Hi!

CONTINUOUS-TIME MARKOV CHAINS

CONTINUOUS-TIME STOCHASTIC PROCESS
WITH N DISCRETE STATES
TRANSITIONS BETWEEN STATES ARE POISSON

EX

$$\begin{tikzcd}
\textcircled{1} \arrow[r, bend left, "\lambda_{12}"] & \textcircled{2} \arrow[l, bend left, "\lambda_{21}"] \arrow[r, bend left, "\lambda_{23}"] & \textcircled{3} \arrow[l, bend left, "\lambda_{32}"]
\end{tikzcd}$$

$\rightarrow$ (UNITS s^{-1})

$\mathbb{P}_i(t)$ PROBABILITY OF STATE i AT TIME t

$$\vec{\mathbb{P}}(t) = \begin{bmatrix} \mathbb{P}_1(t) \\ \vdots \\ \mathbb{P}_N(t) \end{bmatrix}$$ THEN

$$\frac{d}{dt}\vec{\mathbb{P}} = \begin{bmatrix} -\sum \lambda_{1j} & \lambda_{21} & \cdots & \lambda_{N1} \\ \lambda_{12} & & & \\ \lambda_{13} & & & \\ \vdots & & & \\ \lambda_{1N} & & & \end{bmatrix} \cdot \vec{\mathbb{P}}(t)$$

EX $N(t)$ - # OF POISSON EVENTS

$N(0) = 0$

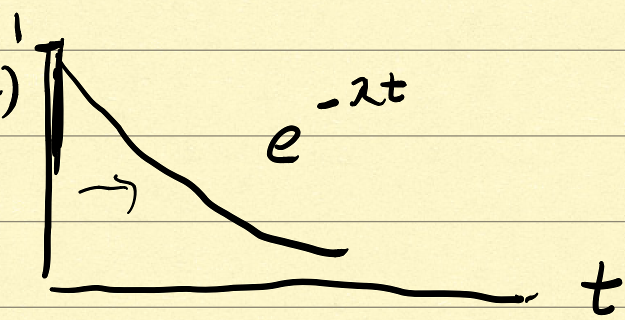
$$\frac{d}{dt}\vec{\mathbb{P}} = \underline{M} \cdot \mathbb{P}$$

PURE BIRTH PROCESS

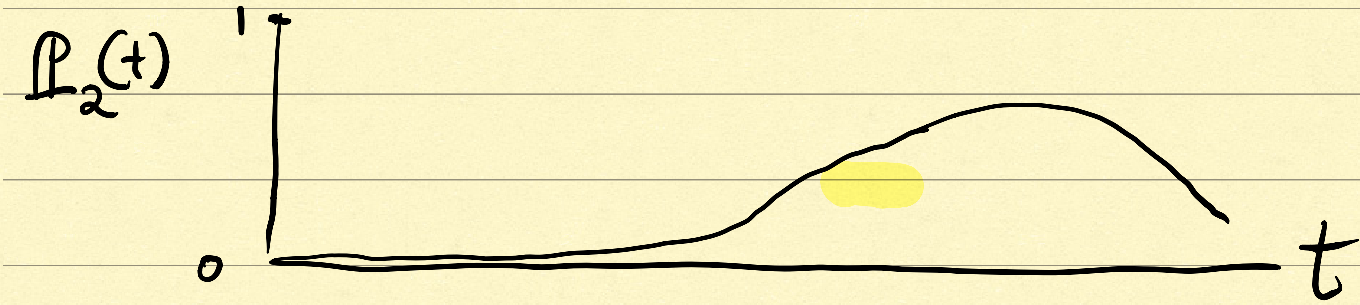
$$\begin{bmatrix} -\lambda & 0 & 0 & \\ \lambda & -\lambda & 0 & \\ 0 & \lambda & -\lambda & \\ 0 & 0 & \lambda & \\ 0 & \vdots & \vdots & \\ \vdots & & & \end{bmatrix}$$

$$\vec{\mathbb{P}}(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \end{bmatrix}$$

$\mathbb{P}_0(t)$

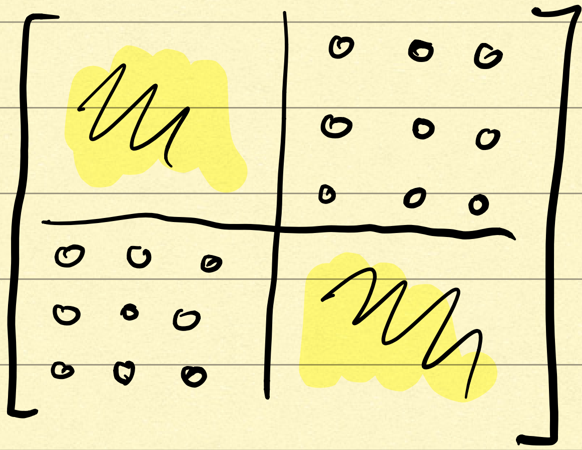


$\mathbb{P}_1(t)$

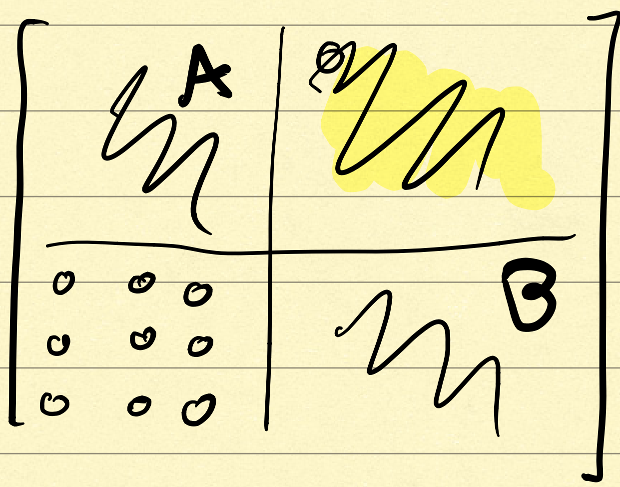


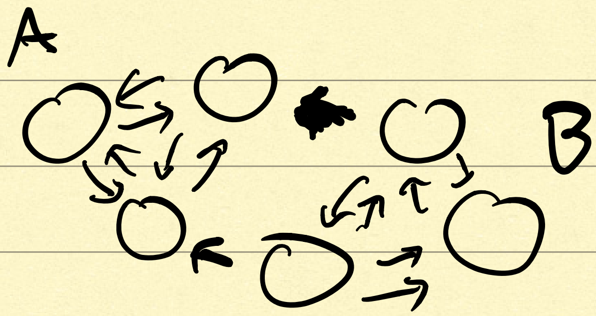
EX

M =



M =





EX WAITING FOR 2 MUTATIONS

$\Rightarrow$ MAX TWO EVENTS

STATES: (W, W)

(M, W)

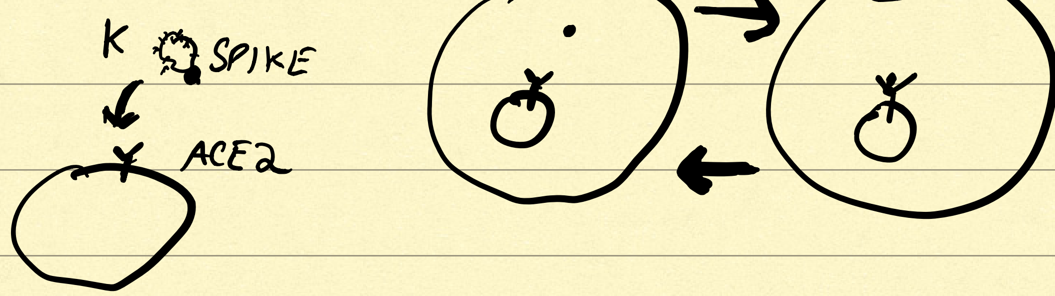
(W, M)

(M, M)

$$
\begin{tikzcd}
 & MW \arrow[dr, "\lambda"] & \\
WW \arrow[ur, "\lambda"] \arrow[dr, "\lambda"'] & & MM \\
 & WM \arrow[ur, "\lambda"'] &
\end{tikzcd}
$$

$$
M = \begin{bmatrix} -2\lambda & 0 & 0 & 0 \\ \lambda & -\lambda & 0 & 0 \\ \lambda & 0 & -\lambda & 0 \\ 0 & \lambda & \lambda & 0 \end{bmatrix}
$$

PS 5 A



JONATHAN VU

$\alpha = 0, \quad \mu_2 = \lambda = 1$

$\mathbb{P}_{FREE} = \frac{1}{2}$

$\mathbb{P}_{ANTAGONIST} = \frac{1}{2}$

$\mathbb{P}_{AGONIST} = 0$

VINCENT HU : $\quad \mathbb{P}_{FREE} \cancel{=} \dfrac{\lambda}{\alpha\mu_1 + (1-\alpha)\mu_2 + \lambda}$

$\lambda \to 0 \qquad \mathbb{P}_{FREE} \to 1 \quad ?$

$$\mathbb{P}_{FREE} \cancel{=} 1 - \frac{\lambda}{\alpha\mu_1 + (1-\alpha)\mu_2 + \lambda}$$

NARGES $\quad \mathbb{P}_{FREE} = \dfrac{\frac{1}{\lambda}}{\alpha \frac{1}{\mu}}$

$$\mathbb{P}_{FREE} \cancel{=} 1 - \frac{\alpha \frac{\cancel{\lambda}}{\mu_1} + (1-\alpha)\frac{\cancel{\lambda}}{\mu_2}}{\cancel{\lambda}}$$

VINCENT HU 2

$\alpha = 0, \ \mu_2 = \lambda = 1$

?

$$\mathbb{P}_{FREE} \stackrel{?}{=} \frac{\frac{1}{\lambda}}{\frac{\alpha}{\mu_1} + \frac{1-\alpha}{\mu_2} + \frac{1}{\lambda}} = \frac{\frac{1}{1}}{0 + \frac{1}{1} + \frac{1}{1}} = \frac{1}{2}$$

$\checkmark$

$\lambda \to 0$

$$\mathbb{P}_{FREE} \to \frac{\frac{1}{\lambda}}{\frac{1}{\lambda}} \to 1$$

$$T_{BOUND} = \alpha T_{BOUND}^{AG} + (1-\alpha) T_{BOUND}^{ANT}$$

PS 5 A

$$\frac{d}{dt} \begin{bmatrix} \mathbb{P}_{FREE} \\ \mathbb{P}_{AG} \\ \mathbb{P}_{ANT} \end{bmatrix} = \begin{bmatrix} -\lambda & +\mu_1 & +\mu_2 \\ \alpha\lambda & -\mu_1 & 0 \\ (1-\alpha)\lambda & 0 & -\mu_2 \end{bmatrix} \begin{bmatrix} \mathbb{P}_{FREE} \\ \mathbb{P}_{AG} \\ \mathbb{P}_{ANT} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbb{P}_{FREE} + \mathbb{P}_{AG} + \mathbb{P}_{ANTAG} = 1$$

MEAN FIRST PASSAGE TIMES

EX

$$\begin{tikzcd} A \arrow[r, bend left, "k_{AB}"] & B \arrow[l, bend left, "k_{BA}"] \arrow[r, "k_{BC}"] & C \end{tikzcd}$$

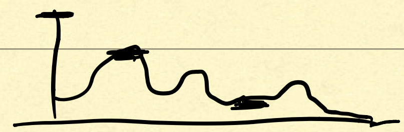
$$M = \begin{bmatrix} -k_{AB} & k_{BA} & 0 \\ k_{AB} & -(k_{BA}+k_{BC}) & 0 \\ 0 & k_{BC} & 0 \end{bmatrix} \qquad \frac{d\vec{\mathbb{P}}}{dt} = M \cdot \vec{\mathbb{P}}$$

C - ABSORBING

A, B - TRANSIENT

MEAN TIME IN A BEFORE LEAVING (FOREVER)

$$T_A = \int_0^\infty \mathbb{P}_A(t)\, dt$$



MEANTIME TO C: $E[T_{A\to C}] = T_A + T_B$

$$\int_0^\infty \frac{d\mathbb{P}_A}{dt} dt = -k_{AB}\int_0^\infty \mathbb{P}_A\, dt + k_{BA}\int_0^\infty \mathbb{P}_B\, dt$$

$$\int_0^\infty \frac{d\mathbb{P}_B}{dt} dt = +k_{AB}\int_0^\infty \mathbb{P}_A\, dt - k_{BA}\int_0^\infty \mathbb{P}_B\, dt - k_{BC}\int_0^\infty \mathbb{P}_B\, dt$$

$$\frac{d\mathbb{P}_c}{dA} = \text{~~~~}$$

$$\mathbb{P}_A(t)\Big|_0^\infty = 0 - 1 = -k_{AB}\,T_A + k_{BA}\,T_B$$

$$\mathbb{P}_B(t)\Big|_0^\infty = 0 - 0 = +k_{AB}\,T_A - k_{BA}\,T_B - k_{BC}\,T_B$$

$$-1 = -k_{AB}\,T_A + k_{BA}\,T_B$$
$$0 = k_{AB}\,T_A - k_{BA}\,T_B - k_{BC}\,T_B$$

THEN $E[T_{A\to C}] = T_A + T_B$

IN GENERAL

LET M_{-j} - TRANSITION MATRIX WITH jTH ROW + COLUMN REMOVED

$$\begin{array}{r l} & \\ i\text{TH ROW} \to & \end{array}\begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 0 \end{bmatrix} = M_{-j} \cdot \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ \vdots \\ T_{N-1} \end{bmatrix}$$

$$E[T_{i\to j}] = \sum_k T_k$$