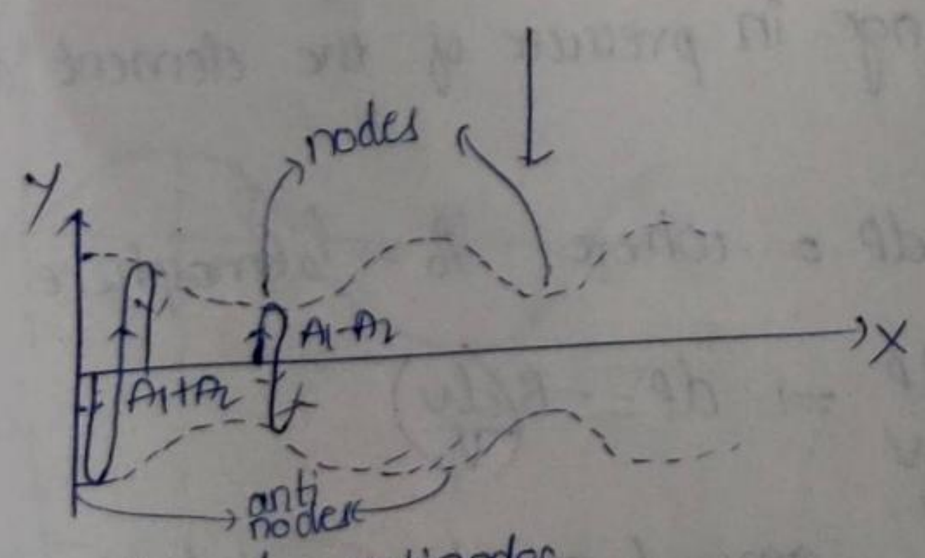
* $y = y_i + y_r$

$$= A_1\sin(\omega t-kx)+A_2\sin(\omega t+kx)$$

$$= A_1\sin(\omega t-kx)+A_2\sin(\omega t+kx)-A_2\sin(\omega t-kx)+A_2\sin(\omega t-kx)$$

$$= \underbrace{(A_1-A_2)\sin(\omega t-kx)}_{\text{progressive wave}} + \underbrace{2A_2\cos kx\sin\omega t}_{\text{pure standing wave}}$$





* At $x=0$, i.e at antinodes

$$y = (A_1-A_2)\sin\omega t + 2A_2\sin\omega t$$

$$= (A_1+A_2)\sin\omega t$$

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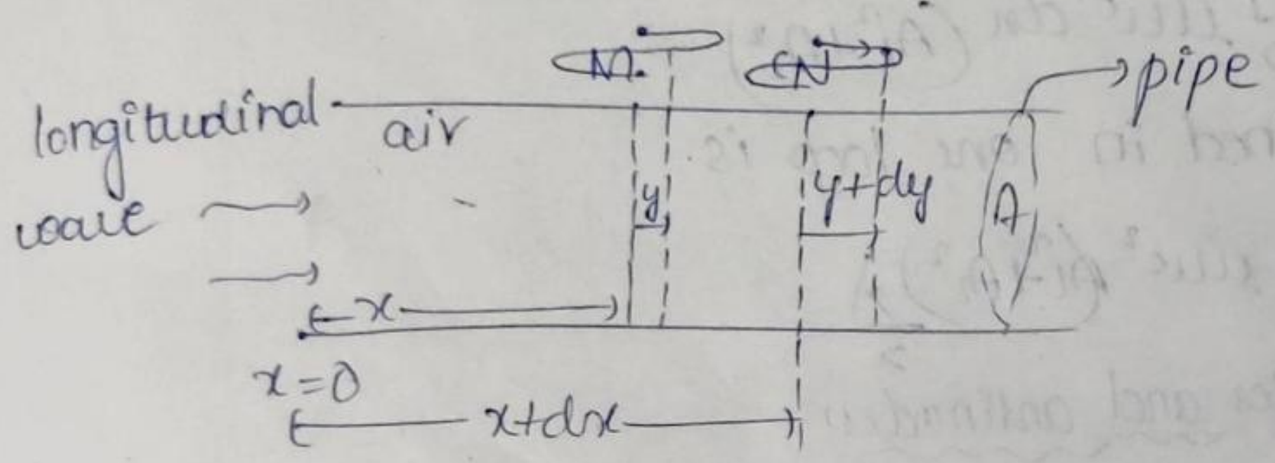
Pressure Waves

* When a longitudinal wave propagates through air, the elements of air compress and expand adiabatically as a consequence, pressure of that air element also varies ~~period~~ adiabatically
* If the wave is described in terms of pressure variations, it is called pressure wave. Similarly if the wave is described in terms of displacements of particles from their respective mean positions, then the wave is called displacement wave.
* Pressure wave and displacement wave are two descriptions of same longitudinal wave



* Let progressive wave equation is $y = A\sin(\omega t - kx)$

$x \rightarrow y$

$x+dx \rightarrow y+dy$

volume of air element within cross sections M and N in the absence of wave is $V = A(dx)$

When air is present, change in volume at time 't' is

$$dV = A(dy)$$

volume strain $\frac{dV}{V} = \frac{A(dy)}{A(dx)} = \frac{\partial y}{\partial x}$ = slope of y-x graph

Let dP be the change in pressure of the element due to change in volume

$$P_{element} = P_0 + dP \quad \text{where} \quad P_0 = P_{atmosphere}$$

Bulk modulus $B = \frac{dP}{-\frac{dV}{V}} \Rightarrow dP = -B\left(\frac{dV}{V}\right)$

$\boxed{dP = -B\left(\frac{\partial y}{\partial x}\right)}$ ⇒ general equation valid for any type of wave travelling in any direction

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* If $\frac{\partial y}{\partial x}$ is +ve, dP is -ve $\Rightarrow P < P_0$. This disturbance is called rare faction

* If $\frac{\partial y}{\partial x}$ is -ve, dP is +ve $\Rightarrow P > P_0$. This disturbance is called compression

* If $\frac{\partial y}{\partial x} = 0$ then $dP = 0$, $P = P_0$

* In case of a harmonic wave, $dP = -B\left(\frac{\partial y}{\partial x}\right)$

$$dP = -B\left[-AK\cos(\omega t - Kx)\right]$$

$$= \underbrace{+BAK}_{(dP)_m = \text{Pressure Amplitude}}\cos(\omega t - Kx)$$

$$dP = (dP)_m \sin\left(\omega t - Kx + \frac{\pi}{2}\right)$$

dP is a function of x and t

* Phase difference b/w pressure wave and displacement wave is $\frac{\pi}{2}$

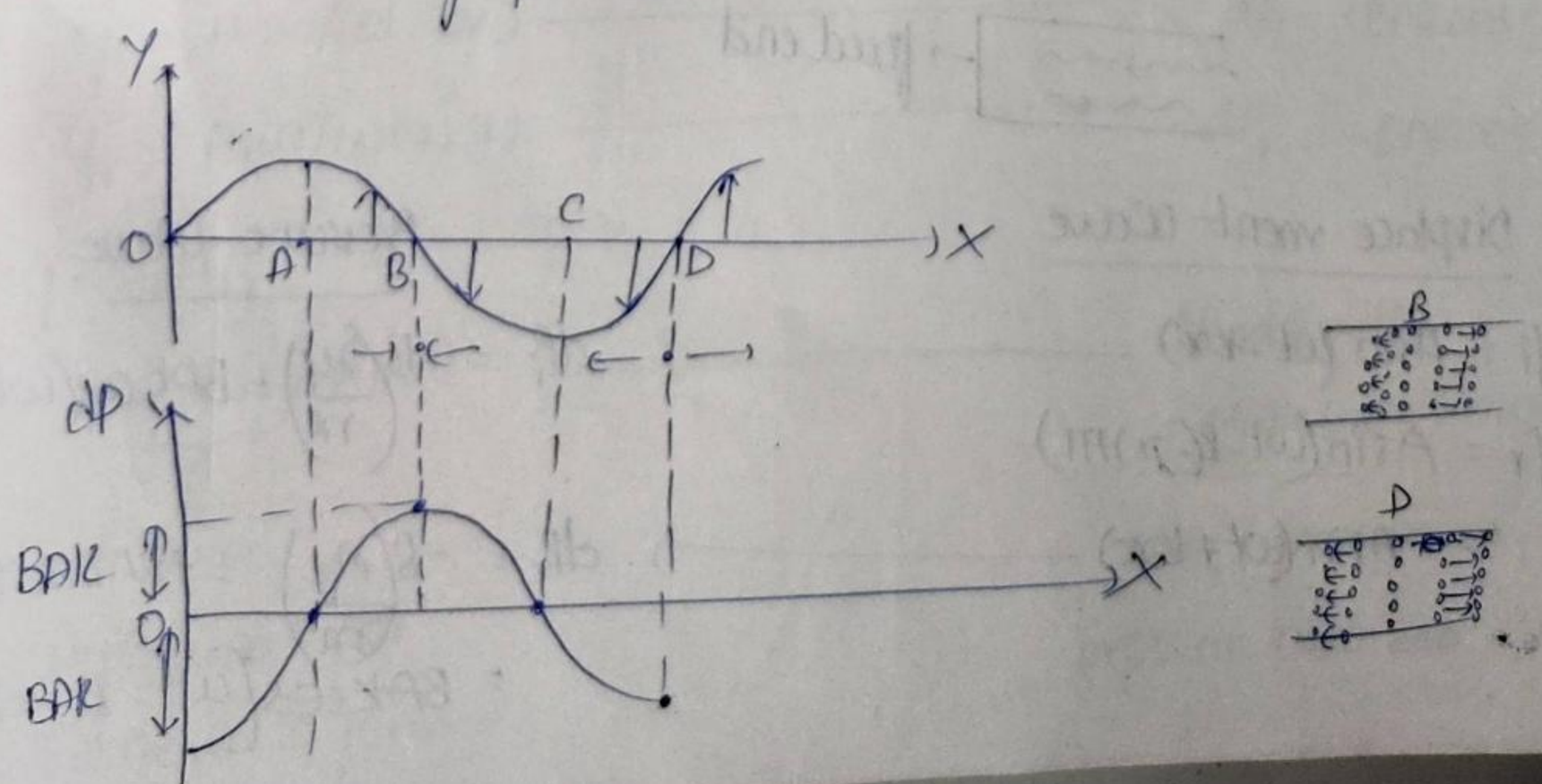
* Again $V_p = -V_s$, $\Rightarrow \frac{\partial y}{\partial x} = \frac{-1}{V}\frac{\partial y}{\partial t}$

$$dP = -B\left(\frac{\partial y}{\partial x}\right) = \frac{B}{V}\left(\frac{\partial y}{\partial t}\right) \longrightarrow$$ when the wave is travelling along +ve x-axis

* If the wave travels along -ve x-axis, $dP = \frac{-B}{V}\left(\frac{\partial y}{\partial t}\right)$

Finding compressions and rarefractions from wave diagrams:

case(i): From Y-X graph at time t



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At A & C, $\frac{\partial y}{\partial x} = 0 \Rightarrow dP = 0 \Rightarrow P = P_0$

O→A, C→D: $\frac{\partial y}{\partial x}$ is +ve ⇒ dP is −ve

rare faction region

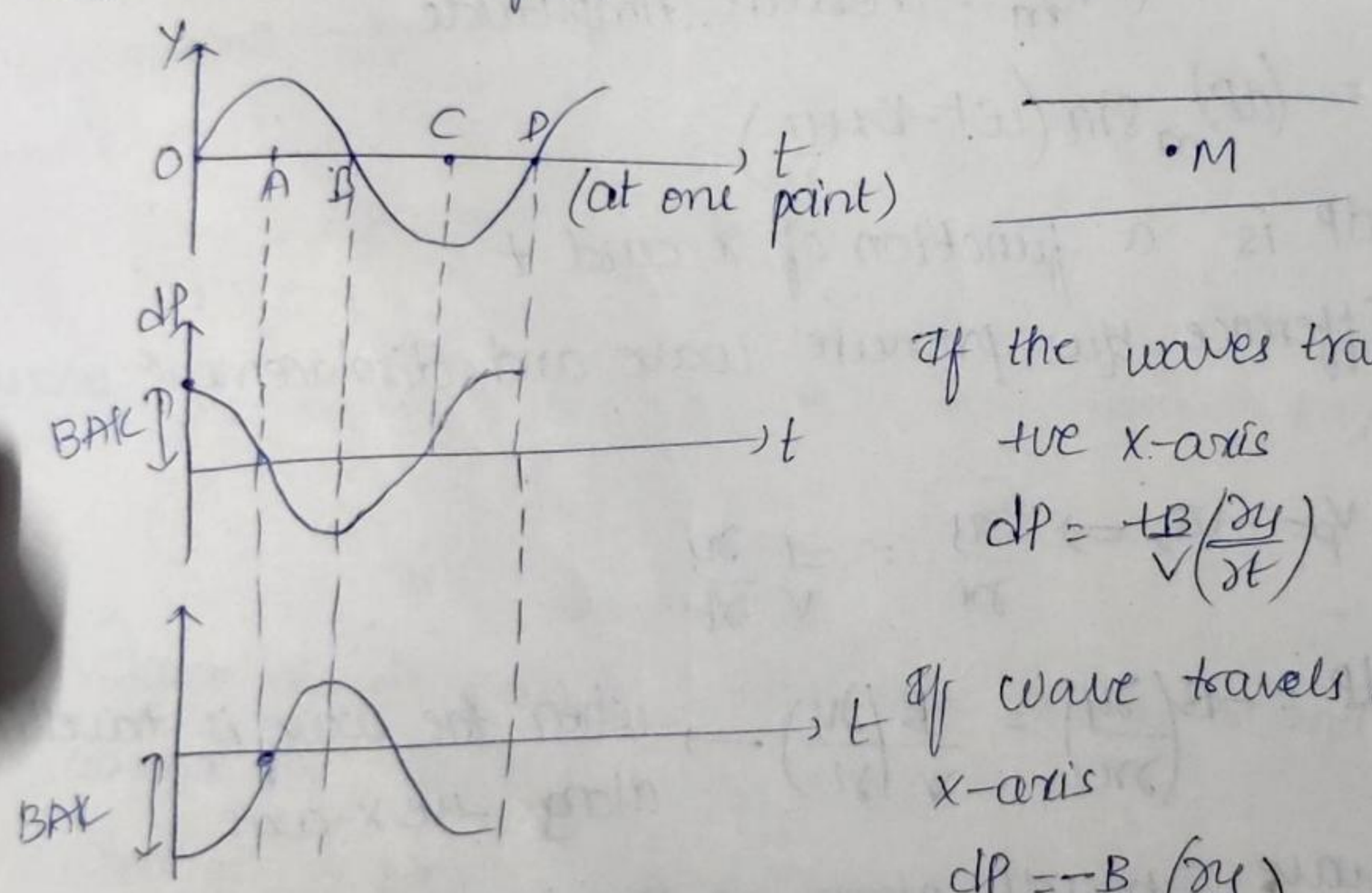
A→C: $\frac{\partial y}{\partial x}$ is −ve ⇒ dP is +ve

compression region

$P_0 = P_D = P_{atm} - BAK = P_{min}$

$P_B = P_{atm} + BAK = P_{max}$

case (ii): From y-t graph



If the waves travel along +ve x-axis

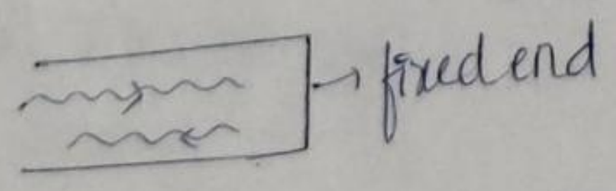
$dP = \frac{+B}{v}\left(\frac{\partial y}{\partial t}\right)$

If wave travels along −ve x-axis

$dP = -\frac{B}{v}\left(\frac{\partial y}{\partial t}\right)$

Reflection of Pressure Waves:

1) Fixed end Reflection:



Displacement wave	Pressure wave
$y_i = A\sin(\omega t - kx)$	$dP_i = -B\left(\frac{\partial y}{\partial x}\right) = BAK\cos(\omega t - kx)$
$y_r = A\sin(\omega t - k(-x) + \pi)$	
$= -A\sin(\omega t + kx)$	$dP_r = -B\left(\frac{\partial y}{\partial x}\right) = +BAK\cos(\omega t + kx)$
	$= BAK\cos[\omega t - k(-x) + 0]$

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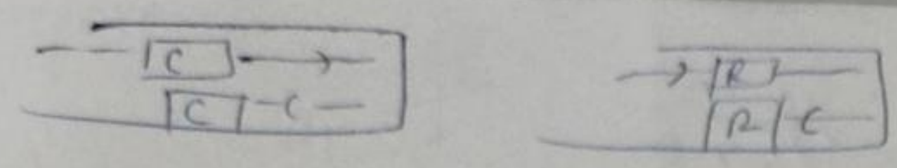
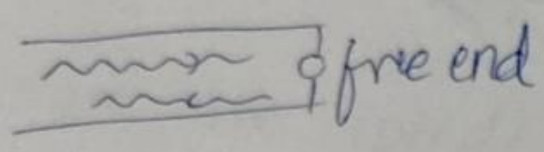


* Pressure pulses are not inverted when reflected at fixed end i.e. compression reflects as compression and rare faction reflects as rare faction

2) Free end Reflection:



Displacement wave

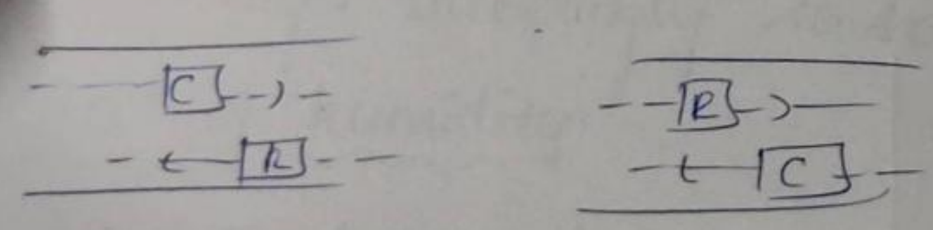
$y_i = A\sin(\omega t - kx)$ ⟶ $dP_i = -B\left(\frac{\partial y}{\partial x}\right) = BAk\cos(\omega t - kx)$

$y_r = A\sin(\omega t + kx)$ ⟶ $dP_r = -B\left(\frac{\partial y}{\partial x}\right) = -BAk\cos(\omega t + kx)$

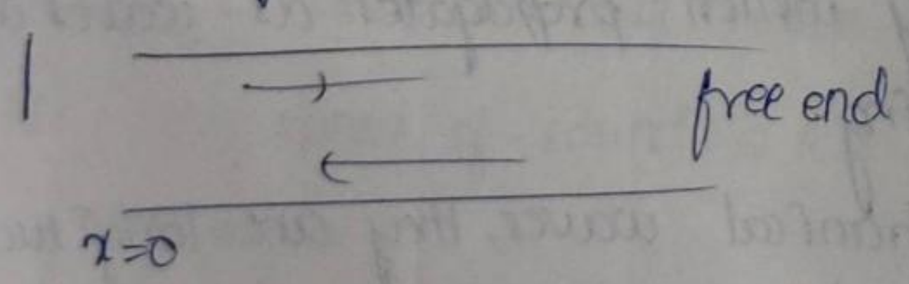
Pressure wave

$= BAk\cos(\omega t + kx + \pi)$

* π phase is added
* i.e, compressions are reflected as rare factions and rarefactions are reflected as compressions



Pressure standing waves:



Displacement wave

let $y_i = A\sin(\omega t - kx)$ ⟶ $dP_i = +BAk\cos(\omega t - kx)$

$y_r = A\sin(\omega t + kx)$ ⟶ $dP_r = -BAk\cos(\omega t + kx)$

Pressure wave

$y = y_i + y_r$

$= \underbrace{2A\cos kx}\sin\omega t$ ⟶ $dP = dP_i + dP_r$

$= \underbrace{2BAk\sin kx}\sin\omega t$

at $x = 0$

displacement $= 2A$

Antinode is formed

$x = 0$, pressure amplitude $= 0$

pressure node is formed

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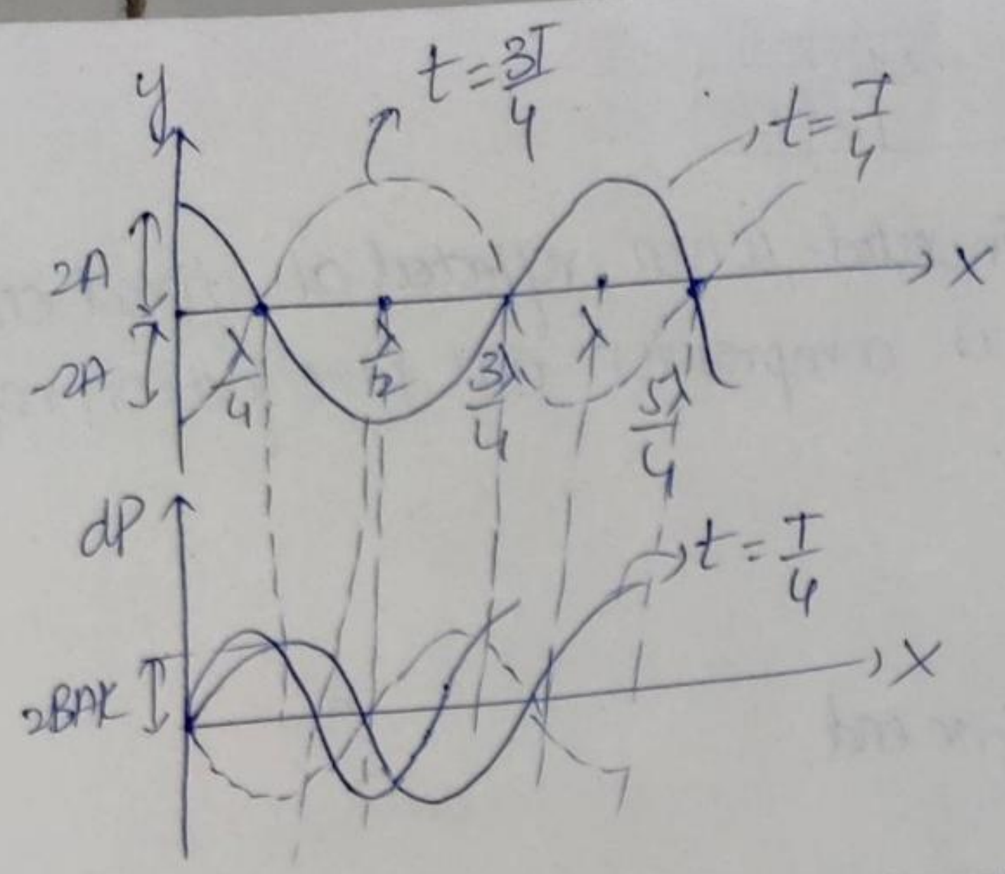

* At $x = 0, \frac{\lambda}{2}, \lambda, \ldots$ displacement antinodes, pressure nodes are formed

* At $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$ displacement nodes pressure antinodes are formed

* At pressure nodes $dP = 0$ and $P = P_0$ always, compressions, rarefactions never form

* At pressure antinodes, pressure varies b/w $P_{min} = P_0 - 2BAk$ and $P_{max} = P_0 + 2BAk$ i.e, compressions and rarefactions both are formed alternately

* $P_{min} > 0 \Rightarrow 2BAk < P_0$

Sound waves:

* It is the form of energy which propagates as waves and can cause sensation of hearing

* Sound waves are mechanical waves, they are longitudinal in liquids and gases but can be transverse (or) longitudinal in solids

* Speed of sound in air $v = \sqrt{\frac{B}{\rho}}$. Here B = adiabatic bulk modulus of air $= \gamma P$. Therefore $v = \sqrt{\frac{\gamma P}{\rho}}$

* But $\frac{P}{\rho} = \frac{RT}{M}$. Therefore $v = \sqrt{\frac{\gamma RT}{M}}$

Effect of Temperature:

* For a given gas, γ and M are constant. Therefore $v \propto \sqrt{T}$

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Therefore $\cancel{\frac{V_1}{T_1} = \sqrt{\frac{V_2}{T_2}}}$ $\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}$

Ex: What is the temp. in °C at which speed of sound is twice of its value at 0°C?

Sol: $\frac{V_1}{2V_1} = \sqrt{\frac{273}{T_2}} \Rightarrow \boxed{T_2 = 273(4)\,K}$

$\boxed{T_2 = 819°C}$

* If V_0, V_t are the speeds of sounds at 0°C and t°C respectively, then $\frac{V_t}{V_0} = \sqrt{\frac{t+273}{273}} = \left(1+\frac{t}{273}\right)^{\frac{1}{2}}$

If $\frac{t}{273} << 1$, then $V_t = V_0\left(1+\frac{t}{546}\right)$

Effect of pressure:

* $\frac{P}{\rho} = \frac{RT}{M}$. If T is maintained constantly, then V will be independent of P. If P is changed in this way then ρ also changes accordingly so that $\frac{P}{\rho}$ remains constant

* Effect of humidity:

* At a given P, on T if the humidity of air increases, then M_{eff} (or ρ decreases and hence, the speed of sound increases

* In this way, speed of sound is less in humid hydrogen than in dry hydrogen

Intensity in terms of pressure amplitude:

$I_{avg} = \frac{1}{2}\rho A^2\omega^2 v \longrightarrow$ For harmonic progressive wave

$dP_m = BAK$

$I_{avg} = \frac{1}{2}\rho\left(\frac{dP_m}{BK}\right)^2 \omega^2 v$

$= \frac{\rho}{2}\frac{dP_m^2}{B^2K^2}\cdot\omega^2 v = \frac{\rho}{2}\frac{dP_m^2}{v^4\rho^2}\cdot v^2 v$

$\boxed{I_{avg} = \frac{dP_m^2}{2\rho v}}$

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Sound level:

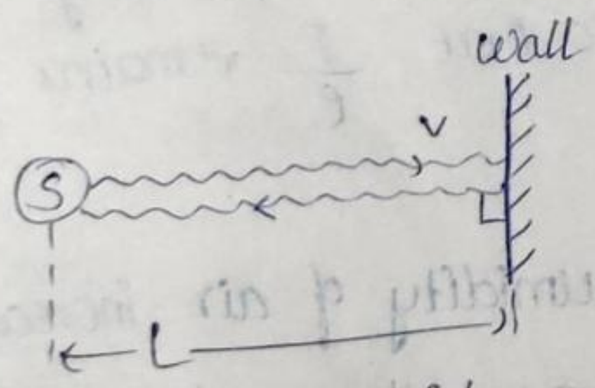
* If I is the intensity of sound, then its sound level is defined as $S.L = 10\log\left(\frac{I}{I_0}\right)$ where $I_0 = 10^{-12}\ W/m^2$ this is called threshold of audibility. This is the least intensity of sound that we can hear

units of sound level = Decibel (dB)

* For $I = I_0$, $S.L = 0\,dB$.
* Maximum intensity of sound that we can hear is $1\,W/m^2$. This is called threshold of pain. Its sound level = 120 dB
* If I_1, I_2 are intensities of two sounds, then difference in their sound levels $S.L_2 - S.L_1 = 10\log\left(\frac{I_2}{I_0}\right) - 10\log\left(\frac{I_1}{I_0}\right) = 10\log\left(\frac{I_2}{I_1}\right)$

Echo:

* If a sound is given by a person, and it is received after reflection. Then the received sound is called echo



with normal incidence,

time of echo, $t = \frac{2d}{v}$

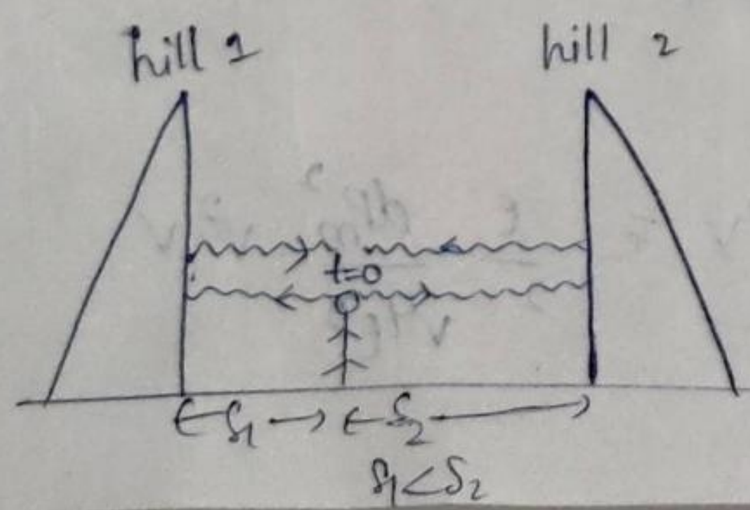
v = speed of sound in air

$v = 330\,m/s$

* Persistance of hearing = $\frac{1}{10}$ sec. To distinguish echo from direct sound $\frac{2d}{v} > \frac{1}{10}\,sec \Rightarrow d > \frac{v}{20}$

$d > 16.5\,m$

Ex1)



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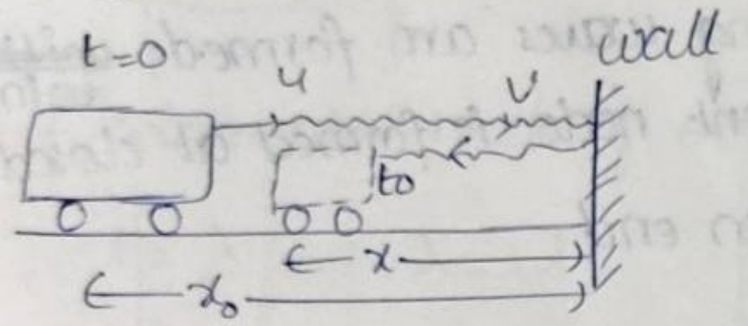
* Sound is released at $t=0$

* First echo is heard after time $t_1 = \frac{2s_1}{v}$. Second echo is heard after time $t_2 = \frac{2s_2}{v}$. $t_3 = \frac{2}{v}(s_1+s_2)$.

* At time t_3, two echos are heard. Each one is twice reflected. At this time, situation is similar to that at $t=0$

* 4th echo is heard after time $t_4 = t_3 + t_1$. and $t_5 = t_3 + t_2$.
$t_6 = t_3 + t_3$

Ex1)



Sol: Distance travelled by sound is

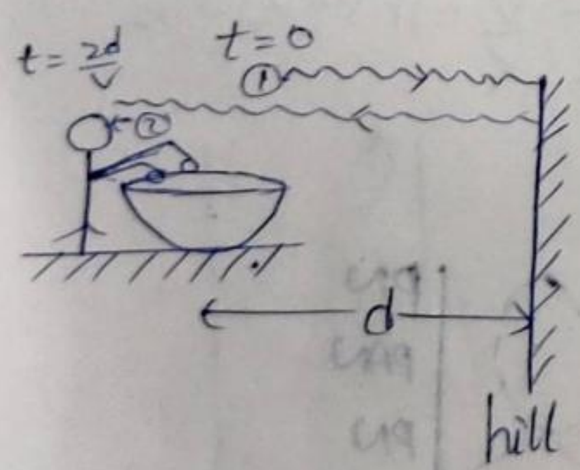
$$x_0 + x = vt_0$$

Distance travelled by vehicle is

$$x_0 - x = ut_0$$

$$\boxed{x_0 = \frac{(v+u)}{2} t_0} \quad \boxed{x = \left(\frac{v-u}{2}\right) t_0}$$

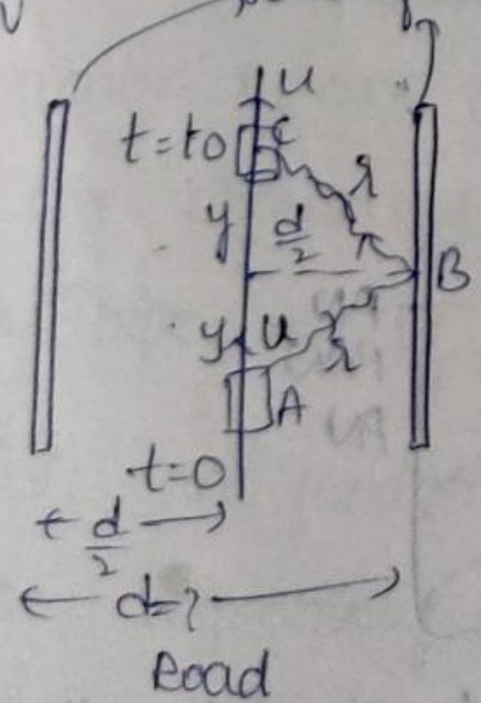
Ex2)



Let T is the time gap b/w consecutive strokes. If this time equals $\frac{2d}{v}$ then echo of first ~~sound~~ stroke and direct sound of second stroke are received simultaneously. He cannot distinguish echo from direct sound.

So $T = \frac{2d}{v}$

Ex3)



$$2y = ut_0 \Rightarrow y = \frac{ut_0}{2} \quad -(1)$$

$$2r = vt_0 \Rightarrow r = \frac{vt_0}{2} \quad -(2)$$

$$\frac{d}{2} = \sqrt{r^2 - y^2} \Rightarrow d = 2\sqrt{\frac{v^2t_0^2}{4} - \frac{u^2t_0^2}{4}}$$

$$\boxed{d = t_0\sqrt{v^2 - u^2}}$$

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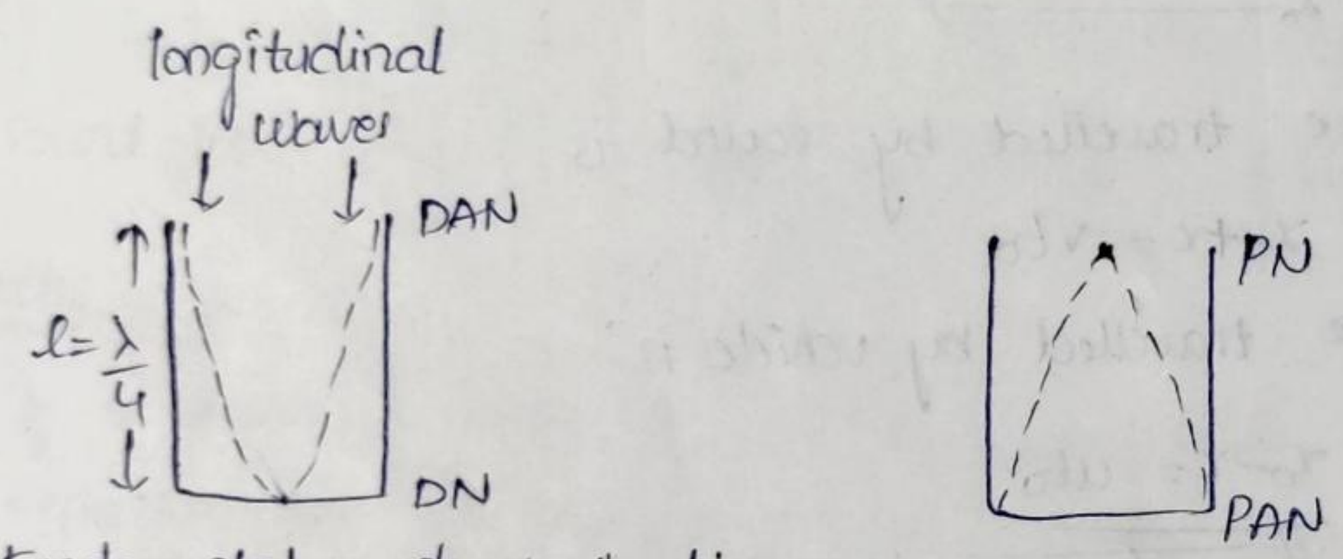
* longitudinal wave velocity in thin rod $v = \sqrt{\frac{Y}{\rho}}$ (Y = young's modulus)

transverse wave velocity in thin rod $v = \sqrt{\frac{\eta}{\rho}}$ (η = rigidity modulus)

Organ Pipes:

* Closed pipe:

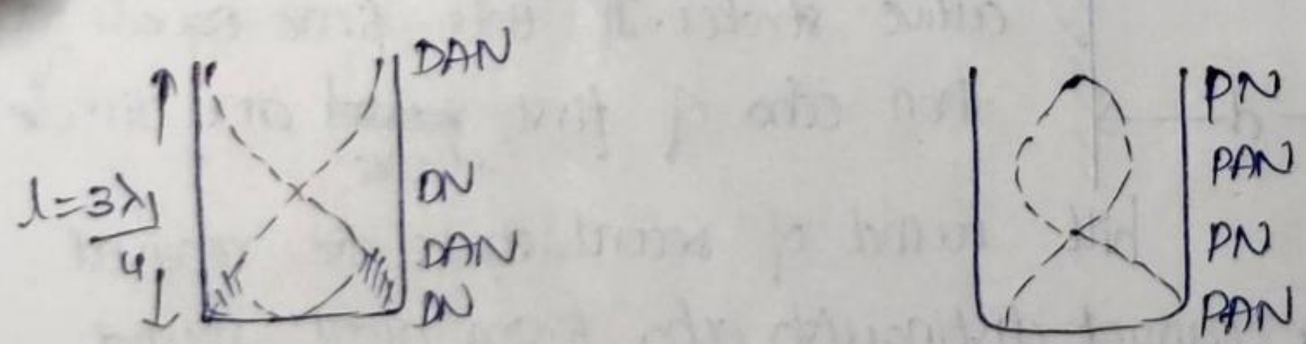
It is the pipe closed at one end and open at other end when longitudinal standing waves are formed ~~with~~ in the air of pipe, always displacement node is formed at closed end and anti node is formed at open end



Fundamental mode of vibration

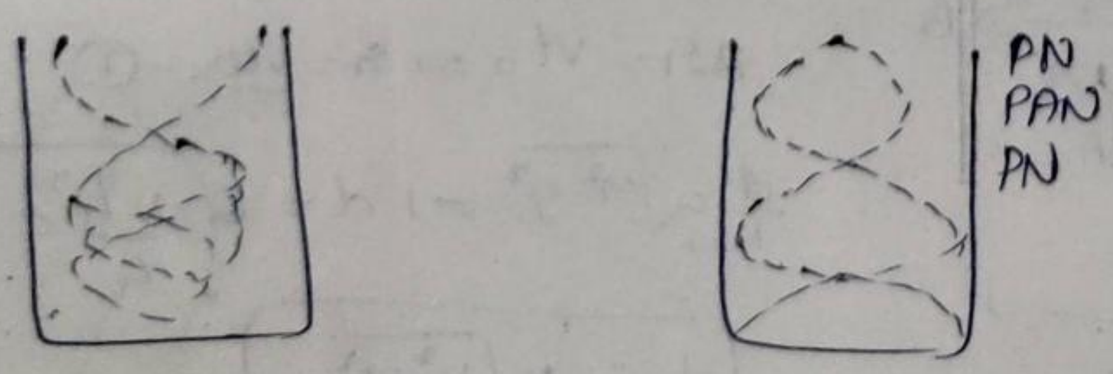
$$l = \frac{\lambda}{4} \Rightarrow \lambda = 4l$$

$$\boxed{n = \frac{v}{\lambda} = \frac{v}{4l} = \frac{1}{4l}\sqrt{\frac{\gamma RT}{M}}}$$



$$l = \frac{3\lambda_1}{4} \Rightarrow \lambda_1 = \frac{4l}{3}$$

$$n_1 = \frac{v}{\lambda_1} = 3n = 3\text{rd Harmonic}$$



* These frequencies are similar to that of string fixed at one end and free at other end.

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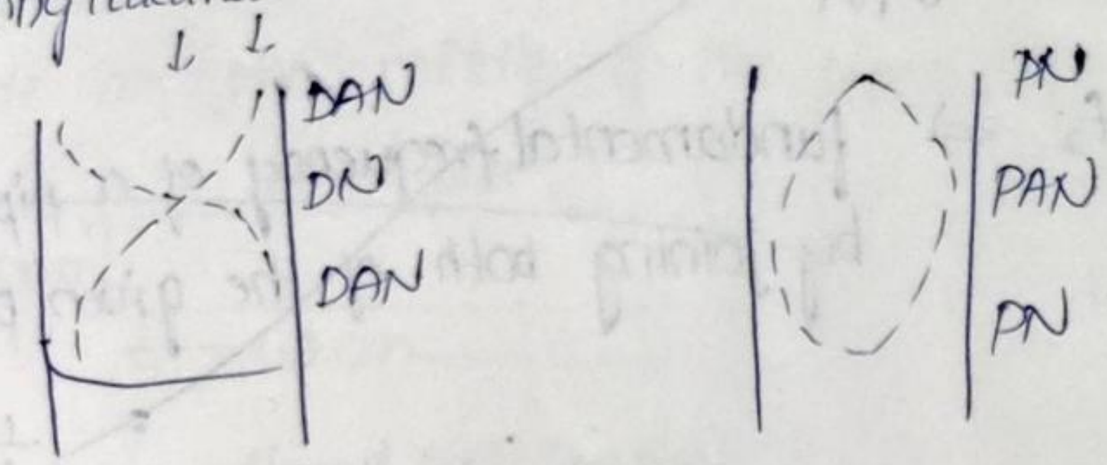
Open pipe:

* It is the pipe open at both ends. Displacement antinodes are formed at both ends

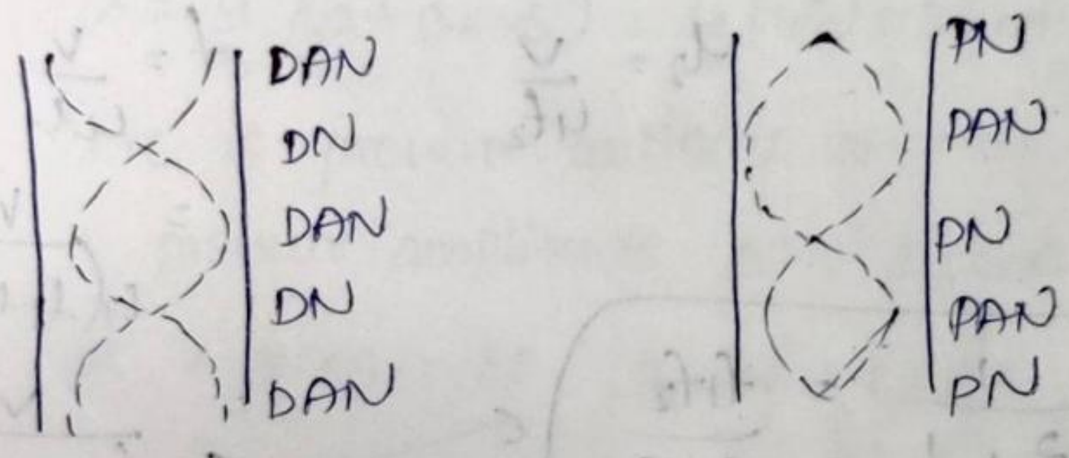
longitudinal waves



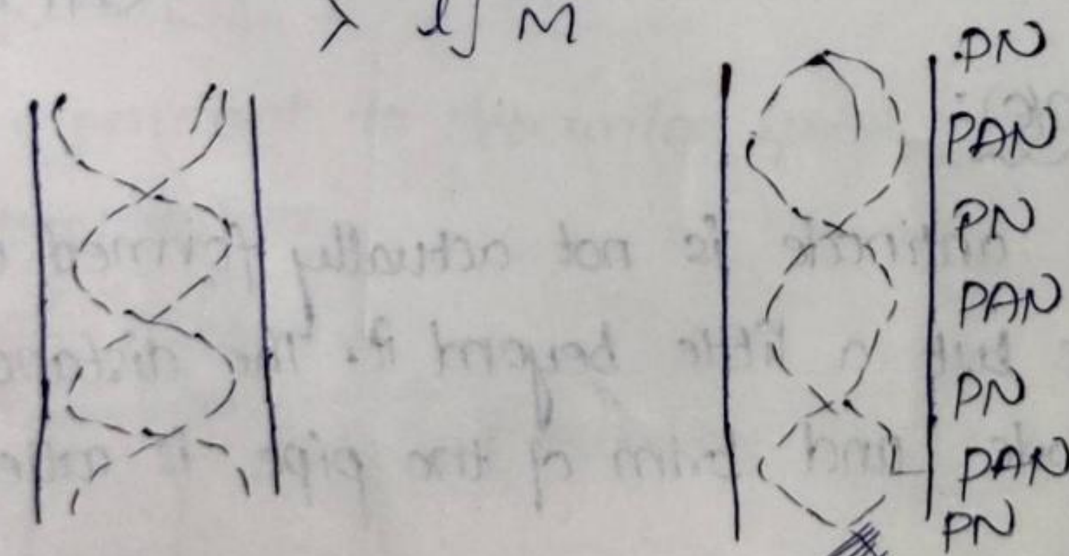
In fundamental mode of oscillation

$$l = \frac{\lambda}{2} \Rightarrow \lambda = 2l$$

$$n = \frac{v}{\lambda} = \frac{1}{2l}\sqrt{\frac{\gamma RT}{M}}$$



$$l = \lambda \Rightarrow n = \frac{v}{\lambda} = \frac{1}{l}\sqrt{\frac{\gamma RT}{M}}$$



* open pipe is more useful than closed pipe as both even and odd harmonics are allowed in open pipe and only odd harmonics are allowed in closed pipe

* In any pipe, $n \propto \frac{\sqrt{T}}{l}$ where T = absolute temperature.

Ex1) Let f_1, f_2 are fundamental frequencies of an open and a closed organ pipe respectively. Find the fundamental frequency of a closed pipe formed by joining the above two pipes

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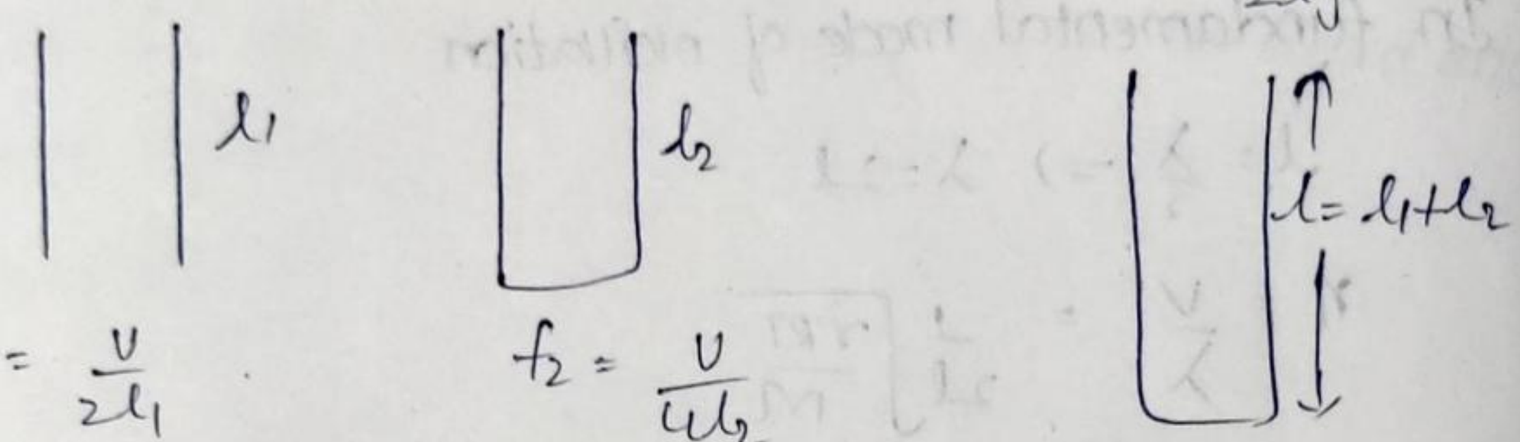
Sol: $N_1 f_1 = N_2 f_2$

$$\frac{f_2}{f_1} = \frac{\frac{1}{2l}\sqrt{\frac{\gamma RT}{M}}}{\frac{1}{4l}\sqrt{\frac{\gamma RT}{M}}} = 2$$

$2f_1 = f_2 \Rightarrow$ fundamental frequency of a pipe obtained by joining both of the given pipes $= 2f_1$

$= \frac{1}{2l}\sqrt{\gamma RT}$



$f_1 = \frac{v}{2l_1}$ $\quad f_2 = \frac{v}{4l_2}$

$l_1 = \frac{v}{2f_1}$ $\quad l_2 = \frac{v}{4f_2}$ $\quad f = \frac{v}{4l}$

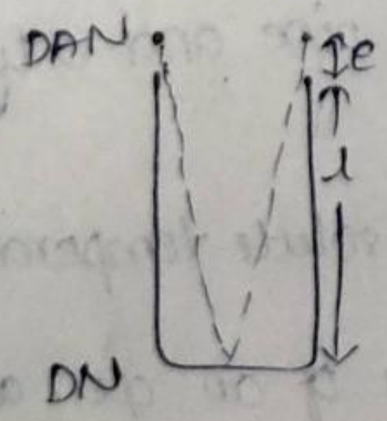
$= \frac{v}{4(l_1 + l_2)}$

$= \frac{v}{4\left(\frac{v}{2f_1} + \frac{v}{4f_2}\right)}$

$$\boxed{f = \frac{1}{\frac{2}{f_1} + \frac{1}{f_2}} = \frac{f_1 f_2}{2f_2 + f_1}}$$

End Correction(e):

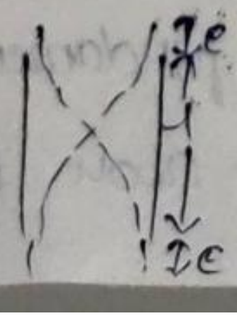
* Displacement antinode is not actually formed at free end of the pipe but a little beyond it. The distance b/w displacement antinode and brim of the pipe is called end correction (e).
* It was observed that $e = 0.6r$ where r = radius of pipe



$l + e = \frac{\lambda}{4} \Rightarrow \lambda = 4(l+e)$

$$\boxed{n = \frac{v}{\lambda} = \frac{v}{4(l + 0.6r)}}$$

For open pipe,



$l + 2e = \frac{\lambda}{2}$

$$\boxed{n = \frac{v}{\lambda} = \frac{v}{2(l + 1.2r)}}$$

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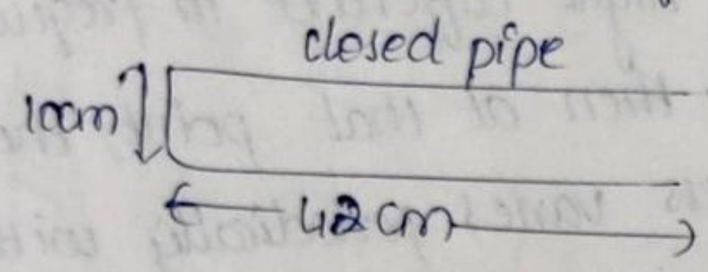
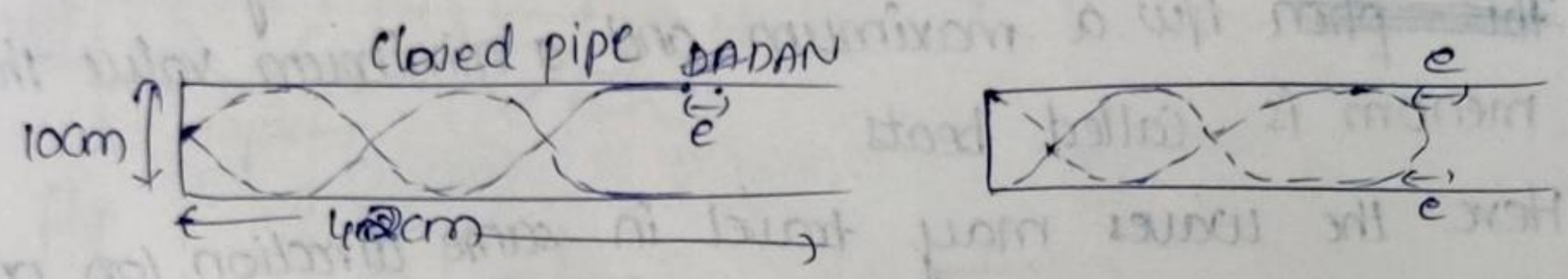
* Natural frequencies are more for narrower tubes than for wider tubes

Ex) If the air column in the pipe shown vibrates in ~~1st~~ 2nd over tone with maximum amplitude ΔP_0. what is the pressure amplitude at the middle of the pipe?



Sol:



$$42 + e = \frac{5\lambda}{4}$$

$$\lambda = \frac{4}{5}(42 + 0.6 \times 5) = \frac{4}{5}(45) = 36cm$$

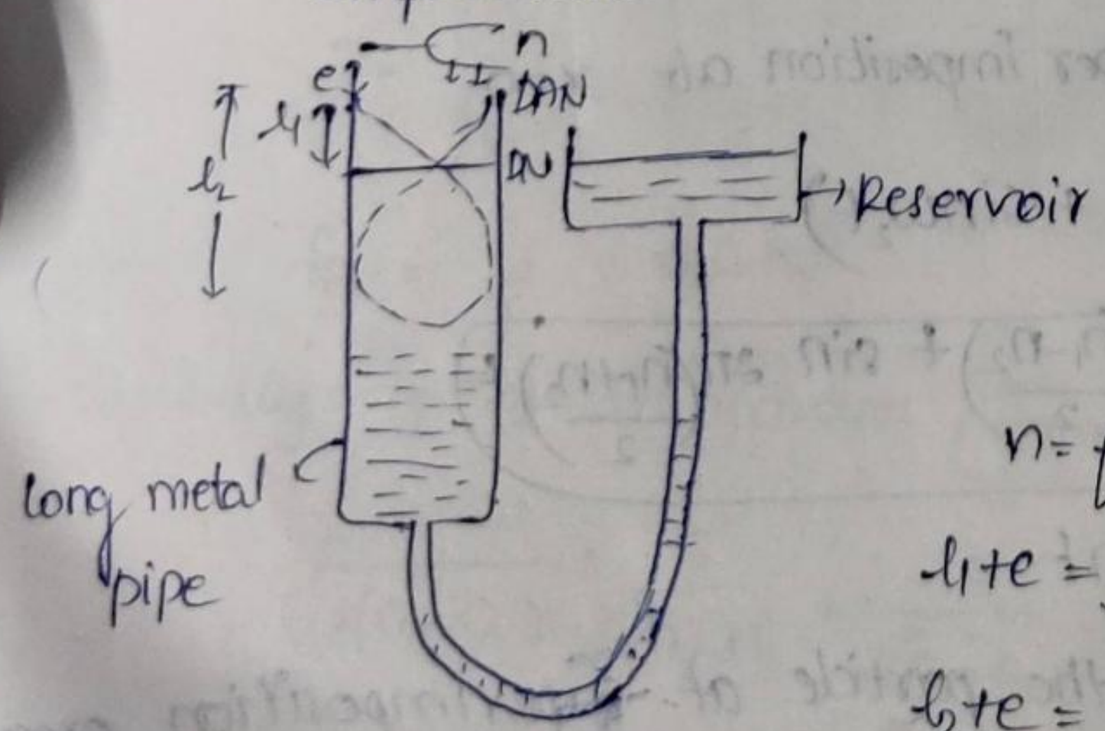
There is pressure antinode at $x=0$

pressure amplitude $\Delta P = \Delta P_0 \cos kx$

at $x = 21cm$, $\Delta P = \left|\Delta P_0 \cos\left(\frac{2\pi}{36}(21)\right)\right| = \frac{\sqrt{3}}{2}\Delta P_0$

Resonating air columns experiment:

* It is the experiment to determine speed of sound in air at room temperature.



l_1, l_2 are first, second resonating lengths of air column

n = frequency of tuning fork

$$l_1 + e = \frac{\lambda}{4} \quad \text{—} \ (1)$$

$$l_2 + e = \frac{3\lambda}{4} \quad \text{—} \ (2)$$

$(2) - (1) \Rightarrow l_2 - l_1 = \frac{\lambda}{2} \Rightarrow \lambda = 2(l_2 - l_1)$

$v = n\lambda = 2n(l_2 - l_1)$

$\frac{(2)}{(1)} \Rightarrow 3 = \frac{l_2 + e}{l_1 + e} \Rightarrow \boxed{e = \frac{l_2 - 3l_1}{2}}$

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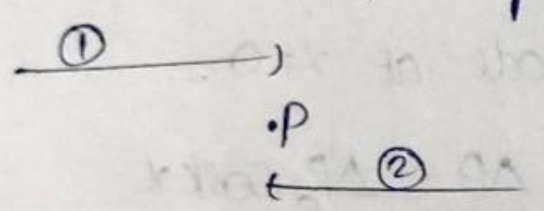
* 3rd resonance occurs at a length $l_2 + \frac{\lambda}{2} = l_3$ and $l_4 = l_3 + \frac{\lambda}{2}$
$= l_2 + \lambda$.

* It can happen until $l_n \leq$ total length of pipe

Beats:

* When two sound waves of slight difference in frequencies are super imposed at a point. then at that point, the sound intensity and hence loudness varies periodically with time ~~This phen~~ b/w a maximum and a minimum value. this phenomenom is called beats
* Here the waves may travel in same direction (or) opposite direction
* No. of maximas heard per second (or) minimas heard per second is called beat frequency



$$\left.\begin{aligned} y_1 &= A\sin(\omega_1 t - k_1 x) \\ y_2 &= A\sin(\omega_2 t \pm k_2 x) \end{aligned}\right\} \quad \frac{\omega_1}{k_1} = \frac{\omega_2}{k_2}$$

$$y = y_1 + y_2$$
$$= A\sin(\omega_1 t - k_1 x) + A\sin(\omega_2 t \pm k_2 x)$$

let us discuss super imposition at $x=0$

$$y = A(\sin\omega_1 t + \sin\omega_2 t)$$

$$\boxed{y = 2A\cos 2\pi\left(\frac{n_1 - n_2}{2}\right)t \sin 2\pi\left(\frac{n_1 + n_2}{2}\right)t}$$

$$y = A_r \sin 2\pi n t$$

As the resultant, the particle at superimposition executes SHM with

resonant frequency $n = \frac{n_1 + n_2}{2}$

amplitude $A_r = 2A\cos 2\pi\left(\frac{n_1 - n_2}{2}\right)t$

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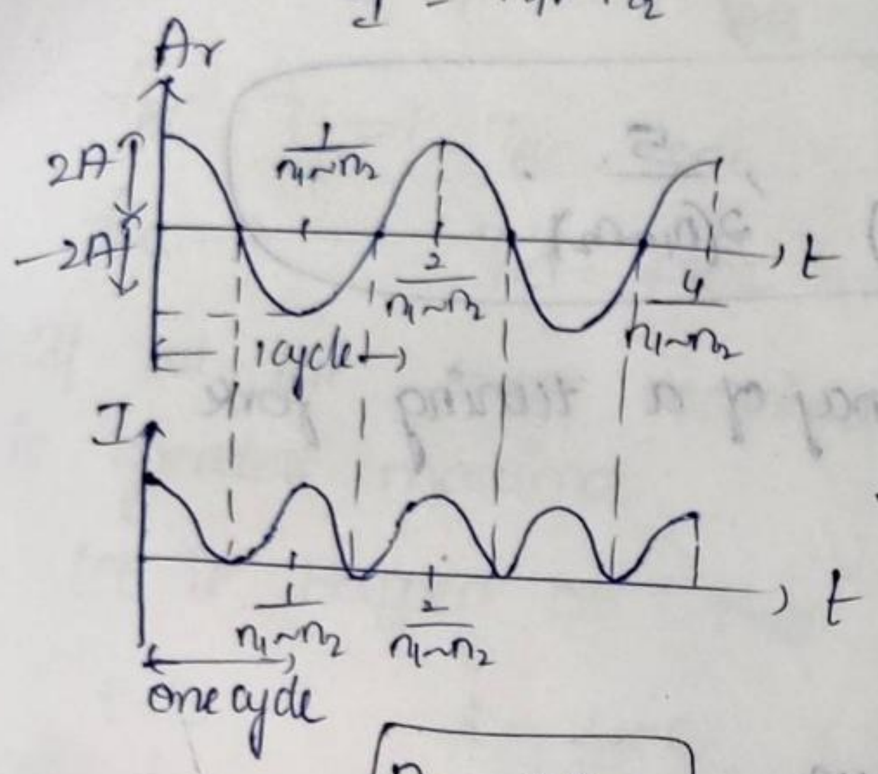
Amplitude varies periodically with a frequency

$$n_A = \frac{n_1 - n_2}{2} < \text{"}n\text{" always}$$

As intensity $I \propto A^2$

I varies periodically with a frequency

$$n_I = n_1 - n_2$$



$$T = \frac{2}{n_1 - n_2}$$

$$T = \frac{1}{n_1 - n_2}$$

$$\boxed{n_I = 2n_A}$$

* I is maximum when $\cos 2\pi\left(\frac{n_1 - n_2}{2}\right)t = \pm 1$

$$\pi(n_1 - n_2)t = m\pi$$

$$t = \frac{m}{n_1 - n_2} = 0, \frac{1}{n_1 - n_2}, \frac{2}{n_1 - n_2}, \frac{3}{n_1 - n_2}, \ldots$$

* Time difference b/w consecutive maximas

$$T = \frac{1}{n_1 - n_2}$$

$$f_B = \frac{1}{T} = n_1 - n_2$$

Similarly "I" is minimum $[I_{min} = 0]$ at

$$t = \frac{1}{2(n_1 - n_2)}, \frac{3}{2(n_1 - n_2)}, \frac{5}{2(n_1 - n_2)}, \ldots$$

$$\Delta T = \frac{1}{n_1 - n_2}$$

no. of minimas heard per second

$$f_B = \frac{1}{\Delta t_{min}} = n_1 - n_2$$

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Intensity Minima:

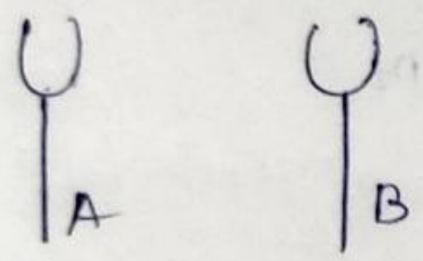
$I_{min} = a$ when $\cos 2\pi\left(\frac{n_1-n_2}{2}\right)t = 0$

$$\pi(n_1-n_2)t = (2m+1)\frac{\pi}{2}$$

$$t = \frac{2m+1}{2(n_1-n_2)}, \quad m = 0, 1, 2, 3, \ldots$$

$$\boxed{t = \frac{1}{2(n_1-n_2)}, \frac{3}{2(n_1-n_2)}, \frac{5}{2(n_1-n_2)}, \ldots}$$

Ex 1) Finding unknown frequency of a tuning fork



n_A is known, n_B is unknown

let $f_{beat} = 6$, when A and B are sounded together

$n_A - n_B = 6$ — ① (or) $n_B - n_A = 6$ — ②

* When the prongs of tuning fork are filed, it's frequency increases and as a consequence, beat frequency changes

(i) When A is filed, if f_{beat} is decreased (observed)

then ② is correct i.e, $n_B = n_A + 6$

(ii) When A is filed, if f_{beat} is increased (observed)

then ① is correct i.e, $n_B = n_A - 6$

(iii) When B is filed, if f_{beat} is decreased (observed)

then ① is correct i.e, $n_A = n_B + 6 \Rightarrow n_B = n_A - 6$

(iv) When B is filed, if f_{beat} is increased (observed)

then ② is correct i.e, $n_B = n_A + 6$

* Instead of filing, if wax is attached to the prongs, then frequency of that tuning fork decreases and hence f_{beat} also changes. The above four cases will again arised

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f_B with 3 ~~seconds~~ sounds:

$n_A = n-1 \quad (400)$

$n_B = n \quad (401)$

$n_C = n+1 \quad (402)$

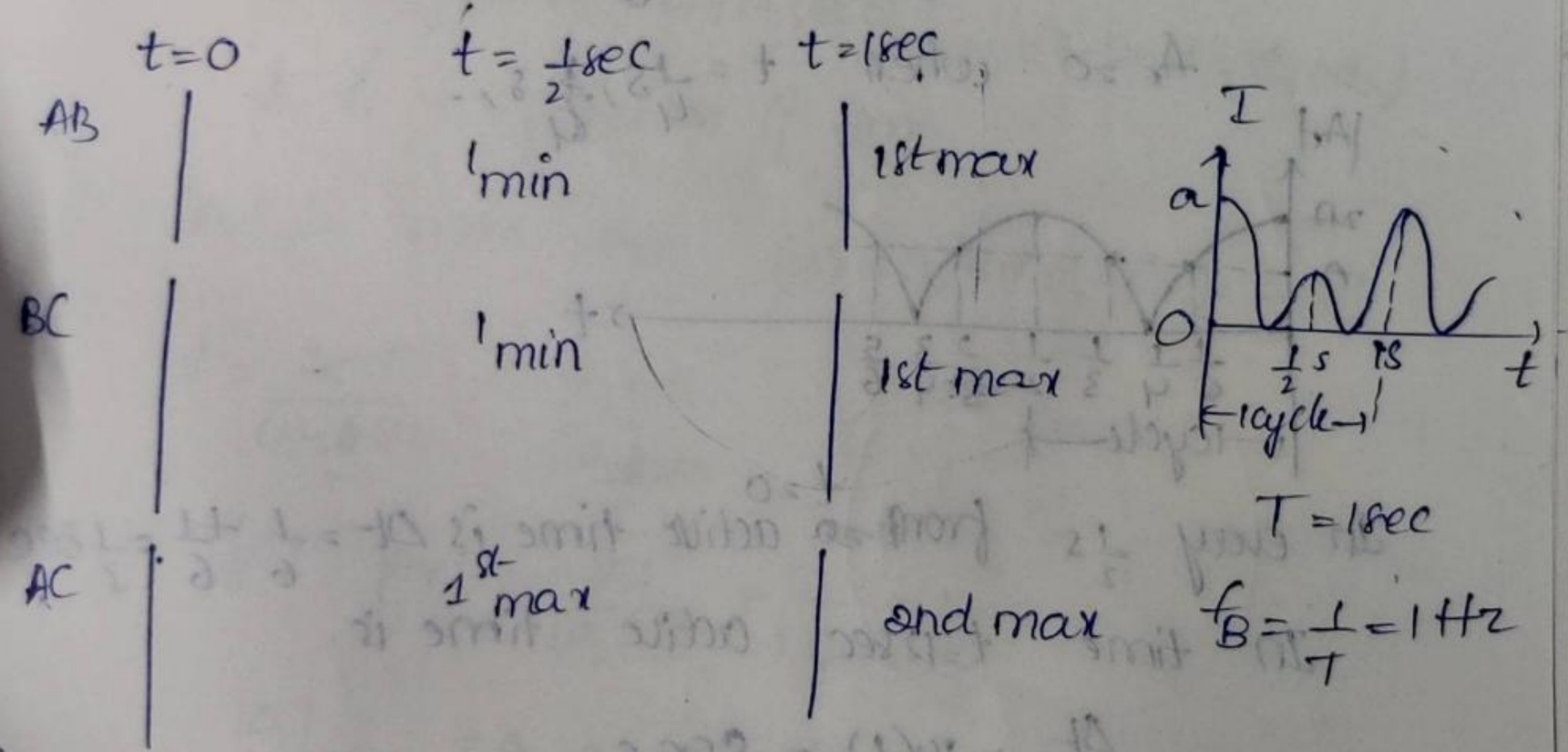
$f_{AB} = n_B - n_A = 1 \Rightarrow T_{AB} = \frac{1}{f_{AB}} = 1\,sec$

$f_{BC} = 1 \Rightarrow T_{BC} = 1\,sec$

$f_{AC} = 2 \Rightarrow T_{AC} = \frac{1}{2}\,sec$

If all the three maximas formed at an instant, it is greatest maxima

let it happen at $t=0$



$f_B = \frac{1}{T} = 1\,Hz$

Ex)

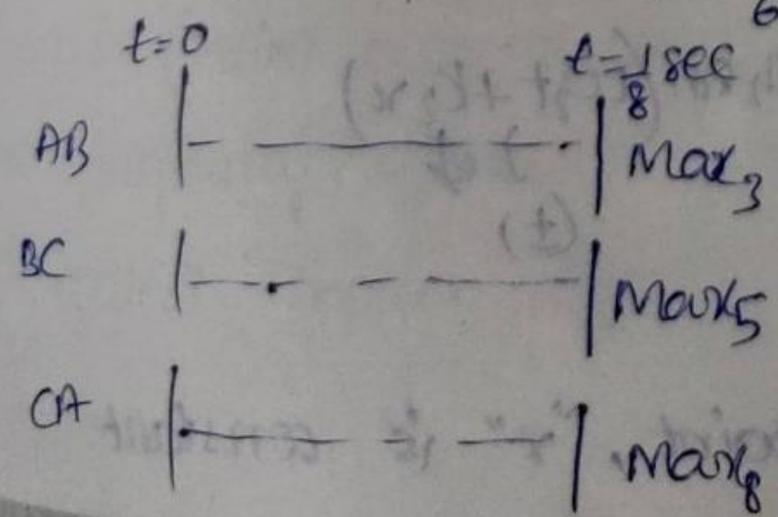
$n_A = 300\,Hz$
$n_B = 324\,Hz$
$n_C = 364\,Hz$
} Find f_B

Sol:

$f_{AB} = 24 \Rightarrow T_{AB} = \frac{1}{24}\,sec$

$f_{BC} = 40 \Rightarrow T_{BC} = \frac{1}{40}\,sec$

$f_{CA} = 64 \Rightarrow T_{CA} = \frac{1}{64}\,sec$



ΔT b/w greatest maximas $= \frac{1}{8}\,sec$

$f_B = \frac{1}{\Delta T} = 8\,Hz$

$= G.C.D$ of n_{AB}, n_{BC}, n_{CA}

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Ex13) Two sound waves of $f_1 = 100\,Hz$ and $f_2 = 102\,Hz$ having same amplitude are interfering. A detector which can detect the waves of amplitude $\geq A$ is kept at rest. In a time interval of 12 seconds, find total duration in which detector is active

Sol:

$$A_r = 2A\cos 2\pi\left(\frac{n_1 - n_2}{2}\right)t$$

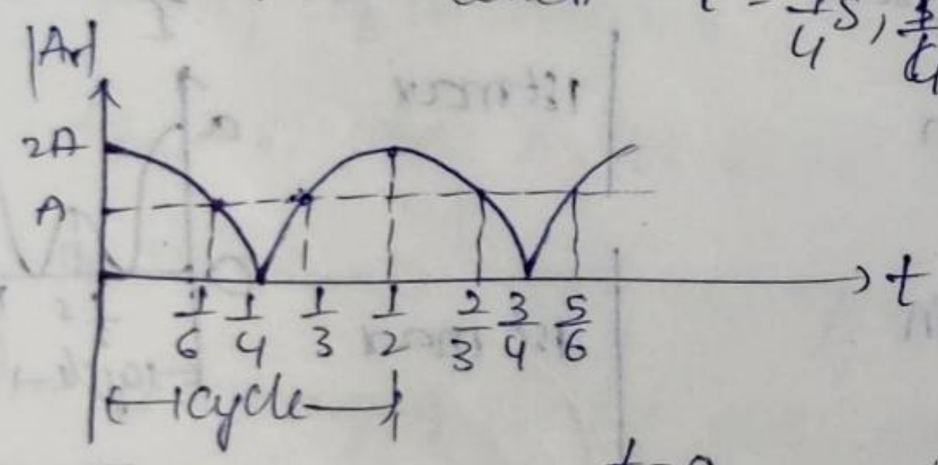
$$A_r = 2A\cos 2\pi t$$

$|A_r| = A$ when $\cos 2\pi t = \pm\frac{1}{2}$

$$2\pi t = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \ldots$$

$$t = \frac{1}{6}s, \frac{1}{3}s, \frac{2}{3}s, \frac{5}{6}s \ldots$$

$A_r = 0$ when $t = \frac{1}{4}s, \frac{3}{4}s, \ldots$



In every $\frac{1}{2}s$ from $t=0$ active time is $\Delta t = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$ sec

In time $t = 12$ sec, active time is

$$\Delta t_a = 24\left(\frac{1}{3}\right) = 8\,sec$$

Note: (i) As sound detectors sense the pressure variations, sound waves are described with pressure waves only.

(ii) In the above discussion of beads, I_{max} means sound intensity maxima

(iii) In the beats equation, if the amplitudes of waves are different and the observation is made at $x=0$

$$y = A_1\sin(\omega_1 t - k_1 x) + A_2\sin(\omega_2 t + k_2 x)$$

$(\pm)$

$$\frac{\omega_1}{k_1} = \frac{\omega_2}{k_2} = v$$

As we observe at one point, "x" is constant

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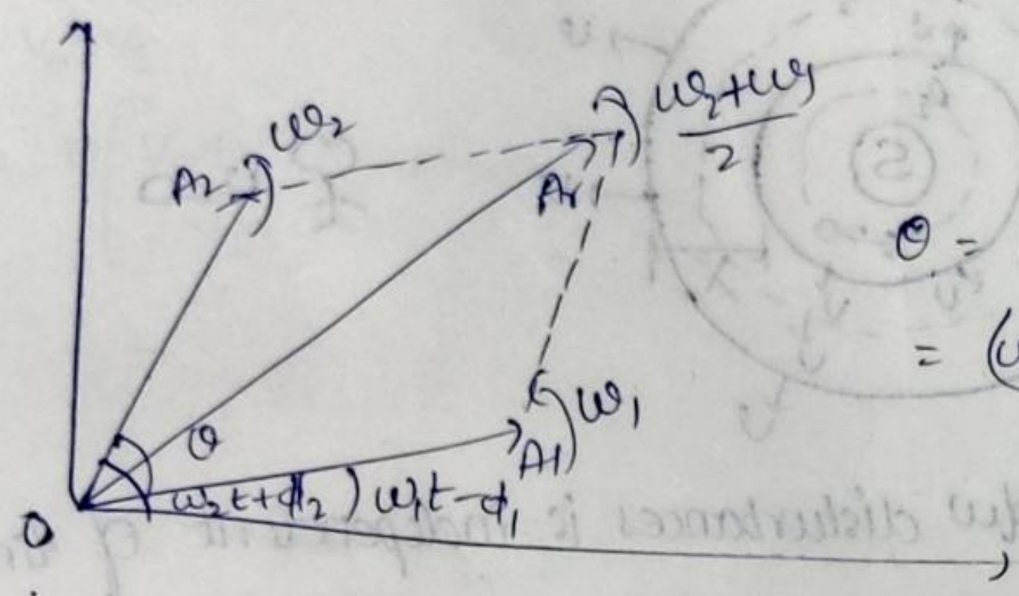
$k_1 x$ and $k_2 x$ are constants

let $k_1 x = \phi_1$, $k_2 x = \phi_2$

$y_1 = A_1 \sin(\omega_1 t - \phi_1) + A_2 \sin(\omega_2 t + \phi_2)$

= super imposition of two SHM's at one point



$\theta = (\omega_2 t + \phi_2) - (\omega_1 t - \phi_1)$

$= (\omega_2 - \omega_1) t + (\phi_2 + \phi_1)$

$|A_r| = \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos\theta}$ $\longrightarrow$ $(\omega_2 - \omega_1) t + (\phi_2 + \phi_1)$

A_r is a function of time

$|A_r|$ is max when $(\omega_2 - \omega_1) t + (\phi_2 + \phi_1) = 2m\pi$

$t = \frac{2m\pi}{\omega_2 - \omega_1} - \frac{\phi_2 + \phi_1}{\omega_2 - \omega_1}$

T for $|A_r| = \frac{2\pi}{\omega_2 - \omega_1} = \frac{1}{n_2 - n_1}$

$|A_r|_{max} = A_1 + A_2$

$|A_r|_{min} = A_1 \sim A_2 \longrightarrow$ when $\cos\theta = -1$

$(\omega_2 - \omega_1) t + (\phi_2 + \phi_1) = (2m+1)\pi$.

Doppler Effect:

* When there is relative motion b/w source and observer along the line joining them, frequency of sound received by the observer (called apparent frequency (n')) will be different from the frequency of sound emitted by source. This phenomenon is called Doppler Effect

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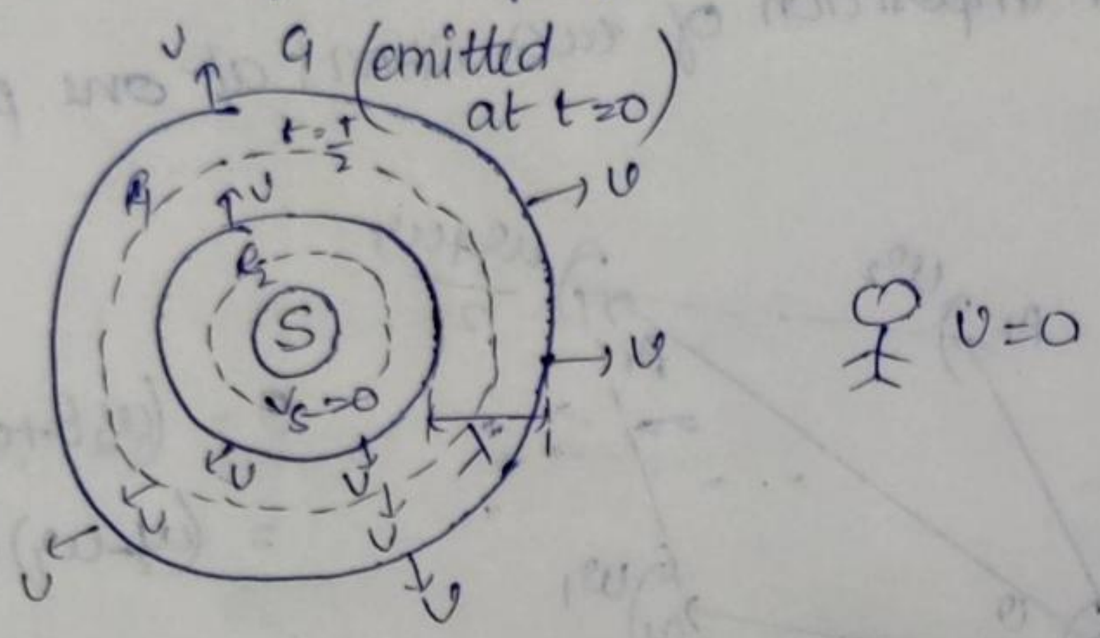
Expression for n':

case(i): When source S and observer O are at rest

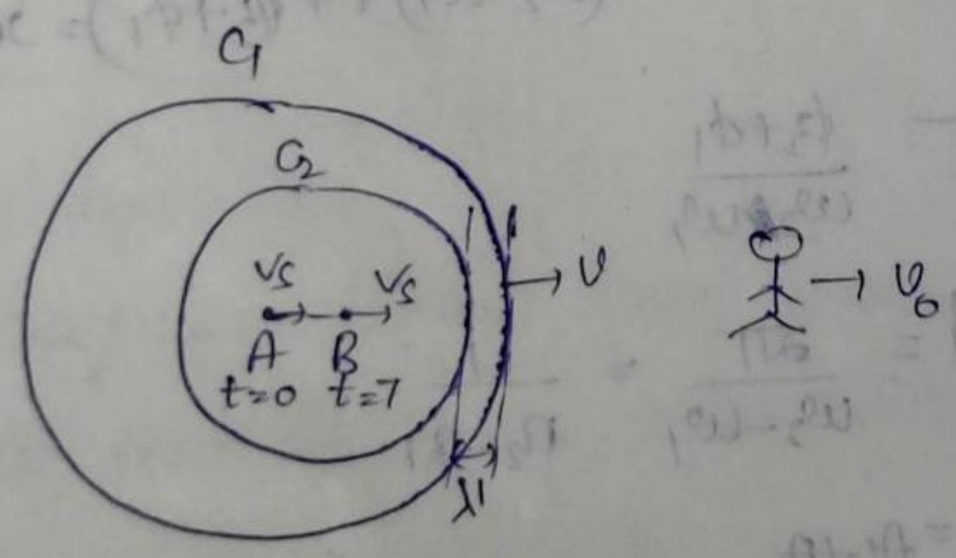
n = frequency of sound emitted by source



Gap b/w disturbances is independent of time

$$n' = \frac{\text{speed of sound w.r.t observer}}{\text{wave length}} = \frac{v}{\lambda} = n$$

case(ii): If "S" moves with speed v_s and "O" moves with a speed v_o



A and B are centres of G_1 and G_2 emitted at time t=0, t=T

Apparent wave length $\lambda' = AB = \lambda - v_s T = \frac{v}{n} - \frac{v_s}{n}$

$$\lambda' = \frac{v - v_s}{n}$$

$$n' = \frac{v_{rel}}{\lambda'} = \frac{v - v_o}{\frac{(v - v_s)}{n}} = n\left(\frac{v - v_o}{v - v_s}\right)$$

* The above formula for n' is used when observer moves away from source and source moves towards the observer. If the source moves away from observer, then replace v_s with $-v_s$. Similarly, if the observer moves towards source, replace v_o

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with $-v_o$

1) $S \rightarrow v_s$ $\quad$ $v_o \leftarrow O$

$$n' = n\left[\frac{v+v_o}{v-v_s}\right], \quad n'>n$$

2) $v_s \leftarrow S$ $\quad$ $\leftarrow O$

$v_o > v_s$

$$n' = n\left[\frac{v_o+v}{v+v_s}\right], \quad n'>n$$

3) $v_s \leftarrow S$ $\quad$ $O \rightarrow v_o$

$$n' = n \frac{\cancel{v_o+v}}{\cancel{v}}\left[\frac{v-v_o}{v+v_s}\right], \quad n'<n$$

4) $v_s \leftarrow S$ $\quad$ $\leftarrow O$

$v_o < v_s$

$$n' = n\left[\frac{v+v_o}{v+v_s}\right], \quad n'<n$$

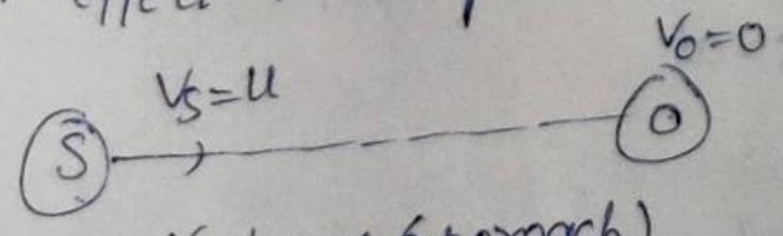
Conclusions:

1) when ever, there is relative velocity of approach b/w s and o, $n'>n$. Similarly when ever there is relative velocity of separation, $n'<n$

2) n' is independent of distance b/w s and o but depends on v_o, v_s and v. If v_o, v_s are constant with time then n' is constant with time

Ex: If a train approaching a plat form with a constant velocity gives a horn, then the observer on the platform receives the sound with constant higher frequency but gradually increasing loudness

5) Doppler effect is assymmetric in sound



$V_{rel} = u$ (approach)

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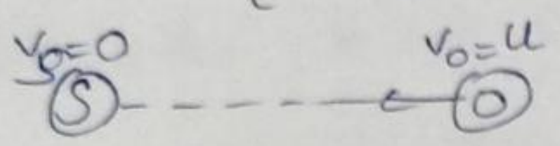
$$n_1 = n\left[\frac{v}{v-u}\right] > n$$



$V_{rel} = u$ (approach)

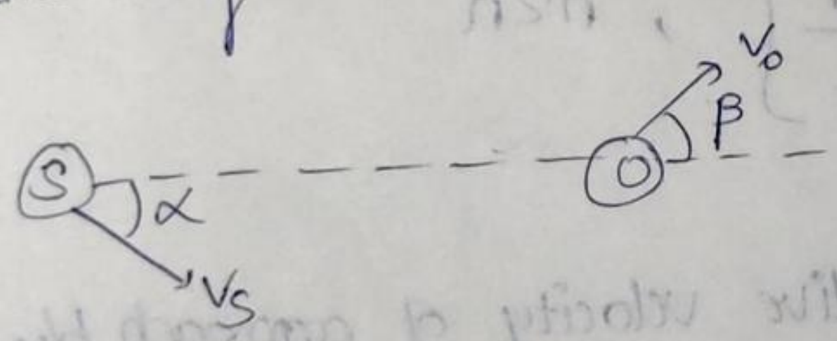
$$n_2 = n\left(\frac{v+u}{v}\right) > n$$

* In these two cases, n' is different even though relative velocity of approach is same. It indicates that effect of motion of source and motion of observer are not same on n'

* $n' = \frac{V_{rel}}{\lambda'}$

* λ' is effected by motion of S. V_{rel} is effected by motion of O. These two donot produce same effect on the ratio

* If V_o, V_s are not along the line joining them, then their components along SO have to be substituted in the formula of n'

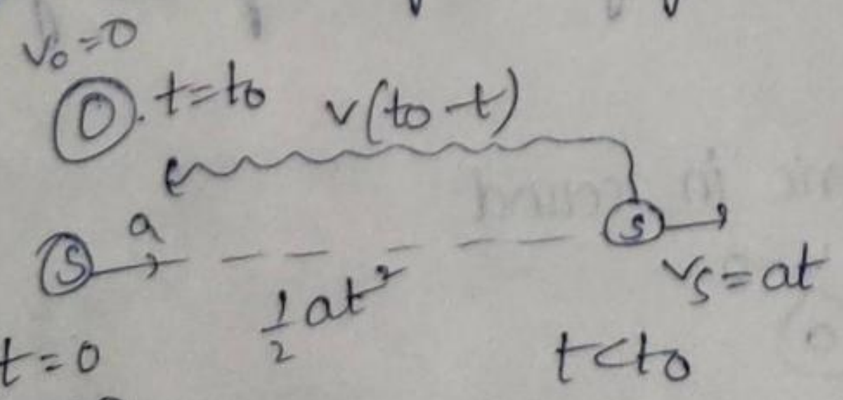


$$n' = n\left[\frac{v - V_o\cos\beta}{v - V_s\cos\alpha}\right]$$

* If V_s, V_o along line joining them change with time, then V_o at the instant of receiving the sound and V_s at the instant when this sound was emitted

Ex 1) A source of sound S and a detector O are at rest at a point ~~observer~~ source starts moving with a constant acceleration a. Find the frequency of sound registered by observer at a time t_0?

Sol:



let the given sound was emmitted by "S" at time $t<t_0$

$$\frac{1}{2}at^2 = v(t_0 - t)$$

$$at^2 + 2vt - 2vt_0 = 0$$

$$t = \frac{-2v \pm \sqrt{4v^2 + 8vt_0 a}}{2a}$$

$$at = v\left[\sqrt{1 + \frac{2t_0 a}{v}} - 1\right] \Rightarrow at + v = v\sqrt{1 + \frac{2t_0 a}{v}}$$

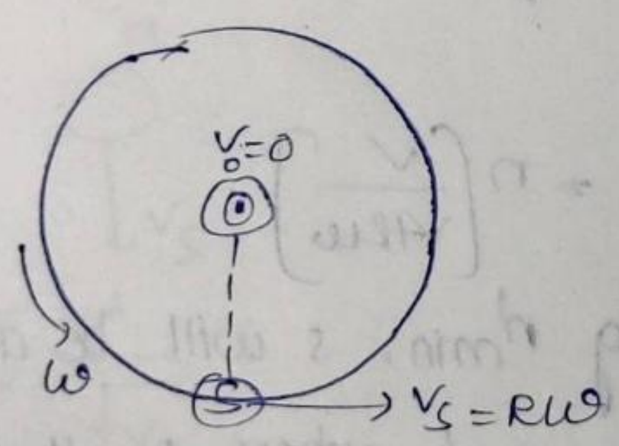
$$n' = n\left[\frac{v}{v + v_s}\right]$$

$$= n\,\frac{v}{v + at}$$

$$\boxed{n' = \frac{n}{\sqrt{1 + \frac{2t_0 a}{v}}}}$$

Source moving along circle with constant angular velocity,

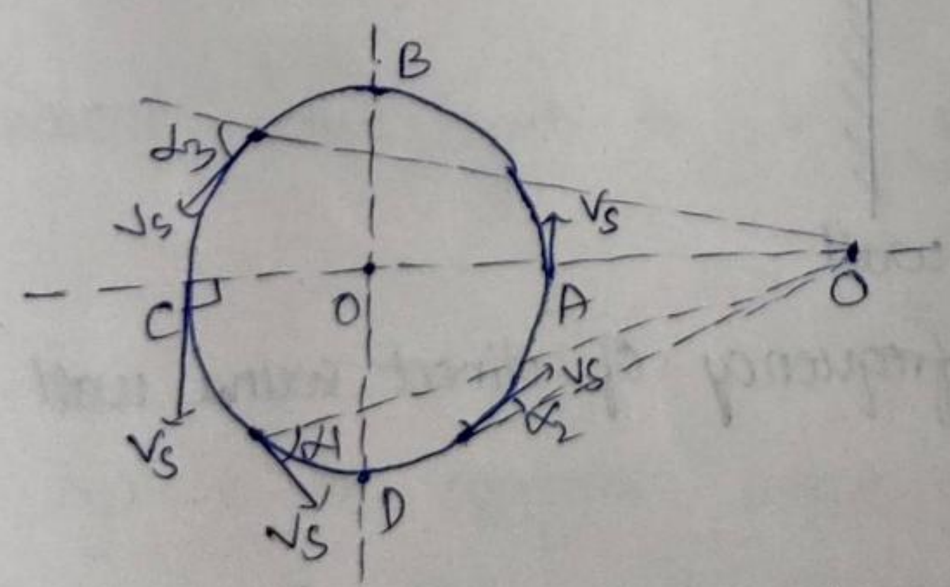
Case-1: "O" is at rest at centre of circle



$$n' = n\left[\frac{v}{v \pm v_s \cos 90^\circ}\right]$$

$$= n$$

Case-2: If "O" is outside the circle



with the sounds emitted when S is

a) at A (or) C, $n' = n$

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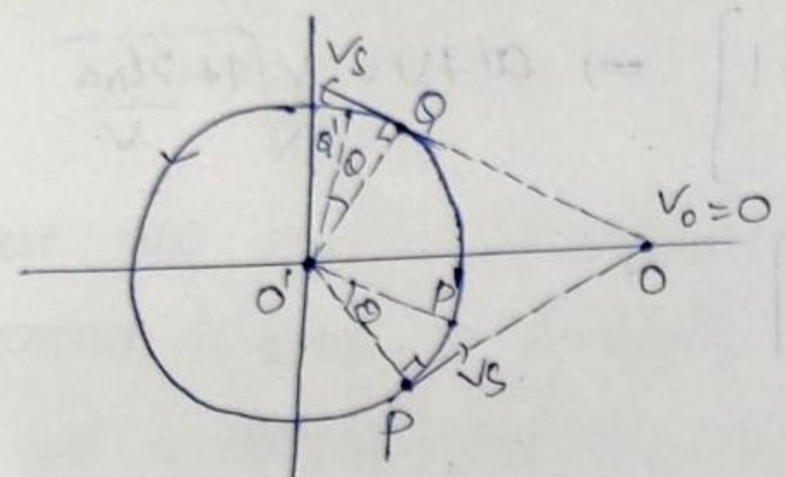
b) In the region CDA, $n' > n$ as V_{rel} is approach type

c) In the region ABC, $n' < n$ as V_{rel} is separation type

* n' is maximum (or) minimum when $\alpha = 0$ i.e, line joining S and O is along tgt to the circle



* n' is max corresponding to the sound emitted by source from P

$$n'_{max} = n\left[\frac{V}{V - V_S}\right] = n\left[\frac{V}{V - R\omega}\right]$$

At the instant of receiving n'_{max}, S will be at P' but not at P

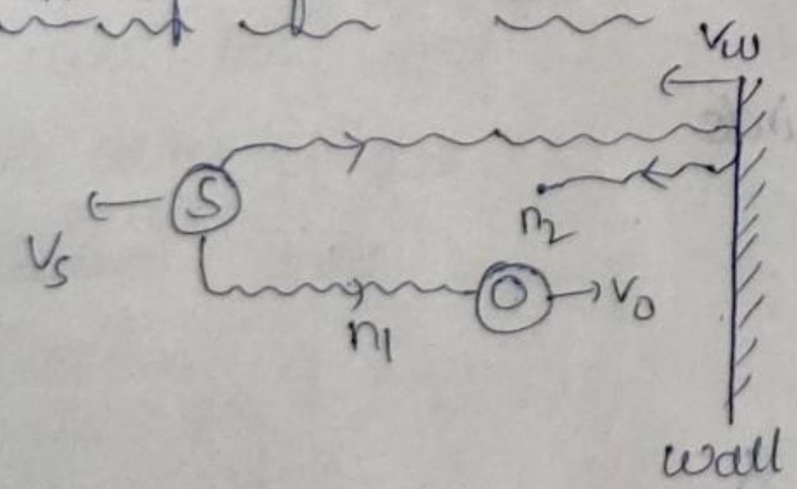
* n' is minimum corresponding to the sound emmitted by the source Q

$$n'_{min} = n\left[\frac{V}{V + V_S}\right] = n\left[\frac{V}{V + R\omega}\right]$$

* At the instant of receiving n'_{min}, S will be at Q' but not at Q

Angle $\angle POP'$ = Angle $\angle QO'Q'$ = ωt where t = time of travel of sound from P to O (or) from Q to O

* n' of reflected sound:



n_1 = apparent frequency of direct sound

$$= n\left[\frac{V - V_0}{V + V_S}\right]$$

To find n_2, let us split the path of reflected sound

into two parts

In 1st part, wall is the reciever, S is the source

$$n' = n\left(\frac{v+v_w}{v+v_s}\right)$$

In 2nd part, wall is the source of frequency n', "O" is the recei

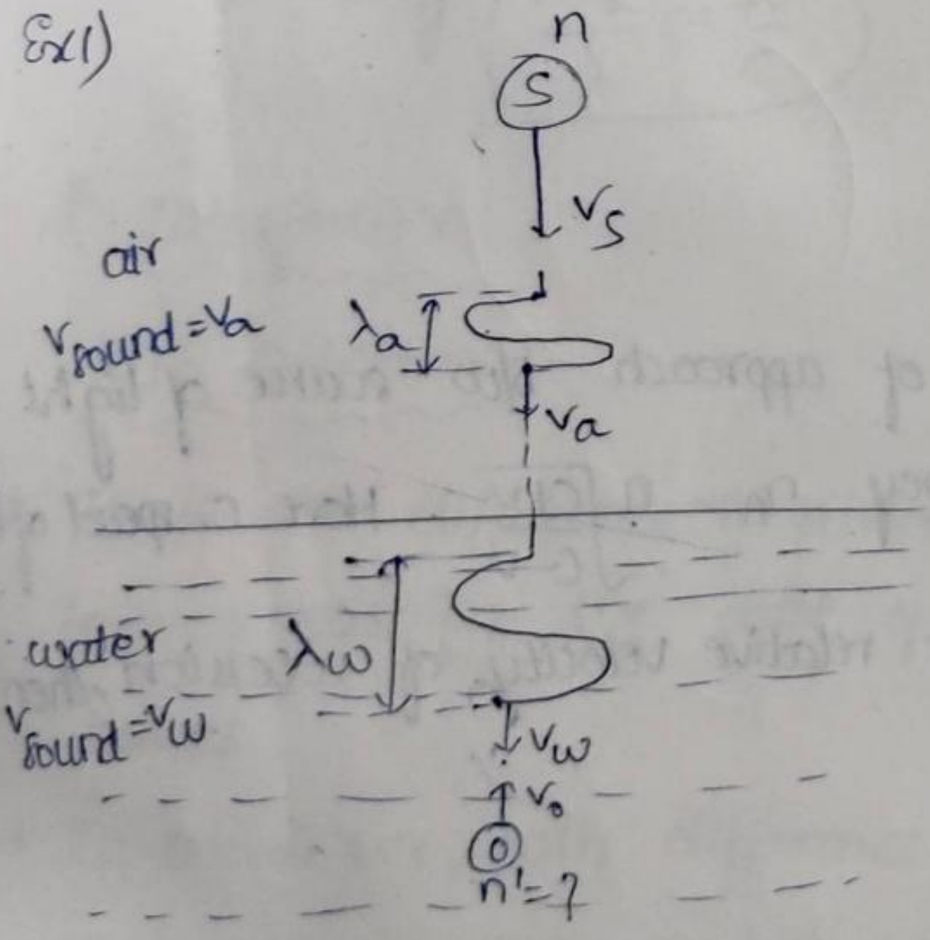
$$n_2 = n'\left(\frac{v+v_o}{v-v_w}\right) = n\left(\frac{v+v_w}{v+v_s}\right)\left(\frac{v+v_o}{v-v_w}\right)$$

* $n_1 \sim n_2$ = beat frequency if $v_o = 0$
* apparent wave length of n_1 is $\lambda_1 = \frac{v+v_s}{n}$
* Apparent wave length of n_2 is $\lambda_2 = \cancel{\frac{v+v_w}{n_2}}\ \frac{v_{rel}}{n_2} = \frac{v+v_o}{n_2}$

$$= \frac{(v+v_s)(v-v_w)}{n(v+v_w)}$$

If S and O are in different media

Ex1)



Apparent wave length in air $= \frac{v-v_s}{n}$

$$= \frac{v_a - v_s}{n}$$

If λ_w is the apparent wave length in water,

$$n_a = n_w$$

$$\frac{v_a}{\lambda_a} = \frac{v_w}{\lambda_w}$$

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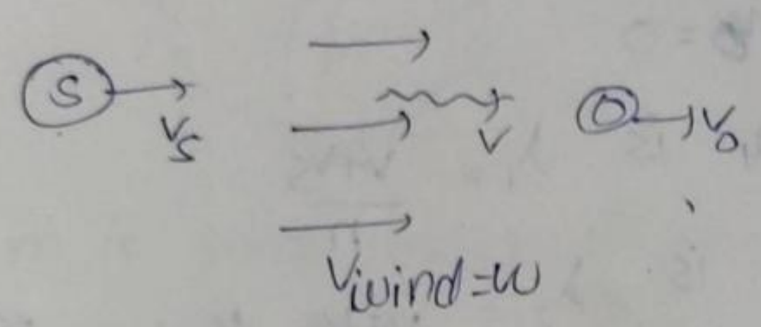
$$\lambda_w = \lambda_a \left(\frac{v_w}{v_a}\right)$$

$$= \left[\frac{v_a - v_s}{n}\right]\left[\frac{v_w}{v_a}\right]$$

$$n' = \frac{v_{rel}}{\lambda'} = \frac{v_w + v_o}{\lambda_w}$$

If air blows:



* If the wind blows towards the observer, then replace v with v+w in the formula of n'
* If the wind blows away from the observer, then replace v with v-w in the formula of n'

$$n' = n\left[\frac{v \pm w - v_o}{(v \pm w) - v_s}\right]$$

Doppler effect in light:

* If v is the relative velocity of approach b/w source of light & observer, then apparent frequency $n_A = n\sqrt{\frac{c+v}{c-v}}$. Here c = speed of light ($n_A > n$). Similarly, if v is the relative velocity of recession, then $n_R = n\sqrt{\frac{c-v}{c+v}}$.

In general, $v << c$

$$n_A = n\left[\frac{1+\frac{v}{c}}{1-\frac{v}{c}}\right]^{\frac{1}{2}}$$

$$= n\left[1+\frac{v}{c}\right]^{\frac{1}{2}}\left[1-\frac{v}{c}\right]^{-\frac{1}{2}}$$

$$= n\left[1+\frac{v}{2c}\right]\left[1+\frac{v}{2c}\right]$$

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$$n_A = n\left[1+\frac{v}{c}\right]$$

$$\frac{n_A - n}{n} = \frac{v}{c} \quad \text{i.e,} \quad \frac{dn_A}{n} = +\frac{v}{c}$$

Similarly $\frac{dn_R}{n} = -\frac{v}{c}$

Again,

$$\lambda = \frac{c}{n}$$

$$d\lambda = -c\frac{dn}{n^2}$$

$$\frac{d\lambda}{\lambda} = -\frac{c\,dn}{n^2\lambda} = -\frac{dn}{n}$$

$$\frac{d\lambda_A}{\lambda} = -\frac{dn_A}{n}$$

$$\boxed{\frac{d\lambda_A}{\lambda} = \frac{-v}{c}}$$

Similarly $\boxed{\frac{d\lambda_R}{\lambda} = \frac{+v}{c}}$

Interference:

Path Difference:

* It is the difference in distances travelled by two waves before reaching the point of superimposition from their sources.

* In the figure, path difference at P is $\Delta r = S_2P - S_1P = r_2 - r_1$. Δr will be different for different points in space. S_1, S_2 are two point sources of sound producing waves of frequencies ω_1, ω_2

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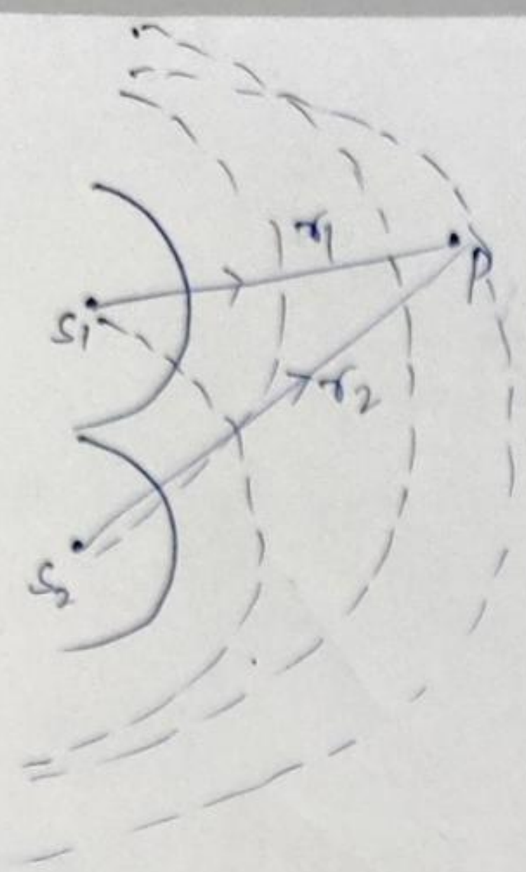

At P: $y_1 = A_1 \sin(\omega_1 t - k_1 r_1 + \phi_1) = A_1 \sin\left(\omega_1 t - \frac{\omega_1}{v} r_1 + \phi_1\right)$

$y_2 = A_2 \sin(\omega_2 t - k_2 r_2 + \phi_2) = A_2 \sin\left(\omega_2 t - \frac{\omega_2}{v} r_2 + \phi_2\right)$

$\frac{\omega_1}{k_1} = \frac{\omega_2}{k_2} = v$ as the waves are in same medium

A_1, A_2 are amplitudes of wave at P Amplitude varies from point to point

phase difference $\phi = \left(\omega_1 t - \frac{\omega_1}{v} r_1 + \phi_1\right) - \left(\omega_2 t - \frac{\omega_2}{v} r_2 + \phi_2\right)$

$= (\omega_1 - \omega_2)t + \frac{1}{v}(\omega_2 r_2 - \omega_1 r_1) + (\phi_1 - \phi_2)$

At one point r_1, r_2 are constant but ϕ depends on time

If $\omega_1 = \omega_2$ then $\phi = \frac{\omega}{v}(r_2 - r_1) + (\phi_1 - \phi_2)$

$\boxed{\phi = k(\Delta r) + (\phi_1 - \phi_2)}$ → independent of time

* Here y_1, y_2 are the pressure variations of the waves.
*

* Here $k(\Delta r)$ is the phase difference arising due to path difference
* $(\phi_1 - \phi_2)$ is the phase difference arising due to initial phases at the sources
* If the sources emit the waves continuously, then $\phi_1 - \phi_2$ is constant

Coherent waves:

* If the phase difference b/w the two waves at the point of

superimposition is independent of time, then they are called as coherent waves and their sources are called coherent sources

* Two fixed sources are coherent if $\omega_1 = \omega_2$ & $\phi_1 - \phi_2 = \text{constant}$
If $\phi_1 = \phi_2$, then $\phi = k(\Delta r)$ i.e, path difference will be the only origin of phase difference. Unless specified otherwise, we consider this case.

* At any point P, two SHM's are superimposed. Hence the resultant amplitude $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$.

* Intensity $(I) \propto A^2$. Therefore $I = k_0 A^2$, $I_1 = k_0 A_1^2$, $I_2 = k_0 A_2^2$

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$$

$$\frac{I}{k_0} = \frac{I_1}{k_0} + \frac{I_2}{k_0} + \frac{2\sqrt{I_1 I_2}}{k_0}\cos\phi$$

$$\boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi}$$

* If $A_1 = A_2 = A_0$, then $A = 2A_0\cos\left(\frac{\phi}{2}\right)$. Now $I_1 = I_2 = I_0 \Rightarrow I = 4I_0\cos^2\left(\frac{\phi}{2}\right)$
* When there is coherence, ϕ and hence I is independent of time at every point in space
* We can observe stable loudness pattern

Interferences

* It is the modification of intensity distribution in the space when two (or) more coherent waves are superimposed

Constructive Interference:

* I and A are maximum when $\cos\phi = +1$

$$\phi = 2m\pi = 0, 2\pi, 4\pi, \ldots$$

$$\Delta r = \frac{\phi}{k} = \frac{\phi(\lambda)}{2\pi} = \frac{2m\pi\lambda}{2\pi} = m\lambda$$

$$= 0, \lambda, 2\lambda, 3\lambda, \ldots$$

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$$I_{max} = (\sqrt{I_1}+\sqrt{I_2})^2$$

$$A_{max} = A_1 + A_2$$

* Interference of waves when produced I_{max} is called constructive interference

Destructive Interference:

* It is the modification of intensity distribution in space when

I and A minimum when $\cos\phi = -1$

$$\phi = (2m+1)\pi = \pi, 3\pi, 5\pi, \ldots$$

$$\Delta x = \frac{\phi}{k} = \frac{\phi(\lambda)}{2\pi} = \frac{(2m+1)\pi\lambda}{2\pi} = \left(\frac{2m+1}{2}\right)\lambda$$

$$= \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots$$

$$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$$

$$A_{min} = A_1 - A_2$$

* $$I_{avg} = \frac{\int_0^{2\pi} I \cdot d\phi}{\int_0^{2\pi} d\phi} = \frac{\int_0^{2\pi} (I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi)\cdot d\phi}{2\pi}$$

$$= \frac{(I_1+I_2)2\pi + \left[2\sqrt{I_1 I_2}\sin\phi\right]_0^{2\pi}}{2\pi}$$

$$= I_1 + I_2$$ (Assumed that I_1, I_2 are uniform)

This is true in a small space at far distance from the sources)

Excess of Intensity at I_{max} points over I_{avg}:

$$I = I_{max} - I_{avg} = 2\sqrt{I_1 I_2}$$

Deficiency of intensity at I_{min} points below I_{avg} is

$$I_{deficiency} = I_{avg} - I_{min} = 2\sqrt{I_1 I_2}$$

* In interference, total energy is conserved but redistributed

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