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A Mathematical formulation based on the connectedness of stones for the game of Go

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Abstract

We propose the mathematical model for the connectedness in the game of Go. This game has graph theoretical structure and the connectedness of the stones is very important feature to evaluate positions of stones. In this paper, we construct the mathematical model which represents the connectedness of the stones, and propose the recursion of extended liberties for mathematical evaluations of various positions in the game of Go..

Keywords: graph theory, connectedness, the game of Go.

1. Introduction

The game of Go originates in China and spreads to Japan. And many people play this old board game in the world now.

This game is played two persons. Two players put black or white stones at intersections of grids on a game board by turns. The same colored stones on a game board make blocks. A block on a game board is a group of the same stones connected through lines as neighbors. If an opponent block is closed by the different colored stones then stones in the block are captured by another player. And the enclosed region is occupied by the player who captured this block. Finally, the player who has more captured stones and occupied regions wins a game.

The strategies to win in the game of Go are complex even though the rule is simple. In this paper, we focus the connectedness of stones which is one of complexities in the game of Go.

The connectedness of stones is very important feature for evaluation of strategies and can give relations between stones and intersections in a game board. We will introduce a mathematical model for representation of various features based on the connectedness of stones.

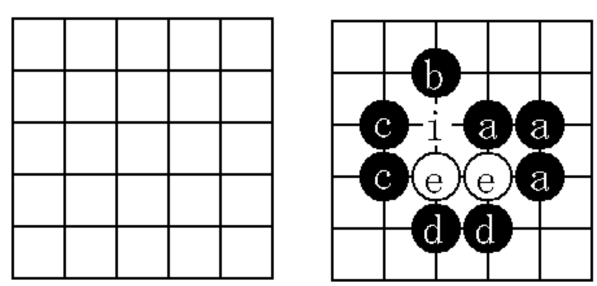
In previous studies, Graepel et. al. introduced a mathematical model named the "common fate graph(CFG)" in [3]. CFG is a (weighted) graph which uses blocks as nodes. A *block* is one of the units in the game of Go. Because stones in a same block have "common-fate", never be distinguished until they are captured.

Benson [1] proposed a mathematical model for the graph theoretic static analysis of the board arrangement. The purpose of [1] is to define the condition "unconditional alive" mathematically. It is the condition in which stones can never be captured. In [1], Benson used mappings from blocks to sets of intersections in order to represent relations between stones.

Berlekamp and Wolfe [2] classified local patterns and constructed the rules to evaluate them by using the combinatorial game theory. At nearly the end of a game, positions can be devide to some independent local subgames. Each local subgame consists of unconditional alive stones and narrow regions, and has a value which is not integer. Using this method, we can calculate the value of whole board from values of local games. Berlekamp and Wolfe gave values to many patterns and classified them with their values in the appendix of [2].

Our purpose of this paper is to propose a mathematical model for the game of Go named the "BW graph model". Our model is based on graphs and mappings and clearly represents various features such as the "connectedness" and so forth. In addition, we will efficiently evaluate various strategies by using "BW graph model".

In Section 2, we survey related works on some mathematical models of the game of Go. In Section 3, we define a mapping **ex-liberty**, which represents the



(a) (b)

figure 1:(a):The position at the beginning of the 6-by-6 board. (b):An example of positions for 6-by-6 board.

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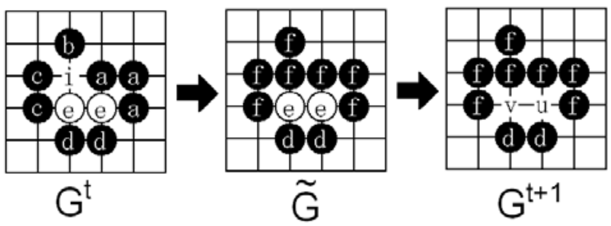


figure 2:An example of move processes.

connectedness of stones. In Section 4, we prove the theories that must be satisfied by the connectedness of stones. We also construct a mathematical model and its recursion. We introduce the coordinates on the Go board as an example of applications of our model in Section 5. Finally, in Section 6, we give a conclusion and future works.

2. Realated works

2.1 The rules of the game of Go

We refer Thorp and Walden [5], and Benson [1] for the rules and notations of the game of Go.

The game of Go is one of the two players, perfect information, zero sum board games. The board is a square grid. A n-by-n board has n^2 intersections, and they are adjacent to neighbors. Note that two intersections are **adjacent** if they are on the same horizontal or vertical line and there are no intersections between them. The official board is 19-by-19 board. But for like mini-games, 6, 9, or 13 are used. In this paper, we explain with 6-by-6 board. The game of Go can be played in the same rule regardless of its size.

At the beginning of a game, there are no stones on the board (see (a) in figure 1). The first player has black stones, and second has white. Each player chooses an intersection and places a stone in his/her turn. It is called a **move**.

Each intersection is in one of the following three states; a black stone is placed, a white stone is placed, or no stone is placed. We call these states, **black, white** and **null**, respectively. When black (white) player places a stone on an intersection, the state of that intersection is changed from null to black (white). If two same colored stones are adjacent to each other, then they are said to be connected. A connected set of same colored stones is called a **block**. An example is shown in (b) in figure 1, in which there are 4 black blocks and 1 white block.

A block is **captured** when it is enclosed by opponent stones. If a black (white) block is captured, then stones in the block are removed and that intersections are changed from black (white) to null. Figure 2 is an example of such process of a move. Black player places a black stone on $\boldsymbol{i}$ of the left position $\mathbf{G}^{\boldsymbol{t}}$. At first, 3 blocks are combined to one block $\boldsymbol{f}$ with the new black stone $\boldsymbol{i}$ as a relay point , and

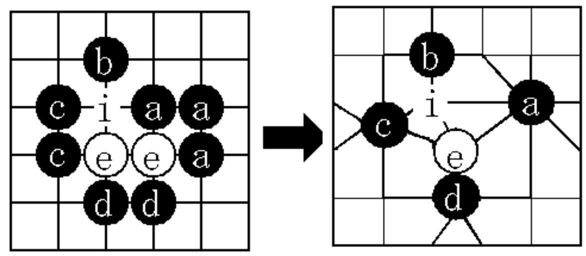


figure 3:An example of CFG.

white stones $\boldsymbol{e}$ are enclosed in the middle position. Finally, two null itersections $\boldsymbol{u}$ and $\boldsymbol{v}$ appear in exchange of removing $\boldsymbol{e}$.

The null intersection that it is adjacent to a block is called a **liberty** of that block. In (b) of figure 1, the null intersection $\boldsymbol{i}$ is a liberty of the blocks $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$ and $\boldsymbol{e}$. In other words, a stone is captured and removed when it loses all its liberties.

2.2 The common fate graph

Graepel et. al. [3] visualized blocks by common fate graph (CFG). CFG is a graph model whose nodes are blocks including the null blocks. Figure 3 is an example of CFG. Graepel et. al. defined the transformation that combines two nodes to one node if they are the same color and neighbors. CFG is given by a repeated application of this transformation. When once stones are in a same block, they continue being same state, same colored and having same liberties, until they are captured. Graepel et. al. called this fact *common fate.* Stones in a same block are in a same state, so values of them are also same.

2.3 The arrangement

Benson [1] proposed a board position as a mapping named **arrangement**. An arrangement is a mapping from the set of intersections to the set of labels {black, white, null}.

Let $\boldsymbol{V}$ be a set of intersections. An arrangement $\boldsymbol{\alpha}$ is

$$\boldsymbol{\alpha}: \boldsymbol{V} \to \{\boldsymbol{black}, \boldsymbol{white}, \boldsymbol{null}\}.$$

Each arrangement corresponds to a position, hence a game can be represented as a sequence of arrangements $\{\boldsymbol{\alpha}^{\mathbf{0}}, \boldsymbol{\alpha}^{\mathbf{1}}, \cdots, \boldsymbol{\alpha}^{\boldsymbol{n}}\}$.

Benson defined a block using the concept of the **state-connectedness**. Two intersections are state-connected if they are in the same state and there is a path between them. The state-connectedness is an equivalence relation. A block is defined as the element of the quotient set of intersections by this. In [1], it was proved that a block having two

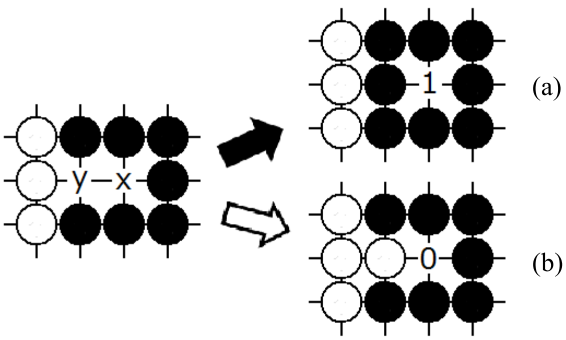


figure 4:An example of corridors

liberties which opponent cannot place stones on is never captured.

2.4 A method for local patterns

Berlekamp and Wolfe [2] constructed a game tree model with combinatorial game theory. At nearly the end of a game, a position can be divided into the independent local subgames. Berlekamp and Wolfe made a method to calculate these values. Figure 4 is an example of local patterns.

This is called a (blocked) **corridor**. A corridor consists of unconditional alive stones, a dead-end null intersection and some streets. For example, there are a dead-end $\boldsymbol{x}$ and a street $\boldsymbol{y}$ in figure 4. Berlekamp and Wolfe gave to it a value $\{\mathbf{1}|\mathbf{0}\}$. $\{\mathbf{1}|\mathbf{0}\}$ means that black player can get one point by his/her move but if opponent moves first then its value is zero. In (a) of the figure 4., when black stones is placed on $\boldsymbol{x}$ then $\boldsymbol{y}$ is enclosed by black stones and it becomes to a black territory. But if white stone is placed on $\boldsymbol{x}$, $\boldsymbol{y}$ is adjacent to both of black and white stones, so $\boldsymbol{y}$ is neither's territory((b) of figure 4).

Values of local subgames are defined with advantages and cannot be represented as a number. Berlekamp and Wolfe defined non-number values "chilling" for local subgames, and also defined the summation of them.

3. A Model of Positions

The patterns on the arrangement of stones are on the square grid, because the board is square. However, the patterns of blocks are not a square grid. The detection algorithms of connectedness to make blocks needs a repeated application.

We propose a model to represent the patterns with the connectedness of stones, which we call "BW graph model" [4]. Also, we propose the recursions of the connectedness of stones. This gives the state of connectedness in the next position from one before without the repeated application.

The expansion of liberty is used in our model. We call it the **ex-liberty**. It will be defined in this section. Also we formularize the followings.

Blocks A block is a component of same colored stones.
Liberty A Liberty of a block is a null intersection that is adjacent to that block.
Captured Being captured is equivalent to having no liberty. If a block is captured then it is removed.
Legal If a move makes its own block to lose all its liberty, then it is illegal.
Atari *Atari* is one of the states of blocks, in which a block has only one liberty. Blocks must be in the state "*atari*" before captured.

3.1 Position Graphs

A position of a game is defined as follows:

Definition 1

Let $\boldsymbol{L} = \{\boldsymbol{black}, \boldsymbol{white}, \boldsymbol{null}\}$ be a label set. An undirected graph $\boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{E}, \boldsymbol{\alpha})$ is called a **position** if and only if

$\boldsymbol{V}$: a set of intersections.
$\boldsymbol{E}$: a set of edges between two adjacent intersections and self-edges of all intersections.
$\boldsymbol{\alpha}$: a mapping from $\boldsymbol{V}$ to $\boldsymbol{L}$.
And no stone satisfies the captured condition that will be defined in 4.1.

Note that there are two type edges. Some edges lie between two adjacent intersections. Others have an intersection as both endpoints. Each intersction has an edge in the second type.

And a **game** is defined as a sequence of positions $\{\mathbf{G}^t\} = \{(\boldsymbol{V}, \boldsymbol{E}, \boldsymbol{\alpha}^t)\}$. $\boldsymbol{G}^t$ is the position after $\boldsymbol{t}$-th move. $\boldsymbol{G}^0 = (\boldsymbol{V}, \boldsymbol{E}, \boldsymbol{\alpha}^0)$ refers to an initial position. Here, $\boldsymbol{\alpha}^0(\boldsymbol{v}) = \boldsymbol{null}$ for any $\boldsymbol{v} \in \boldsymbol{V}$.

Note that $\boldsymbol{\alpha}^t$ is similar to an arrangement defined in [1]. In this paper, we use the following notations,

$$\boldsymbol{B}^t = (\boldsymbol{\alpha}^t)^{-1}(\boldsymbol{black}),$$
$$\boldsymbol{W}^t = (\boldsymbol{\alpha}^t)^{-1}(\boldsymbol{white}),$$
$$\boldsymbol{N}^t = (\boldsymbol{\alpha}^t)^{-1}(\boldsymbol{null}).$$

Then it is easily verified that the folllowings hold.

For any $\boldsymbol{t}$,

$$\boldsymbol{V} = \boldsymbol{B}^t \cup \boldsymbol{W}^t \cup \boldsymbol{N}^t,$$
$$\boldsymbol{B}^t \cap \boldsymbol{W}^t = \boldsymbol{W}^t \cap \boldsymbol{N}^t = \boldsymbol{N}^t \cap \boldsymbol{B}^t = \emptyset,$$

$$\boldsymbol{B^0 = W^0 = \emptyset},$$
$$\boldsymbol{N^0 = V}.$$

We will define a position with $\boldsymbol{B^t}$, $\boldsymbol{W^t}$ and $\boldsymbol{N^t}$, and will not use $\boldsymbol{\alpha^t}$ in our model **BW Graph Model**.

3.2 Blocks

First, we define a relation $\boldsymbol{M}$ as follows,

Definition 2

Let $\boldsymbol{G = (V, E, \alpha)}$ be a position. Then $\boldsymbol{M}$ is called a relation cooresponding to $\boldsymbol{G}$ if and only if

$\forall \boldsymbol{a, b \in V}$,

$$\boldsymbol{aMb} \Leftrightarrow \begin{cases} \boldsymbol{a = b} \\ \text{or} \\ \text{a path from } \boldsymbol{a} \text{ to } \boldsymbol{b} \text{ exists on } \boldsymbol{\alpha^{-1}(black)} \\ \text{or} \\ \text{a path from } \boldsymbol{a} \text{ to } \boldsymbol{b} \text{ exists on } \boldsymbol{\alpha^{-1}(white)} \end{cases}$$

where "the existence of a path from $\boldsymbol{x}$ to $\boldsymbol{y}$ on $\boldsymbol{A}$" means

$$\exists \{\boldsymbol{z_i}\}_{i=1}^{n} s.t. \boldsymbol{xz_1, z_1z_2, \cdots, z_iz_{i+1}, \cdots, z_ny \in E}$$
$$\text{and } \boldsymbol{x, y, \forall z_i \in A}.$$

Each path has only one color; black or white. And all null intersections can only satisfy the first proposition $\boldsymbol{a = b}$ because they cannot be in any black or white path. Hence the following lemma holds,

Lemma 3

$$\forall \mathbf{x} \in \boldsymbol{\alpha^{-1}(null)}, \forall \boldsymbol{z \in V, xMz \Rightarrow x = z}.$$

This lemma means that all null intersections are on the same level as blocks.

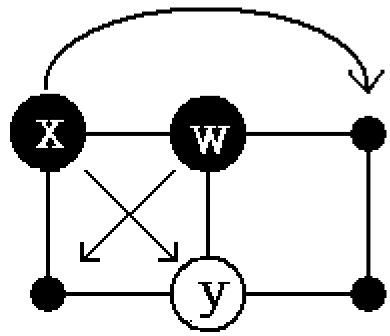
$\boldsymbol{M}$ is an equivalence relation, so

$$\boldsymbol{xMx},$$
$$\boldsymbol{xMy \Leftrightarrow yMx},$$
$$\boldsymbol{xMy, yMz \Rightarrow xMz}.$$

$\boldsymbol{M}$ is similar to the state-connected in [1] and the repeated application of the transformation in [3]. Then, a component (maximal connected subset) for $\boldsymbol{(\alpha^t)^{-1}(black)}$ and $\boldsymbol{(\alpha^t)^{-1}(white)}$ is represented by $\boldsymbol{M}$. For instance, if $\boldsymbol{x \in (\alpha^t)^{-1}(black)}$ then $\{\boldsymbol{y | xMy}\}$ is the only component of $\boldsymbol{(\alpha^t)^{-1}(black)}$ such that $\boldsymbol{x}$ is in it.

3.3 Liberties and Ex-Liberties

By the rule, stones having no liberty are removed or else it is an illegal move.



Let $\mathbf{G^t} = \boldsymbol{(V, E, \alpha^t)}$ be a position. The set of liberties of $\boldsymbol{x}$, $\boldsymbol{Liberty(x)}$ is defined

$$\boldsymbol{Liberty(x)} = \{\boldsymbol{y \in N} | \exists \boldsymbol{z \in V} \; s.t. \boldsymbol{yz \in E, zMx}\}.$$

We expand this definition as follows,

(9)

figure 5:An example of $\boldsymbol{F}$ on a position.

Definition 4

The set of ex-liberties of $\boldsymbol{x}$, $\boldsymbol{F^t(x)}$ is defined as

$$\boldsymbol{F(x)} = \{\boldsymbol{y \in V} | \exists \boldsymbol{z \in V} \; s.t. \boldsymbol{yz \in E, zMx}\}.$$

Figure 5 is an example of $\boldsymbol{F}$ on a position. If $\boldsymbol{y \in F(x)}$ then we draw an arrow from $\boldsymbol{x}$ to $\boldsymbol{y}$.

Remark

The arrow from $\boldsymbol{x}$ to $\mathbf{y}$ means $\boldsymbol{x}$ can reach to $\boldsymbol{y}$ through $\mathbf{w}$. (10) On other hand, $\mathbf{y}$ cannot reach to $\boldsymbol{x}$, because the block that $\boldsymbol{y}$ belongs to is not adjacent to $\boldsymbol{x}$.

Remark

$\boldsymbol{v \in F(v)}$ is held for any intersection $\boldsymbol{v}$ and this proposition should be represented by using self-edges. But (11) we omit them to avoid complexity.

(12)

(13)

The liberty of a black (or white) stone $\boldsymbol{x}$ can be written as

$$\boldsymbol{Liberty(x) = F(x) \cap N}.$$

And $\boldsymbol{F}$ satisfies below.

$$\boldsymbol{xMz} \Leftrightarrow \mathbf{x \in F(z)}$$
$$\text{and } \boldsymbol{x, z \in B}, \boldsymbol{x, z \in W} \text{ or } \boldsymbol{x, z \in N}.$$

$\boldsymbol{x \in F(x)}$ is trivial. But $\boldsymbol{F(x)}$ is not symmetric

$$\boldsymbol{x \in F(y) \not\Leftrightarrow y \in F(x)},$$

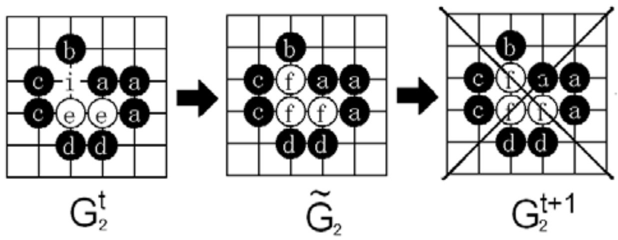


figure 6:An example of illegal moves for the white player.

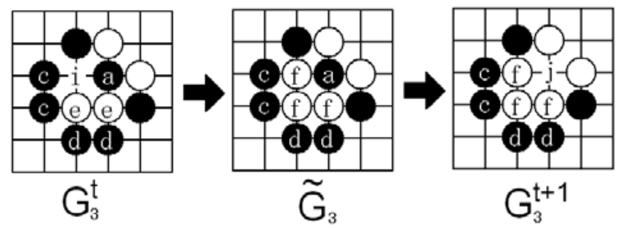


figure 7:An example of legal moves by capturing a stone.

because the relation between $\boldsymbol{x}$ and $\boldsymbol{z}$ is different from that between $\boldsymbol{y}$ and $\boldsymbol{z}$. $\boldsymbol{F}$ is symmetric in the following condition.

Theorem 5

$$\boldsymbol{xy} \in \boldsymbol{E} \Rightarrow \boldsymbol{y} \in \boldsymbol{F}(\boldsymbol{x}), \boldsymbol{x} \in \boldsymbol{F}(\boldsymbol{y}),$$
$$\boldsymbol{xMy} \Rightarrow \boldsymbol{y} \in \boldsymbol{F}(\boldsymbol{x}), \boldsymbol{x} \in \boldsymbol{F}(\boldsymbol{y}).$$

Proof of Theorem 5

These are trivial because we can choose $\boldsymbol{x}$ and $\mathbf{y}$ as $\boldsymbol{z}$ in eq. (15).

■

And the following holds.

$$\forall \boldsymbol{x} \in \boldsymbol{N}, \forall \boldsymbol{y} \in \boldsymbol{V}, \boldsymbol{y} \in \boldsymbol{F}(\boldsymbol{z}) \Rightarrow \boldsymbol{xy} \in \boldsymbol{E}.$$

Because there is an intersection $\boldsymbol{z}$ such as $\mathbf{yz} \in \boldsymbol{E}, \boldsymbol{zMx}$ and $\boldsymbol{x} = \boldsymbol{z}$ by lemma 3 and definition 4.

4. The BW Graph Model

In this section, we consider differences between the two positions $\boldsymbol{G^t}$ and $\boldsymbol{G^{t+1}}$ in a game $\{\boldsymbol{G^t}\}$. There are changes in the states of intersections and connectedness of stones. We propose the recursions instead of the detection of the connectedness. And we define the mathematical model "BW graph model", which is a representation of a position as a pair of states of intersections and its connectedness.

4.1. Captured Condition

Let $\{\boldsymbol{G^t}\} = \{(\boldsymbol{V}, \boldsymbol{E}, \boldsymbol{\alpha^t})\}$ be a game. We construct the representation of $\boldsymbol{G^{t+1}}$, which is a position after $\boldsymbol{G^t}$ with a **move**.

A **move process** consists of three steps,

1. Choose a null intersection, and place a stone.
2. Search for opponent stones satisfying the **captured condition**. If they exist, remove them.
3. Check if the move is legal or not. If it is illegal, then backtrack to step 1.

At first, we formularize the rule of **capturing**. If a stone has no liberty then it is captured. Using $\boldsymbol{F}$, the **captured condition** of a stone $\boldsymbol{x}$ is described as

$$\boldsymbol{F}(\boldsymbol{x}) \cap \boldsymbol{N} = \emptyset.$$

(19)
(20)

This condition is satisfied only in the middle of a move process, steps 2 and 3. We indicate this temporary position by $\widetilde{\mathbf{G}} = (\boldsymbol{V}, \boldsymbol{E}, \widetilde{\boldsymbol{\alpha}})$. If a stone satisfies (22) in $\widetilde{\boldsymbol{G}}$, then it is captured immediately or the move is cancelled. So $\widetilde{\boldsymbol{G}}$ is not a position of $\{\boldsymbol{G^t}\}$. In other words, no stone satisfy (22) in $\boldsymbol{G^t}$ for any $\boldsymbol{t}$. On the other hand,

$$\forall \boldsymbol{e} \in \boldsymbol{N}, \boldsymbol{F}(\boldsymbol{e}) \cap \boldsymbol{N} \neq \emptyset,$$

by $\boldsymbol{x} \in \boldsymbol{F}(\boldsymbol{x})$. Therefore,

(21)

$$\forall \boldsymbol{t}, \forall \boldsymbol{x} \in \boldsymbol{V}, \boldsymbol{F^t}(\boldsymbol{x}) \cap \boldsymbol{N^t} \neq \emptyset.$$

4.2 Move Processes

Let $\boldsymbol{G^t}$ be a position in black's turn, and assume that black player place a stone on an intersection $\boldsymbol{i}$. $\widetilde{\boldsymbol{G}}$ is a position after adding a stone and before capturing stones. The states of stones in $\widetilde{\boldsymbol{G}}$ is defined for $\boldsymbol{G^t}$ as follows

$$\widetilde{\boldsymbol{B}} = \boldsymbol{B^t} \cup \{\boldsymbol{i}\},$$
$$\widetilde{\boldsymbol{W}} = \boldsymbol{W^t},$$
$$\widetilde{\boldsymbol{N}} = \boldsymbol{N^t} \setminus \{\boldsymbol{i}\}.$$

Step 1 causes only one null intersection $\boldsymbol{i}$ to turn into a black stone. But the change causes the stones satisfying the captured condition. Figure 2 is an example of move processes satisfying the captured condition. In the middle position $\widetilde{\boldsymbol{G}}$ in figure 2, two white stones $\boldsymbol{e}$ satisfy the captured condition by the move $\boldsymbol{i}$ on the left position $\boldsymbol{G^t}$. After removing them (the right position $\boldsymbol{G^{t+1}}$), no stone satisfies that condition. So the black move $\boldsymbol{i}$ is legal.

The second example is in figure 6. In this case, the white move $\boldsymbol{i}$ on $\boldsymbol{G_2^t}$ in figure 6 makes no black stone satisfying the condition. But the white stones $\boldsymbol{f}$ have no liberty. So $\boldsymbol{i}$ is an illegal move for the white player.

Another case is in figure 7. Both black $\boldsymbol{a}$ and white stones $\boldsymbol{f}$ satisfy the condition in $\widetilde{\boldsymbol{G}_3}$ by a white move $\boldsymbol{i}$. In this case, removing the black (opponent) stone $\boldsymbol{a}$ is done before decision of illegality of the white move.

Let $\boldsymbol{D^t(i)}$ be a set of captured stones by the black move $\boldsymbol{i}$ on the position $\boldsymbol{t}$. Then the states of intersections in the position $\boldsymbol{G^{t+1}}$ can be written as

$$\begin{aligned}
&\boldsymbol{B^{t+1}} = \widetilde{\boldsymbol{B}} = \boldsymbol{B^t} \cup \{\boldsymbol{i}\},\\
&\boldsymbol{W^{t+1}} = \widetilde{\boldsymbol{W}} \setminus \boldsymbol{D^t(i)} = \mathbf{W^t} \setminus \boldsymbol{D^t(i)},\\
&\boldsymbol{N^{t+1}} = \widetilde{\boldsymbol{N}} \cup \boldsymbol{D^t(i)} = \mathbf{N^t} \setminus \{\boldsymbol{i}\} \cup \boldsymbol{D^t(i)}.
\end{aligned}$$

And if there exists a black stone satisfying (22) then the move is illegal.

4.3 Recursion Theorem

We formularize the discriminant of the legal moves and the recursion of $\mathbf{F^t}$.

Theorem 6

Let $\{\boldsymbol{G^t}\} = \{(\boldsymbol{V}, \boldsymbol{E}, \boldsymbol{\alpha^t})\}$ be a game. The position $\boldsymbol{G^t} = (\boldsymbol{V}, \boldsymbol{E}, \boldsymbol{\alpha^t})$ is changed to $\{\boldsymbol{G^{t+1}}\} = \{(\boldsymbol{V}, \boldsymbol{E}, \boldsymbol{\alpha^{t+1}})\}$ with the black player's move $\boldsymbol{i}$ as follow

$$\begin{aligned}
&\boldsymbol{B^{t+1}} = \boldsymbol{B^t} \cup \{\boldsymbol{i}\},\\
&\boldsymbol{W^{t+1}} = \mathbf{W^t} \setminus \boldsymbol{D^t(i)},\\
&\boldsymbol{F^{t+1}(x)} = \begin{cases} \boldsymbol{F^t}\left(\boldsymbol{A^t(i)}\right) & \boldsymbol{x} \in \boldsymbol{A^t(i)} \\ \boldsymbol{F^0(x)} & \boldsymbol{x} \in \boldsymbol{D^t(i)} \\ \boldsymbol{F^t(x)} & otherwise \end{cases},
\end{aligned}$$

where

$$\begin{aligned}
&\boldsymbol{A^t(i)} = (\boldsymbol{F^t})^{-1}(\boldsymbol{i}) \cap (\boldsymbol{B^t} \cup \{\boldsymbol{i}\}),\\
&\boldsymbol{D^t(i)} = (\boldsymbol{F^t})^{-1}(\boldsymbol{i}) \cap \boldsymbol{W^t} \cap \boldsymbol{H^t},\\
&\boldsymbol{H^t} = \{\boldsymbol{z} \in \boldsymbol{V} \mid \#(\boldsymbol{F^t(z)} \cap \boldsymbol{N^t}) = \mathbf{1}\}.
\end{aligned}$$

At the beginning position $\boldsymbol{G^0} = (\boldsymbol{V}, \boldsymbol{E}, \boldsymbol{\alpha^0})$, $\boldsymbol{B^0}, \boldsymbol{W^0}$ and $\boldsymbol{F^0(x)}$ satisfy

$$\begin{aligned}
&\boldsymbol{B^0} = \emptyset,\\
&\boldsymbol{W^0} = \emptyset,\\
&\boldsymbol{F^0(x)} = \{\boldsymbol{z} \mid \boldsymbol{zx} \in \boldsymbol{E}\}.
\end{aligned}$$

And the move $\boldsymbol{i}$ must be chosen in the set of legal moves $\boldsymbol{L^t}$

$$\boldsymbol{i} \in \boldsymbol{L^t} = \boldsymbol{F^t}(\overline{\boldsymbol{W^t} \oplus \boldsymbol{H^t}}) \cap \boldsymbol{N^t}.$$

A position changes to a next position with a new added stone $\boldsymbol{i}$ and some captured stones $\boldsymbol{D^t(i)}$. In the first change, one stone is added and is formularized with the new block $\boldsymbol{A^t(i)}$. And in the second change, captured blocks are removed and is formulated with the removed blocks $\boldsymbol{D^t(i)}$.

$\widetilde{\boldsymbol{F}}$ of $\widetilde{\boldsymbol{G}}$ is given as follows.

Lemma 7

$$\widetilde{\boldsymbol{F}}(\boldsymbol{x}) = \begin{cases} \boldsymbol{F^t}(\boldsymbol{A^t(i)}) & \boldsymbol{x} \in \boldsymbol{A^t(i)} \\ \boldsymbol{F^t(x)} & otherwise \end{cases}$$

where $\boldsymbol{A^t(i)} = (\boldsymbol{F^t})^{-1}(\boldsymbol{i}) \cap (\boldsymbol{B^t} \cup \{\boldsymbol{i}\})$.

(28)

(29)

(30)

Proof of Lemma 7

Remark that $\boldsymbol{i} \notin \boldsymbol{B^t}$ and $\boldsymbol{i} \in \widetilde{\boldsymbol{B}}$. We consider $\widetilde{\boldsymbol{M}}$. By $\boldsymbol{i} \in \boldsymbol{N^t}$ and (21),

$$\forall \boldsymbol{x} \in \boldsymbol{F^t(i)} \Rightarrow \boldsymbol{ix} \in \boldsymbol{E}.$$

And $\boldsymbol{i} \in \widetilde{\boldsymbol{B}}$ and $\boldsymbol{x} \in \widetilde{\boldsymbol{B}} \cap \boldsymbol{F^t(i)}$ leads $\boldsymbol{i}\widetilde{\boldsymbol{M}}\boldsymbol{x}$. The step 1 cuts no paths, because $\boldsymbol{M^t}$ does not use any null intersection as a relay point,

$$\forall \boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{V}, \boldsymbol{yM^tx} \Rightarrow \boldsymbol{y}\widetilde{\boldsymbol{M}}\boldsymbol{x}.$$

So

$$\begin{aligned}
&\forall \boldsymbol{y} \in \boldsymbol{V}, \exists \boldsymbol{x} \in \boldsymbol{F^t(i)} \cap \boldsymbol{B^t}\ s.t.\ \boldsymbol{yM^tx}\\
&\Rightarrow \boldsymbol{y}\widetilde{\boldsymbol{M}}\boldsymbol{x}, \boldsymbol{i}\widetilde{\boldsymbol{M}}\boldsymbol{x}\\
&\Rightarrow \boldsymbol{y}\widetilde{\boldsymbol{M}}\boldsymbol{i}.
\end{aligned}$$

(31)

(32)

On the other hand, for any $\boldsymbol{x}$ and $\boldsymbol{y}$,

(33)

$$\boldsymbol{y} \in \boldsymbol{V}, \boldsymbol{x} \in \boldsymbol{B^t}, \boldsymbol{xM^ty} \Leftrightarrow \boldsymbol{y} \in \boldsymbol{B^t}, \boldsymbol{x} \in \boldsymbol{V}, \boldsymbol{xM^ty},$$

and

$$\forall \boldsymbol{y} \in \boldsymbol{B^t}, \boldsymbol{i} \in \boldsymbol{F^t(y)} \Leftrightarrow \boldsymbol{y} \in (\boldsymbol{F^t})^{-1}(\boldsymbol{i}) \cap \boldsymbol{B^t}.$$

(34)

(35)

(36)

So

$$\boldsymbol{y} \in (\boldsymbol{F^t})^{-1}(\boldsymbol{i}) \cap \boldsymbol{B^t} \Rightarrow \boldsymbol{y}\widetilde{\boldsymbol{M}}\boldsymbol{i}.$$

Clearly if $\boldsymbol{i} \notin \boldsymbol{F^t(y)}$ then there is no path from $\boldsymbol{y}$ to any neighbor of $\boldsymbol{i}$. As the result,

(37)

(38)

$$\forall \boldsymbol{y} \in \boldsymbol{V} \setminus \{\boldsymbol{i}\}, \boldsymbol{y} \in (\boldsymbol{F^t})^{-1}(\boldsymbol{i}) \cap \boldsymbol{B^t} \Leftrightarrow \boldsymbol{y}\widetilde{\boldsymbol{M}}\boldsymbol{i}.$$

(39)

Let $\boldsymbol{A^t(i)}$ be the new block, then

$$\boldsymbol{A^t(i)} = (\boldsymbol{F^t})^{-1}(\boldsymbol{i}) \cap (\boldsymbol{B^t} \cup \{\mathrm{i}\}).$$

(40)

And $\widetilde{\boldsymbol{F}}$ is

$$\begin{aligned}
\forall \boldsymbol{x} \in \boldsymbol{A^t(i)}, \widetilde{\boldsymbol{F}}(\boldsymbol{x}) &= \{\boldsymbol{z} \mid \exists \boldsymbol{y} \in \boldsymbol{A^t(i)}, \boldsymbol{yz} \in \boldsymbol{E}\}\\
&= \boldsymbol{F^t}(\boldsymbol{A^t(i)}).
\end{aligned}$$

It is easy to show that

Lemma 8

$$\forall \boldsymbol{x}, \boldsymbol{F}^{\boldsymbol{t}}(\boldsymbol{x}) \subset \widetilde{\boldsymbol{F}}(\boldsymbol{x}).$$

It means step 1 gather up the move $\boldsymbol{i}$ and neighbor stones to a new block, and increase connections.

The captured stones $\boldsymbol{D}^{\boldsymbol{t}}(\boldsymbol{i})$ is given by the next lemma.

Lemma 9

$$\boldsymbol{D}^{\boldsymbol{t}}(\boldsymbol{i}) = \boldsymbol{H}^{\boldsymbol{t}} \cap \boldsymbol{W}^{\boldsymbol{t}} \cap (\boldsymbol{F}^{\boldsymbol{t}})^{-1}(\boldsymbol{i})$$

where

$$\boldsymbol{H}^{\boldsymbol{t}} = \{\boldsymbol{z} \in \boldsymbol{V} \mid \#(\boldsymbol{F}^{\boldsymbol{t}}(\boldsymbol{z}) \cap \boldsymbol{N}^{\boldsymbol{t}}) = \boldsymbol{1}\}.$$

Proof of Lemma 9

Any removed white stones must satisfy the captured condition (22) in $\widetilde{\boldsymbol{G}}$. Let $\boldsymbol{w} \in \boldsymbol{W}^{\boldsymbol{t}}$ satisfy

$$\widetilde{\boldsymbol{F}}(\boldsymbol{w}) \cap \widetilde{\boldsymbol{N}} = \emptyset.$$

Before the move $\boldsymbol{i}$, $\boldsymbol{w}$ does not satisfy (22) with $\boldsymbol{F}^{\boldsymbol{t}}$ and $\boldsymbol{N}^{\boldsymbol{t}}$. So there exists an intersection $\boldsymbol{u} \in \boldsymbol{F}^{\boldsymbol{t}}(\boldsymbol{w}) \cap \boldsymbol{N}^{\boldsymbol{t}}$.

By lemma 8,

$$\boldsymbol{u} \in \boldsymbol{F}^{\boldsymbol{t}}(\boldsymbol{w}) \cap \boldsymbol{N}^{\boldsymbol{t}} \subset \widetilde{\boldsymbol{F}}(\boldsymbol{w}) \cap \boldsymbol{N}^{\boldsymbol{t}}.$$

On other hand, $\widetilde{\boldsymbol{F}}(\boldsymbol{w}) \cap (\boldsymbol{N}^{\boldsymbol{t}} \setminus \{\boldsymbol{i}\}) = \emptyset$ by (27) and (54).

$$\begin{aligned}\boldsymbol{u} &\in (\widetilde{\boldsymbol{F}}(\boldsymbol{w}) \cap \boldsymbol{N}^{\boldsymbol{t}}) \setminus (\widetilde{\boldsymbol{F}}(\boldsymbol{w}) \cap (\boldsymbol{N}^{\boldsymbol{t}} \setminus \{\boldsymbol{i}\})) \\ &= \{\boldsymbol{i}\}\end{aligned}$$

So $\boldsymbol{u} = \boldsymbol{i}$ and $\boldsymbol{F}^{\boldsymbol{t}}(\boldsymbol{w}) \cap \boldsymbol{N}^{\boldsymbol{t}} = \{\boldsymbol{i}\}$.

$$\begin{aligned}&\boldsymbol{F}^{\boldsymbol{t}}(\boldsymbol{w}) \cap \boldsymbol{N}^{\boldsymbol{t}} = \{\boldsymbol{i}\} \\ &\Leftrightarrow \#(\boldsymbol{F}^{\boldsymbol{t}}(\boldsymbol{w}) \cap \boldsymbol{N}^{\boldsymbol{t}}) = \boldsymbol{1} \ \wedge \ \boldsymbol{i} \in \boldsymbol{F}^{\boldsymbol{t}}(\boldsymbol{w}).\end{aligned}$$

Hence (52) is proved. ■

We define the set **one-liberty set $\boldsymbol{H}$** by (53). In the game of Go, the word ***atari*** means the precondition of being captured. $\boldsymbol{H}^{\boldsymbol{t}}$ includes *atari* stones and also the "isolate" null intersections at $\boldsymbol{G}^{\boldsymbol{t}}$.

Each stone in $\boldsymbol{H}^{\boldsymbol{t}}$ has only one liberty. So they will be captured when an opponent stone is placed on its last (not ex) liberty. On the other hand, the null intersection in $\boldsymbol{H}^{\boldsymbol{t}}$ has no other null intersection as its neighborhood. Because any intersection has a self-edge, any intersection in $\boldsymbol{H}^{\boldsymbol{t}} \cap \boldsymbol{N}^{\boldsymbol{t}}$ has itself as the only one liberty.

The white intersections in $\boldsymbol{D}^{\boldsymbol{t}}(\boldsymbol{i})$ becomes null. So for any $\boldsymbol{w} \in \boldsymbol{D}^{\boldsymbol{t}}(\boldsymbol{i})$,

$$\forall \boldsymbol{x} \in \boldsymbol{V}, \boldsymbol{x}\boldsymbol{M}^{\boldsymbol{t+1}}\boldsymbol{w} \Rightarrow \boldsymbol{x} = \boldsymbol{y}.$$

(51)

In other word,

$$\forall \boldsymbol{w} \in \boldsymbol{D}^{\boldsymbol{t}}(\boldsymbol{i}), \boldsymbol{F}^{\boldsymbol{t+1}}(\boldsymbol{w}) = \boldsymbol{F}^{\boldsymbol{0}}(\boldsymbol{w}).$$

As the result,

$$\boldsymbol{F}^{\boldsymbol{t+1}}(\boldsymbol{x}) = \begin{cases}\boldsymbol{F}^{\boldsymbol{0}}(\boldsymbol{x}) & \boldsymbol{x} \in \boldsymbol{D}^{\boldsymbol{t}}(\boldsymbol{i}) \\ \widetilde{\boldsymbol{F}}(\boldsymbol{x}) & otherwise\end{cases}.$$

(52)

Remark that $\boldsymbol{A}^{\boldsymbol{t}}(\boldsymbol{x}) \subset \boldsymbol{B}^{\boldsymbol{t}} \cup \boldsymbol{N}^{\boldsymbol{t}}$ and $\boldsymbol{D}^{\boldsymbol{t}}(\boldsymbol{x}) \subset \boldsymbol{W}^{\boldsymbol{t}}$. So (33) holds.

4.4 Legal Move

(53)

The illegal move has only one liberty for the same reason of captured stones. Similarly, the set of legal moves $\boldsymbol{L}^{\boldsymbol{t}}$ can be formulated with $\boldsymbol{H}^{\boldsymbol{t}}$.

There are two reasons of this illegality. The first reason is that the move will have no liberty after the move. Second is that the move will capture no stones. The illegal move satisfies both. It must satisfy the followings, (54)

1. is not adjacent to $\boldsymbol{B}^{\boldsymbol{t}} \setminus \boldsymbol{H}^{\boldsymbol{t}}$.
2. is not adjacent to $\boldsymbol{N}^{\boldsymbol{t}} \setminus \boldsymbol{H}^{\boldsymbol{t}}$.
3. is not adjacent to $\boldsymbol{W}^{\boldsymbol{t}} \cap \boldsymbol{H}^{\boldsymbol{t}}$.

(55)

1 and 2 means the new block $\boldsymbol{A}^{\boldsymbol{t}}$ has no liberty, and 3 means no white stone is captured ($\boldsymbol{D}^{\boldsymbol{t}} = \emptyset$). So the set of illegal moves $\boldsymbol{IL}$ is

$$\begin{aligned}\boldsymbol{IL} &= \overline{\boldsymbol{F}^{\boldsymbol{t}}((\boldsymbol{B}^{\boldsymbol{t}} \cup \boldsymbol{N}^{\boldsymbol{t}}) \setminus \boldsymbol{H}^{\boldsymbol{t}}) \cup \boldsymbol{F}^{\boldsymbol{t}}(\boldsymbol{W}^{\boldsymbol{t}} \cap \boldsymbol{H}^{\boldsymbol{t}})} \\ &= \overline{\boldsymbol{F}^{\boldsymbol{t}}(\overline{\boldsymbol{W}^{\boldsymbol{t}}} \setminus \boldsymbol{H}^{\boldsymbol{t}}) \cup \boldsymbol{F}^{\boldsymbol{t}}(\boldsymbol{W}^{\boldsymbol{t}} \cap \boldsymbol{H}^{\boldsymbol{t}})} \\ &= \overline{\boldsymbol{F}^{\boldsymbol{t}}(\overline{\boldsymbol{W}^{\boldsymbol{t}}} \cap \overline{\boldsymbol{H}^{\boldsymbol{t}}}) \cup \boldsymbol{F}^{\boldsymbol{t}}(\boldsymbol{W}^{\boldsymbol{t}} \cap \boldsymbol{H}^{\boldsymbol{t}})} \\ &= \overline{\boldsymbol{F}^{\boldsymbol{t}}(\overline{\boldsymbol{W}^{\boldsymbol{t}} \oplus \boldsymbol{H}^{\boldsymbol{t}}})}\end{aligned}$$

(56)

(57)

where $\mathbf{A} \oplus \mathbf{B} = (\boldsymbol{A} \setminus \mathbf{B}) \cup (\boldsymbol{B} \setminus \boldsymbol{A})$. The legal moves are in $\boldsymbol{N}^{\boldsymbol{t}}$ and not in $\boldsymbol{IL}$, so $\boldsymbol{L}^{\boldsymbol{t}}$ is

$$\boldsymbol{L}^{\boldsymbol{t}} = \boldsymbol{F}^{\boldsymbol{t}}(\overline{\boldsymbol{W}^{\boldsymbol{t}} \oplus \boldsymbol{H}^{\boldsymbol{t}}}) \cap \boldsymbol{N}^{\boldsymbol{t}}.$$

As a result, any position of $\{\boldsymbol{G}^{\boldsymbol{t}}\}$ is given by its recursion and initial values. So we define a position $\boldsymbol{G}^{\boldsymbol{t}}$ of a sequence $\{\boldsymbol{G}^{\boldsymbol{t}}\}$ as a triplet $(\boldsymbol{B}^{\boldsymbol{t}}, \boldsymbol{W}^{\boldsymbol{t}}, \boldsymbol{F}^{\boldsymbol{t}})$ as follows.

Definition 10

Let $\{\boldsymbol{G}^{\boldsymbol{t}}\}$ be a game, a position $\boldsymbol{G}^{\boldsymbol{t}}$ is defined as

$$\boldsymbol{G}^{\boldsymbol{t}} = (\boldsymbol{B}^{\boldsymbol{t}}, \boldsymbol{W}^{\boldsymbol{t}}, \boldsymbol{F}^{\boldsymbol{t}}; \boldsymbol{V}, \boldsymbol{E}),$$

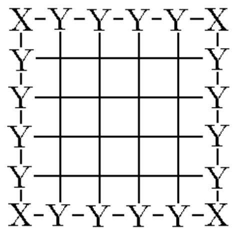


figure 8:The **sides** $\boldsymbol{Y}$ and the **corners** $\boldsymbol{X}$.

with its initial position $(\emptyset, \emptyset,\ \boldsymbol{F^0})$.

We call the triplet $(\boldsymbol{B^t}, \boldsymbol{W^t}, \boldsymbol{F^t})$ the **BW graph**, and the sequence of BW graphs with its recursion the **BW graph model**.

5. Applications

5.1 Coordinates

A **coordinate** is useful only when the board is rectangle. Our model represents the **corner** and the **side** by degrees (see figure 8) which can be used for non-rectanglar parts.

Corners have less neighbors than inners. Based on the definition $\boldsymbol{H^t}$, the set of corners $\boldsymbol{Co}$ and the set of sides $\boldsymbol{Si}$ are defined as

Definition 11

$$\boldsymbol{Co} = \{\boldsymbol{z} \in \boldsymbol{V} \mid \#\left(\boldsymbol{F^0}(\boldsymbol{z})\right) = \boldsymbol{3}\}, \tag{67}$$

$$\boldsymbol{Si} = \{\boldsymbol{z} \in \boldsymbol{V} \mid \#\left(\boldsymbol{F^0}(\boldsymbol{z})\right) = \boldsymbol{4}\}. \tag{68}$$

Furthermore, we generalize this definition as follows

Definition 12

$$\boldsymbol{H^t}_{\boldsymbol{n}} = \{\boldsymbol{z} \in \boldsymbol{V} \mid \#(\boldsymbol{F^t}(\boldsymbol{z}) \cap \boldsymbol{N^t}) = \boldsymbol{n}\}. \tag{69}$$

Then $\boldsymbol{H^0}_{\boldsymbol{3}} = \boldsymbol{Co}$, $\boldsymbol{H^0}_{\boldsymbol{4}} = \mathbf{Si}$, and coordinates $\{(\boldsymbol{i}, \boldsymbol{j})\}$ are represented by $\boldsymbol{H^t}_{\boldsymbol{n}}$ as

$$\{(\boldsymbol{i},\boldsymbol{j})\} = ((\boldsymbol{F^0})^{\boldsymbol{i+j}}(\boldsymbol{H^0}_{\boldsymbol{3}}) \setminus\ (\boldsymbol{F^0})^{\boldsymbol{i+j-1}}(\boldsymbol{H^0}_{\boldsymbol{3}})) \\ \cap\ ((\boldsymbol{F^0})^{\boldsymbol{min(i,j)}}(\boldsymbol{H^0}_{\boldsymbol{4}}) \setminus\ (\boldsymbol{F^0})^{\boldsymbol{min(i,j)-1}}(\boldsymbol{H^0}_{\boldsymbol{4}})), \tag{70}$$

where

$$(\boldsymbol{F^t})^k(\cdot) = \boldsymbol{F^t}\left((\boldsymbol{F^t})^{k-1}(\cdot)\right), \tag{71}$$

$$(\boldsymbol{F^t})^1(\cdot) = \boldsymbol{F^t}(\cdot), \tag{72}$$

$$(\boldsymbol{F^t})^0(\cdot) = \emptyset. \tag{73}$$

In (70), $\{(\boldsymbol{i},\boldsymbol{j})\}$ is a set of intersections whose distance from corners and from sides are $\boldsymbol{i+j}$ and $\boldsymbol{min}\ (\boldsymbol{i},\boldsymbol{j})$, respectively. Note that definitions of corners, sides and coordinates do not depend on shapes of parts. In other words, the meaning of the **corner** is extended by this definition.

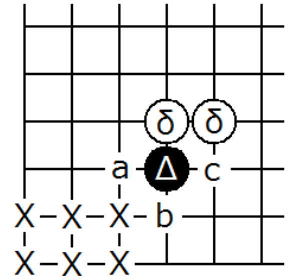
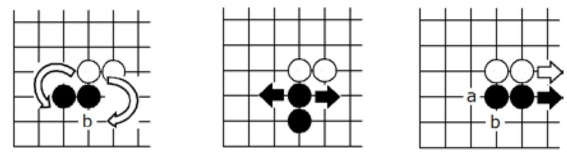


figure 9:Examples of directions



(a) (b) (c)

figure 10:differences between each moves.

In 5.2 and 5.3, we introduce some examples of features that use $\boldsymbol{H^t}_{\boldsymbol{n}}$.

5.2 Directions

In the game of Go, *direction* is named with its intention, like *nobi*, *sagari*, *hane*, etc. We classify moves with $\boldsymbol{H^t}_{\boldsymbol{n}}$ and distances from other stones.

See figure 9. It is a left down part of a position. We assume that a black stone Δ and white stones δ fight for the region. Black player wants left down field **X**, and white player prevent black stones from invading out. Intersections $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are examples of next black moves. They are neighbors of a black stone samely. But moves to each intersection have different intentions, so we must distinguish them.

Move $\boldsymbol{a}$ is one of defensive moves, it will be the *roof* (a space between a black stone and left side intersections) of the territory. Move $\boldsymbol{b}$ make a *wall* which devides the territory and the right space. Move $\boldsymbol{c}$ is an offensive move to expand own territory to right space.

We propose following subsets as examples.

$$\boldsymbol{a} \in \boldsymbol{F}(\boldsymbol{B}) \setminus \left(\boldsymbol{F}(\boldsymbol{W}) \cup \boldsymbol{F}(\boldsymbol{H^0}_{\boldsymbol{4}})\right),$$
$$\boldsymbol{b} \in \boldsymbol{F}(\boldsymbol{B}) \cap \boldsymbol{F}(\boldsymbol{H^0}_{\boldsymbol{4}}),$$
$$\boldsymbol{c} \in \boldsymbol{F}(\boldsymbol{B}) \cap \boldsymbol{F}(\boldsymbol{W}).$$

The move $\boldsymbol{c}$ compete with white stones for right space. If Both players place stones straightly then black player will get the down side space and white player make a wall between the down side and the center space. $\boldsymbol{F}(\boldsymbol{B}) \cap \boldsymbol{F}(\boldsymbol{W})$

means intersections of it are on the *border* between black and white powers.

If black player places a black stone on $\boldsymbol{b}$ then the black block is expanded to a down side and only one move $\boldsymbol{d}$ in figure 10(b) will devide left territory and right space.

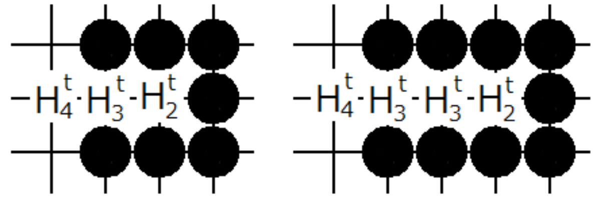


figure 11:Examples of local patterns, corridors.

According to circumstances, black player encloses either the left terrritory or the right space. We represent it as the nearest neighborhood from sides. Only the intersetion $\boldsymbol{b}$ is adjacent to an intersection of sides $\boldsymbol{H^0}_4$.

The roof space of the teritory is wider than right side space. If black player wants the left territory certainly, he/she must close this roof space hastly. In fact, that move make a distance between the territory and white stones. $\setminus \left(\boldsymbol{F}(\boldsymbol{W}) \cup \boldsymbol{F}(\boldsymbol{H_4})\right)$ means *otherwise* in this representation.

These are not general representations. Because the meanings of $\boldsymbol{F}(\boldsymbol{H^0}_4)$ and $\boldsymbol{F}(\boldsymbol{W})$ are depend on arragenment of white stones and distance from sides. These are only a proof that we can distinguish them. In this paper, we propose the way to represent directions in our model.

5.3 A mathematical formulation of corridors

Berlekamp and Wolfe [2] showed many patterns of corridors and described its features. They defined two type of corridors, blocked and unblocked. But they did not represent mathematically. In this paper, we provide a mathematical formulation of blocked corridors.

Blocked corridors have two parts, a dead-end and some streets. A dead-end has only one neighbor which is the entrance and the exit, so all dead-end intersections are in $\boldsymbol{H^t}_2$. Streets are in $\boldsymbol{H^t}_3$ because they have just two neighbors. Intersections which have more than three neighbors are junctions.

We can find corridors by using following steps.

1. Find dead-ends as $\boldsymbol{H^t}_2$.
2. Find first street as $\boldsymbol{H^t}_3 \cap \boldsymbol{F}(\boldsymbol{H^t}_2)$. Denote it $\boldsymbol{S_1} = \boldsymbol{H^t}_3 \cap \boldsymbol{F}(\boldsymbol{H^t}_3)$.
3. Find (n+1)-th street as $\boldsymbol{S_{n+1}} = \boldsymbol{H^t}_3 \cap \boldsymbol{F}(\boldsymbol{S_n}) \setminus \boldsymbol{S_n}$ by using n-th street $\boldsymbol{S_n}$.

$\boldsymbol{S_n}$ is the set of n-th streets of any corridors. This is a meta-rule of corridors, and clasifies them. $\boldsymbol{S_n}$ can be described

$$\boldsymbol{S_n} = (\boldsymbol{H_3} \cap \boldsymbol{F})^{\boldsymbol{n}}(\boldsymbol{H_2}) \setminus \boldsymbol{S_{n-1}}, \tag{77}$$
$$\boldsymbol{S_0} = \emptyset \tag{78}$$

where $(\boldsymbol{H_3} \cap \boldsymbol{F})^{\boldsymbol{n}}$ is defined as same as $(\boldsymbol{F^t})^{k}$.

Patterns which have length are represented with repeated application of some mappings.

6. Conclusion and future works

We introduced a mathematical model for the game of Go called the “BW graph model” and constructed the recursion of position sequences. Furthermore we provided mathematical formulations for some position in the game of Go such as coordinates, directions and corridors.

One of advantages for our model is that we can locally evaluate various positions in the game of Go. In addition, our model is able to be used for non rectangular parts in a game board.

Our future works are to give mathematical formulations of various patterns and complex strategies by our model. And we would like to develop algorithms to classify strategies by professional players.

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References

[1] David B. Benson, “Life in the game of go”, *Infomation Services*, Vol. 10, pp 17-29, 1976.
[2] Elwyn Berlekamp and David Wolfe. “Mathematical Go: Chilling Gets the Last Point”, *A K Peters Ltd*, Natick, MA, 1994.
[3] Thore Graepel and Mike Goutrie and Marco Kruger and Ralf Herbrich, “Learning on Graphs in the Game of Go”, *in Proc. of the Ninth International Conference on Artificial Neural Networks*, pp. 347-352,2001.

[4] Masafumi Satou and Koichi Anada and Masayoshi Tsutsumi, “A Mathematical Model for the game of Go and its Application”(in Japanese). *In The 16th Game Programming Workship 2011*, pp. 100-103, 2011.

[5] Edward Thorp and William E. Walden, “A computer assisted study of Go on M × N boards”, *American Elsevier Publishing Company, Inc.*, 1972.

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The Desirable Requirements of Cross Platform Mobile Development Tools

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Abstract

The ubiquity of mobile devices is the result of increasing popularity of mobile applications that are becoming more diverse and intuitive. The mobile device market is composed of several mobile platforms such as Android and iOS, so it is important to be able to deliver applications available for more than one platform in less time. But it is becoming increasingly difficult with the incompatibility between platforms and their SDK. To solve that, cross platform mobile development solutions have been investigated. Although each of such tools allows developing cross-platform mobile applications, the result can sometimes be unsatisfactory in comparison to a native application. In this paper, we present existing cross platform approaches. We discuss the general architecture of cross platform mobile applications. Then, we propose the desirable requirements in cross platform frameworks. Some existing solutions will be evaluated based on these requirements. We conclude the paper by some future perspectives.

Keywords: Cross-platform tools, mobile application, cross-platform development, requirements, standardization.

1. Introduction

Our lives have become more mobile and the near ubiquity of mobile devices (e.g. smart phones and tablets) and the internet have contributed to this effect. Due to the features they offer (GPS, accelerometer, Music, Camera, etc.), mobile devices have becoming more a necessary than need. These kinds of functionalities are provided by all the major mobile Operating Systems such as Android, iOS and Windows Phone. In addition, the many and varied applications available make smart devices the engine for innovation. The increasing demand for mobile applications requires to developers and companies several points to consider. One influencing point is the fragmentation of the landscape of mobile platforms. Android and iOS are the leaders with a large part of the market, while other platforms as Blackberry and Windows Phone are less present [12]. Thus, it is of prime importance for applications vendors to provide their solutions as many platforms as possible in order to attract more users. Implementing applications for multiple mobile platforms requires considerable efforts in term of, development time, resource, tools, maintenance and deployment. Indeed, programming language and development environment of each OSs differ from one to another [5]. Then, application developed for specific OS cannot be deployed on all other OS, which forces developers to rebuild the same application for other OS using, for each one, its own SDK.

To address platform and device coverage problems, cross-platform mobile development tools allow producing mobile applications which can be deployed on multiple platforms. They reduce time and cost development by allowing developers to write applications once, using adaptable cross platform framework. These tools are based on different cross platform approaches that have emerged in recent years. They differ in term of implementation technology on which they are based. When some of such tools use web technologies as HTML, CSS and JavaScript (e.g. Sencha Touch), others use some abstraction layers as JavaScript API to access device specific features (e.g. PhoneGap). More and more cross-platform solutions appearing on the market and it is no wonder they are gaining popularity knowing the customer expectations are becoming increasingly challenging. Cross platform solutions can be recommended in general, but they differ in term of performance, usability and development environment provided.

After introduction, we present in this paper the five approaches of cross platform mobile development in section 2. Then, section 3 gives the general architecture of a cross-platform mobile application. After what, we describe some related works in section 4. Section 5 provides the definition of the desirable requirements of cross platform mobile development tools. Based on these requirements, we analyze five selected frameworks (one per approach) in section 6.

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The paper concludes in section 7 with some future perspectives.

2. Existing approaches of cross platform mobile development

There are two ways to develop mobile applications for mobile devices: the native and the cross-platform approach [5]. The native approach which permits to create native applications consists of developing the same application as many as there's platform using, for each platform, its own Software Development Kit (SDK) and frameworks. For example, applications for Android are programmed in Java and XML, the platform functionality is accessed using the framework provided by Android and the user interface is rendered using platform provided elements. In distinction, applications for iOS are developed using the language programming Objective-C and Apple's frameworks.

In contrast to native approach, we present in this section an overview of the cross-platform approaches. Cross platform approach proposes to create single application which can be used across multiple platforms. They are designed to save time and costs by allowing developers to write an application once in a language they know and using a framework which is adaptable for multiple platforms. For that purpose, different ways can be borrowed [14][11]:

The web approach consists to produce web applications, using web technologies as HTML, CSS and JavaScript, designed to be executed in the web browser of mobile devices,. Applications are accessed using an URL and the mobile device will not have any application specific components installed. Hence, web applications cannot access the mobile device features as calendar or GPS sensors and can suffer for a lake of performance due to connection and network delays. Also, web applications using this approach usually do not look and feel like a native application. Several mature tools are dedicated to mobile web applications development, we can find: jQuery Mobile[d], jQTouch[e], Dojo Mobile[f], AppsBuilder[g], iBuildApp[h] and Sencha Touch[i] which is the tool that we have selected for our study.

To address the lack of native features but to still allow using common web technologies, the hybrid approach has emerged as a combination of web technologies and native features. Hybrid approach uses the browser engine of the mobile device which renders and displays the HTML content in native container on the device that is a full screen Web view control. Yet the application may lack native look and feel. Hybrid applications are installed on the device and its capabilities are accessed through an abstraction layer such as JavaScript APIs. However, the execution through the browser engine can affect application performances. The most popular exponent of this approach is PhoneGap[j] that we have chosen for our study. Among the existing tools based on this approach we can also cite frameworks as MoSync[k], mgwt[l] or vaadin TouchKit[m].

The Interpreted approach follows a different way. Application code is deployed in the mobile device and at runtime the interpreter interprets the source code across different platforms. The main advantage of this approach is that the user interface is made using platform-specific native elements. The device hardware and platform features are wrapped with specific framework API, providing access to the interpreted application. The main disadvantages of this approach are the runtime interpretation that could degrade the performance of the application, and development that generally depends on the set of features provided by the selected framework. Appcelerator Titanium[n] is among tools based on this approach. We will describe it in the appropriate section. Other tools that help to create interpreted applications are Adobe Flash Builder[o] and Rhodes[p].

In the case of cross-compiled approach, the cross-compiler compiles the source code into particular native code generating the native binaries. A platform-specific version of the application is created for each target platform and runs with native performances. Device features are accessible and all the native user interface components can be used. The main inconvenient of this approach is the complexity to identifying and rectifying the cross-compilation phase issues. The cross platform solutions currently available in the market are not mature enough. A very prominent example for a cross-compiler tool is Xamarin[q] which is the framework that we chose for that approach. We can also add QTMobile[r] as cross-compiler tool. The tools based on this approach are dependent on the efficiency and the reliability of their cross compiler.

The last but not least, model driven approach bases the development on models to describe the applications. The models are expressed using DSLs (Domain Specific Languages) and UML (Unified Modeling Language).

[d] http://jquerymobile.com/
[e] http://jqtjs.com/
[f] http://dojotoolkit.org/
[g] http://www.apps-builder.com/fr/accueil
[h] http://ibuildapp.com/
[i] http://www.sencha.com/products/touch/

[j] http://phonegap.com/
[k] http://www.mosync.com/
[l] http://www.m-gwt.com/
[m] https://vaadin.com/add-ons/touchkit
[n] http://www.appcelerator.com/titanium/
[o] http://www.adobe.com/fr/products/flash builder.html
[p] http://www.motorolasolutions.com/US-EN/RhoMobile+Suite/Rhodes
[q] http://xamarin.com/
[r] http://www.qt.io/mobile-app-development/

Automatic transformations generate source code for the supported target platforms from the defined models. The application generated is truly native and respect the native look & feel. In addition, it runs on native environment without intermediate layout.

Even if most of model driven tools are still in early stage and they are not yet very popular, we have still selected the AutoMobile Project[s] as representative of model driven cross platform development solutions for our study. Most existing model-driven solutions like canappi mdsl[t] or applause[u] have not been much development progress lately, or not relevant in general practice like mobl[v], or also not active anymore like iPhonical[w]. However, model-driven approaches appears as the most promising approach for cross-platform mobile development and gives rise to various research projects such as Xmob [18], MD² [15] or AXIOM [16].

3. General architecture of cross-platform mobile application

In this section, we provide definitions of terms and background about profiling. We discuss both edge and path profiling including a greedy algorithm for computing edge profile information. We conclude this section with a discussion of phases and phased behavior, including the impact of phased behavior on superblock scheduling.

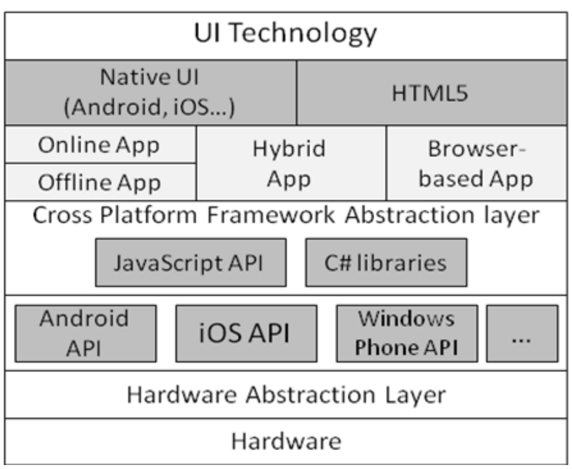

Figure 1. General architecture of cross-platform mobile application.

Mobile application is kind of client-server applications where the user interface is hosted on a mobile device. In term of GUI components, we distinguish between applications that use native elements (specific for each platform), applications based on HTML5 technology who intended to run in the browser, and hybrid solutions thereof. Native applications can run online and offline too, in contrast to web HTML5 applications.

The cross platform framework allows access to device features as Camera, Contact, Sensors, etc., using JavaScript API or C# libraries (depends on the framework used) which interact with the native API of a mobile platform.

Then, the application is built separately, for each target platform, to generate the corresponding executables.

4. Related work

Researches addressing the area of mobile development have evolved to head towards cross platform mobile development. Until recently, papers only discussed mobile platforms as we can see with the works presented in [6] and [20]. For existing work dealing with cross-platform mobile development, we can refer to [2] that have compared mobile platforms with regard to the openness of their architectures. The authors in [5] compare the development of native applications and Web applications. Even if these works address more than one platform, they introduce the cross platform perspective only marginally.

While most articles dealing with cross-platform mobile development provide comparison criteria of cross-platform mobile development approaches or evaluation criteria for implementation technology of these approaches [23][1]. In this paper, we focus on how to define the desirable requirements for cross platform mobile development tools. Indeed, with the emergence of more than 100 tools for this field, we think it is time to ask the question about the requirements that such tools are required to provide.

We can find some related works that can be identified whenever we look for the desirable requirements of the cross platform tools. Even if the authors in [7] discuss the desirable requirements in a cross platform framework, their aim is different. In fact, they have developed Android application with four cross platform tools (PhoneGap only, PhoneGap + JQuery Mobile, PhoneGap + Sencha Touch 2.0, Titanium) to evaluate the performance of such tools. The work presented in [23] is an evaluation of three frameworks (Titanium, Rhodes, PhoneGap + Sencha Touch) versus two native SDKs (Android SDK, iOS SDK) in criteria pertaining to functionality, usability features, performance and other categories. In [14], the authors elaborate on a list of 14 criteria for evaluating cross-platform development approaches. These criteria have been structured into infrastructure and development perspective. "*The infrastructure perspective sums up criteria relating to the life-cycle of an app, its usage, operation and functionality/functional range. The development perspective covers all criteria that are directly related to the*

[s] http://automobile.webratio.com/
[t] http://www.canappi.com/pages/mdsl
[u] https://github.com/applause/
[v] http://www.mobl-lang.org/
[w] https://code.google.com/p/iphonical/

development process of the app, e.g. topics like testing, debugging and development tools”. These criteria can be seen as requirements of cross platform tools but this is not explicitly mentioned because it is not the purpose of this article. In addition, all approaches are not addressed in the evaluation (Just PhoneGap and Titanium are evaluated). Despite the topic seems to be similar to our work, our aim is different. In our work, we classify cross platform development approaches into five distinct categories. We include, in our analysis, one tool of each approach for an overall assessment. The evaluation of such tools will be based on the requirements that we define in the next section.

As we can see, previous research in this area is sparse and often focuses on comparing a set of existing frameworks. In this paper, we would like to provide the desirable requirements of cross platform tools. The aim is to identify the required elements in the process of developing a cross platform mobile application, and thus, such required elements that should provide a tool to claim the title of cross platform tool. Thus, our work can be considered as a complementary work.

5. The desirable requirements of cross platform tools

The main “objective” of cross platform mobile development approaches is to provide a mobile application that can execute on multiple platforms. However, we must not lose sight that the main “motivation” is to provide a cross-platform application which is closest to the concept of native application.

The question of supporting multiple platforms is not new. The same problem occurred, 20 years ago, for PC platforms (Windows, Unix, Mac OS, etc.). With the omnipresence of mobile devices in our daily life and fragmentation of platforms, it makes sense that developers are turning to multi-platform development and thus that framework vendors provide appropriate solutions [1].

Based on literature and our own analysis, we have identified the desirable requirements of any cross-platform technology presented as follow:

5.1 Mobile platform supported

Cross platform approaches by definition must support several platforms. This requirement takes into account the number and importance of supported mobile platforms, e.g. iOS, Android and Windows Phone are practically mandatory since they are the largest shared mobile platforms. In addition, the equality of the support between the platforms should also be considered.

5.2 Development environment

Sherwood Development environment covers various parameters. In this requirement, we consider the features of the development environment offered with the framework such as an Integrated Development Environment (IDE,) debugger, compiler, emulator, etc. and functionalities as source code editor, intelligent auto-completion, which usually accompanies the IDE. In addition to source code editor, the opportunity to create the graphical user interface through a WYSIWYG editor and to test the application without having to deploy it to a device or an emulator constantly is greatly appreciated. Also, the maturity of the development environment reflects the maturity of the framework.

5.3 API and documentation

In this requirement, we discuss the documentation quality of the framework and APIs available. The influence of the documentation on the quality and ease of learning is reflected in the progress of a developer during his training of a framework. The use of APIs determines the feature of the application developed. APIs are specific to devices/platform and their availability varies from one framework to another.

5.4 Security

Smartphone are considered as easy to trap objects whose flaws are already known and exploited (tracks conversations, data recovery, scams). Applications developed with cross platform tools are not highly secure [7]. When it is considered that each mobile OS and each mobile device has its own flaws, it is difficult to apply a security policy to an application designed to run on multiple platforms/mobile device. The ideal would be to introduce the concept of security in the heart of the development process of the application using for example the security by design concept [10][4][20]. Proper research needs to be carried out to secure the tools and applications.

5.5 Access to device specific features

The kind of application determines its capacity to access the features of the mobile device. There is a difference between the features according to native application and web site application. The functionality requirements of an application can be identified as follow: 1) informational requirements, where the user primarily consumes content 2) transactional requirements, where the user primarily interacts with the application to accomplish a task and 3) Device-specific requirements, e.g. offline transaction entry or file access. Most frameworks support standard device features, e.g. camera, GPS, Accelerometer, etc. and provide access to such features through intermediate layer as APIs.

5.6 Resource consumption

Resource consumption relates to the application developed with the cross platform framework. This requirement include in order: memory usage, CPU usage and power consumption [7]. Mobile phones like other pervasive devices suffer from resource shortages. These resources can influence each other, for example CPU utilization affects battery consumption. Several studies have been undertaken on this subject [25][22][21]. The memory usage may increase for various reasons, this can be due to the addition of features to generate user interface or to the use of HTML and JavaScript files. Several research works [9][3][8] [19] have appeared recently dealing with the power consumption of mobile applications. In mobile devices, the power is the most important resource [17] and the applications developed using cross-platform tools must use the battery of mobile devices effectively.

5.7 Look and feel

Success on an application depends in large part on user experience. Currently, the cross platform tools try to reproduce as closely as possible the look and feel of the native application. Most users seek applications that resemble to native applications, in term of graphic user interface and reactivity of the application. Indeed, mobile is a device when applications have to be interrupted by event as a call or SMS. When users want to react to this event, after that, he wants returning to the application where he left it. For that purpose, the support for backend communication protocols and data formats becomes mandatory.

5.8 Visibility

The way to distribute the application will determine the visibility of this application. Generally, users are turning to app stores of mobile platforms to obtain an application, when web site applications are accessible only through URL and internet connection. In addition, this requirement determines also the way of how to update and maintain the application, i.e. any application update in the mobile device is required for web application since data and application are hosted on the server.

6. The desirable requirements of cross platform tools

In this section, we have selected five cross platform tools, one per approach, in order to provide a description of such tools, according to the desirable requirements that we have detailed in the previous section. The main criteria of selecting these tools were their popularity and extensive use, especially for the first four tools. For the last one, even if model driven tools are not mature enough, Automobile Project was selected because it appears as the most promising tool in its category.

We do not aim to compare these selected tools. But, our objective is to provide, for each approach, an overview of the existing cross platform tools based on what we have presented as the necessary requirements that must be met by any cross platform tool.

6.1 Example of web approaches: Sencha Touch

Cross Web applications are most often obtained through frameworks such as Sencha Touch that allows creating free web applications. Sencha Touch is an HTML5 mobile application framework for building web applications. Sencha Touch 2.3.1, which is the latest version at the time of this writing supports Android browser, Google Chrome for Android, BlackBerry 10, Bada Mobile Browser, Kindle Fire Browser, Windows Phone 8 and Mobile Safari. In fact, this version of Sencha Touch only targeted at webkit browsers.

Sencha Touch can be used with Apache Cordova/PhoneGap or Sencha's native packager. Either enables to package the application in a native container and enables access to select device-level APIs unavailable to traditional web applications. However, the application suffers from the lack of performances due to the execution on the browser.
Sencha Touch does not provide IDE for developing Touch applications but we can look to IDE as NetBeans, Webstorm and Aptana. But none of them provides a great experience programming. In addition, Touch provides a plugin for Eclipse environment.

6.2 Example of hybrid approaches: PhoneGap

This framework is based on the open source Cordova project. PhoneGap supports as for now (version 3.3.0) 7 mobile platforms: Android, iOS, BlackBerry, webOS, Windows Phone 7 and 8, Symbian and Bada. It allows developers to create mobile applications using modern web technologies as HTML5, CSS3 and JavaScript.

PhoneGap is a "web code wrapper", i.e., PhoneGap operates by packaging the web page with the specific PhoneGap engine which is specific for each supported platform. This engine displays the page in a regular web view. Additionally, PhoneGap provides an access to device functionality as Accelerometer, Camera, Contact, GPS, etc.

through JavaScript API's. The API is implemented differently for each platform. This is why the application result is hybrid, i.e., it is not purely native neither purely web [1]. The product result is a binary application archive that can be distributed through the application market.

PhoneGap does not allow having a centralized development environment because it does not provide an IDE to develop applications. Instead, it provides a service called PhoneGap Build that allows developers to compile their applications in the cloud. Developers can also choice an IDE to write the source code and take it to an appropriate IDE for the target platform, e.g. Eclipse for Android, to additional code modifications.

Even if applications developed with PhoneGap are elaborate, they have more look and feel as web application than native application. PhoneGap is more adaptable for projects where cross platform reach is more important than high performances and user experience [1].

6.3 Example of interpreted approaches: Appcelerator Titanium

Appcelerator Titanium is a development environment for creating native applications across different platforms, developed by Appcelerator Inc. Titanium applications are developed using web technologies as HTML5, JavaScript and CSS3. It uses JavaScript APIs and platform APIs to build the interface and to access native device features.

Titanium links JavaScript to native libraries, compiles it to bytecode and then the platform SDK builds the package for the desired target platform. The code is then packaged with Titanium's engine. At runtime, this engine interprets the JavaScript code and creates the user interface. The final application resembles to the typical platform appearance due to using native elements to build the user interface. However, application performances can be disappointing due to the interpretation of the source code every time the application runs.

Titanium also includes the Titanium Studio which is an Appcelerator's IDE. It is allows the writing, testing, and debugging of mobile applications. It is also offer the possibility to run and deploy the applications [13][24].

Small numbers of platforms are supported by Titanium: iOS, Android, Windows Phone, BlackBerry OS, and Tizen with, by the latest stable release version 3.2.0 of the framework.

6.4 Example of cross-compiled approaches: Xamarin

Xamarin is a development environment for cross-compiled applications that allows developing applications using the C# language. Xamarin 3, the latest version of the framework, supports iOS, Windows Phone and Android.

To run the application on the device, the cross compiler Xamarin translates the code of the application writing in C# to binary code that can run on the target platform. Xamarin applications are built with native user interface controls. Thus, applications not only look like native application, they behave that way too. Xamarin also allows the access to all native devices and platforms features by providing all the native APIs of the supported platforms as C# libraries. Xamarin applications leverage platform-specific hardware acceleration, and are compiled for native performance.

Xamarin offers also its own IDE named Xamarin Studio. Studio provides a convivial development environment with functionalities as code completion, debugger, etc. Xamarin Studio allows also distributing the applications through the application stores.

6.5 Example of model driven approaches: AutoMobile

AutoMobile Project appears as one of the most active and most promising in its category. AutoMobile is funded by the European Commission.

The AutoMobile project exploits the modern paradigm of Model-Driven Engineering and code generation. It is based on abstraction, modeling and code generation to represent applications in a platform-independent manner, and then, generate the code to deploy the application and interaction logic onto the target platform. AutoMobile relies on modeling languages such as IFML (Interaction Flow Modeling Languages) and on tools like WebRatio.

What proposes AutoMobile: 1) a platform independent modeling language based on OMG standards (MDA, UML, IFML) for modeling applications 2) a set of software components and an architectural framework acting as technical building blocks based on HTML5 and also target native applications (iOS and Android) 3) a model-to-code generator, which consists of a set of model transformations integrated in the existing WebRatio platform. AutoMobile targeted only iOS and Android platform for now.

Table 1 summarizes the result of the evaluation. Our discussion let appear that although the user experience is not as good as native applications, the cross platform applications can be deployed in several platforms at once to reach out most of the potential users, which is essential for the application vendors:

Table 1. Simulation Configuration

	Sencha Touch	PhoneGap	Appcelerator Titanium	Xamarin	Automobile
Mobile platform supported	Only targeted at webkit browsers (Android browser, BlackBerry 10, WP8…)	Android, iOS, BlackBerry, webOS, WP 7 & 8, Symbian, Bada	iOS, Android, WP, BlackBerry OS, Tizen	iOS, WP, Android	iOS, Android
Development environment	NetBeans, Webstorm, Aptana, plugin for Eclipse	PhoneGap Build	Titanium Studio	Xamarin Studio	Not defined
API and documentation	Extensive documentation with large community of developers			Significantly smaller than for native and HTML5 based apps	Not defined
Security	Depends on browser security		Not good		Not defined
Access to device specific features	Access through Apache Cordova APIs	Access through JavaScript APIs	Access through JavaScript APIs and platform APIs	Access through C# libraries	Acces to all device features
Resource Consumption	Execution on browser engine may degrade the performances		Can be disappointing due to the interpretation of the code each execution	Significantly better with the use of native user interface controls	Same as native apps
Look & feel	More web application than native application		Resembles to the typical platform appearance due to using native elements		Pure native application
visibility	distributed through the application store				

8. Conclusions

The cross platform mobile development follows the concept: "*Develop once and run anywhere*". In other words, it promotes code reuse which reduces the development cost and time. Even if the user experience does not equal those of native applications, cross platform mobile applications can be deployed in several platforms. Given the competition that animates the market for mobile platforms, more and more developers are turning to cross-platform mobile applications solutions.

In this paper, we have distinguished between five cross platform development approaches. Then, we have described the general architecture of cross platform mobile applications. We have identified the desirable requirements of cross platform technologies. Based on these aspects, we have provided a detail survey that covers five cross platform tools (one per approach) allowing developers to gain insight about cross platform solutions. We have observed that the cross-platform tools can be recommended in general, even if none of such tool satisfied all the requirements.

We present this study as a first step toward defining standards, allowing the specification of cross platform mobile application development process. We are aware that there is still much work to do. More experimental studies are required to collect more data and support our findings. In the future studies, we will extend our analysis to additional cross platform tools. At the same time, we will further investigate requirements that call for more attention, such as security and resource consumption described in this paper.

References

[1] S. Allen, V. Graupera and L. Lundrigan, Pro Smartphone Cross-Platform Development: iPhone, BlackBerry, Windows Mobile, and Android Development and Distribution, Apress,

2010.

[2] M. Anvaari and S. Jansen, "Evaluating architectural openness in mobile software platforms", *In Proc. of the Fourth European Conference on Software Architecture (ECSA '10)*, ACM, New York, NY, USA, pp. 85-92, 2010.

[3] R. Bala and A. Garg, "Battery Power Saving Profile with Learning Engine in android Phones", *International Journal of Computer Applications*, vol. 69, no. 13, pp. 38-41, 2013.

[4] A. Cavoukian and M. Chanliau. Privacy and Security by Design: A Convergence of Paradigms. Last Accessed On: 28 March 2014. Available: https://www.privacybydesign.ca/content/uploads/2013/01/pbd-convergenceofparadigms.pdf

[5] A. Charland and B. Leroux, "Mobile application development: web vs. native", *Commun. ACM*, vol. 54, no. 5, pp. 49-53, 2011.

[6] Y. C. Cho and J. W. Jeon, "Current software platforms on mobile phone", *In Proc. of the International Conference on Control, Automation and Systems (ICCAS 2007)*, IEEE Computer Society, Washington, DC, USA, pp.1862–1867, 2007.

[7] I. Dalmasso, S. K. Datta, C. Bonnet, N. Nikaein, "Survey, Comparison and Evaluation of Cross Platform Mobile Application Development Tools", In *Proc. of the 9th International Wireless Communication and Mobile Computing Conference (IWCMC)*, IEEE Computer Society, Washington, DC, USA, pp.323 - 328. 2013.

[8] S. K. Datta, C. Bonnet, N. Nikaein, "Android power management: Current and future trends", In *Proc. of the First IEEE Workshop on Enabling Technologies for Smartphone and Internet of Things (ETSIoT 2012)*, IEEE Computer Society, Washington, DC, USA, pp.48-53, 2012.

[9] S. Kanti Datta, "Android stack integration in embedded systems", In *Proc. of the International Conference on Emerging Trends in Computer & Information Technology (ICETCIT 2012)*, 2012.

[10] Denimgroup.com. Secure mobile application development reference. Last Accessed On: 28 March 2014. Available: http://www.denimgroup.com/know_artic_secure_mobile_application_development_reference.html

[11] P. Friese. Overview | Cross-Platform Mobile Development. Last Accessed On: 16 March 2014. Available: http://cross-platform.mobi/documents/01-overview/01-introduction.html

[12] Gartner.com. Gartner Says Smartphone Sales Accounted for 55 Percent of Overall Mobile Phone Sales in Third Quarter of 2013. Last Accessed On: 28 March 2014. Available: http://www.gartner.com/newsroom/id/2623415

[13] G. Hartmann, G. Stead, A. DeGani. "Cross-platform mobile development", *Mobile Learning Environment*, 2011.

[14] H. Heitkötter, S. Hanschke, T. A. Majchrzak, "Comparing cross-platform development approaches for mobile applications", In *Proc. of the meeting of the Web Information Systems and Technologies (WEBIST 2013)*, Scitepress DL, pp. 120–138, 2013.

[15] H. Heitkötter, T. A. Majchrzak, H. Kuchen, "Cross-platform model-driven development of mobile applications with md2", In *Proc. of the 28th Annual ACM Symposium on Applied Computing (SAC '13)*, ACM, New York, NY, USA, pp.526-533, 2013.

[16] X. Jia and C. Jones, "Cross-platform application development using AXIOM as an Agile Model-Driven Approach", In *Proc. of the 7th International Conference (ICSOFT)*, Springer Berlin Heidelberg, Berlin, pp.36-51, 2012.

[17] R. Kravets and P. Krishnan, "Power management techniques for mobile communication", In *Proc. of the 4th annual ACM/IEEE international conference on Mobile computing and networking (MobiCom '98)*, ACM, New York, NY, USA, pp.157-168, 1998.

[18] O. Le Goaer and S. Waltham, "Yet another DSL for cross-platforms mobile development", In *Proc. of the First Workshop on the Globalization of Domain Specific Languages (GlobalDSL '13)*, ACM, New York, NY, USA, pp.28-33, 2013.

[19] J Lee, H. Joe, H. Kim, "Smart phone power model generation using use pattern analysis", In *Proc. of the IEEE International Conference on Consumer Electronics (ICCE 2012)*, IEEE Computer Society, Washington, DC, USA, pp.412-413, 2012.

[20] F. Lin and W. Ye, "Operating System Battle in the Ecosystem of Smartphone Industry", In *Proc. of the International Symposium on Information Engineering and Electronic Commerce (ISIEEC 2009)*, IEEE Computer Society, Washington, DC, USA, pp.617-621, 2009.

[21] R. Murmuria, J. Medsger, A. Stavrou, J. M. Voas, "Mobile Application and Device Power Usage Measurements", In *Proc. of the 2012 IEEE Sixth International Conference on Software Security and Reliability (SERE '12)*, IEEE Computer Society, Washington, DC, USA, pp.147-156, 2012.

[22] R. Rawassizadeh. "Mobile Application

Benchmarking Based on the Resource Usage Monitoring". *IJMCMC* vol. 1, no. 4, pp. 64-75, 2009.

[23] A. Sommer and S. Krusche, "Evaluation of cross-platform frameworks for mobile applications", In *Proc. of the 1st European Workshop on Mobile Engineering (ME'13)*, 2013.

[24] Visionmobile.com. Cross-Platform Developer Tools 2012: Bridging the worlds of mobile apps and the web. Last Accessed On: 28 March 2014. Available: http://www.visionmobile.com/product/cross-platform-developer-tools-2012/

[25] W. Yuan and K. Nahrstedt, "Energy-efficient soft real-time CPU scheduling for mobile multimedia systems", In *Proc. of the nineteenth ACM symposium on Operating systems principles (SOSP '03)*, ACM, New York, NY, USA, pp.149-163, 2003.

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Home Birth in Bangladesh: Possible Reasons, Problems and Perceptions

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***Abstract*— Data Mining is the process of finding pattern or useful information from large volume of data. The goal of this paper is to find the reason behind the unusual high birth rate by applying data mining techniques, e.g., decision tree, neural network, Bayes Classifier, Ripper and Support Vector Machine. The dataset was collected from the baseline survey conducted by the maternal neonatal and child health programme by ICDDR,B. If we could find the reason(s), high birth rate at home could be avoided in future. Giving birth at home is very dangerous as many complications may arise during pregnancy as well during birth. We analyze child's health after birth and reveal the fact that the number of complications in home birth is higher than that of in non-home birth. We also present the perceptions of mothers about home birth and drawbacks or shortcomings of a rural health care center reported by the mothers. From the opinions of experts and professionals, it could be said that the risk of mortality of new born during home birth is quite alarming and birth at hospital/clinics seemed to be the safest place to protect the health and well-being of the woman and her baby.**

Keywords—Home birth; Place of delivery, Data mining, Decision Tree, Neural Network, Probabilistic Neural Network.

I. Introduction

ICDDR,B [1] is the international center for diarrheal disease research in Bangladesh. It is an international health research organization located in Bangladesh. It is dedicated to save lives through translation of research into treatment, training and policy advocacy. It also addresses some of the most critical health concerns facing the world today.

The baseline survey we used in this research was conducted on mothers in the Matlab, Bangladesh region, which is the primary rural field site for ICDDR,B and the world's longest running health project. The survey is an analysis of current situation to identify the starting points for a program or project.

The survey was carried out from 1st. November 2005 to 30th. September 2006 under ICDDR,B. Maternal Neonatal and Child Health programme, commonly known as MNCH. This programme's main purpose is to improve the quality and availability of all maternal, newborn and child health services. Ensuring optimal health for girls of reproductive age, improving the health and nutrition of mothers-to-be, and providing quality reproductive health services including ante- and post-natal care are pivotal to ensure safe motherhood.

Maternal, newborn and child health is an area where aid can make a huge difference. By training midwives we could prevent deaths during childbirth, increase access to life-saving vaccines to help providing better nutrition to reduce child deaths and stunting. MNCH is one of the highest impact areas of effective development aid.

Many of the non-government and government organizations are also carrying out MNCH programs both locally and overseas, e.g., Australia, Canada, Pakistan and etc.

This paper is organized as follows: the literature review related to existing research by ICDDR,B is described in Section II. Section III describes the methods used in this research in detail. The results and analysis of results is also discussed in this Section. Finally conclusion and future work is presented in Section IV.

II. LITRETATURE REVIEW

Decision Tree models generated using decision tree induction algorithms are very comprehensive to the end-users.. Hospital Surveillance data has been historically used for early detection of emerging epidemics for example influenza or Cholera etc. using time series analysis. Historical data of surveillance system is used to detect recognizable wave patterns or cycles in the earlier epidemic attacks to forecast future probable attacks.

In [11] the authors have used Decision Tree induction algorithm to generate decision tree models from hospital surveillance data to classify hospital patients according to their physical conditions and personal disease history on admission to hospital. Decision tree models are generated using ICDDR,B hospital surveillance data. From the decision tree generated based on earlier cases stored in the surveillance data decision rules are generated. These rules are used to classify patients into three classes according to their criticality: High, Mid, Low so that hospital can take prudent actions for the patients. Different preprocessing and formatting activities have been carried out on the data to make it ready for the model building. Different decision tree models have been generated to find out an optimized model which can classify new patients more accurately i.e., the prediction accuracy is higher. Different optimization techniques have been employed. Lastly performance of different decision tree models have been measured and compared using different performance metrics.

The heart disease accounts to be the leading cause of death worldwide. It is difficult for medical practitioners to predict the heart attack as it is a complex task that requires experience and knowledge. The health sector today contains hidden information that can be important in making decisions. Mesthe et al.[15] used data mining algorithms, J48, Bayes Net, and Naive Bayes, Simple Cart, and REPTREE algorithm to classify and develop a model to diagnose heart attacks in the patient data set from medical practitioners. The objective of the research is to predict possible heart attacks from the patient dataset using data mining techniques and determine the model that gives the highest percentage of correct predictions for the diagnoses.

The main objective of the project [16] was to develop a prototype intelligent Heart Disease Prediction System (IHDPS) using three data mining modelling techniques, namely, Decision Tree, Naïve Bayes and Neural Network. IHDPS can discover and extract hidden knowledge (patterns and relationships) associated with heart disease from a historical heart disease database. It can answer complex queries for diagnosing heart disease and thus assist healthcare practitioners to make intelligent clinical decisions which traditional decision support system cannot. By providing effective treatments, it also helps to reduce treatment costs.

While the recent advances in neonatal medicine has greatly increased the chance of survival of infants born after 20 weeks of gestation, these infants still frequently suffer from lifelong handicaps, and their care can exceed a million dollars during the first year of life as cited in [17]. As a first step for preventing preterm birth, decision support tools are needed to help doctors predict preterm birth. A number of popular classification algorithms are applied to the dataset for predicting preterm birth, and their prediction performance is compared with the associative classifier. The applied classifiers are Logistic Regression, Naive Bayes, C4.5 decision trees, Support Vector Machines, and Neural Networks. They implemented the algorithms with default parameters provided in Weka, an open source data-mining package [13]. For Naive Bayes and C4.5 Decision Trees, the numerical attributes are recoded into a set of categorical attributes. Three-fold cross validation is used to evaluate the performance of each classifier.

The survey [18] has reviewed standard algorithms that are well known in research community and has discussed the criterion for these algorithms, which are classification, regression, segmentation, association, and sequence analysis. These data mining classifications are subsets of standard algorithms and are used by data mining software vendors for their data analysis services. Depressive symptoms are common amongst pregnant women following anemia and it could predict subsequent maternal mortality and morbidity and fetal abnormalities.

III. METHOD OF ANALYSIS

A. Choosing the Class Attribute:

The data we acquired was in .sav or in SPSS format. We used the software SPSS as well as KNIME to handle our dataset. In SPSS we went through most of the attributes to get an appropriate class for our analysis and through rigorous searching we saw an outlier, 58.5% of the baby born were in home and it seemed like a perfect classifier for our analysis. It is also represented in Table 1.

TABLE I. BIRTH AT DIFFERENT PLACES

	Frequency	Percent	Cumulative %
01	2752	58.5	58.5
02	66	1.4	59.9
03	410	8.7	68.6
04	780	16.6	85.2
05	58	1.2	86.5
06	36	0.8	87.2
07	12	0.3	87.5
08	342	7.3	94.7
09	54	1.1	95.9
10	7	0.1	96.0
11	106	2.3	98.3
12	44	0.9	99.3
98	35	0.7	100
Total	4703	100	

In Table I, 01 represent Home Birth which has a frequency of 2752 whereas in other places like 03(MATLAB SUB-CENTER) and 04(MATLAB HOSPITAL) seem to be very low.

B. Preprocessing:

In the dataset we collected, there are about 6000 participants. The survey questionnaire was a very detailed set; it contains a total of 549 attributes. The various fields that are asked from the participants are: general information, previous birth information, pregnancy, delivery, and postpartum care for mother, immediate newborn care, and newborn care, perception of local facility, attitude and perception, etc.

As our main objective was to focus on exploring the unusually high rate of HOME BIRTH and the several factors leading to the decision, we finally concentrated on the following data sets: checkups during pregnancy, who did checkups, plans for birth during pregnancy, complications and seek treatment, type of treatment received, planned for transport, saved money, parent’s education background, mother’s perception (knowledge), perception of local facility, and attitude, etc.

We trimmed data’s in two ways: *Vertically* and *Horizontally*

Vertically we considered the factors after various omissions from the original data set. Firstly we omitted the tuples which had missing data. Doing that our total number of tuples has been declined in size from 5248 to 4703.

Horizontally we focused on the sections which were irrelevant to birth of the child such as:

i) Postpartum care for mother
ii) Immediate newborn care
iii) Newborn care

All these sections deal with care of the mother or child after the delivery of the child.

Postpartum care for mother:

Postpartum care for mother asks the individual about, complications she faced after child birth, the person who checked her health checkup after delivery. We omitted this section which comprised of questions Q401 to Q406. With this our total attributes are decreased from 549 to 531.

Immediate newborn care:

Immediate newborn care is about, health problems of the baby after birth, basic cares about the baby, etc. We trimmed Q501 to Q527 which were all in this section. With this our total attributes are further decreased from 531 to 446.

Newborn care:

New born care and first month is about: was the baby with mother or relative, was there any checkup performed on baby, did the baby have any problems. Q601 to Q631 were trimmed from this section. With this our attributes are further decreased from 446 to 311.

The general information of the participants was also removed from the attributes list for example: name, id, date, etc. With this we were left with attributes156 from 311.

156 attributes seem to be huge but according to our dataset, the question's answers were provided with their own attribute field, for example Q302 question has 11 choices and in the dataset it is given as Q302A, Q302B and so on. The answers are filled according to their choices on that attribute.
Therefore there are many questions in the dataset that take up more than 10 attribute spaces.
Considering those 10 attributes as 1 (with different values) we actually have an attribute count of 49.

C. Classification Methods

We used SPSS to filter out the attributes that were unnecessary to our objective. SPSS did not have direct access to any data mining algorithm thus we decided to use software for our data mining purpose, KNIME.

KNIME, the Konstanz Information Miner, is an open source data analytics, reporting and integration platform. KNIME integrates various components for machine learning and data mining through its modular data pipelining concept. A graphical user interface allows assembly of nodes for data preprocessing (ETL: Extraction, Transformation, Loading), for modeling and data analysis and visualization.

For every analysis we used the color RED as Home Birth and the color GREEN as Other.

At first we partitioned the dataset according to 60-40 percentile, 60% used as the training set and the rest 40% as test set. According to Figure 1.0 the 60% training set goes to each of the classifiers learner node and after learning the 40% is used for the Predictor node. As shown in Figure 1, we created our data mining nodes. For our analysis we used 5 classifiers,

- Decision Tree
- Naïve Bayes
- Probabilistic Neural Network
- Multilayer Perceptron
- Support Vector Machine

1. Decision Tree:

KNIME **decision tree learner node** uses C4.5 [2] which is an algorithm used to generate a decision tree developed by Ross Quinlan. C4.5 is an extension of Quinlan's earlier ID3 algorithm. The decision trees generated by C4.5 [5] can be used for classification, and for this reason, C4.5 is often referred to as a statistical classifier. Pre-pruning a decision tree involves using a 'termination condition' to decide when it is desirable to terminate some of the branches prematurely as the tree is generated. Post-pruning[3,4] a decision tree implies that we begin by generating the tree to its full length completely and then adjust it with the aim of improving the classification accuracy on unseen instances.

For our dataset, the decision tree was given few specific attributes such as, Quality Measure with respect to Gini Index and Pruning method as Minimum Description Length (MDL) and we got a Confusion matrix as shown in Table II.

We found out that the factors or the attributes that played the most important roles in determining the Home Birth Class are:

- Birth delivery Kit available
- Receiving Checkup during last Pregnancy
- How many times Checkup during pregnancy
- Make plans for pregnancy
- Where they planned to give birth
- Who they selected as their birth attendant

For our decision tree we have correctly classified 88.2% of the data and the remaining 11.8% are incorrectly classified.

2. Naïve Bayes:

Naive Bayes is a simple technique for constructing classifiers. Naïve Bayes model assigns class labels to problem instances, represented as vectors of feature values, where the class labels are drawn from some finite set. Figure 2 shows a part of Naïve Bayes learner view in KNIME and also the Confusion Matrix is shown in Table II. More details of this technique could be found elsewhere [6,8].

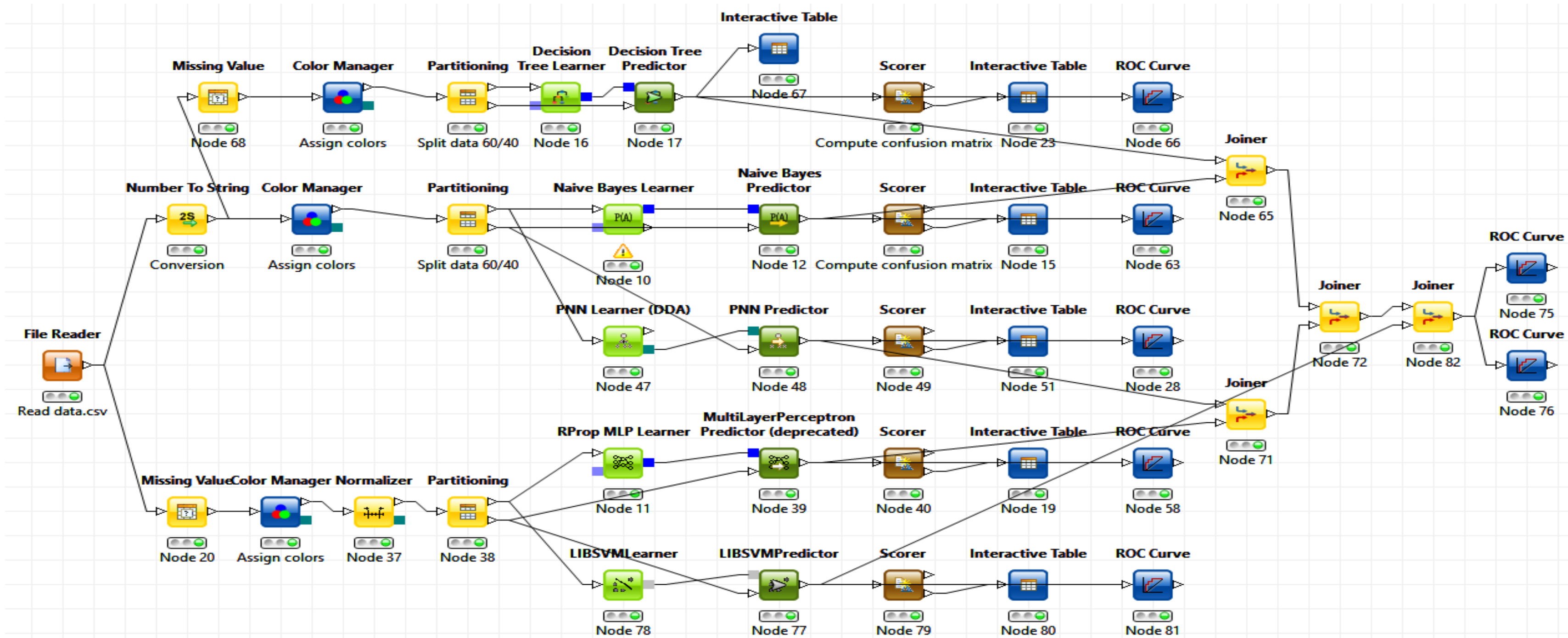

Figure 1. KNIME Data Mining Node Structure

TABLE II. ACCURACY OF DIFFERENT CLASSIFIERS

Classifier	Recall	Precision	Sensitivity	Specifity	F-measure	Accuracy	Cohen's Kappa
Decision Tree	0.898	0.798	0.898	0.873	0.845	0.882	0.75
Naïve Bayes	0.873	0.983	0.873	0.987	0.924	0.935	0.868
Probabilistic Neural Network	0.735	0.731	0.735	0.599	0.733	0.68	0.335
Multilayer Perceptron	0.919	0.9	0.919	0.859	0.91	0.894	0.782
Support Vector Machine	0.884	0.906	0.884	0.873	0.895	0.879	0.754

Gaussian distribution for Q204 per class value

	No	Yes
Count:	1045	1140
Mean:	2.76077	4.32281
Std. Deviation:	4.44466	8.11004
Rate:	37%	40%

Gaussian distribution for Q205 per class value

	No	Yes
Count:	1045	1140
Mean:	6.00478	5.77018
Std. Deviation:	7.2924	10.12736
Rate:	37%	40%

Figure 2. Representation of Naïve Bayes in KNIME

3. Probabilistic Neural Network:

Probabilistic Neural Network (PNN) is trained based on the DDA (Dynamic Decay Adjustment) method on data using Constructive Training of Probabilistic Neural Networks as the underlying algorithm. More about this algorithm could be found [7, 13, 14].

The PNN learner statistics from KNIME is as shown below,

Learner Statistics

- Number of epochs: 6
- Number of classes: 2
- Number of rules learned per class: (in total 825)
    - Yes: 400
    - No: 425
- Number of training instances per class: (in total 2821)
    - Yes: 1192
    - No: 1629

4. Multilayer Perceptron Neural Network:

A multilayer perceptron (MLP) is a feed forward artificial neural network model that maps sets of input data onto a set of appropriate outputs. A MLP consists of multiple layers of nodes in a directed graph, with each layer fully connected to the next one. Except for the input nodes, each node is a neuron (or processing element) with a nonlinear activation function. MLP utilizes a supervised learning technique called back propagation for training the network.

For our MPNN we used 50 learning iterations, 1 hidden layer and 10 hidden neurons per layer using our homebirth attribute as class column. The Confusion Matrix of MLP is as shown in Table II.

5. Support Vector Machine:

In machine learning, support vector machines (SVM) are supervised learning models with associated learning algorithms that analyze data and recognize patterns, used for classification and regression analysis. Given a set of training examples, each marked as belonging to one of two categories, an SVM training algorithm builds a model that assigns new examples into one category or the other, making it a non-probabilistic binary linear classifier.

Like the authors in [20]we also used Sequential Minimal Optimization with RBF (radial basis function) containing Sigma = 0.5 as our kernel in SVM with an overlapping penalty of 1, which basically determines the penalty assigned to each point that is misclassified.

The Confusion Matrix of SVM is as shown in Table II.

D. Measuring the performance of a Classifier

We measure the performance of the classifiers with respect to different performance metrics. The performance is represented in Table II. The confusion matrices that we acquired also had the performance of the classifier which was made by the Scorer node and represented by the Interactive table as shown in the Figure 1.
According to that we formed the ROC curve for each classifier and joined them to determine the best classifier.

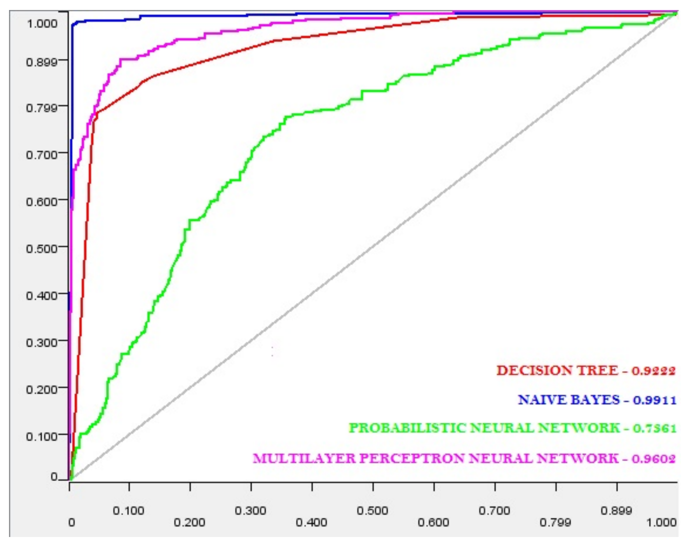

Figure 3. ROC Curve for Classifiers

Figure 3 represents the ROC curves for the classifiers. From here we can determine that Naïve Bayes classified the dataset most accurately with an accuracy of 93.5%.

After investigating the reasons behind home birth we wanted to delve a little further and for the next step we wanted to determine the effects of the children, the mother's perception with respect to home and non-home birth.

TABLE III. HOME BIRTH CHILD PROBLEMS COUNT

Problems	Yes	No	Don't remember/ Don't know
Fever	643	2040	68
Trouble Breathing	355	2331	65
Jaundice	556	2131	64
Diarrhea	10	2667	64
Umbilical infection or discharge	277	2410	64
Convulsion	19	2668	64
Feeding	82	2605	64

TABLE IV. NON-HOME BIRTH CHILD PROBLEMS COUNT

Problems	Yes	No	Don't remember/ Don't know
Fever	337	1540	74
Trouble Breathing	225	1653	73
Jaundice	380	1498	73
Diarrhea	21	1856	74
Umbilical infection or discharge	253	1625	73
Convulsion	23	1854	74
Feeding	60	1817	74

We analyzed further to determine the effects of home birth and non-home birth on the children. For this purpose, we used an attribute (question 609) that indicates the diseases of a newborn, similar use and representation can be observed here [19].

The question we used was number 609, which describes any complications the child had in the first month after birth. The number of cases with complication is significantly less compared to the cases without any complication, but the data shows interesting facts when we compare attribute 609 with the place of birth (attribute 302). Fever, Jaundice, trouble breathing and umbilical infection have the highest numbers among the listed problems, the other options have very insignificant numbers and will not be discussed. Table III-IV depict the results. Below is the listed comparison of occurrence of complications in the first month between home birth and non-home birth newborns:

- Fever: 337 cases of fever were recorded in children not having home birth while 643 cases of children having home birth and fever in the first month were recorded. This clearly shows that the cases with fever is higher, approximately double in that of home birth.

- Jaundice: There was also a higher number of cases of Jaundice in home birth than non-home birth. 556 recorded counts in home birth while 380 counts in non-home birth. So approximately 1.5 times more cases are seen in home birth.

- Trouble breathing: 355 cases found in home birth while 225 cases in non-home birth. Higher number of children who had trouble breathing found in home birth, exactly 130 cases more in home birth than in non-home birth with trouble breathing.

- Umbilical infection: In this instance both home birth and non-home birth have very similar numbers, 277 cases for home birth and 253 cases for non-home birth. This can mean either of two things; either the hospitals need to improve their standard and maintenance of hygiene or in the cases of home birth proper precautions were taken and so the number cases with infection are similar to that of non-home birth.

Altogether we can see that for fever, jaundice and trouble breathing higher number complications for home birth as opposed to non-home birth. We can conclude that chances of complications even after birth are higher in home birth than non-home birth and another reason as to why home birth should be avoided.

As this survey was conducted on a rural area in a developing country we also decided to check whether their perception and attitude towards local facility played any roles towards their choices in these matters. This is represented in Table V.

TABLE V. QUESTION ABOUT MOTHERS ATTITUDE AND PERCEPTION

Questions	1 (Strongly Disagree) no. of responses	2 (Disagree) no. of responses	3 (Agree) no. of responses	4 (Strongly Agree) no. of responses
1. A woman should plan ahead of time where she will give birth to her baby.	7	121	3559	1015
2. A woman should plan ahead of time how she will get to the place where she will give birth.	16	198	3765	723
3. A woman should avoid going to the hospital during labor, because they always want to do a Caesarian / Operation.	517	3836	330	18
4. I think that the health provider will cut my vagina at the health center or hospital.	299	3995	388	19
5. A woman should avoid going to hospital even if she is very sick because her family might have to pay a lot of money.	266	3820	579	37
6. If I go to the health center, I will have to see a male doctor.	188	3130	1328	56
7. The health providers sometimes do not treat me nicely at the health center or hospital.	103	2886	1644	69
8. If my husband doesn't give me permission, I cannot go to the health center or hospital.	198	2376	2017	111
9. The health center is far away, and it is hard for me to get transportation.	211	3327	1124	40
10. A woman should avoid going to hospital, because they might want to give a blood transfusion.	425	4019	247	10
11. I am concerned that no one will be at home to do my work if I go to a clinic or hospital.	269	3749	656	28
12. I am concerned my mother-in-law will say there is no need for me to go to a clinic or hospital.	296	3685	688	33

The last section of the datasets was designed for recording the attitude and perception of the mothers on pregnancy and childbirth whom the whole survey was carried on. To understand the attitude and perception, every mother was asked twelve questions and they were given four choices (4=Strongly Agree; 3=Agree; 2=Disagree; 1=Strongly Disagree).

After analyzing the responses for all the questions we can conclude that most people have the correct perception about the questions asked. But for some questions the responses reveal that there are some problems in attitude and conditions which need to be looked into and solved. For example in question no.7 (The health providers sometimes do not treat me nicely at the health center or hospital) 2886 no. of person says that they disagree whereas 1466 no. of person Agrees. The high number of negative responses show that the attitude and personality of the health care providers need to be more caring and friendly towards the patients and not discourage them from seeking professional medical help.

In question no. 8 (If my husband doesn't give me permission, I cannot go to the health center or hospital) 2376 no. of person disagree whereas 2017 no. of person agrees. This shows that husband in the families are not properly aware of the risks and dangers involved of home birth and child birth in general. More attention to increase awareness in fathers should be given when conducting awareness programs.

In question no. 9 (The health center is far away, and it is hard for me to get transportation.) 3327 people disagree and 1124 people agree. For this 1124 people it is difficult to get to the health center and this reflects the need for improvement in roads and transportation system. It should be made easier for expecting mothers to reach the health centers for giving birth and regular checkups as well.

Though most of them have clear attitude and perception regarding the topic but there are some who are suffering from misconceptions and incorrect attitudes.

TABLE VI. QUESTION ABOUT WHAT MOTHERS DID NOT LIKE ABOUT THE CARE PROVIDED BY THE STAFF OF THE SUB-CENTRE/HOSPITAL

Question	Mothers agreed to the problem
Bad Behavior	445
Non-availability of medicine	297
Started treatment late	869
Very expensive/ Cannot afford	63
Not willing to answer question	126
Not clean	110
Not friendly and communicating	49
Treatment not effective/ Not cured	31

Finally, and might be one of the most important reason is that the rural area has a lot of disadvantages compared to cities. If we add bad services of health care professionals of rural medical centers on the top of it, the risk of home birth might not be avoided at all.

Table VI presents the drawbacks or shortcomings of a rural health care center reported by the mothers. We can see that 869 people were not properly taken care of and resulted in late treatment which might not always be fatal but from our point of view that issue should never rise in the first hand.

445 mothers were faced with bad behaviors which might have forced them or just simple arrogance might have been the cause of them carrying out home birth.

Non-availability of medicine is always a concern for any village area of a country and it can also lead to difficult situations.

There are other factors that are reported in Table VI. More than one-third of our mothers are faced with the problems in the place where they are supposed to get solutions. The concerned departments of Government should look into this and we believe the situation will be improved in coming days.

IV. CONCLUSION

This paper has analyzed the Matlab Maternal Neonatal and Child Health Programme's baseline survey. After analyzing we saw that there was an anomaly of high home birth. According to MONDAY, Feb. 3, 2014 (HealthDay News) -- The number of pregnant women who elect to deliver their baby at home is increasing, but home delivery can lead to problems.

The risk of a baby dying is nearly four times higher when delivered by a midwife at home than by a midwife in a hospital, according to a new study.
Even though in our dataset the death rate of new born was insignificant it is still a concern as a child may lose their life due to carelessness of the parents or not knowing the actual process of home birth.

Normally, the reasons for home birth are as follows [10]:
-A desire to give birth in a familiar, relaxing environment surrounded by people of your choice
-A desire to wear your own clothes, take a shower or bath, eat, drink and move around freely during labor
-A desire to control your labor position or other aspects of the birthing process

-A desire to give birth without medical intervention, such as pain medication
-Cultural or religious norms or concerns
-A history of fast labor

From our findings we can also observe that the desire to control the time of delivery is the most important part for the mother and as such from our analysis we saw that most mothers had a birth delivery kit as well as a birth attendant of their choice.

Labor position was also an important factor for home birth.
But there are situations where a home birth is not recommended,
-Have diabetes, chronic hypertension, a seizure disorder or any chronic medical condition
-Previously had a C-section
-Develop a pregnancy complication, such as preeclampsia
-Are pregnant with multiples or your baby doesn't settle into a position that allows for a headfirst delivery
-Less than 37 weeks or more than 41 weeks pregnant.

Moreover analysis of child's health after birth and during the first month after birth revealed that the number of complications in home birth was higher than that of in non-home birth. These complications include fever, Jaundice and breathing trouble. It is evident by comparing the statistics of home and non-home birth that a greater level of risk is present to the child after birth in home births.

As the region where the survey was conducted has a literacy rate less than 70%, everyone there will not be aware of the problems they might face during home birth. Thus, the main goal from this baseline survey should be to initiate a campaign about the dangers of home birth among the local people and also let them aware about the precautions necessary for home birth.

Analysis also reveals that although most of the mothers are aware of actual problems and precautions related to child birth, there is still presence of wrong beliefs and judgment. To overcome this situation more projects dealing with increasing awareness of risks involved in child birth must be conducted. These projects must not be limited to women and mothers only but should also target fathers and men in general.

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REFERENCES

[1] ICDDR,B, Available at: http://www.icddrb.org/who-we-are/our-mission/about-us (Accessed: 11 January 2015).

[2] Data Mining , Available at: http://www.cs.ccsu.edu/~markov/ccsu_courses/DataMining-7.html (Accessed: 12 January 2015).

[3] Mansour Y., Pessimistic decision tree pruning based on tree size, Proceedings of the 14th International Conference on Machine Learning (1997): 195-201, Weblink: http://citeseer.ist.psu.edu/76752.html.

[4] Ignizio J.P. An Introduction to Expert Systems: The Development and Implementation of Rule Based Expert Systems. – McGraw – Hill, Inc., 1991, 402 p.

[5] C4.5 Algorithm, Available at: http://en.wikipedia.org/wiki/C4.5_algorithm (Accessed: 12 January 2015).

[6] Geisser, Seymour. Predictive Inference. New York, NY: Chapman and Hall, 1993.

[7] Kohavi, Ron . "A study of cross-validation and bootstrap for accuracy estimation and model selection". Proceedings of the Fourteenth International Joint Conference on Artificial Intelligence (San Mateo, CA: Morgan Kaufmann), 1995.

[8] Devijver, Pierre A.; Kittler, Josef. Pattern Recognition: A Statistical Approach. London, GB: Prentice-Hall, 1982.

[9] "Newbie question: Confused about train, validation and test data!". Retrieved 2013-11-14.

[10] Mayo Clinic Staff Home birth: Know the pros and cons, Available at: http://www.mayoclinic.org/healthy-living/labor-and-delivery/in-depth/home-birth/art-20046878 (Accessed: 12 January 2015), 2014.

[11] Rashedur M Rahman, Fazle Rabbi Hassan, "Using and comparing different decision tree classification techniques for mining ICDDR, B Hospital Surveillance data". Expert Syst. Appl. 38(9): 11421-11436 (2011) .

[12] A. Aziz, N. Ismail, and F. Ahmad, "Mining Student's Academic Performance", Journal of Theoretical & Applied Information Technology, vol. 53, no. 3, 2013.

[13] Feldman W.E. and Wood B., The economic impact of high risk pregnancies. Journal of Health Care Finance, 24 64-71, 1997.

[14] Goodwin, L. and Maher, S. Data mining for preterm birth prediction. In Proceedings of the 2000 ACM Symposium on Applied Computing - Volume 1 (Como, Italy). J. Carroll, E. Damiani, H. Haddad, and D. Oppenheim, Eds. SAC '00. ACM, New York, NY, 46-51, 2000

[15] Hlaudi Daniel Masethe, Mosima Anna Masethe, "Prediction of Heart Disease using Classification Algorithms", Proceedings of the World Congress on Engineering and Computer Science 2014 Vol II WCECS 2014, 22-24 October, 2014, San Francisco, USA.

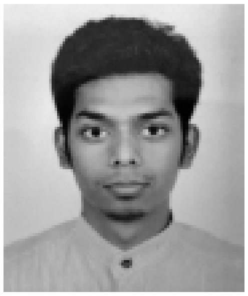
[16] Ms. Ishtake S.H , Sanap S.A., "Intelligent Heart Disease Prediction System Using Data Mining Techniques", International J. of Healthcare & Biomedical Research, Volume: 1, Issue: 3, April 2013, pp. 94-101.

[17] Yavar Naddaf, Mojdeh Jalali Heravi and Amit Satsangi, "Predicting Preterm Birth Based on Maternal and Fetal Data", http://yavar.naddaf.name/downloads/Predicting%20Preterm%20Birth%20Based%20on%20Maternal%20and%20Fetal%20Data.pdf.

[18] Fayyaz Ahmed, Adibah Sitara, "Exploration of Co-Relation between Depression and Anaemia in Pregnant Women using Knowledge Discovery and Data Mining Algorithms and Tools", Cyber Journals: Multidisciplinary Journals in Science and Technology, Journal of Selected Areas in Health Informatics (JSHI), September Edition, 2012

[19] Kanazawa, Akinori et al. 'Development of a Digital Signage and MR Contents to Promote Hand Hygiene'. *ACIS International Journal of Computer & Information Science,* Volume 15, Isuue 2, pp. 36-45, 2014.

[20] E. Sorour, Shaymaa et al. 'Comment Data Mining For Student Grade Prediction Considering Differences In Data For Two Classes'. *ACIS International Journal of Computer & Information Science,* Volume 15, Isuue 2, pp.12-25, 2014.



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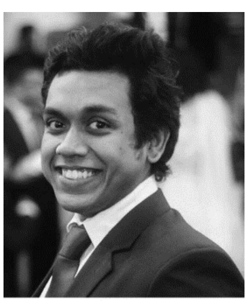

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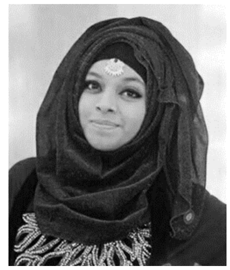

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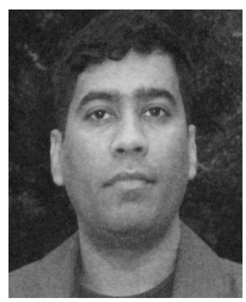

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Allocating Resources based on Multiple Bid Declaration with Preference*

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Abstract

In this paper, we discuss solutions to a problem called single-object resource allocation with preferential orders and propose a number of efficient algorithms for semi-optimal allocations. Resource allocation is a fundamental problem that is widely used in many applications. Although many studies on allocation with the associated utilities are conducted to maximize social welfare or total utilities, they have rarely taken into account agents' individual preferential orders that may have interfered with one another. Our proposed framework allocates one unit of resources to individual users but allows them to declare multiple resources with their own preferential orders. It then tries to allocate a resource to each agent by not only maximizing the total values but also considering the agent's preferences, at least, by ensuring that no or few dissatisfactions are reported. This is obviously a combinatorial problem to find optimal solutions. Thus, we propose efficient methods for semi-optimal solutions (allocations) that satisfy as many user preferences as possible. Finally, we analyze the quality of solutions and computation time by comparing them with the solutions obtained by CPLEX. Then, we experimentally demonstrate that the proposed methods are extremely efficient, while the reduced quality of their solutions is quite small.

Keywords: resource allocation, assignment problem, intelligent agent, preference

1. Introduction

With advances in computers and communications, many computerized systems have been introduced and used in our daily life such as electronic commerce and computerized cooperate transactions. In such applications, formalizations of resource allocation are widely used because they are quite fundamental in computer science [1]. The formalizations are also used in the computer systems providing Internet services in service oriented programming, in order to allocate and schedule individual tasks to agents appropriately, cooperatively, and distributedly (e.g., Spohere and Maglio [2]). Because resource allocation usually imposes high computational costs, many allocation algorithms for semi-optimal solutions have been studied thus far, such as a variety of market- and auction-based methods [3]–[8]. However, studies on resource allocation methods have often aimed at finding solutions to maximize social welfare, and only a few have focused on the preferences of individual agents that correspond to users who buy resources (goods) on the Internet.

Agents (or users that are owners of agents) often have their own preferences that may be affected by the internal and external conditions of problem settings and thereby are not correlated with social values or market prices [9]. For example, let us consider tickets for a concert. Seats in front are usually expensive, since many customers want to buy them. However, even a concert freak sometimes prefers a cheaper seat if he also intends to go to another concert with a limited budget. Agents' values regarding resources and attitudes to buy them differ between agents and may vary over time. This suggests that social values, such as the total values of monetary prices, and social benefits and priority, are often not compatible with individual preferences, and thus, the assumption of shared utility cannot be applied to a number of real-world applications.

This paper introduces a resource allocation problem called single-object resource allocation with preferential order (SORA/PO). The basic concept underlying this problem setting is that agents can only obtain one resource from the set of resources, but can declare a number of possible candidates with their values and preferential orders. The values here have a number of different aspects, such as monetary prices that agents can accept to pay, benefits obtained by receiving the required resources, and utilities in mechanism design, and these should be maximized from the social viewpoint. Moreover, the preferential order belongs to individuals and is not common in society. Thus, higher values (prices or benefits) do not always express higher preferences from individual agents. In SORA/PO, solutions must first reflect the agents' preferences although larger social welfare is better.

In this paper, we formulate SORA/PO as an integer-programming problem to maximize the total value of

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participating agents while taking into account as many individual preferences as possible. Then, we propose a number of efficient algorithms to find semi-optimal solutions. We then compare the quality of semi-optimal solutions derived by using the proposed algorithms with that of solutions obtained by CPLEX. We also compare their efficiency in finding solutions. We already proposed and discussed the semi-optimal solutions [10], but the discussion was limited due to the page limitations. In this paper, we added extensive experimental results to evaluate our methods in various situations and to analyze their characteristics of their solutions.

This paper is organized as follows. Section II discusses a number of related studies and Section III formalizes SORA/PO and explains a number of examples. We then propose efficient exploration algorithms and slightly sophisticated algorithms in which some additional options are intensively explored in Section IV. Section V presents results from experimental evaluations and describes the features of solutions derived from the proposed exploration algorithms and CPU time required to find the solutions. Finally, we provide a conclusion and discuss future challenges.

2. Related Work

There have been a number of studies related to resource allocation like the SORA/PO problem. One major approach to the problem within a multi-agent context is the use of game theoretic and auction-like protocols [1], [11], and many studies have been conducted to achieve semi-optimal allocations that have been incentive compatible and strategy-proof. For example, Sakurai et al. [8] proposed an efficient algorithm to determine winners in combinatorial auctions using limited discrepancy search. Edelman et al. [5] proposed generalized second-price auctions that could be used in search engines to allocate online advertisements. Devanur and Hayes [12] formalized the AdWords problem that is an extension of keyword auctions by adding the feature of random keywords arrivals and they proposed an efficient algorithm that was motivated by probably approximately correct (PAC) learning. Cavallo [9] took into account two types of resources, which were plentiful and scarce in situations where there were no money flows since money is often an inappropriate medium to achieve user incentives. These approaches are quite effective for multi-agent settings, but since they rely on game theory, they usually take social welfare into consideration by assuming that participants' preferences are consistent with prices.

Another approach is more heavily based on features of application structures [4], [13]–[16]. Ho and Vaughan [13], for example, proposed a learning method for task assignment for crowd-sourcing where their work was different and they had unknown skill sets. Mailler et al. [14] proposed a negotiation protocol for distributed task allocation to incrementally improve the quality of solutions. Although this was efficient and could be applied to realtime applications, they assumed that agents would be cooperative. Market-based approaches have been used to apply resource/task allocation problems to self-interested agents (e.g. [3], [4], [6]). An et al. [3], for example, proposed market-based resource allocation in environments where only incomplete information was available. In our research, agents are self-interested and have their own preferences that may be inconsistent with their values. However, we believe that our algorithms can be used in these methods.

3. Formulation

3.1 Single-Object Resource Allocation with Preferential Orders

Let us assume that $A = \{1, ..., n\}$ is a set of agents and $G = \{g_1, . . . , g_m\}$ is a set of resources. The (unit) allocation of G to A is the subset, L , of direct product $G \times A$ s. t. for $\forall g \in G$ and $\forall i \in A$, g and i appear at most once in L . Let $\boldsymbol{\Lambda}$ be the set of all allocations of G to A. Obviously, we also can see that L is the allocation of A to G. Thus, for $\forall L \in \boldsymbol{\Lambda}$, we can canonically define two functions

$$\boldsymbol{g_L : A \to G \cup \{null\}} \text{ and}$$

$$\boldsymbol{a_L : G \to A \cup \{null\}},$$

where $g_L(i) = null$ and $a_L(g) = null$ when i and g do not appear in L .

Agent i has a sequence of disjoint subsets of resources, $G_1^i, ..., G_N^i$ ($\subset G$), and the associated value, V_m^i, for each G_m^i. Note that the values defined here express a number of aspects, such as the monetary price that i accepts to pay and utility by being allocated the resource, which may not mean "utility" in utility value theory. As subscript m is i's preferential number of G_m^i, G_1^i and G_2^i, for example, are the sets of the first and second choices, respectively, so i prefers resource $x \in G_1^i$ over anything in G_2^i. Note that smaller preferential numbers mean higher preferential orders. We also define $d_L(i) = m_i$ if $g_L(i) \in \boldsymbol{G^i_{m_i}}$ for $L \in \boldsymbol{\Lambda}$.

Let us consider a simple example for this framework that we briefly described in Section 1. Suppose that the tickets of a pop-music group in a small concert venue would be sold with an (sealed-bid) auction-like method [16]. Agents know from the past pricing data that seats near the stage are more expensive and those near the rear are cheaper; these prices have usually been determined based on the order of popularity. However, agent i absolutely wants to go to the concert but prefers a cheaper ticket. This is because i could not go to another concert due to his/her budget if s/he could not buy a cheaper ticket. In this case, G_1^i is the set of tickets for seats around the rear, G_2^i is that for seats near the stage, and $G_3^i = G \backslash G_1^i \cup G_2^i$ (since these seats are more expensive than G_1^i, i cannot go to another concert and if so, a good seat is better). V_m^i is the price that i can pay for $x \in G_m^i$ and $V_1^i < V_3^i < V_2^i$. This example indicates that we cannot compare the seat prices with the degrees of willingness to go to a concert in a simple way because the values and preference for tickets are not always compatible.

We consider the allocation with which i is dissatisfied using the previous example. Suppose that resource $g \in G_{m_0}^i$ is allocate d to i, $g' \in G_{m_1}^i$ is done to $j \in A$ and $m_0 > m_1$. When $g' \in G_{m_2}^j$, then j will pay $V_{m_2}^j$. However, if $V_{m_2}^j < V_{m_1}^i$, i complains about this allocation. Now, we can define the SORA/PO using integer-programming formulation.

Definition 1. *Let* G *be the set of resources and* $B = \{B^i\}_{i \in A}$ *be the collection of the ordered sets of the declared resources with preferences by* $i \in A$*, where* $B^i = \{(G_1^i, V_1^i), \ldots, (G_{N_i}^i, V_{N_i}^i)\}$ *is the ordered set. A SORA/PO is defined as a problem to find allocation* $L \in \Lambda$ *that maximizes*

$$TV(G,A,B,L^*) = \sum_{(g,i) \in L^*} V_{d_{L^*}(i)}^i, \qquad (1)$$

subject to

$$\begin{aligned} &\forall (g,i) \in L^* \wedge 0 < \forall j < d_{L^*}(i) \\ &\text{if } \exists g' \in G_k^i \text{ s. t. } a_{L^*}(g') = j \wedge i \neq j \qquad (2) \\ &\qquad \text{then } V_{d_{L^*}(i)}^j \geq V_k^j \end{aligned}$$

This problem is denoted by SORA/PO(G,A,B).

The value of term (1) is referred to as the *total value* by allocation L^* and the total value for $\forall L \in \Lambda$ is denoted by $TV(G, A, B, L)$. For example, for $|A| = 3$ and $G = \{g_1, g_2\}$, if $G_1^1 = \{g_2\}$, $V_1^1 = 8000$, $G_2^1 = \{g_1\}$, $V_2^1 = 10000$, $G_1^2 = \{g_2\}$, $V_1^2 = 7500$, $G_2^2 = \{g_1\}$, $V_2^2 = 9500$, $G_1^3 = \{g_1\}$, $V_1^3 = 9900$, and $G_2^3 = \{g_2\}$, $V_2^3 = 8200$, then allocation $L^* = \{(g_1, 1), (g_2, 3)\} \subset G \times A$ is the solution to SORA/PO and $TV(G, A, B, L^*) = 18200$.

We define that agent i is *dissatisfied with allocation L* if the satisfiable conditions (2) are not held, i. e., $\exists$ (g, i) and $\exists (g', j) \in L$, $\exists k < d_L(i)$ s. t. $g' \in G_k^i$ and $V_{d_L(j)}^j < V_k^i$. The number of agents dissatisfied with allocation L is denoted by $Dis(G, A, B, L)$. Obviously, Conditions (2) express a kind of externality, since agents concern the resources allocated to others. Note that since an agent's values do not change according to the allocations, the externality here is different from value externality often discussed in game theoretic approaches.

Naive algorithms for optimal solutions to SORA/PO require exponential time with respect to the size of A and G to find a solution in the worst case. For the given possible solution, L, we can verify Conditions (2) in polynomial time according to the size of $|G|$ and $|A|$. However, we have to check all other possible allocations to verify whether it provides the maximal total value (Condition (1)).

We want to define more intuitive subclass of SORA/PO.

Definition 2. *Problem SORA/PO(G, A, B) is consistent when values in B are always amenable to the associated preferences, i.e., the following conditions are satisfied:* $\forall$ *I* $\in A$*, k, k'* $\in \mathbf{N}$ *(**N** is the set of natural numbers), if* $k < k'$ *then* $V_k^i \geq V_{k'}^i$

Note that the consistency between values and preferences in this definition is only stated inside a single agent, and non-local consistency is not included.

3.2 Examples

We will now explain a number of examples that can be formalized using SORA/PO.

Example 1. (CONCERT TICKETS USING BIDDING-BASED ALLOCATION [16])

The ticket purchase for a pop-music group described in Section 3.1 is a simple example. We added another example for purchasing tickets to a classical music concert. Regular customers want to buy seats in the front and center regions due to better acoustic conditions. Thus, the prices of seats of concert venues are usually determined in a fixed way; seats in the front and center regions are the most expensive, and the prices reduce cheaper if they are close to the rear and side edges. However, if this is a piano-solo concert, quite a few customers try to buy seats in the left-side regions to see the pianist and his/her fingers, even if the seats are near the left edge and backward to the left. Thus, there are two types of customers who have different preferences, and the price system is not consistent with the latter types of customers. For standard customer $i \in A$, i's preference and the associate prices are, e.g., $B^i = \{(G_1^i, V_1^i), (G_2^i, V_2^i), (G_3^i, V_3^i)\}$, where G_1^i is the set of front seats, G_2^i is that of the around-center and non-front seats, and G_3^i is that of the seats near the rear and side edges. Of course, agents of the same types also have the different preferences, so $G_k^i \neq G_k^{i'}$ even if $i' \in A$ is also a standard agent. The preferences and the associated prices of the second types of agents may become more complicated; for $j \in A$, they will be $B^j = \{(G_1^j, V_1^j), (G_2^j, V_2^j), (G_3^j, V_3^j)\}$, where G_1^j, for example, is the set of the front seats on the left- hand side. After all the prices and preferences are declared, SORA/PO generates allocation L^* that maximizes the sales earnings with no or fewer customer complaints. Note that we can assume that $V_k^i > V_{k'}^i$ if $k < k'$ in this example, so that this problem is a consistent SORA/PO.

Example 2. (ALLOCATING ADS ON WEB PAGES [5])

Let us assume that a number of advertisements (ads) are provided on the result pages of a search engine for given search keywords. The positions of ads are often determined by using auction-like protocols. Let us consider allocating ads to five spaces arranged in tandem. Agents of advertisers bid certain prices for one of the spaces, usually assuming that their ads will be placed in the top space. Thus, if their ads appear in other spaces, they may be dissatisfied because the click-through rates are smaller. $G = \{s_1, \ldots, s_5\}$ is a set of five advertisement spaces in our framework, and agent i can submit the bid for each G with difference prices. Agents may prefer a cheaper or lower position even in this example because their budgets are tight or their click-through rates do not differ very much. So this is not consistent SORA/PO. If i does not care about its the position, it can submit the same price by bidding $\{(G, V_1^i)\}$.

4. Proposed Method for Semi-Optimal Solutions

4.1. Efficient Explorations

Since the original SORA/PO(G,A,B) is the combinatorial problem, we propose a number of methods of obtaining semi- optimal solutions by relaxing Conditions (1) and (2) to soft constraints. First, we propose two algorithms that are simple and efficient and we will then propose the *bounded eager exploration* (BEE), which is the more elaborate version of these algorithms.

These algorithms allocate resources one by one according to the values and preferences declared by agents. We introduce a number of functions: for $g \in G$ and $S \subset A$,

$$p_S(g) = \min\{j \mid \forall i \in S, g \in G_j^i\}, \tag{3}$$

$$v_{1st}(g) = \max\{V_j^i \mid \forall i \in A, g \in G_j^i\}, \text{ and} \tag{4}$$

$$v_{2nd}(g) = \max\{V_j^i \mid \forall i \in A \setminus \{a_{first}(g)\}, g \in G_j^i\}, \tag{5}$$

where $a_{first}(g) \in A$ is one of the agents who declared the highest value for g. Functions $p_S(g)$, $v_{1st}(g)$, and $v_{2nd}(g)$ correspond to the smallest preferential number, the first (largest) value and the second (second largest) value declared by agents for g. We can also define $v_{nth}(g)$ to describe the n-th value for resource g. Function $N_{bid}(g)$ outputs the number of agents who bid for g. If no agents bid for g, $p_A(g) = v_{1st}(g) = v_{2nd}(g) = v_{nth}(g) = \text{undef}$.

We focus on the resources whose highest value is declared by agents with the higher preferences and one of these resources is allocated first. Let $H(g)$ be the set of agents who declare the highest values for $g \in G$. Then $G_j = \{g \in G \mid p_{H(g)}(g) = j\} \setminus G_{j-1}$, where we set $G_0 = \varnothing$. We denote K' as one of $G_1, G_2, \ldots$ that is not empty and whose associated subscript is the smallest. Then,

$$K = \{g \in K' \mid v_{1st}(g) = m(K')\},$$

where $m(K') = \max_{g \in K'}\{v_{1st}(g)\}$. If K is the singleton, let g_0 be the element in K. Otherwise, element $g_0 \in K$ is selected using one of two strategies.

4.1.1 Lowest next value first, LNVF

Select element g_0 whose second value is the lowest:

$$g_0 \in K_2^{LNVF} = \operatorname*{argmin}_{g \in K}(v_{2nd}(g)).$$

If K_2^{LNVF} is a singleton, g_0 is determined. Otherwise, g_0 is determined on the third and the lower number values, i.e.,

$$g_0 \in K_n^{LNVF} = \operatorname*{argmin}_{g \in K_{n-1}^{LNVF}}(v_{nth}(g)).$$

If $K_n^{LNVF} = \varnothing$ and $K_{n-1}^{LNVF} \neq \varnothing$, then g_0 is randomly selected from K_{n-1}^{LNVF}.

4.1.2 Smallest request number first, SRNF

Select g_0 whose number of bids is the smallest;

$$g_0 \in K^{SRNF} = \operatorname*{argmin}_{g \in K}(N_{bid}(g)).$$

If K^{SRNF} is not a singleton, g_0 is randomly selected from K^{SRNF}. SRNF allocates resource g first in which the number of agents that required g is smaller, to avoid no allocations to the maximum extent possible.

Then, g_0 is allocated to agent $i_0 \in A$ who declared the largest value with the highest preference (i.e., the smallest number of preferences) for g_0. If multiple agents have declared it with the highest preference, one of them is randomly selected. Then, the request data that are declared by i_0 and concerned with g_0 are eliminated i.e., $G_j^i = G_j^i \setminus \{g_0\}$, $\forall i \in A$, $A \leftarrow A \setminus \{i_0\}$ and $G \leftarrow G \setminus \{g_0\}$. Then, if G_k^i has become empty for i ($\neq i_0$), i's requests are moved over, so $N_i \leftarrow N_i - 1$, $G_k^i \leftarrow G_{k+1}^i$, and $V_k^i \leftarrow V_{k+1}^i$ for $k = j, \ldots, N_i$. Then, this process is repeated until $G = \varnothing$ or no resources can be allocated to agents.

The allocation method using LNVF or using SRNF is refereed to as *LNVF exploration* and *SRNF exploration* (or simply LNVF and SRNF). Note that we can consider other strategies such as the *largest next value first* and the *largest request number first*, but as their qualities of solutions were not as advantageous as those obtained by LNVF and SRNF in our experimental settings below, we omitted them.

4.2 Bounded Eager Exploration (BEE)

Set $K \subset G$ is determined as described in the previous section. Then, the elements in K are sorted by using the evaluation function, $e(g)$. The evaluation function, e, in BEE is derived by applying SRNF first and then applying LNVF in second. Then, the first n_e elements are selected in the n_e processes in which the subsequent resources are concurrently allocated, where n_e is the bounded number of processes.

We define non-negative integers *rank* and the *deviation number* of an exploration process to bound the number of concurrent processes. We also introduce the non-negative integer, E_{max}, called the *eagerness number*, that specifies the number of processes that eagerly explore the E_{max} best solutions concurrently. Suppose that $g^0, \dots, g^{|K|-1}$ is the list of the elements of K sorted by $e(g)$. Then, we define the rank of $g \in K$ as:

$$rank(g^k) = k.$$

Note that if K is a singleton ($K = \{g_0\}$), then $rank(g_0) = 0$.

Then, we define the *deviation number*, $dev(p)$, of the initial explore process p to zero. Then, g^0 that is the first element in K is allocated agent i_0 in the current process p who declared the largest value for g^0 (if such agents are multiple, one is randomly selected, like that in LNVF and SRNF). For $\exists g^l \in K$ s. t. $g^l \in \boldsymbol{G_k^{i_0}}$ (and $l \neq 0$), if

$$dev(p) + rank(g^l) \leq E_{max} ,$$

then another process p' is launched and g^l is allocated to i_0 in p (instead of allocating g^0 to i_0). Then,

$$dev(p') = dev(p) + rank(g^l).$$

Then, data declared by i_0 are eliminated from B and resources g^0 (g^l) are also eliminated in process p (p'); this elimination process is identical to that in LNVF and SRNF explorations.

After a number of allocations have been generated by these concurrent processes, BEE finally selects allocation L_1^* that provides the maximal total value, $TV(G, A, B, L_1^*)$. This type of BEE is denoted by BEE/MTV. We also considered another option by selecting allocation L_2^* whose dissatisfied number, $Dis(G, A, B, L_2^*)$ was the smallest. This type of BEE is denoted by BEE/SDN.

We only focused on ordered set,

$$K^{dev} = \{g \in K \mid dev(p) + rank(g) \leq E_{max}\}$$

in BEE, and these resources are allocated concurrently (or sequentially with backtracking according to the order, $e(g)$). Note that when $E_{max} = 0$, BEE is almost identical to that of SRNF. It is obvious that the computational order of LNVF, SRNF, and BEE are polynomial, although those of LNVF and SRNF are much lower

4.3 Characteristics of Proposed Algorithm*s*

Although LNVF and SRNF cannot generate optimal solutions, they are applicable to the situations where agent's declarations B^i arrived intermittently, because resources are allocated one by one in these algorithms according to the currently arrived declarations and available resources. This feature is very useful in the real applications where complete information cannot be obtained such as user's requests in the Internet, and thus, optimal solutions at a certain time will be not optimal at the next time.

Furthermore, and if a SORA/PO problem is consistent, LNVF, SRNF, and BEE can efficiently generate solutions with no dissatisfied agents.

Proposition 1. *For any consistent SORA/PO(G, A, B) problem, LNVF, SRNF, and BEE generate solution, L, s. t. Dis(G, B, A, L)* = 0.

Proof: Suppose that $Dis(G, B, A, L) > 0$. Then agent $i \in A$ that is dissatisfied with L exists. Therefore, $\exists\, j \in A$, $\exists\, k \in \boldsymbol{N}$ (natural number), $d_L(i) > k \geq 1$, $g_L(j) \in G_k^i$, and $V_k^i > \boldsymbol{V_{d_L(j)}^j}$ are held (and thus, i is dissatisfied with the allocation of $g_L(j) \in G_k^i$ to j). First, we assume that $g_L(j)$ was allocated to j before $g_L(i)$ was allocated to i. Since $V_k^i > \boldsymbol{V_{d_L(j)}^j}$, $g_L(j) \in G_k^i$, and $g_L(j) \in \boldsymbol{V_{d_L(j)}^j}$, $g_L(j)$ was never allocated to j. Therefore, g_L(j) was not allocated to j yet when $g_L(i)$ had just been allocated to i. Let A' be the set of agents to which no resources were allocated at the time g_L(i) was going to be allocated to i. At this time, $g_L(j)$ was not allocated to i, so $\exists j_1 \in A' \setminus \{i, j\}$ s. t. $g_L(j) \in \boldsymbol{G_{k_1}^{j_1}}$ and j_1's value for $g_L(j)$ is the largest.

$$\boldsymbol{V_{k_1}^{j_1} > V_k^i > V_{d_L(j)}^i}$$

Note that $\boldsymbol{V_{k_1}^{j_1} \neq V_k^i}$ since g_1 was not allocated to j_1. However, $g_L(j)$ was not allocated to j_1 either, so $k_1 > d_L(i) > k \geq 1$. Thus, $\exists g_1 \in \boldsymbol{G_1^{j_1}}$ and $\boldsymbol{V_1^{j_1} \geq V_{k_1}^{j_1}}$. However, as g_1 was not allocated to j_1, $\exists j_2 \in A' \setminus \{i, j, j_1\}$, $g_1 \in \boldsymbol{G_{k_2}^{j-2}}$ s. t. $k_2 > d_L(i)$ and $\boldsymbol{V_{k_2}^{j_2} > V_1^{j_1}}$. By repeating these steps, infinite inequations

$$\cdots > V_{k_{n+1}}^{j_{n+1}} > V_1^{j_n} \geq V_{k_n}^{j_n} > \cdots > V_{k_2}^{j_2} > V_1^{j_1} \geq V_{k_1}^{j_1} > V_k^i$$

must be held. However, A and G are finite, and this concludes a contradiction. □

5. Experimental Evaluation

5.1 Experimental Setting

We investigated the qualities of solutions derived by the algorithms proposed in Section 4 by comparing them with the solutions derived with IBM's ILOG CPLEX. We also compared the efficiency of our algorithms using our implemented programs with Java, by comparing them with those of CPLEX.

For resources, $G = \{g_1, \dots, g_m\}$, and agents, $A = \{1, \dots, n\}$, the agents' preferences for these resources were generated as follows: Four integers, $1 \leq k_i \leq m$, $0 \leq l_1^i \leq 2$, $0 \leq l_2^i \leq 3$, and $0 \leq l_3^i \leq 3$ were randomly selected for agent

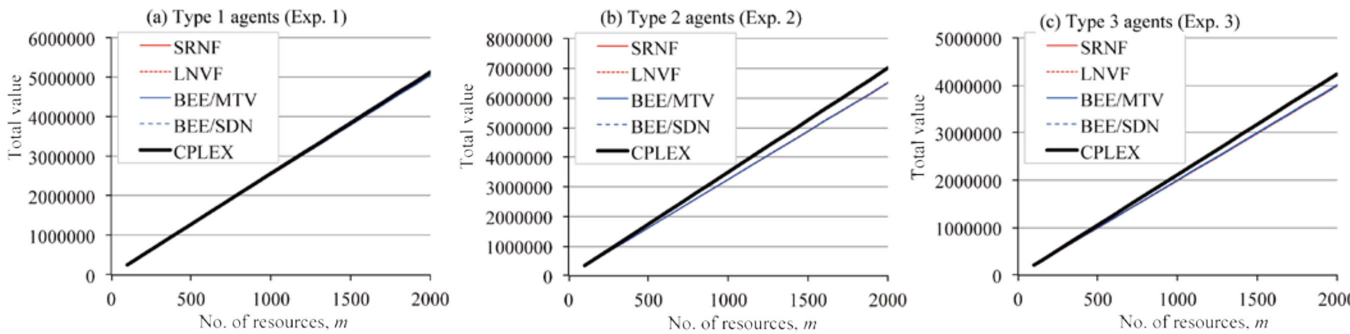

Fig. 1. Total values in Exps. 1, 2 and 3.

Table 1. Total values when m = 2000

	Exp. 1	Exp. 2 ($n = m$)	Exp. 2 ($n = 2m$, $m = 900$)	Exp. 2 ($n = m/2$)	Exp. 3
SRNF	5057491	6505668	3980777	3319174	3991825
LNVF	5057492	6533475	4002166	3333040	3988782
BEE/MTV	5060382	6520629	3983388	3323674	4016280
BEE/SDN	5060371	6507622	3981680	3319678	3991580
CPLEX	5136768	7014802	4032483	3925612	4241508

$\forall i \in A$. Then, i's required resources with the preferences were defined as

$$G_1^i = \{g_{k_i-t_1^i}, \cdots, g_{k_i}, \cdots, g_{k_i+t_1^i}\}, \quad (6)$$

$$G_2^i = \{g_{k_i-t_1^i-t_2^i}, \cdots, g_{k_i+t_1^i+t_2^i}\} \setminus G_1^i, \text{ and} \quad (7)$$

$$G_3^i = \{g_{k_i-t_1^i-t_2^i-t_3^i}, \cdots, g_{k_i+t_1^i+t_2^i+t_3^i}\} \setminus G_1^i \cup G_2^i \quad (8)$$

for j = 1, 2, and 3. Note that element g_k is not included in G_j^i when $k \le 0$ or $k > m$, and if $G_2^i = \varnothing$, we set $G_2^i \leftarrow G_3^i$ and $G_3^i \leftarrow \varnothing$.

Let us consider three types of agents based on the relationships between values and preferences. The first (type-1) agent i has values consistent with its preferences; thus, $V_k^i \ge V_{k'}^i$ if $k < k'$. Its value for G_1^i, V_1^i, was determined according to the normal distribution, $N(3000, 500)$, where 3000 and 500 are the average and variance. Then, integers, $10 \le b_1 \le 20$ and $10 \le b_2 \le 20$ were randomly selected and V_2^i was set to $(1 - b_1/100)\cdot V_1^i$ and V_3^i was set to $(1 - b_2/100) \cdot V_2^i$. Conversely, type-3 agent i has its values in reverse order; V_1^i was defined in the same way, and V_2^i was set to $(1 + b_1/100) \cdot V_1^i$ and V_3^i was set to $(1 + b_2/100) \cdot V_2^i$. A type-2 agent has values not correlated with preferences, and their values, V_k^i for k = 1, 2, and 3 were randomly selected between 1000 and 5000.

Let us define a number of notations to measure the quality of solutions to SORA/PO. Suppose that SORA/PO(G, A, B) is given and L^* is its optimal solution derived by CPLEX. Then,

(1) The *dissatisfaction ratio for allocation* L, DR_L, is defined as $DR_L = Dis(G, A, B, L)/\min(|A|, |G|)$. Of course, $DR_{L^*} = 0$ since $Dis(G, A, B, L^*) = 0$.

(2) The *ratio of difference in the total values for allocation* L, δ_L (= $\delta_L(G, A, B)$), is defined as

$$\delta_L = \frac{TV(G,A,B,L^*) - TV(G,A,B,L)}{TV(G,A,B,L^*)}$$

We call δ_L the *differential ratio* of the total value of L.

(3) The *unallocation ratio* for allocation L, $R_L^{unalloc}$, is defined as

$$R_L^{unalloc} = \begin{cases} |g_L^{-1}(\{null\})|/n & (n \le m) \\ |a_L^{-1}(\{null\})|/m & (n \ge m) \end{cases}$$

where $g_L^{-1}(\{null\}) = \{i \in A \mid g_L(i) = null\}$ and $a_L^{-1}(\{null\}) = \{g \in G \mid a_L(g) = null\}$. Note that $n = |A|$ and $m = |G|$.

We conducted three experiments; all agents were type-1 in Exp. 1, type-2 in Exp. 2 and type-3 in Exp. 3. We varied the value of m from 100 to 2000 by increasing it in increments of 100. We set $n = m$ in all experiments, and we also examined the cases when $n = 2m$ and $n = m/2$ in Exp. 2 to investigate how unbalance numbers of agents and resources affected the performance and quality. We set E_{max} = 1 in BEE. All data plotted below were the averages of 50 independent trials in the same machine.

5.2 Total Values (social welfare)

First, we compared the total values of solutions with four methods, i.e., $TV(G, A, B, L)$, where L is the allocation derived by SRNF, LNVF, BEE/MTV or BEE/SDN with those with CPLEX. Note that the solutions we obtained by CPLEX are optimal.

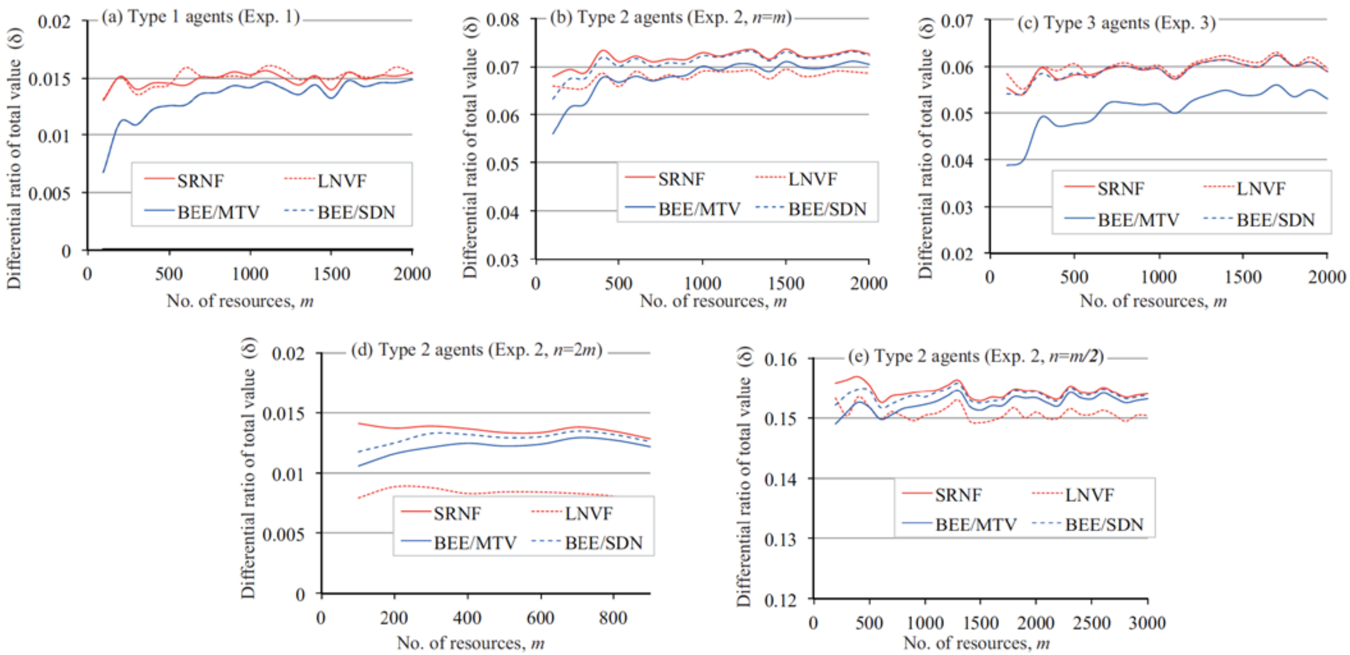

Fig. 2. Differential ratios of total values.

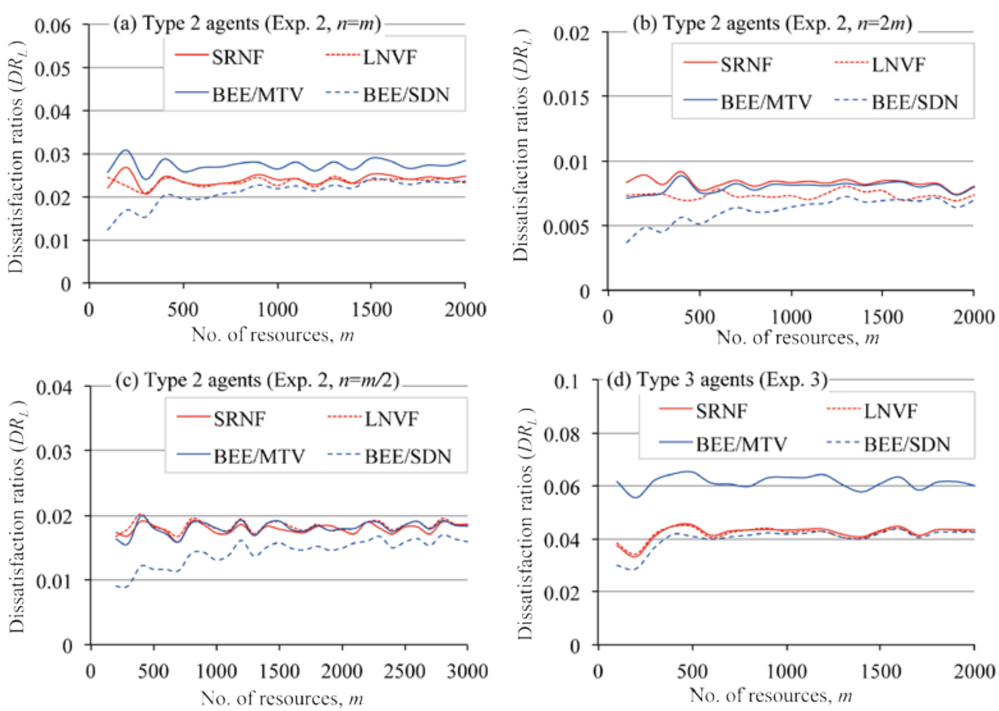

Fig. 3. Dissatisfaction ratios (DR_L).

Figure 1 that plots the total values when m = 100 to 2000 (m = n) in Exps. 1, 2 and 3 indicates that their differences quite small. To further clarify any differences, we plotted the differential ratios of total values in Fig. 2, and also listed their total values when m = 2000[§] the in Table I. We can see from Fig. 2 that the differential ratios, δ_L, are small in all the experiments, especially those in Exp. 1 and Exp. 2 when n =

[§] When n = 2m, we indicated the data when m = 900 because we could obtain the data for CPLEX due to long computation time.

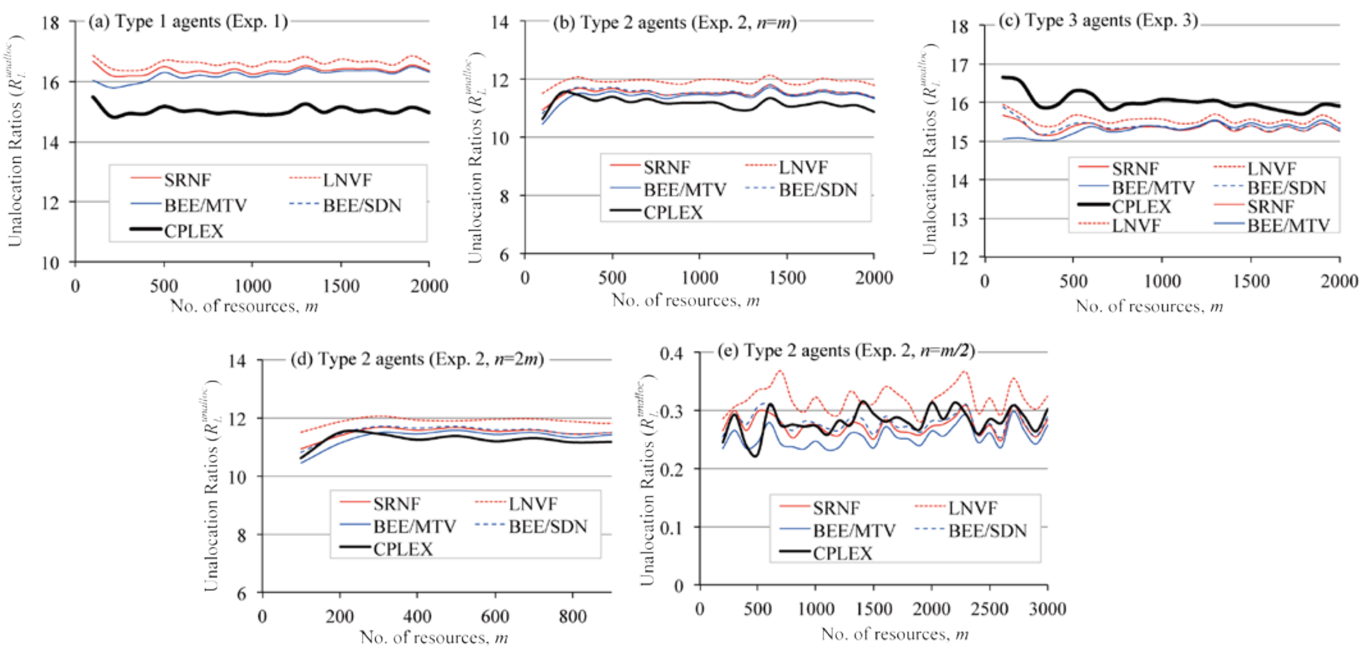

Fig. 4. Unallocation ratios $(R_L^{unalloc})$

2m (Fig. 2 (a) (d)). However, When $n = m/2$, the values for δ_L in Exp. 2 are slightly larger (Fig. 2 (e)). We also expected that BEE/MTV would achieve better total values than the others, since BEE/MTV eagerly explored allocations that could generate higher total values. However, this was true only in Exps. 1 and 3 in which agents' preferential orders and values were positively or negatively correlated. Otherwise, especially in the case when $n = 2m$, LNVF produced more total values than BEE/MTV although it is the simpler version of BEE/MTV.

5.3. Dissatisfaction Ratios

The dissatisfaction ratio is an important measure in SORA/PO because solutions must generate no (or fewer) dissatisfied agents. We plotted the dissatisfaction ratios, $Dis(G, A, B, L)$ in Exps. 2 and 3 in Fig. 3; we did not present the data obtained from in Exp. 1 because $Dis(G, A, B, L) = 0$ was proved in Section 4.3; actually we could not find any dissatisfied agents in Exp. 1 because all agents were type 1.

Figure 3 indicates that the dissatisfaction ratios were also quite small even in Exp. 3 which we think that was a unusual case; thus, we can say that the quality of the solutions generated by the proposed methods were not very degraded from this viewpoint. These graphs indicate that the allocations generated by BEE/SDN had relatively smaller dissatisfaction ratios in all the experiments, because it tried to explore the allocations with lower dissatisfied agents. On the other hand, Fig. 3 indicates that the allocations generated by BEE/MTV had relatively higher dissatisfaction ratios, especially in Exp. 3. Because it tried to explore the allocations providing higher total values, this may have resulted in more dissatisfied agents.

It also shows that if we compare the graphs in Exp. 2, the dissatisfaction ratios when $n = 2m$ and $n = m/2$ were smaller than those in others. When $n = 2m$, the number of agents are larger so the proposed four methods could find the agents that bided the higher values with higher preferential orders for any resources in G. Conversely, when $n = m/2$ the number of agents was smaller so their aggregated conditions for preferential orders were relatively simper, so could find the allocation with less dissatisfaction. Finally, in all cases, BEE/SDN could contribute to lowering the dissatisfied agents.

5.4 Unallocation Ratios

Figure 4 indicates the unallocation ratios of allocations generated by these methods. Note that even in the CPLEX solutions, a number of agents were not allocated since they request the resources with low values. The unallocation ratios of CPLEX were always the smallest in Exp. 1 but those are the largest in Exp. 3; since agents' values were reverse to the preferences, some resources cannot be allocated. However, the proposed four methods try to take into account the agents' preferences even in Exp. 3, their unallocation ratios were relatively small than those by CPLEX. The unallocation ratios of all methods in Exp. 2 were quite similar because they were intermediary situations between Exps. 1 and 3. We can also see that in all situations,

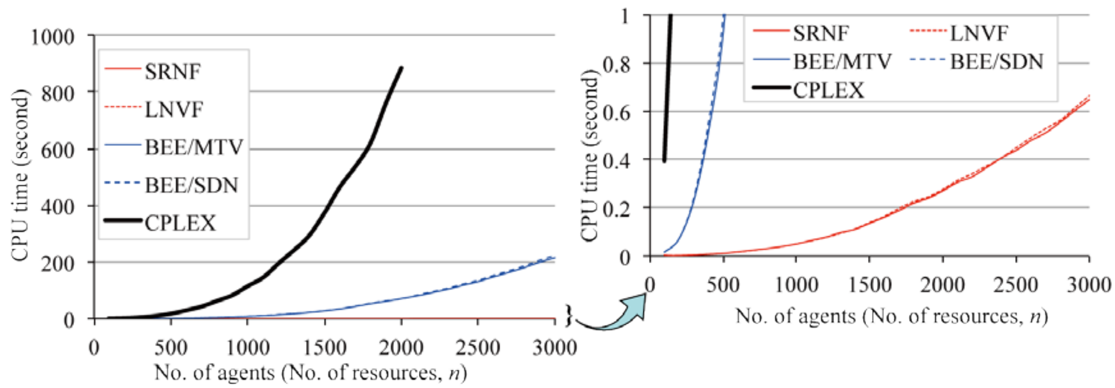

Fig. 5. CPU time in Experiment 2.

Table 2. CPU time when m=900 (second).

	Exp. 1	Exp. 2	Exp. 2($n = 2m$)	Exp. 2($n = m/2$)	Exp. 3
SRNF	0.047	0.040	0.073	0.012	0.047
LNVF	0.056	0.039	0.077	0.016	0.048
BEE/MTV	6.33	5.61	13.21	1.11	6.43
BEE/SDN	7.48	5.96	13.60	1.14	6.77
CPLEX	24.82	83.01	941.43	8.80	66.20

Table 3. CPU time when m=2000 (second).

	Exp. 1	Exp. 2	Exp. 2($n = 2m$)	Exp. 2($n = m/2$)	Exp. 3
SRNF	0.327	0.270	0.385	0.069	0.285
LNVF	0.344	0.276	0.426	0.092	0.293
BEE/MTV	85.92	70.89	119.56	13.52	77.85
BEE/SDN	88.04	69.60	135.17	13.80	80.29
CPLEX	286.37	884.96	–	63.38	650.73

the unallocation ratios of LNVF were the largest and those of BEE/MTV were the smallest of the proposed four methods. However, these differences were smaller than those in Fig. 2, since BEE/MTV explored more combinations for higher total value and did not try to allocate more resources. Another observation is that when $n = m/2$, the unallocation ratios were very small. Because the number of agents is half of the number of resources, most resources were allocated to agents.

5.5 Efficiency

Finally but most importantly, we compared the CPU time required to find the solutions. Figure 5 plots the required CPU time when m = 100 to 3000 in Exp. 2 when $m = n$ (the CPU time required for CPLEX was shown until m = 2000 since it required much time). Because the CPU times required in SRNF and LNVF were very small, their data are plotted in the close-up graph on the right in Fig. 5. We did not plot the graphs for Exps. 1 or 3 and Exp. 2 for other cases because their curves were quite similar to those in Fig. 5. Instead, we listed the CPU time when m = 900 and m = 2000 in Tables 2 and 3. Note that in the case when $n = 2m$ and $m = 900$, we omit the data for CPLEX since it could not find the solution in a reasonable time. These data were measured in MAC Pro with 3.5GHz (6 core) CPU and 48GB memory. Note again that our proposed methods were implemented with Java.

Figure 5 indicates that SRNF and LNVF were much more efficient than the other three. If we take into account that the solution qualities derived by SRNF and LNVF were fairly good, SRNF and LNVF were quite powerful in finding reasonable solutions more quickly, which is especially useful in realtime applications. This figure also shows that BEE/MTV and BEE/SDN required more CPU time than SRNF and LNVF, although this was much less than that for CPLEX. If we had set a higher value for E_{max}, its solution quality would have improved, but it would have required more CPU time than CPLEX.

Tables 2 and 3 indicate that when $n = m$, CPLEX took longer time in Exp. 2 whereas other methods took more time in Exps. 1 and 3.

5.6 Discussion

Our experimental results indicate that the difference in the qualities of solutions between SRNF, LNVF, BEE/MTV, and BEE/SDN were quite small. Since the computational costs for SRNF and LNVF are quite small, these are reasonable to use in the actual applications, especially when they are busy. From Figs. 2 and 3, BEE/MTV may be better method in applications that require higher total values and BEE/SDN is better when fewer dissatisfied agents are required, although its computational cost is high. Since the difference in the qualities of solutions between SRNF, LNVF, and CPLEX were small, SRNF and LNVF are very useful. However, if users wanted to pursue more improved quality solutions, BEE/MTV and BEE/SDN would be useful.

Another important feature of SRNF and LNVF is that resources are allocated one by one, and thus, new declarations by other agents can be added when they arrived. Of course, the resources that are already allocated must be excluded from the new declarations, but this feature is meaningful in multi-agent systems where complete information is not available in a timely manner.

6. Conclusion

We proposed a resource allocation problem, SORA/PO, by taking into account agents' preferences. Agents can declare a number of resources with their values and preferences in the framework of SORA/PO. The optimal solution is an allocation that provides the maximum total values subject to conditions in which no dissatisfactions are reported. Because agents take into account the resources allocated to other agents and their declared values, SORA/PO includes externality in allocation to some degree. Since this problem requires exponential time to find the optimal solution using naive methods, we proposed two efficient algorithms, SRNF and LNVF, and their sophisticated versions of other algorithms, i.e., BEE, in which better allocation to provide larger total values or less dissatisfied agents was eagerly explored. We experimentally compared the solutions obtained with these algorithms with the solutions obtained with CPLEX. Our experiments revealed that although the reduced qualities of the solutions derived with SRNF and LNVF were quite small, their required CPU times were extremely small. We next plan to pursue better algorithms that generate allocations with lower dissatisfaction ratios.

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References

[1] Y. Shoham and K. Layton-Brown, *Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations,* Cambridge University Press, 2009.

[2] J. Spohrer and P. P. Maglio, "The emergence of service science: Toward systematic service innovations to accelerate co-creation of value," *Production and Operations Management*, vol. 17, no. 3, pp. 238- d246, 2008.

[3] B. An, C. Miao, and Z. Shen, "Market based resource allocation with incomplete information." in *Proc. of Int. Joint Conference on Artificial Intelligence*, vol. 7, 2007, pp. 1193-1198.

[4] J. Bredin, D. Kotz, D. Rus, R. Maheswaran, C. Imer, and T. Basar, "Computational markets to regulate mobile-agent systems," *Autonomous Agents and Multi-Agent Systems*, vol. 6, no. 3, pp. 235-263, 2003.

[5] B. Edelman, M. Ostrovsky, and M. Schwarz, "Internet Advertising and the Generalized Second-Price Auction: Selling Billions of Dollars Worth of Keywords," *American Economic Review*, vol. 97, no. 1, pp. 242-259, 2007.

[6] K. Murakata, S. Takahashi, and T. Matsuo, "Reassuring Electronic Commerce Mechanism based on User Evaluation Controlling," *ACIS International Journal of Computer & Information Science,* vol. 12, no. 2, pp. 1-7, 2011.

[7] R. G. Smith, "The Contract Net Protocol: High-Level Communication and Control in a Distributed Problem Solver," *IEEE Transactions on Computers*, vol. C-29, no. 12, pp. 1104-1113, 1980.

[8] Y. Sakurai, M. Yokoo, and K. Kamei, "An efficient approximate algorithm for winner determination in combinatorial auctions," in *Proc. of the 2nd ACM Conference on Electronic commerce*, pp. 30-37, ACM, 2000.

[9] R. Cavallo, "Incentive compatible two-tiered resource allocation without money," in *Proc. of the 2014 Int. Conference on Autonomous Agents and Multi-agent Systems*, pp. 1313-1320, IFAAMAS, 2014.

[10] K. Saito and T. Sugawara, "Single-Object Resource Allocation in Multiple Bid Declaration with Preferential Order," in *Proc. of the 14th IEEE/ACIS International Conference on Computer and Information Science* (ICIS 2015), pp. 341–348, IEEE Xplore, 2015.

[11] P. Krysta, O. Telelis, and C. Ventre, "Mechanisms for Multi-unit Combinatorial Auctions with a Few Distinct Goods," in *Proc. of the 2013 International Conference on Autonomous Agents and Multi-agent Systems*, IFAAMAS, pp. 691–698, 2013.

[12] N. R. Devanur and T. P. Hayes, "The adwords

problem: Online keyword matching with budgeted bidders under random permutations," in *Proc. of the ACM Conference on Electronic Commerce*, pp. 71-78, ACM, 2009.

[13] C-J. Ho and J. W. Vaughan, "Online task assignment in crowdsourcing markets." in *Proc. of the 25th AAAI Conference on Artificial Intelligence*, 2012.

[14] R. Mailler, R. Vincent, V. Lesser, J. Shen, and T. Middlekoop, "Soft real-time, cooperative negotiation for distributed resource allocation," in *Proc. of AAAI Fall Symposium on Negotiation Methods for Autonomous Cooperative Systems*, pp. 63–69, 2001.

[15] H. Tofig, N. Oka, and M. Ozeki, "Dynamically Created Decision Support Systems in a Multi-Agent Environment," *ACIS International Journal of Computer & Information Science*, vol. 13, no. 1, pp. 19–26, 2012.

[16] N. Nishino, K. Fukuya, and K. Ueda, "Service Design in Movie Theaters Using Auction Mechanism with Seat Reservations," in *Proceedings of the International Conference of Soft Computing and Pattern Recognition*, IEEE, pp. 528–533, 2009.

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