Educational Material: Math and Physics Application

Maverick S. H. Oh

Most recent update: November 21, 2024

Abstract

This material is developed to provide interesting and useful application of mathematics and physics for high school and undergraduate students. The problems are related to other professional fields, such as biology, chemistry, and everyday life. The author decided to archive the material such that it is easy to share and more widely used.

Move to the next page for the problems and solutions.

1 Calculus + Biology Problems

1. PCR Optimization Problem

   You are running a biology lab business and you need to find the right temperature for an enzyme (polymerase) to work.

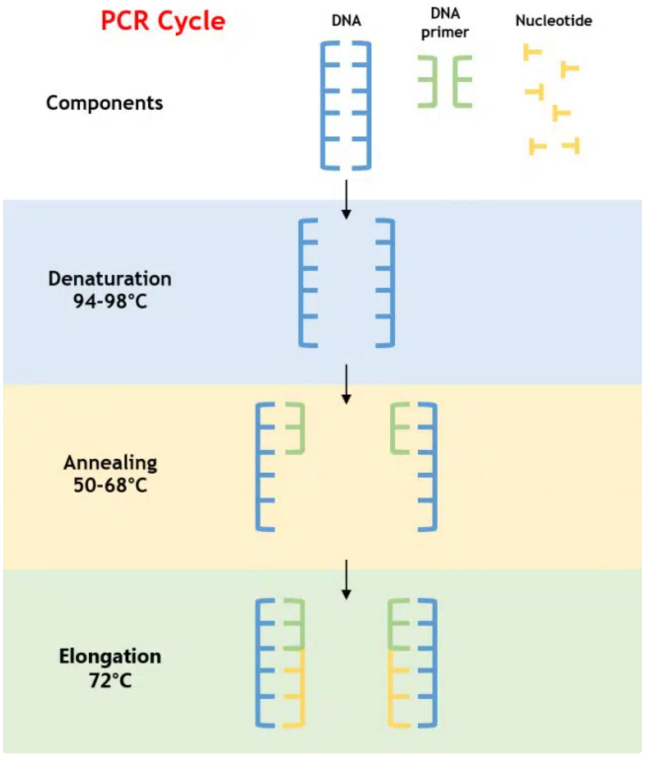

Figure 1: PCR process diagram. Source: clinisciences

Polymerase Chain Reaction (PCR) is a widely used technique in biology and bioengineering to duplicate DNA copies and amplify the quantity of DNA. It requires multiple components, but one key ingredient is polymerase. We will focus on polymerase to simplify the problem.

The most widely-used polymerase for PCR is Taq polymerase. This

protein is special in the sense that it doesn't get deformed easily in high temperature. This makes it the best polymerase for PCR.

Bio Quiz: Do you know why it is good for the polymerase to be heat-resistent for PCR? If you know, try explaining 😀. If you don't, you can look up online too.

(a) We want to find the best temperature for the "annealing" process, where primers attach to single-stranded DNA helix. Let's say the efficiency of annealing process A is given as a function of temperature T. The functional form is as follows:

$$A(T) = e^{-(T-50)^2/100}$$

Find the temperature value $T_{\text{best,A}}$ that maximizes the annealing efficiency $A(T)$. What is the best efficiency in this case?

(b) Now, we want to find the best temperature for the "elongation" process, where polymerase extends DNA strands by putting nucleotides. The efficiency of elongation E is also given as a function of temperature T. The functional form is as follows:

$$E(T) = e^{-(T-75)^2/1000}$$

Find the best temperature value $T_{\text{best,E}}$ that maximizes the efficiency of elongation $E(T)$. What is the best efficiecy in this case?

(c) Emergency! Our PCR machine is broken and we cannot set two different temperatures for annealing and elongation separately. We can only set one single temperature for annealing and elongation. In this case, we would like to maximize the overall efficiency $O(T)$, defined as the product of $A(T)$ and $E(T)$:

$$O(T) = A(T) \cdot E(T) = e^{-(T-50)^2/100} \cdot e^{-(T-75)^2/1000}$$

Find the best temperature value $T_{\text{best,O}}$ that maximizes the overall efficiency $O(T)$. What is the best efficiency in this case?

(d) Where does $T_{\text{best,O}}$ fall compared to $T_{\text{best,A}}$ and $T_{\text{best,E}}$? Does it make sense?

(e) What is the efficiency loss in the emergency situation?

(f) Note that this problem simplified the problem a lot. If you can improve the problem, what would it be?

The solution of the problem can be found in: page 5 of the following link/QR code:

maverick-oh.github.io/assets/pdf/Educational_Material.pdf

2 Calculus + Biology Solutions

1. PCR Optimization

   I haven't made the solution yet. Sorry. Visit later...