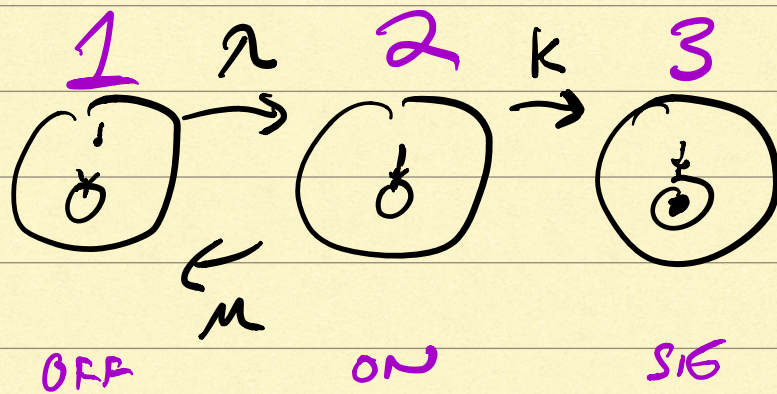


Hi!

PS 5 Bii



$$\frac{d\vec{P}}{dt} = M \cdot \vec{P}$$

$$M = \begin{bmatrix} -\lambda & +\mu & 0 \\ +\lambda & -(\mu+k) & 0 \\ 0 & +k & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -k & 0 \\ 0 & +k & 0 \end{bmatrix}$$

$$T_{2 \rightarrow 3} = ?$$

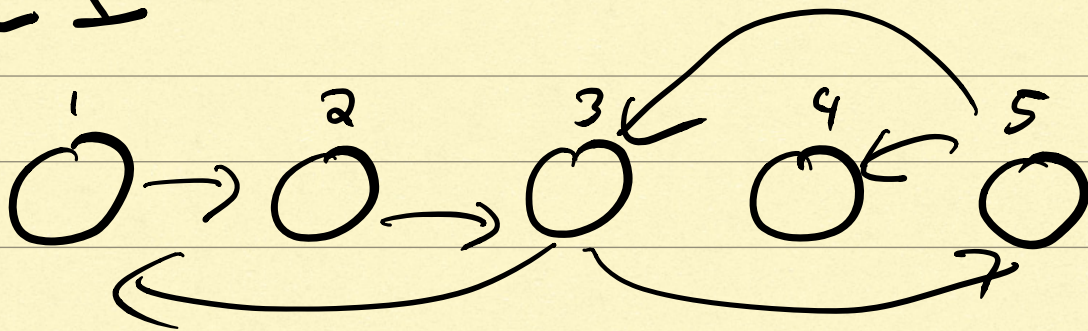
$$M_{-3} = \begin{bmatrix} -\lambda & +\mu \\ +\lambda & -(\mu+k) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -k \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -\lambda & +\mu \\ +\lambda & -(\mu+k) \end{bmatrix} \cdot \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} \quad \vdots$$

$$\dots T_1, T_2$$

$$T_{2 \rightarrow 3} = T_1 + T_2 = \frac{1}{k} \left(1 + \frac{\mu}{\lambda} \right)$$

ASIDE 1

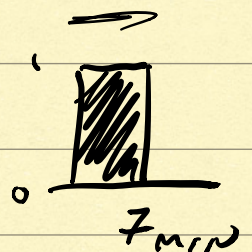


$T_{3 \rightarrow 5}$

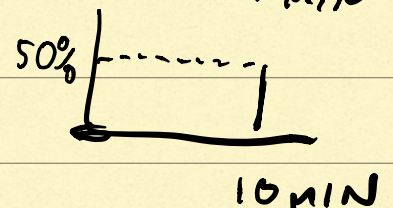
$$M_{-5} = \begin{bmatrix} & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} = M_{-5} \cdot \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix}$$

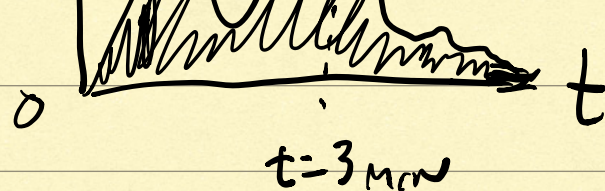
$$T_{3 \rightarrow 5} = T_1 + T_2 + T_3 + T_4$$



ASIDE 2

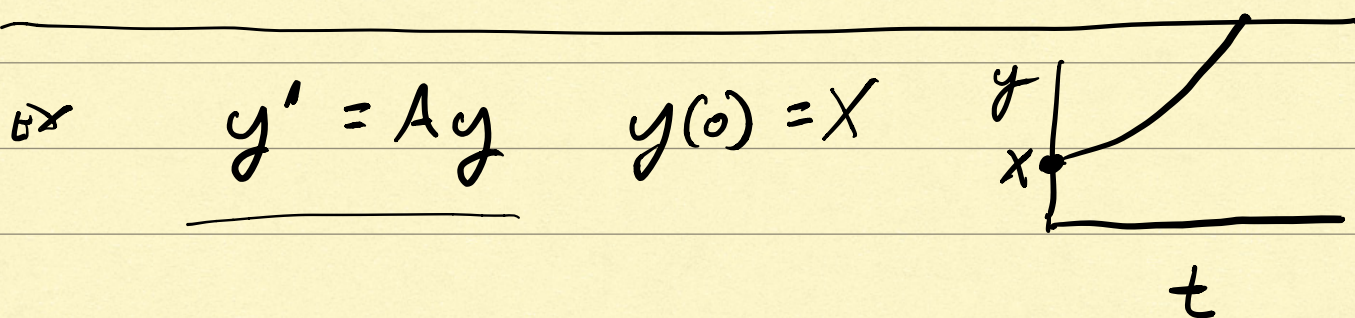


$$P_i(t)$$

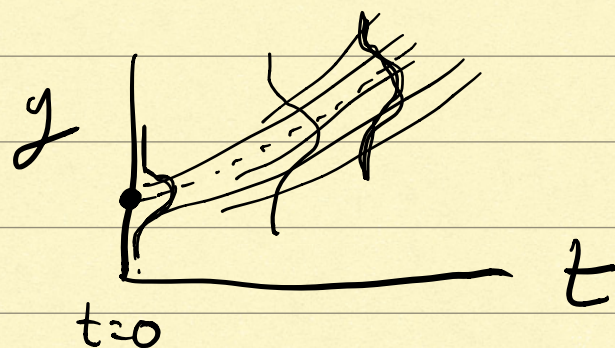


$$\int_0^{\infty} P_i(t) dt$$

PARAMETRIC NOISE ~ HETEROGENEITY



suppose $X \sim p_X(x)$



$$y(t) = x e^{At}$$

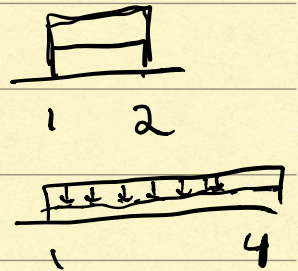
WHAT IS $y \sim p_Y(y)$?

GIVEN A RANDOM VARIABLE $X \sim p_X(x)$,

SUPPOSE $Y = g(X)$ THEN WHAT IS $p_Y(y)$?

SUPPOSE $Y = cX$

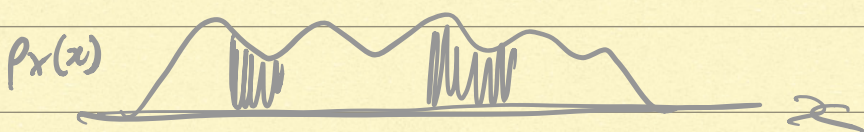
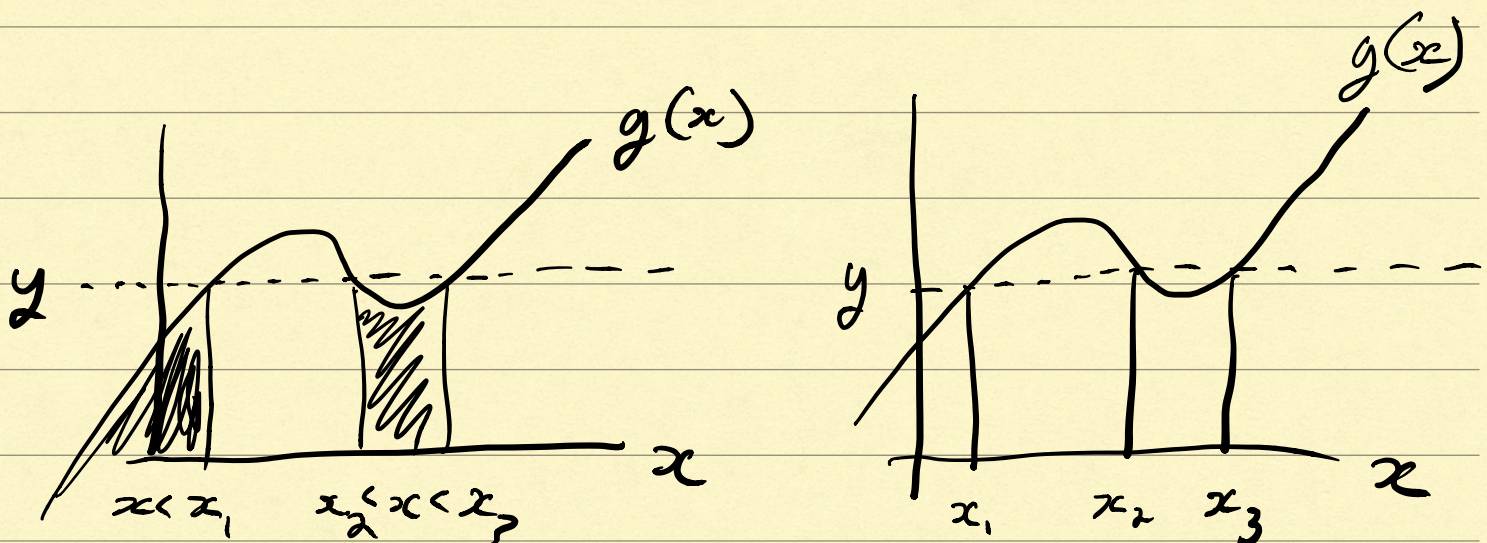
NOT $P_Y(y) = c \cdot P_X(x)$



FOR A GENERAL FUNCTION $g(x)$,
DEFINE

$I_y = \{x : g(x) \leq y\}$ INDEX SET OF y

$\{x_k\} = \{x : g(x) = y\}$ PRE-IMAGE OF y

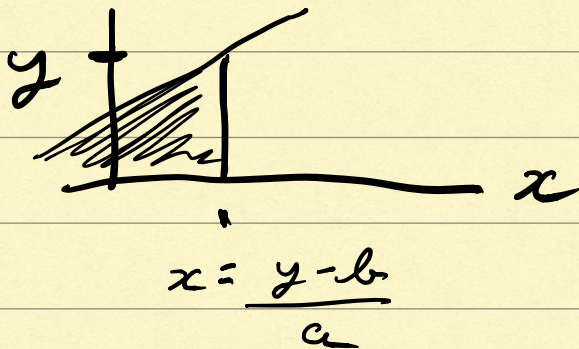


$$F_Y(y) = \mathbb{P}(Y \leq y)$$

$$= \mathbb{P}(x \in I_y) = \int_{I_y} p_x(x) dx$$

ex $y = ax + b$

$$a > 0$$

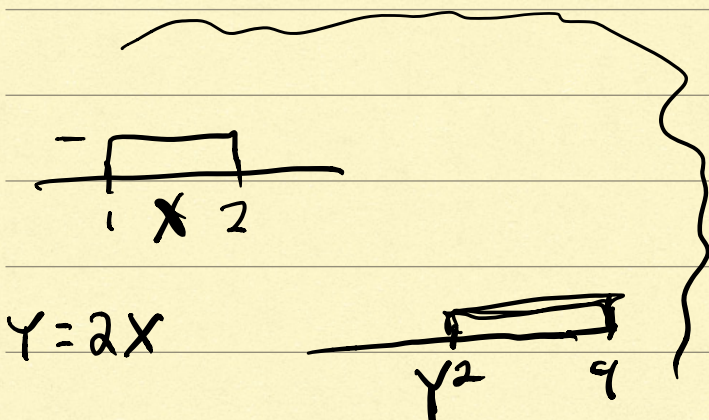


$$I_y = \left\{ x \leq \frac{y-b}{a} \right\}$$

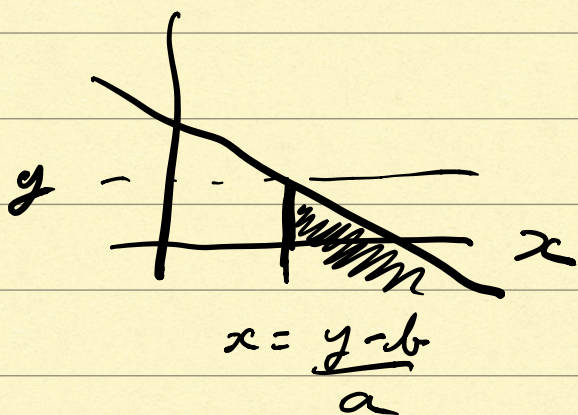
GIVEN $F_x(x)$ THEN

$$F_Y(y) = F_x\left(\frac{y-b}{a}\right)$$

$$p_Y(y) = \frac{d}{dy} F_Y(y) = \frac{1}{a} p_x\left(\frac{y-b}{a}\right)$$



EX $a < 0$



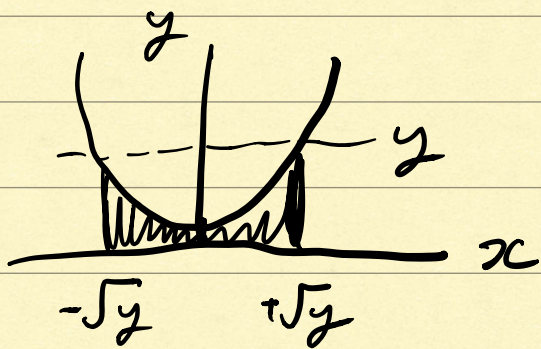
$$I_y = \left\{ x \geq \frac{y-b}{a} \right\}$$

$$F_Y(y) = 1 - F_X\left(\frac{y-b}{a}\right)$$

$$p_Y(y) = -\frac{1}{a} p_X\left(\frac{y-b}{a}\right)$$

$$p_Y(y) = \left| \frac{1}{a} \right| p_X\left(\frac{y-b}{a}\right) \quad a \in \mathbb{R}$$

EX $Y = X^2$



$$g(x) = x^2$$

$$I_y = \begin{cases} (-\sqrt{y}, +\sqrt{y}) & y \geq 0 \\ \emptyset & y < 0 \end{cases}$$

$$F_Y(y) = \begin{cases} F_X(\sqrt{y}) - F_X(-\sqrt{y}) & y \geq 0 \\ 0 & y < 0 \end{cases}$$

$$p_Y(y) = \begin{cases} p_X(\sqrt{y}) \cdot \frac{1}{2\sqrt{y}} + p_X(-\sqrt{y}) \cdot \frac{1}{2\sqrt{y}} & y \geq 0 \\ 0 & y < 0 \end{cases}$$

GENERAL FORMULA

LET $X \sim p_X(x)$ AND $Y = g(X)$

$$p_Y(y) = \sum_k p_X(x_k(y)) \cdot \left| \frac{dx_k}{dy} \right|$$

WHERE $\{x_k\}$ IS THE PRE-IMAGE OF y

IF $\frac{dg}{dx} \neq 0$, THIS SIMPLIFIES TO

$$p_Y(y) = \sum_k p_X(g_k^{-1}(y)) \cdot \left(\left| \frac{dg}{dx} \right| \right)^{-1}$$
