Power System Harmonic Analysis using the Kalman Filter

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Abstract—**This paper explores the practical application of the Kalman filter to the analysis of harmonic levels in power systems. The merits and limitations of different possible implementations are investigated and the effect of fundamental frequency variation is examined. The tuning of the Kalman filter for desired dynamic response is discussed and an adaptive tuning algorithm derived for the improved convergence of nonlinear models. The effectiveness of the resulting schemes are tested under a variety of typical power system operating conditions.**

I. Introduction

The significant growth of the semiconductor industry in recent decades has lead to a corresponding increase in harmonic levels in power systems worldwide. Concurrently, increasing demands upon electrical resources have made the subject of power quality an important issue in modern electric utility operation. Negative effects of harmonic currents and voltages, such as increased I^2R losses and the reduction of the lifespan of sensitive equipment, has prompted the establishment of a number of standards and guidelines regarding acceptable harmonic levels. Accurate analysis of power system measurements is essential to determine harmonic levels and effectively design mitigating filters. Harmonic levels and fundamental frequency are load dependant. The problem thus becomes one of determining the frequency content of a non-stationary signal. The effectiveness of standard Fourier transform techniques such as the FFT and the STFT is limited by averaging, spectral leakage and the picket fence effect, prompting the search for alternative methods. A wide variety of techniques have been investigated. Modified versions of the standard DFT analysis have been proposed by Kamwa et al [1], whilst wavelet analysis has been successfully applied by Huang et al in [2] and Pham and Wong in [3]. Neural networks have also been extensively studied as a means of harmonic extraction [4] [5] [6].

One method in particular which has attracted much interest is the Kalman filter. The Kalman filter offers the possibility of dynamic tracking of the harmonic content, increasing the accuracy of the analysis, and providing a possible input to a feedback control scheme, such as an active filter. One of the first papers to apply the Kalman filter to the harmonic estimation problem was Girgis et al [7]. Since then, a variety of work has been published in the area investigating a number of different models and approaches [8] [9] [10]. Whilst the potential of the Kalman filter as a tool for harmonic analysis has been demonstrated, practical application has been limited by implementation difficulties. The response of the filter is governed by the error covariance matrices $\mathbf{Q}$ and $\mathbf{R}$, which act as "tuning" parameters for the estimation, balancing between accurate speed of tracking and filter divergence. In practice, choosing appropriate parameters for desired filter operation can be an arduous task; limiting the success of the application.

This paper explores the application of the Kalman filter to the estimation of power system harmonics. Different possible implementations of the filter are presented and issues such as the effect of fundamental frequency variation are addressed. Some of the difficulties encountered in the practical application of the filter are discussed and possible solutions investigated. Finally, the effectiveness of the Kalman filter analysis is assessed during typical power system operation and also during disturbances such as a sudden frequency dip and a line fault.

II. Application of the Kalman Filter to Power System Harmonic Estimation

The Kalman filter is a model based optimal estimator which requires a model of the form:

$$\underline{x}_{k+1} = \mathbf{\Phi}\underline{x}_k + \mathbf{G}\underline{w}_k \quad (1)$$

$$z_k = \mathbf{H}\underline{x}_k + v_k \quad (2)$$

Where $\underline{x}_k$ is the state vector and z_k is the measurement at time t_k. $\underline{w}_k$ and v_k are model and measurement errors respectively whose covariance matrices are defined as:

$$E[\underline{w}_i\underline{w}_j^T] = \mathbf{Q}\delta_{ij} \quad (3)$$

$$E[v_iv_j^T] = \mathbf{R}\delta_{ij} \quad (4)$$

For power system applications the harmonic signal is modelled and desired information, such as the amplitude and phase of each frequency component, is assigned as states in the state

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vector. The state vector $\underline{x}_k$ is then recursively estimated $\hat{\underline{x}}_k$:

$$\hat{\underline{x}}_k^- = \Phi_k \hat{\underline{x}}_{k-1} \tag{5}$$

$$\mathbf{P}_k^- = \Phi_k \mathbf{P}_{k-1} \Phi_k^T + \mathbf{G}\mathbf{Q}\mathbf{G}^T \tag{6}$$

$$K_k = \mathbf{P}_k^- \mathbf{H}_k^T (\mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^T + \mathbf{R})^{-1} \tag{7}$$

$$\hat{\underline{x}}_k = \hat{\underline{x}}_k^- + K_k (z_k - \mathbf{H}\hat{\underline{x}}_k^-) \tag{8}$$

$$\mathbf{P}_k = (I - K_k \mathbf{H}_k)\mathbf{P}_k^- \tag{9}$$

The flexibility of the model structure required by the Kalman filter allows the harmonic signal to be represented in a number of ways based on different assumptions. If the fundamental frequency of the signal is assumed constant, then the following linear model can be derived.

A. Model 1: Linear, constant fundamental frequency

Expressing the harmonic signal as:

$$z_k = \sum_{r \in harms} A_k^r \sin(rwk\Delta t + \theta_k^r) \tag{10}$$

Where $harms$ is the set of n harmonic orders present. This can be rewritten in the form:

$$\begin{aligned} z_k = \sum_{r \in harms} & A_k^r \cos\theta_k^r \cos(rwk\Delta t) \\ & - A_k^r \sin\theta_k^r \sin(rwk\Delta t) \end{aligned} \tag{11}$$

Assigning states as:

$$\begin{aligned} x_k^1 &= A_k^1 \cos\theta_k^1 \\ x_k^2 &= A_k^1 \sin\theta_k^1 \\ & \cdot \\ x_k^{2n-1} &= A_k^n \cos\theta_k^r \\ x_k^{2n} &= A_k^n \sin\theta_k^r \end{aligned} \tag{12}$$

Yielding the following measurement matrix $\mathbf{H}$:

$$\begin{aligned} \mathbf{H}_k = [& \cos(wk\Delta t) \quad -\sin(wk\Delta t) \quad \ldots \\ & \ldots \quad \cos(nwk\Delta t) \quad -\sin(nkw\Delta t)] \end{aligned} \tag{13}$$

It is assumed that the propagation of the states can be described by a random walk model:

$$\underline{x}_{k+1} = \underline{x}_k + \underline{w}_k \tag{14}$$

Hence Φ is the identity matrix. The amplitude and phase of each frequency component can be calculated from the estimated states as:

$$\hat{A}_k^r = \sqrt{(\hat{\underline{x}}_k^{2r-1})^2 + (\hat{\underline{x}}_k^{2r})^2} \tag{15}$$

$$\hat{\theta}_k^r = \arctan\left(\frac{\hat{\underline{x}}_k^{2r-1}}{\hat{\underline{x}}_k^{2r}}\right) \tag{16}$$

This model assumes that the fundamental frequency remains constant. In reality, the fundamental frequency of a power system will fluctuate around the nominal value as the ratio between generated and load power changes. Whilst in normal operation these deviations in frequency are quite small, a loss of load or generation can cause a sudden significant frequency change, affecting the harmonic estimation. Consequently, to maintain accuracy during abrupt frequency changes Model 1 must be adapted. The fundamental frequency of the model must vary with the actual frequency of the signal as calculated by a technique such as the FFT or zero crossing. However, the calculation of the fundamental frequency of a distorted harmonic waveform is in itself a difficult task [11]. An alternative solution is to include the variable fundamental frequency in the harmonic model.

B. Model 2: Nonlinear, variable fundamental frequency

By including the fundamental frequency as state to be estimated, the model becomes nonlinear and the Extended Kalman filter must be used. The following model, [12], [13], [14] is derived by expressing the harmonic signal equation (10) as the sum of instantaneous phase and quadrature components:

$$\begin{aligned} z_{k+1} = \sum_{r \in harms} & A_{k+1}^r \sin(rwk\Delta t + \theta_{k+1}^r)\cos(rw\Delta t) \\ & + A_{k+1}^r \cos(rwk\Delta t + \theta_{k+1}^r)\sin(w\Delta t) \end{aligned} \tag{17}$$

States are assigned as follows:

$$\begin{aligned} x_k^1 &= A_k^1 \cos(wk\Delta t + \theta_k^1) \\ x_k^2 &= A_k^1 \sin(wk\Delta t + \theta_k^1) \\ & \cdot \\ x_k^{2n} &= A_k^n \sin(nwk\Delta t + \theta_k^n) \\ x_k^{2n+1} &= w\Delta t \end{aligned}$$

Assuming the amplitude and phase change by a negligible amount between samples relative to the sinusoid propagation, the following relationship can be derived:

$$\begin{aligned} x_{k+1}^1 &\approx x_k^1 \cos(x_k^{2n+1}) - x_k^2 \sin(x_k^{2n+1}) \\ x_{k+1}^2 &\approx x_k^1 \sin(x_k^{2n+1}) + x_k^2 \cos(x_k^{2n+1}) \\ & \cdot \\ x_{k+1}^{2n+1} &\approx x_k^{2n+1} \end{aligned} \tag{18}$$

In Extended Kalman filter, the nonlinear model and measurement relationships are linearised by means of a first order Taylor series, i.e.

$$\underline{x}_{k+1} = \phi(\underline{x}_k) \tag{19}$$

$$\Phi_k = \left.\frac{\partial\phi(x)}{\partial x}\right|_{x=\hat{x}} \tag{20}$$

$$z_{k+1} = h(\underline{x}_k) \tag{21}$$

$$\mathbf{H}_k = \left.\frac{\partial h(x)}{\partial x}\right|_{x=\hat{x}} \tag{22}$$

The Extended Kalman filter algorithm is formed by substituting equations (19) to (22) into the previous Kalman filter algorithm (5) to (9).

Note that the actual states estimated using Model 2 are the instantaneous rectangular components of each frequency present and the fundamental frequency . The desired harmonic amplitude and phase information must then be extracted via inverse trigonometrical calculations such as (15) and (16) which may amplify any errors in the estimated states.

C. Model 3:Nonlinear, variable fundamental frequency

Similar to Model 2, the following model simultaneously estimates harmonic content and fundamental frequency. However, the amplitude and instantaneous phase of each frequency component are directly estimated by the state vector, defined as:

$$
\begin{aligned}
x_k^1 &= A_k^1 \\
x_k^2 &= (kw\Delta t + \theta_k^1) \\
&\cdot \\
x_k^{2n-1} &= A_k^n \\
x_k^{2n} &= (nwk\Delta t + \theta_k^n) \\
x_k^{2n+1} &= w
\end{aligned}
\tag{23}
$$

The propagation of the amplitude and frequency states can be modelled as a simple random walk, whilst the instantaneous phase propagated is modelled by:

$$\vartheta_{k+1}^r = \vartheta_k^r + r\omega\Delta t \tag{24}$$

Where:

$$\vartheta_k^r = rwk\Delta t + \theta_k^r \tag{25}$$

In state form:

$$x_{k+1}^{2r} = x_k^{2r} + r\Delta t x_k^{2n+1} \tag{26}$$

The nonlinear measurement relationship is give by:

$$z_k = \sum_{r \in harms} \underline{x}_k^{2r-1} \sin(\underline{x}_k^{2r}) \tag{27}$$

Linearising the relationship using equation (22) yields:

$$
\begin{aligned}
\mathbf{H}_k = [\, & \sin(\hat{\underline{x}}_k^2) \;\; \hat{\underline{x}}_k^1 \cos(\hat{\underline{x}}_k^2) \;\; \ldots \\
& \ldots \;\; \sin(\hat{\underline{x}}_k^{2n}) \;\; \hat{\underline{x}}_k^{(2n-1)} \cos(\hat{\underline{x}}_k^{(2n+1)}) \;\; 0]
\end{aligned}
\tag{28}
$$

III. Tuning of the Kalman filter

The practical application of the Kalman filter has been limited by the difficulties involved in accurately tuning the filter for desired performance. The Kalman filter optimally combines model and measurement information to estimate the state vector $\underline{x}_k$. The error covariances $\mathbf{Q}$ and $\mathbf{R}$ determine the relative weighting of the model and measurement information, in essence, acting as tuning parameters balancing the dynamic response of the filter against sensitivity to noise. The effect of $\mathbf{Q}$ and $\mathbf{R}$ on the Kalman filter performance is a topic which has been investigated in detail [14] [15] [16].

Whilst there are a number of excellent texts on the subject, the calculation of the error covariance matrix $\mathbf{Q}$ and measurement error $\mathbf{R}$ is often a non-trivial task[17] [18]. Inaccuracies due to modelling error and linearisation error can often be difficult to quantify. Effects such as cross-correlation between model and measurement error and the coloration of the noise processes $\underline{w}_k$ and v_k must be taken into consideration. Also, research has indicated that in many cases, particularly for nonlinear systems, theoretical values for $\mathbf{Q}$ and $\mathbf{R}$ do not necessarily yield the most accurate results [16]. These factors combined ensure that the tuning of the Kalman filter can often be a challenging task. However, the process can be facilitated by adopting a methodical approach as follows:

A. Calculate theoretical values if possible

While sometimes difficult, if sufficient information is available regarding the propagation of states and measurement, then $\mathbf{Q}$ and $\mathbf{R}$ can be calculated using equations (3) and (4). As shown in [16] theoretical values might not necessarily yield desired results. However they will often offer a good initial starting point from which a desired response can be obtained with minimum adjustment of actual values

B. Examine effect of $\mathbf{Q}/\mathbf{R}$ ratio

Assuming no cross-correlation between states, $\mathbf{Q}$ is typically a diagonal matrix of the form:

$$diag(\mathbf{Q}) = [q^1 q^2 \ldots q^n] \tag{29}$$

while for a single measurement, $\mathbf{R}$ reduces to a scalar value. In many cases it is not the actual value of $\mathbf{Q}$ and $\mathbf{R}$ that determines the filter response, but rather the ratio, $\mathbf{Q}/\mathbf{R}$ [14]. If this is the case, $\mathbf{R}$ can be set to a nominal value e.g. $\mathbf{R} = 1$ and $\mathbf{Q}$ can be tuned accordingly in order to give desired response.

C. Investigate variants of Kalman filter algorithm

In the case of highly nonlinear systems, the use of the $\mathbf{Q}/\mathbf{R}$ ratio as a tuning parameter may not be valid, and it is often difficult to tune the filter effectively. Techniques such as variable forgetting factors [19], optimization [20] and modification of the Extended Kalman filter algorithms [21] have been employed to improve stability and reduce errors due to nonlinearities. There are also a number of algorithms proposing adaptive updating of the $\mathbf{Q}$ and $\mathbf{R}$ parameters [22] [23] [24]. The adaptive algorithm adopted in this paper was first proposed by Jazwinski in [25]. Since then it has been studied and employed by a number of authors [26] [27] [28]. It can be derived by assuming $\mathbf{Q}$ matrix to be of the form $\mathbf{Q} = q_k\mathbf{I}$, with q_k updated each iteration by

$$\hat{q}_k = f\left(\frac{e_k^2 - E[e_k^2 | q_k = 0]}{\mathbf{H}_k\mathbf{H}_k^T}\right) \tag{30}$$

where $f(x)$ is the heaviside function defined as

$$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{31}$$

e_k^2 is the prediction error:

$$e_k = z_k - \mathbf{H}_k\hat{\underline{x}}_k^- \tag{32}$$

and $E[e_k^2|q_k = 0]$ is the predicted error variance assuming that $q_k = 0$, calculated by:

$$E[e_k^2|q_k = 0] = \mathbf{R}_k + \mathbf{H}_k\mathbf{P}_{k-1}\mathbf{H}_k^T \tag{33}$$

In order to increase statistical significance, the estimate is smoothed by:

$$q_k = \alpha q_{k-1} + (1 - \alpha)\hat{q}_k \tag{34}$$

This method estimates a single value for the model error covariance, q. Individual state error variances are determined by means of the matrix $\mathbf{G}$. The measurement error variance r_k can similarly be recursively calculated by:

$$\hat{r}_k = f(e_k^2 - \mathbf{H}_k P_{k-1}^- \mathbf{H}_k^T) \tag{35}$$

$$r_k = \alpha r_{k-1} + (1-\alpha)\hat{r}_k \tag{36}$$

For practical application a lower bound is placed on r_k to prevent the value dropping below minimum measurement noise.

IV. Results

Three test signals were used to test the Kalman filter analysis. Each signal consisted of a non-stationary harmonic signal containing $5^{th}, 7^{th}, 11^{th}$ and 13^{th} harmonics. The measurements were contaminated with gaussian white noise with a standard deviation of 0.01, representing a SNR of 40dB.

1) Test Signal 1; Normal Operating Conditions: The fundamental frequency of the signal was allowed to drift slightly around a nominal 50Hz value. The harmonic amplitudes similarly varied in a pseudo-random fashion. Variation of the harmonic amplitude and fundamental frequency was based on statistical information gathered from actual measurements of the electrical supply in Ireland.

2) Test Signal 2; Sudden drop in fundamental frequency: The second test signal simulates a sudden frequency drop such as that responding to a sudden change in load or generation power. The frequency drops from the nominal 50Hz to 48.95Hz, and remains at the lower frequency for 0.2 seconds before slowly returning to nominal. In this way, the effectiveness of the analysis under a typical power system disturbance is investigated. Again, the test signal is based on actual recorded data.

3) Test Signal 3; Fault Signal: In order to test the Kalman filter analysis under extreme conditions a fault signal was generated; the amplitude drops to zero for approximatly 0.25 seconds before gradually returning to normal.

A. Estimation using linear signal model:Model 1

Based on knowledge of the actual amplitude of each frequency component $[q^1 q^2 \dots q^n]$ could be calculated by:

$$q_{theor}^r = E[(x_{k+1}^r - x_k^r)(x_{k+1}^r - x_k^r)^T] \tag{37}$$

Similarly, $\mathbf{R}_{theor}$ was determined to be $1e^{-4}$ whilst $\mathbf{G}$ was assumed to be the identity matrix.

1) Test Signal 1; Normal Operating Conditions: Initial estimation with $\mathbf{Q}$ and $\mathbf{R}$ at theoretical values showed poor tracking of the harmonic amplitudes. When the value of the $\mathbf{Q}$ diagonal for the harmonic states,$[q^3 \dots q^n]$ was increased by a factor of 20, the dynamic response of the filter increased, improving the harmonic estimation as illustrated by Fig. 1. Fig. 2 demonstrates the corresponding fundamental amplitude estimation. Clearly, despite the assumption of constant fundamental frequency the model still provides accurate amplitude estimation during slight frequency variation

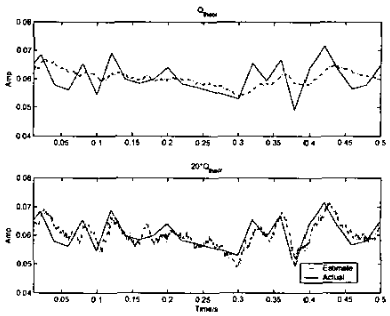

Fig. 1. Comparison of 7^{th} harmonic amplitude estimation based on Model 1 with different $\mathbf{Q}$ values

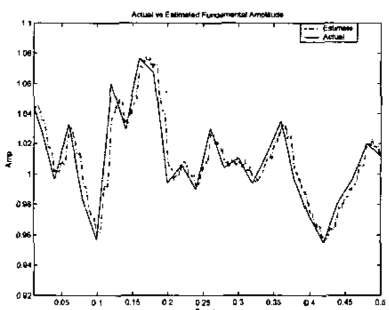

Fig. 2. Estimation of Fundamental Amplitude based on Model 1

2) Test Signal 2; Sudden drop in fundamental frequency: It is shown is Fig. 3 that the estimation deteriorates as the frequency deviates significantly from that assumed by the model. Thus, in order to maintain accurate estimation during large frequency variation, a nonlinear model must be used.

B. Estimation using non linear signal model:Model 2

1) Test Signal 2; Sudden drop in fundamental frequency: It was shown in [14] that it is the ratio of $\mathbf{Q}/\mathbf{R}$, rather than the actual values which determines the estimation response of model 2. Defining λ as $\lambda = \mathbf{Q}/\mathbf{R}$ and $\lambda_{theor} =$

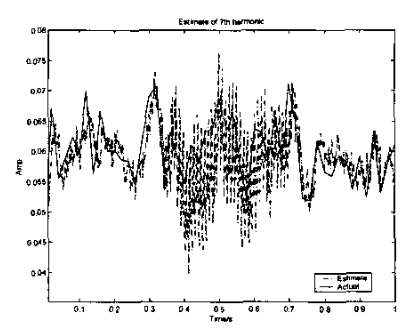

Fig. 3. Estimation of 7^{th} harmonic amplitude based on Model 1 during frequency dip

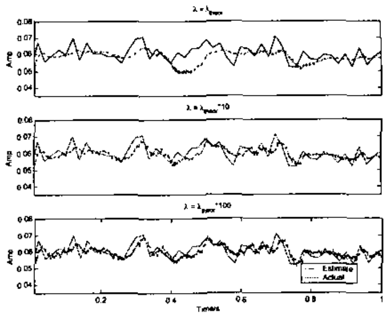

Fig. 4. Estimation of 7^{th} harmonic amplitude during frequency dip based on Model 2 for different λ values

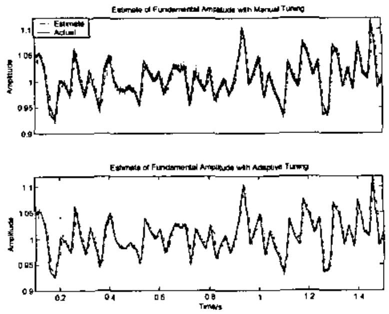

Fig. 6. Comparison of estimation of fundamental amplitude with adaptive and manual tuning of Model 3

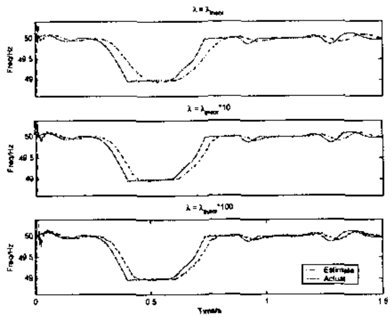

Fig. 5. Estimation of fundamental frequency during frequency dip based on Model 2 for different λ values

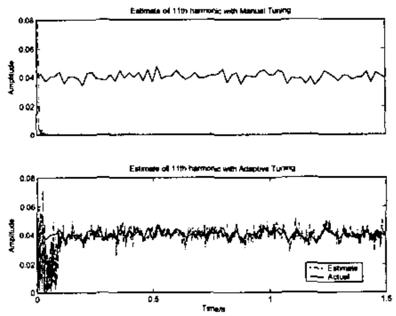

Fig. 7. Comparison of estimation of 11th harmonic amplitude with adaptive and manual tuning of Model 3

$\mathbf{Q}_{theor}/\mathbf{R}_{theor}$, Figs. 4 and 5 illustrate the estimation of the fundamental frequency and 7^{th} harmonic during a sudden frequency change for various values of λ . Clearly, as the ratio of $\mathbf{Q}/\mathbf{R}$ increases, more emphasis is placed on the incoming measurement data, increasing the dynamic tracking capability. However, if the SNR of the signal is increased to 30dB the poor noise rejection of the model becomes evident as shown in Fig. 8.

C. Estimation using non linear signal model:Model 3

Model 3 cannot be tuned using the λ approach making it difficult to choose $\mathbf{Q}$ and $\mathbf{R}$ for convergence. As an alternative, $\mathbf{Q}$ and $\mathbf{R}$ were adaptively estimated as detailed in Section III. As the adaptive approach is virtually independent of initial $\mathbf{Q}$ and $\mathbf{R}$ values beyond initial transients, the task of searching for convergent values is essentially eliminated. The response of the adaptive algorithm can be varied through the modification of the smoothing factor α, and the model error matrix $\mathbf{G}$, which essentially adopt the role of tuning parameters. However, practical experience has indicated that the estimation is relatively robust regarding choice of α and $\mathbf{G}$, ensuring convergence is a relatively simple task

1) Test Signal 2; Sudden drop in fundamental frequency: Figs. 6 and 7 show the results of the adaptive algorithm compared with manual tuning based on theoretical $\mathbf{Q}$ and $\mathbf{R}$. Clearly the adaptive and manual results are comparable for the estimation of the fundamental amplitude however, the $11th$ harmonic estimate fails to converge with manual tuning but is successfully estimated using the adaptive algorithm. Fig. 8 shows the result of the estimation using the adaptive algorithm with the SNR increased to 30dB. The adaptive Model 3 estimation shows a noticeably improved noise rejection when compared against model 2.

2) Test Signal 3; Fault Signal: In previous tests, the actual harmonic amplitudes and fundamental frequency variation were known a priori. Thus theoretical values for $\mathbf{Q}$ and $\mathbf{R}$ could be calculated, providing a reasonable starting point for tuning. However, in practical application, there may be little statistical information to aid the designer. Figure 9 illustrates the estimation of a fault signal with incorrect $\mathbf{Q}$ and $\mathbf{R}$ values. Whilst the estimation initially converges, the occurrence of a sudden change causes the estimation process to diverge. The adaptive estimation however, is unaffected by the original choice of tuning parameters and continues to track the correct state trajectory.

V. Conclusion

In this paper, the application of the Kalman filter to power system harmonic signal analysis was investigated. Different possible implementations were considered and the limitation and difficulties of each explored. An adaptive algorithm was employed to improve convergence of non-linear models. It

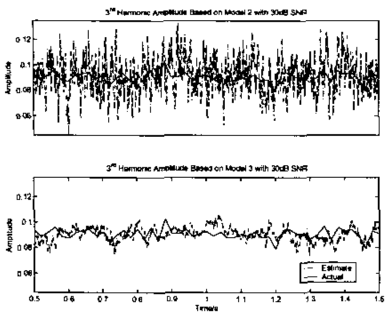

Fig. 8. Comparison of $3rd$ harmonic estimation with SNR of 30dB based on Model 2 and Model 3

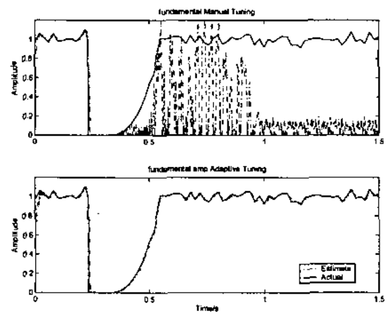

Fig. 9. Fundamental Amplitude Estimation during Fault using the Adaptive and Manual tuning

was demonstrated that by adopting a methodical approach to choosing the error covariances $\mathbf{Q}$ and $\mathbf{R}$ the Kalman filter can be successfully tuned to provide accurate analysis of harmonic content and fundamental frequency even during extreme power system disturbance. Thus indicating the feasibility of the Kalman filter as an input to an harmonic control scheme.

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REFERENCES

[1] I. Kamwa, R. Grondin, and D. Mcnabb, "On-line tracking of changing harmonics in stressed power systems: Application to hydro-quebec network," *IEEE Transactions on Power Delivery*, vol. 11, no. 4, pp. 2020–2027, Oct. 1996.

[2] S.-J. Huang and C.-T. Hsieh, "Visualizing time-varying power system harmonics using a morlet wavelet transform approach," *Electric Power Systems Research*, vol. 58, no. 2, pp. 81–88, 2001.

[3] V. L. Pham and K. P. Wong, "Wavelet-transform-based algorithm for harmonic analysis of power system waveforms," *IEE Proceedings: Generation, Transmission and Distribution*, vol. 146, no. 3, pp. 249–254, 1999.

[4] H. Mori, K. Itou, H. Uematsu, and S. Tsuzuki, "An artificial neural-net based method for predicting power system voltage harmonics," *IEEE Transactions on Power Delivery*, vol. 7, no. 1, pp. 402–409, Jan. 1992.

[5] P. K. Dash, D. P. Swain, A. Routray, and A. C. Liew, "Harmonic estimation in a power system using adaptive perceptrons," *IEE Proceedings Generation, Transmission and Distribution*, vol. 143, no. 6, pp. 565–574, Nov. 1996.

[6] L. L. Lai, C. T. Tse, W. L. Chan, and A. T. P. So, "Real-time frequency and harmonic evaluation using artificial neural networks," *IEEE Transactions on Power Delivery*, vol. 14, no. 1, pp. 52–59, Jan. 1999.

[7] A. A. Girgis and D. Hwang, "Optimal estimation of voltage phasors and frequency deviation using linear and non-linear kalman filtering: Theory and limitations," *IEEE Transaction on Power Apparatus and Systems*, vol. PAS 103, no. 10, pp. 2943–2949, October 1984.

[8] A. A. Girgis, W. B. Chang, and E. B. Makram, "A digital recursive measurement scheme for on-line tracking of power system harmonics," *IEEE Transactions on Power Delivery*, vol. 6, no. 3, pp. 1153–1160, July 1991.

[9] A. Azemi, E. Yaz, and K. Olejniczak, "Reduced-order estimation of power system harmonics," in *IEEE Conference on Control Applications - Proceedings.*, IEEE, Ed. IEEE, Sep 28-29 Location: Albany, NY, USA Sponsors: IEEE 1995, pp. 631–636.

[10] S. A. Soliman, M. H. Abdel-Rahman, and M. E. El-Hawary, "Linear kalman filtering algorithm applied to measurements of power system voltage magnitude and frequency: a constant-frequency model," *Canadian Journal of Electrical and Computer Engineering*, vol. 22, no. 4, pp. 145–153, Oct. 1997.

[11] D. W. P. Thomas and M. S. Woolfson, "Evaluation of frequency tracking methods," *IEEE Trans. Power Deliv.*, vol. 16, pp. 367–371, 2001.

[12] A. A. Girgis and L. Peterson, William, "Adaptive estimation of power system frequency deviation and it's rate of change for calculating sudden power system overloads," *IEEE Transactions on Power Delivery*, vol. 5, no. 2, pp. 585–594, Apr. 1990.

[13] B. F. LaScala and R. R. Bitmead, "Design of an extended kalman filter frequency tracker," *IEEE Trans. Signal Process.*, vol. 44, pp. 739–742, 1996.

[14] S. Bittanti and S. M. Savaresi, "On the parameterization adn design of and extended kalman filter frequency tracker," *IEEE transactions on automatic control*, vol. 45, no. 9, pp. 1718–1724, September 2000.

[15] G. Benmouyal, A. A. Girgis, and R. G. Brown, "Frequency-domain characterization of kalman filters as applied to power-system protection," *IEEE Trans. Power Deliv.*, vol. 7, pp. 1129–1138, 1992.

[16] B. L. Scala and R. Bitmead, "Design of an extended kalman filter frequency tracker," *IEEE trans. signal processing*, vol. 44, no. 3, pp. 739–742, March 1996.

[17] P. S. Maybeck, *Stochastic models, estimation, and contro*, ser. Mathematics in Science and Engineering, 1979, vol. 141.

[18] R. G. Brown, *Introduction to Random Signals and Applied Kalman Filtering*, 3rd ed. New York: Wiley, 1997.

[19] B. E. Ydstie and T. Co, "Recursive estimation with adaptive divergence control," *IEE Proceedings Part D:???*, vol. 132, no. 3, pp. 124–132, May 1985.

[20] J. Y. Jung, "Predictive compensator optimization for head tracking lag in virtual environments," in *IMAGE 2000 Conference Proceedings*, Scottsdale, Arizona, July 2000.

[21] K. R. F. Sonnemann and R. Unbehauen, "An ekf- based nonlinear observer with a prescribed degree of stability," *Automatica*, vol. 24, no. 9, pp. 1119–1123, 1998.

[22] A. L. Maitelli and Y.Yoneyama, "Adaptive control scheme using real time tuning of the paramter estimator," *IEE Proceedings - Control Theory Application*, vol. 144, no. 3, pp. 241–246, May 1997.

[23] B. P. D. M. R. Ananthasayanam and N. V. Vighnesam, "Adaptive kalman filtering technuque for relative orbit estimation for colocated geostationary satellites." *American Institute of Aeronautics and Astronautics Conference*, 2001.

[24] K. A. Myers and B. D. Tapley, "Adaptive sequential estimation with unknown noise statistics," *IEEE Transactions on Automatic Control*, vol. 21, no. 4, pp. 520–523, August 1976.

[25] A. Jazwinski, "Adaptive filtering," *Automatica*, vol. 5, pp. 475–485, 1969.

[26] M. N. JFG de Freitas and A. Gee, "Hierarchical bayesian-kalman models for regularistation and ard in sequential learning," Cambridge University Engineering Department, Tech. Rep., Nov 1998.

[27] W. D. Penny and S. J. Roberts, "Dynamic models for nonstationary signal segmentation," Neural System Research Group, Department o Electrical and Electronic Engineering, Imperial College of Sciemce Technology and Medicine, London, Tech. Rep., Jan 1999.

[28] T. N. Martina Fhal'ova Ilkivova, Boris Rohal Ilkiv, "Comparison of a linear and nonlinear approach to engine misfires detection," *Control Engineering Practice*, vol. 10, pp. 1141–1146, 2002.